Cosmological bounce and Genesis beyond Horndeski

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Null energy condition and its violation

Null Energy Condition (NEC) states that

$$T_{\mu\nu}k^{\mu}k^{\nu} \geq 0 \quad \Longrightarrow \quad \rho + p \geq 0.^{1}$$

Covariant conservation of the stress-energy tensor gives

$$\nabla_{\mu}T^{\mu\nu} = 0 \quad \Longrightarrow \quad \dot{\rho} = -3H(\rho + p).^{2}$$

Einstein equations give

$$G^{\mu}_{\nu} = \kappa T^{\mu}_{\nu} \quad \Longrightarrow \quad \dot{H} = -\frac{\kappa}{2}(\rho + p).^{3}$$

Therefore, if the NEC holds than it follows that

$$\rho + p \geq 0 \quad \Longrightarrow \quad \dot{\rho} \leq 0, \quad \dot{H} \leq 0.$$

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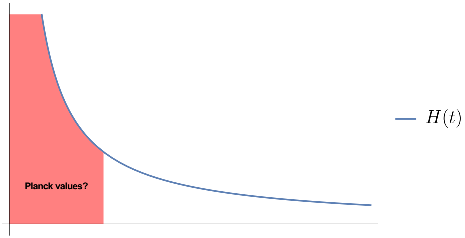
[1] где $T_{\mu\nu}$ is the matter stress-energy tensor, k^{μ} is a null vector, ρ is the energy density, p is the pressure.

[2] $H \equiv \frac{\dot{a}}{a}$ is the Hubble parameter.

[3] $\kappa = 8\pi G$ where G is the gravitational constant.

Singularity problem

In the framework of General Relativity, cosmological model that satisfies NEC inevitably faces with singularity problem: going backwards in time $\rho \sim H^2$ increases and eventually reaches singularity.



Planck values

Moreover, at some point it reaches Planck values and Quantum Gravity and Srting theory come into play.

One can bypass Planck values of energy density during the evolution of the Universe by introducing new peculiar matter that violates NEC

$$\rho + p < 0.$$

Then the Pandora's box of non-standard cosmological scenarios is open.

(a) Cosmological bounce (b) Genesis

During this 'bounce stage,' the negative increasing value of H (well below Planck scale) grow until it reaches a large positive value (well below the Planck scale), at which point the bounce stage ends and H begins to decrease.

Horndeski theory

In Horndesky theory, one can violate NEC without obvious pathologies.

The Lagrangians posses one peculiar feature that makes it possible to violate NEC: they include second derivatives but corresponding equation of motion is second-order differential equation.

For instance, let us consider the case of cubic Galileon. The Lagrangian reads

$$\mathcal{L}_3 = F(\pi, X) + K(\pi, X)\Box\pi,$$

where $X = g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi$, $\Box\pi = g^{\mu\nu}\nabla_\mu\nabla_\nu\pi$. Variation of Lagrangain gives

$$\begin{aligned}
\delta\mathcal{L}_3 &= F_\pi\delta\pi + F_X\delta X + K_\pi\Box\pi\delta\pi + \underline{K_X\Box\pi\delta X + K\Box\delta\pi} = \\
&= ... + K_X\Box\pi\delta\partial_\mu\pi\partial^\mu\pi + K\partial_\mu\partial^\mu\delta\pi \\
&= ... + 2K_X\Box\pi\partial_\mu\pi\partial^\mu\delta\pi + \partial_\mu\partial^\mu K\delta\pi \\
&= ... - \underline{2K_X\partial^\mu\Box\pi\partial_\mu\pi\delta\pi} + \partial_\mu(K_\pi\partial^\mu\pi + \underline{2K_X\partial^\mu\partial_\nu\pi\partial^\nu\pi})\delta\pi \\
&= ... - 2K_X\partial^\mu\partial_\nu\partial^\nu\pi\partial_\mu\pi\delta\pi + 2K_X\partial_\mu\partial^\mu\partial_\nu\pi\partial^\nu\pi\delta\pi \\
&= ...\text{only second derivatives.}
\end{aligned}$$

In Horndeski theory various cosmological scenarios were constructed:

- Genesis model,
- cosmological bounce,
- creating a universe in the laboratory.

However, the task of construction the whole evolution (evolution from $t = -\infty$ to $t = +\infty$) of fully stable Genesis model and cosmological bounce is still unresolved.

No-go

There is no healthy bounce and Genesis in Horndeski theory if one considers the whole evolution from $t = -\infty$ to $t = +\infty$.

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- A. Ijjas and P. J. Steinhardt, Phys. Lett. B 764, 289 (2017) doi:10.1016/j.physletb.2016.11.047 [arXiv:1609.01253 [gr-qc]].

Beyond Horndeski theory

In our work, we consider beyond Horndeski theory with the following action:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_{\mathcal{BH}}\right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right],$$

$$\mathcal{L}_{\mathcal{BH}} = F_4(\pi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}$$
$$+ F_5(\pi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}\pi_{;\sigma\sigma'},$$

where $\pi_{,\mu} = \partial_\mu\pi$.

Horndeski theory is obtained if one sets $F_4(\pi, X) = F_5(\pi, X) = 0$.

One can restore the Einstein-Hilbert gravity by choosing $G_4(\pi, X) = \frac{1}{2\kappa}$, $G_5(\pi, X) = 0$, where $\kappa = 8\pi G$ and G is the gravitational constant.

J. Gleyzes, D. Langlois, F. Piazza and F. Vernizzi, Phys. Rev. Lett. 114, no. 21, 211101 (2015) doi:10.1103/PhysRevLett.114.211101 [arXiv:1404.6495 [hep-th]].

Stability of solutions in beyond Horndeski model

We start from the unperturbed spatially flat FLRW metric (mostly negative signature)

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j.$$

Variation of action with respect to g^{00} and g^{ii} leads to the equations of motion (Einstein equations) wich we will use later.
We choose unitary gauge

$$\delta\pi = 0$$

and adopt the following parametrization:

$$g_{00} = 1 + 2\alpha, \quad g_{0i} = -\partial_i\beta, \quad g_{ij} = -a^2\left(e^{2\zeta}\delta_{ij} + h_{ij}^T\right),$$

where h_{ij}^T denotes tensor perturbations: $h_{ii}^T = 0, \quad \partial_i h_{ij}^T = 0.$
Quadratic action for perturbations then reads:

$$S = \int \mathrm{d}t\mathrm{d}^3x a^3 \Bigg[\left(\frac{\hat{\mathcal{G}}_{\mathcal{T}}}{8}\left(\dot{h}_{ik}^T\right)^2 - \frac{\mathcal{F}_{\mathcal{T}}}{8a^2}\left(\partial_i h_{kl}^T\right)^2 \right) + \Bigg(-3\hat{\mathcal{G}}_{\mathcal{T}}\dot{\zeta}^2 + \mathcal{F}_{\mathcal{T}}\frac{(\nabla\zeta)^2}{a^2}$$

$$-2\mathcal{G}_{\mathcal{T}}\alpha\frac{\triangle\zeta}{a^2} + 2\hat{\mathcal{G}}_{\mathcal{T}}\dot{\zeta}\frac{\triangle\beta}{a^2} + 6\Theta\alpha\dot{\zeta} - 2\Theta\alpha\frac{\triangle\beta}{a^2} + \Sigma\alpha^2\Bigg)\Bigg],$$

where $\hat{\mathcal{G}}_{\mathcal{T}}$, $\mathcal{G}_{\mathcal{T}}$ $\mathcal{F}_{\mathcal{T}}$, Θ, Σ are some expresions of background Galileon functions.

Variation of action with respect to the Lagrange multipliers (α and $\triangle\beta$) leads to the constraint equations.
Substituting these equations into action we obtain quadratic action in the unconstrained form

$$S = \int \mathrm{d}t\mathrm{d}^3xa^3 \left[\frac{\hat{\mathcal{G}}_{\mathcal{T}}}{8} \left(\dot{h}_{ik}^T \right)^2 - \frac{\mathcal{F}_{\mathcal{T}}}{8a^2} \left(\partial_i h_{kl}^T \right)^2 + \mathcal{G}_{\mathcal{S}} \dot{\zeta}^2 - \mathcal{F}_{\mathcal{S}} \frac{\left(\nabla\zeta \right)^2}{a^2} \right],$$

where the coefficients are

$$\mathcal{G}_{\mathcal{S}} = \frac{\Sigma \hat{\mathcal{G}}_{\mathcal{T}}^2}{\Theta^2} + 3\hat{\mathcal{G}}_{\mathcal{T}},$$

$$\mathcal{F}_{\mathcal{S}} = \frac{1}{a} \frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_{\mathcal{T}},$$

$$\xi = \frac{a\mathcal{G}_{\mathcal{T}} \hat{\mathcal{G}}_{\mathcal{T}}}{\Theta} = \frac{a \left(\hat{\mathcal{G}}_{\mathcal{T}} - \mathcal{D}\dot{\pi} \right) \hat{\mathcal{G}}_{\mathcal{T}}}{\Theta},$$

where $\mathcal{D} = 2F_4X\dot{\pi} + 6HF_5X^2$ is purely beyond Horndeski term.
The speeds of sound for tensor and scalar perturbations are, respectively,

$$c_{\mathcal{T}}^2 = \frac{\mathcal{F}_{\mathcal{T}}}{\hat{\mathcal{G}}_{\mathcal{T}}}, \qquad c_{\mathcal{S}}^2 = \frac{\mathcal{F}_{\mathcal{S}}}{\mathcal{G}_{\mathcal{S}}}.$$

A healthy and stable solution requires correct signs for kinetic and gradient terms as well as subluminal propagation:

$$\hat{\mathcal{G}}_{\mathcal{T}} > \mathcal{F}_{\mathcal{T}} > 0, \quad \mathcal{G}_{\mathcal{S}} > \mathcal{F}_{\mathcal{S}} > 0.$$

No-go and bypassing

$$\mathcal{F_S} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F_T} \quad \Longrightarrow \quad \xi(t_2) - \xi(t_1) = \int\limits_{t_1}^{t_2} a(t)\,(\mathcal{F_T} + \mathcal{F_S})\,\mathrm{d}t.$$

For bounce, scale factor is bounded by its minimal value at the moment of bounce $a(t) \geq a_{min} > 0$. Now, suppose that $\xi(t_2) > 0$. We have

$$\xi(t_1) = \xi(t_2) - \int\limits_{t_1}^{t_2} a(t)\,(\mathcal{F_T} + \mathcal{F_S})\,\mathrm{d}t\ .$$

This shows that at early enough times t_1, one has $\xi(t_1) < 0$. Another possibility is that $\xi(t_1) < 0$. Then we write

$$\xi(t_2) = -|\xi(t_1)| + \int\limits_{t_1}^{t_2} a(t)\,(\mathcal{F_T} + \mathcal{F_S})\,\mathrm{d}t,$$

and at large enough t_2 one has $\xi(t_2) > 0$.

Hence, there must be a moment of time when $\xi(t)$ changes sign, i.e, it crosses zero, $\xi(t_0) = 0$.

Let us briefly summarize the proof of no-go theorem for Horndeski and show how to bypass it by introducing beyond Horndeski terms.

$\mathcal{L}_3$:

$$\mathcal{D} = 0, \quad S_{\mathcal{T}} = \frac{1}{8\kappa}\int \mathrm{dtd}^3 x a^3 \left[\left(\dot{h}^T_{ik}\right)^2 - \frac{1}{a^2}\left(\partial_i h^T_{kl}\right)^2\right],$$
$$\hat{\mathcal{G}}_{\mathcal{T}} = \mathcal{G}_{\mathcal{T}} = \frac{1}{\kappa} = const,$$
$$\xi = \frac{1}{\kappa^2}\frac{a}{\Theta}, \quad \xi(t_0) = 0 \Longrightarrow \Theta(t_0) = \infty,$$

$\mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5$:

$$\mathcal{D} = 0, \quad S_{\mathcal{T}} = \int \mathrm{dtd}^3 x a^3 \left[\frac{\mathcal{G}_{\mathcal{T}}}{8}\left(\dot{h}^T_{ik}\right)^2 - \frac{\mathcal{F}_{\mathcal{T}}}{8a^2}\left(\partial_i h^T_{kl}\right)^2\right],$$
$$\hat{\mathcal{G}}_{\mathcal{T}} = \mathcal{G}_{\mathcal{T}} \neq const.,$$
$$\xi = \frac{a{\mathcal{G}_{\mathcal{T}}}^2}{\Theta}, \quad \xi(t_0) = 0 \Longrightarrow \mathcal{G}_{\mathcal{T}} = 0,$$

$\mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_{\mathcal{BH}}$:

$$\mathcal{D} \neq 0, \quad S_{\mathcal{T}} = \int \mathrm{dtd}^3 x a^3 \left[\frac{\hat{\mathcal{G}}_{\mathcal{T}}}{8}\left(\dot{h}^T_{ik}\right)^2 - \frac{\mathcal{F}_{\mathcal{T}}}{8a^2}\left(\partial_i h^T_{kl}\right)^2\right],$$
$$\hat{\mathcal{G}}_{\mathcal{T}} \equiv \mathcal{G}_{\mathcal{T}} + \mathcal{D}\dot{\pi} \neq \mathcal{G}_{\mathcal{T}},$$
$$\xi = \frac{a\hat{\mathcal{G}}_{\mathcal{T}}\mathcal{G}_{\mathcal{T}}}{\Theta}, \quad \xi(t_0) = 0 \Longrightarrow \mathcal{G}_{\mathcal{T}} = 0 \quad and \quad \hat{\mathcal{G}}_{\mathcal{T}} > 0.$$

Coefficient $\mathcal{D}$ plays the crucial role. Let us recall that it reads

$$\mathcal{D} = 2F_4 X\dot{\pi} + 6HF_5X^2$$

end emerges only in the framework of beyond Horndeski.

It is impossible to cook up a Genesis model and cosmological bounce where the Galileon is massless scalar field both at late and early times.

We want to have Einstein–Hilbert gravity at distant past. Thus, at early times one has

$$\mathcal{G}_\mathcal{T} = \frac{1}{\kappa}. \tag{3}$$

However, $\mathcal{G}_\mathcal{T}$ must change sign at some point and is negative at early times, in contradiction with (3). The latter argument assumes that $\mathcal{G}_\mathcal{T}$ crosses zero only once. To see that this is the case we recall that

$$\frac{\mathrm{d}\xi}{\mathrm{d}t} > a\left(\mathcal{F}_\mathcal{T} + \mathcal{F}_\mathcal{S}\right),$$

so the function ξ is always growing. Therefore, it can cross zero only once. $\mathcal{G}_\mathcal{T}$ crosses zero at the same moment as ξ, thus $\mathcal{G}_\mathcal{T}$ crosses zero only once as well.

We construct the bouncing scenario and Genesis with non-trivial Galileon field π in the asymptotic past, which eventually evolves into a conventional scalar field in distant future. In this way we keep $\Theta > 0$ at all times.

Cosmological bounce: an example

At late times, we require the Galileon to become a conventional massless scalar field, whose equation of state is $p = \rho$. Then, the late-time asymptotic of the Hubble parameter is

$$t \to +\infty : \quad H(t) = \frac{1}{3t}.$$

We choose the Hubble parameter at all times equal to

$$H(t) = \frac{t}{3(1+t^2)} \quad \Longrightarrow \quad a(t) = (1+t^2)^{\frac{1}{6}}.$$

so the bounce occurs at $t = 0$.

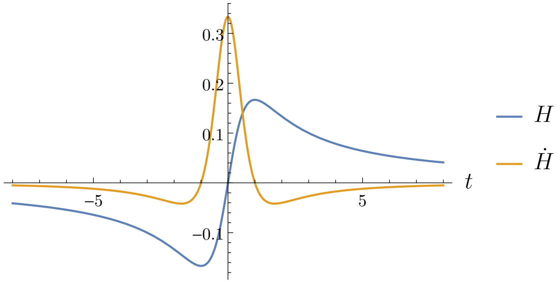
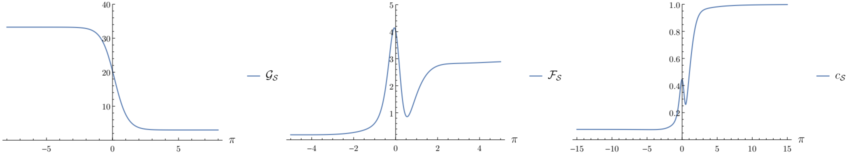


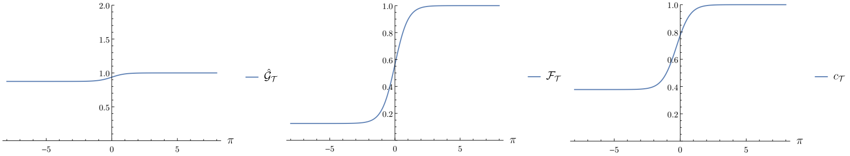
Figure 1: Hubble parameter H and its time derivative $\dot{H}$.

The behavior of scalar and tensor kinetic and gradient terms is shown in Figs. 2 and 3.



(a) (b) (c)

Figure 2: Kinetic and gradient terms $\mathcal{G}_\mathcal{S}$ (a) and $\mathcal{F}_\mathcal{S}$ (b) and speed of scalar perturbations $c_\mathcal{S}$ (c) with parameters given by (13) and $q = 20$. Despite appearance, $\mathcal{F}_\mathcal{S}$ and $c_\mathcal{S}$ are finite as $t \to -\infty$: $\mathcal{F}_\mathcal{S} \approx 0.193$ at $t \to -\infty$; $c_\mathcal{S} \approx 0.08$ at $t \to -\infty$.



(a) (b) (c)

Figure 3: Kinetic and gradient terms for tensor perturbations, $\hat{\mathcal{G}}_\mathcal{T}$ (a) and $\mathcal{F}_\mathcal{T}$ (b), speed of tensor perturbations $c_\mathcal{T}$ (c).

Both ξ and $\mathcal{G}_{\mathcal{T}}$ change sign simultaneously at $\pi = t = arctanh\left(\frac{7}{9}\right) \approx -1$, but $\hat{\mathcal{G}}_{\mathcal{T}}$ stays positive at all times, and our mechanism of evading the no-go theorem works. The behavior of ξ and $\mathcal{G}_{\mathcal{T}}$ is shown in Fig. 4.

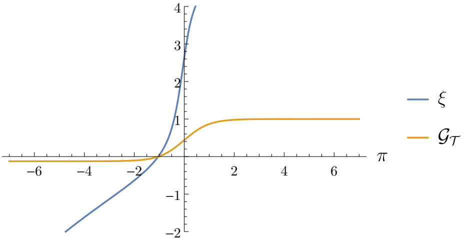


Figure 4: Evolution of ξ and $\mathcal{G}_{\mathcal{T}}$. $\xi = \mathcal{G}_{\mathcal{T}} = 0$ at $\pi = t = arctanh\left[\frac{7}{9}\right] \approx -1$.

Now let us turn to the Lagrangian functions. The Lagrangian at early times has the form (we no longer use the gauge $\dot{\pi} = 1$):

$$\mathcal{L}|_{t=-\infty} = \mathcal{C}_0 \cdot \frac{1}{\pi^2} + \left(\frac{1}{3} + \mathcal{C}_1\right) \frac{(\partial\pi)^2}{\pi^2} + \mathcal{C}_2 \frac{(\partial\pi)^4}{\pi^2}$$
$$+ 2c_1 \frac{(\partial\pi)^2}{\pi} \Box\pi - \left(\frac{1}{2} + 2c_2 + 2c_3(\partial\pi)^2\right) R + 4c_3 \left[(\Box\pi)^2 - \nabla^{\mu\nu}\pi\nabla_{\mu\nu}\pi\right]$$
$$+ 2c_4 \epsilon^{\mu\nu\rho\sigma} \epsilon^{\mu'\nu'\rho'}{}_{\sigma} \nabla_\mu\pi\nabla'_\mu\pi\nabla_{\nu\nu'}\pi\nabla_{\rho\rho'}\pi.$$

where the coefficients $\mathcal{C}_0$, $\mathcal{C}_1$ and $\mathcal{C}_2$ are combinations of coefficients c_1, c_2, c_3 and c_4.

Note that it contains both Horndeski and beyond Horndeski terms.

On the other hand, the Lagrangian at $t = +\infty$ has the form

$$\mathcal{L}|_{t=+\infty} = -\frac{1}{2}R + \frac{1}{3}\frac{(\partial\pi)^2}{\pi^2} = -\frac{1}{2}R + \frac{1}{3}(\partial\phi)^2,$$

where $\phi = \ln(\pi)$.

As we anticipated, Galileon field becomes a free massless scalar field interacting with the Einstein-Hilbert gravity.

Functions f_0, f_1, f_2 are shown in Fig. 5.

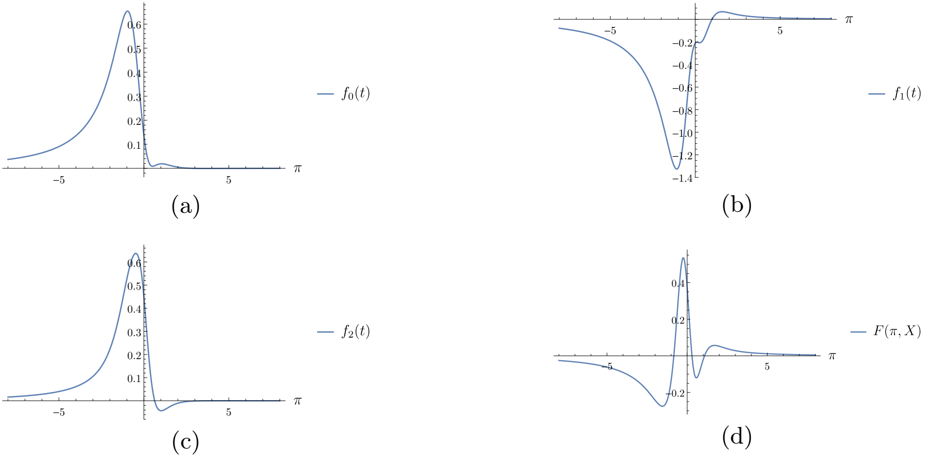


Figure 5: Lagrangian functions $f_0(\pi)$, $f_1(\pi)$, $f_2(\pi)$ and $F(\pi, X)$ in the gauge $\dot{\pi} = 1$. Note that all functions are smooth and without singularities at any point during the entire evolution.

Genesis: an example

Like in the case of the bounce, we require the Galileon to become a conventional massless scalar field at late times. Thus, the late-time asymptotic of the Hubble parameter is

$$t \to +\infty : \quad H(t) = \frac{1}{3t}.$$

We choose the Hubble parameter at all times equal to

$$H(t) = \frac{1}{3\sqrt{1+t^2}} \quad \Longrightarrow \quad a(t) = \left[t + \sqrt{1+t^2}\right]^{\frac{1}{3}}.$$

The evolution of the Hubble parameter and its time derivative are shown in Fig. 6.

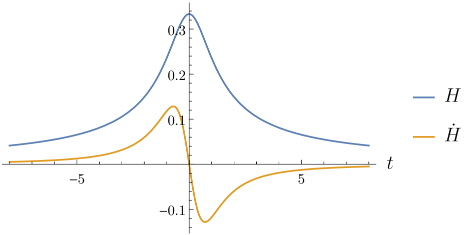


Figure 6: Hubble parameter H and its time derivative $\dot{H}$.

The evolution of kinetic and gradient terms as well as the speed of scalar and tensor perturbations are presented in Fig. 7.

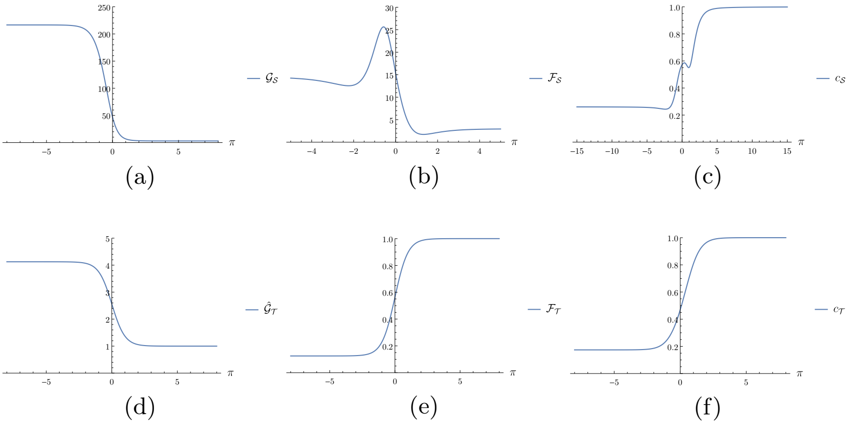


Figure 7: Kinetic and gradient terms $\mathcal{G}_\mathcal{S}$ (a) and $\mathcal{F}_\mathcal{S}$ (b), speed of scalar perturbations $c_\mathcal{S}$ (c). Kinetic and gradient terms $\hat{\mathcal{G}}_\mathcal{T}$ (d) and $\mathcal{F}_\mathcal{T}$ (e), speed of tensor perturbations $c_\mathcal{T}$ (f).

We show in Fig. 8 the behavior of ξ and $\mathcal{G}_{\mathcal{T}}$, which enables one to evade the no-go argument. The functions $\xi(t)$ and $\mathcal{G}_{\mathcal{T}}$ cross zero at $\pi = t = arctanh\left[\frac{1}{17}\right] \approx 0.06$.

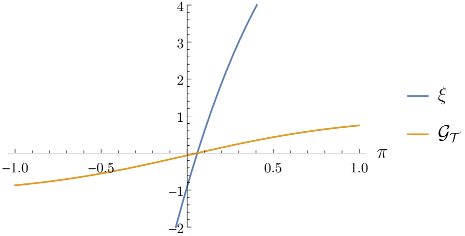


Figure 8: Evolution of ξ and $\mathcal{G}_{\mathcal{T}}$. Zero crossing $\xi = \mathcal{G}_{\mathcal{T}} = 0$ occurs at $\pi = t = arctanh\left[\frac{1}{17}\right] \approx 0.06$.

The functions f_0, f_1, f_2 are shown in Fig. 9. We see that the Genesis solution is completely healthy, with all Lagranian functions smooth at all values of π

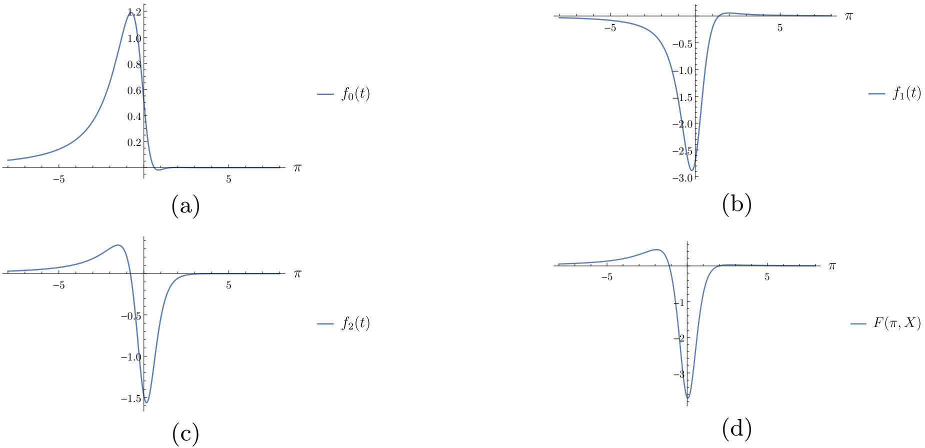


Figure 9: Lagrangian functions $f_0(\pi)$, $f_1(\pi)$, $f_2(\pi)$ and $F(\pi, X)$ in the gauge $\dot{\pi} = 1$. Note that all functions are smooth and without singularities at any point during the entire evolution.

Conclusion

- We constructed an explicit "classical" cosmological bounce that is free of any kind of instabilities and singularities during the whole evolution.
- We also gave an example of fully stable and geodesically complete Genesis.
- We presented the Lagrangian functions and checked that the Einstein and field equations are satisfied.

The characteristic feature of the solutions, namely, the flow of the Galileon field into a conventional massless scalar field at late times, enables one to potentially merge the bouncing and/or Genesis scenario with the conventional evolution at later stages.

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Thank you for your attention!

Appendix

Let us choose the Lagrangian functions as follows:

$$
\begin{aligned}
F(\pi, X) &= f_0(\pi) + f_1(\pi)X + f_2(\pi)X^2, \\
K(\pi, X) &= k_1(\pi)X, \\
G_4(\pi, X) &= \frac{1}{2} + g_{40}(\pi) + g_{41}(\pi)X, \\
G_5(\pi, X) &= 0, \\
F_4(\pi, X) &= f_{40}(\pi), \\
F_5(\pi, X) &= 0.
\end{aligned}
$$

We choose the background field as follows: $\pi = t$.

We first specify the explicit forms of $k_1(t)$, $g_{40}(t)$, $g_{41}(t)$ and $f_{40}(t)$ at all times. Then we express the functions $f_1(t)$ and $f_2(t)$ through $k_1(t)$, $g_{40}(t)$, $g_{41}(t)$ and $f_{40}(t)$ via the Einstein equations, again at all times.

We now turn to functions $k_1(t)$, $g_{40}(t)$, $g_{41}(t)$ and $f_{40}(t)$. We should have the following late-time asymptotics of K, G_4 and F_4:

$$t \to +\infty : \quad K(\pi, X) = 0, \quad G_4(\pi, X) = \frac{1}{2}, \quad F_4(\pi, X) = 0.$$

For Θ one has

$$\Theta = -k_1 + \dot{g}_{40} + 3\dot{g}_{41} - 8g_{41}H + 2\left(\frac{1}{2} + g_{40} + g_{41}\right)H + 10f_{40}H.$$

Considering the latter equation at early times and choosing early-time behavior of G_4 and F_4:

$$t \to -\infty : \quad G_4(\pi, X) = const, \quad F_4(\pi, X) = const., \tag{5}$$

one concludes that $k_1(t)$ is naturally chosen proportional to $H(t)$ to have a simple power-law behavior of Θ.

Let us choose $k_1(t)$, $g_{40}(t)$, $g_{41}(t)$ and $f_{40}(t)$ in the following form

$$\begin{aligned} k_1(t) &= c_1 \frac{t}{1+t^2} \left(1 - \tanh(t)\right), \\ g_{40}(t) &= c_2 \left(1 - \tanh(t)\right), \quad g_{41}(t) = c_3 \left(1 - \tanh(t)\right), \\ f_{40}(t) &= c_4 \left(1 - \tanh(t)\right), \end{aligned}$$

where c_1, c_2, c_3 and c_4 are constant coefficients.

Our main purpose now is to find time dependence of $f_0(t)$, $f_1(t)$ and $f_2(t)$. Note that the only non-vanishing term in the Lagrangian at late times is $f_1(t)$. Thus, $f_0(t)$ and $f_2(t)$ vanish at late times

$$t \to +\infty : \quad f_0(t) = f_2(t) = 0,$$

We define $f_1(t)$ as a sum of two functions

$$f_1(t) = f_{10}(t) + f_{11}(t)$$

with the following late-time asymptotics:

$$t \to +\infty : \quad f_{10}(t) = \frac{p}{t^2}, \quad f_{11}(t) = O(t^{-3}),$$

where p is a constant coefficient. By substituting $H(t) = 1/3t$ and $f_{10}(t) = p/t^2$ into Einstein equations at $t \to +\infty$ with all other Lagrangian functions equal to zero one finds that $p = 1/3$. So, we choose $f_{10}(t)$ equal to

$$f_{10}(t) = \frac{1}{3(1+t^2)}$$

at all times. By solving Einstein equations at all times one has

$$\begin{aligned} f_{11}(t) =& -2f_0 - f_{10} + \dot{k}_1 - 3\ddot{g}_{40} - 3\ddot{g}_{41} + 3H\left(k_1 - 3\dot{g}_{40} - \dot{g}_{41} - 2\dot{f}_{40}\right) \\ & - 6H^2\left(1 + 2g_{40} - 3g_{41} + 4f_{40}\right) - 3\dot{H}\left(1 + 2g_{40} - 2g_{41} + 2f_{40}\right), \\ f_2(t) =& f_0 + \ddot{g}_{40} + \ddot{g}_{41} - H\left(3k_1 - 5\dot{g}_{40} - 7\dot{g}_{41} - 2\dot{f}_{40}\right) \\ & + 3H^2\left(1 + 2g_{40} - 4g_{41} + 6f_{40}\right) + \dot{H}\left(1 + 2g_{40} - 2g_{41} + 2f_{40}\right). \end{aligned}$$

We require that $f_0(t)$ satisfies the equation

$$\Sigma = q(1-\tanh(t))\Theta^2, \tag{8}$$

where q is a constant coefficient. From (8) one has

$$\mathcal{G}_{\mathcal{S}} = q(1-\tanh(t))\hat{\mathcal{G}}_{\mathcal{T}}^2 + 3\hat{\mathcal{G}}_{\mathcal{T}}.$$

From the latter equation it follows that one can vary $\mathcal{G}_{\mathcal{S}}$ by changing coefficient q and, therefore, by changing $f_0(t)$. We will make use of latter equation later. Solving Eq. (8) for $f_0(t)$, one has

$$\begin{aligned} f_0(t) = \frac{1}{4}\Big(&\dot{k}_1 - 3\ddot{g}_{40} - 3\ddot{g}_{41} + 3H\left(k_1 - 5\dot{g}_{40} - \dot{g}_{41} - 2\dot{f}_{40}\right) \\ &- 3H^2\left(3 + 6g_{40} - 6g_{41} - 2f_{40}\right) - 3\dot{H}\left(1 + 2f_{40} + 2g_{40} - 2g_{41}\right) \\ &+ q(1-\tanh(t))\left(-k_1 + \dot{g}_{40} + 3\dot{g}_{41} + H\left(1 + 2g_{40} - 6g_{41} + 10f_{40}\right)\right)^2\Big). \end{aligned}$$

One can check that $f_0(t)$, $f_{11}(t)$ and $f_2(t)$ are smooth functions that rapidly vanish at late times.

The Lagrangian will be fully defined if we specify the values of coefficients c_1, c_2, c_3, c_4 and q.

At early times, we require

$$\Theta|_{t=-\infty} > 0. \tag{9}$$

Now, recall that we want $\mathcal{G}_\mathcal{T}$ to turn sign at some point; thus, we require

$$\mathcal{G}_\mathcal{T}|_{t=-\infty} < 0. \tag{10}$$

1. Tensor perturbations. In Einstein-Hilbert gravity at late times, one has

   $$\hat{\mathcal{G}}_\mathcal{T}|_{t=+\infty} = \mathcal{F}_\mathcal{T}|_{t=+\infty} = 1\ .$$

   We require at early times

   $$\hat{\mathcal{G}}_\mathcal{T}|_{t=-\infty} > \mathcal{F}_\mathcal{T}|_{t=-\infty} > 0. \tag{11}$$

   In addition, we will choose them in such a way that inequality $\hat{\mathcal{G}}_\mathcal{T} > \mathcal{F}_\mathcal{T}$ is satisfied at all times.
2. Scalar perturbations. Inequality $\mathcal{F}_\mathcal{S}|_{t=+\infty} > 0$ is equivalent to $\dot{H}|_{t=+\infty} < 0$ and is satisfied. At early times, we require

   $$\mathcal{F}_\mathcal{S}|_{t=-\infty} > 0. \tag{12}$$

   One possible choice of coefficients c_1, c_2, c_3, c_4, such that the inequalities (9), (10), (11) and (12) are valid, is

   $$c_1 = 1, \quad c_2 = -\frac{1}{4}, \quad c_3 = \frac{1}{32}, \quad c_4 = \frac{1}{4}. \tag{13}$$

   Moreover, by taking $q = 20$, the condition $\mathcal{G}_\mathcal{S} > \mathcal{F}_\mathcal{S}$ is satisfied as well.