Self consistent calculation of nuclear composition in hot and dense stellar matter

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Wok done during Shun Furusawa’s stay at FIAS during

2016-2017

Contents

1. Self-consistent calculation of individual nuclei in stellar environments
2. Equilibrium Nuclear Ensembles (SNE)
3. Equation of State of clusterized nuclear matter

Publications:

S. Furusawa and I. Mishustin PRC95, no.3, 035802 (2017);

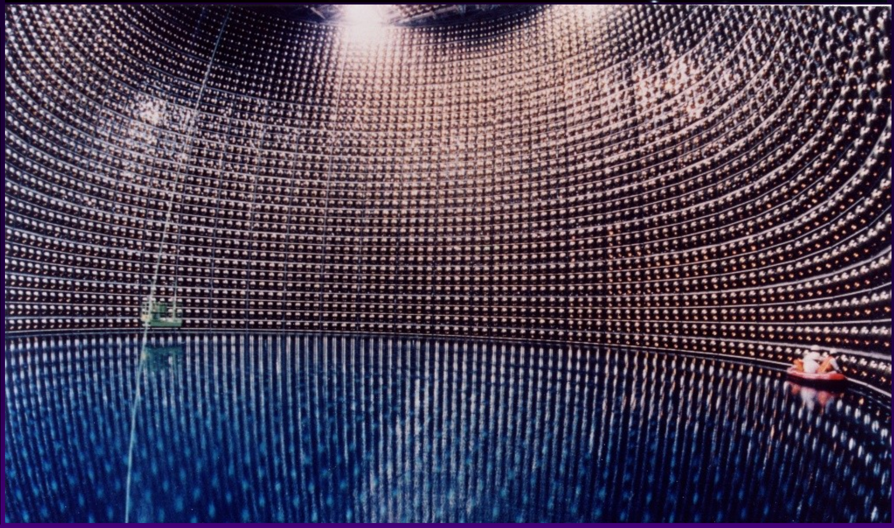
S. Furusawa and I. Mishustin PRC submitted, arXiv:1707.00147 [nucl-th] |

Core Collapse Supernovae

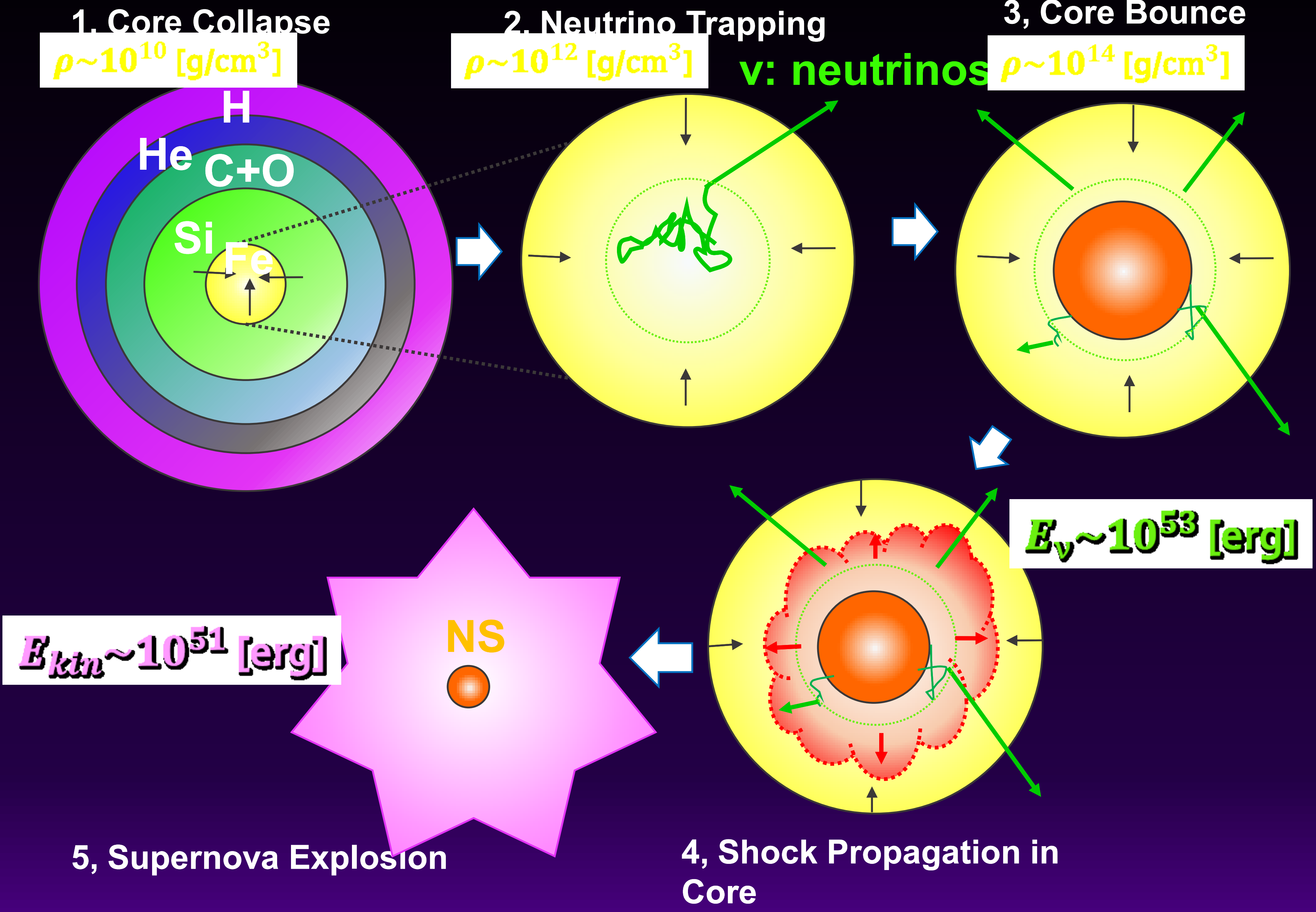
Death of Massive Stars ≥ 10 Msun

- Energetic evets 10^{51-53} erg
- Emissions of neutrinos and gravitational Waves
- Formations of a neutron star or a black hole
- Nucleosynthesis site of heavy elements.


ula (SN1054)



Scenario of Neutrino-Driven Supernovae



1. Core Collapse

$\rho\sim10^{10}\,[\mathrm{g/cm^3}]$

2. Neutrino Trapping

$\rho\sim10^{12}\,[\mathrm{g/cm^3}]$

ν: neutrinos

3, Core Bounce

$\rho\sim10^{14}\,[\mathrm{g/cm^3}]$

$E_\nu\sim10^{53}$ [erg]

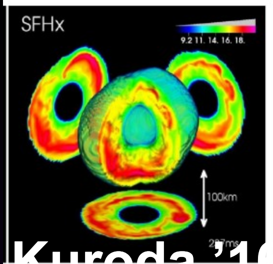
4, Shock Propagation in Core

$E_{kin}\sim10^{51}$ [erg]

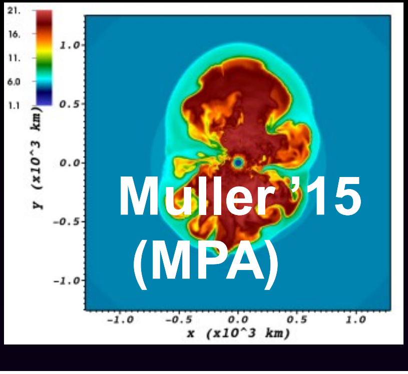
5, Supernova Explosion

2D &3D simulations

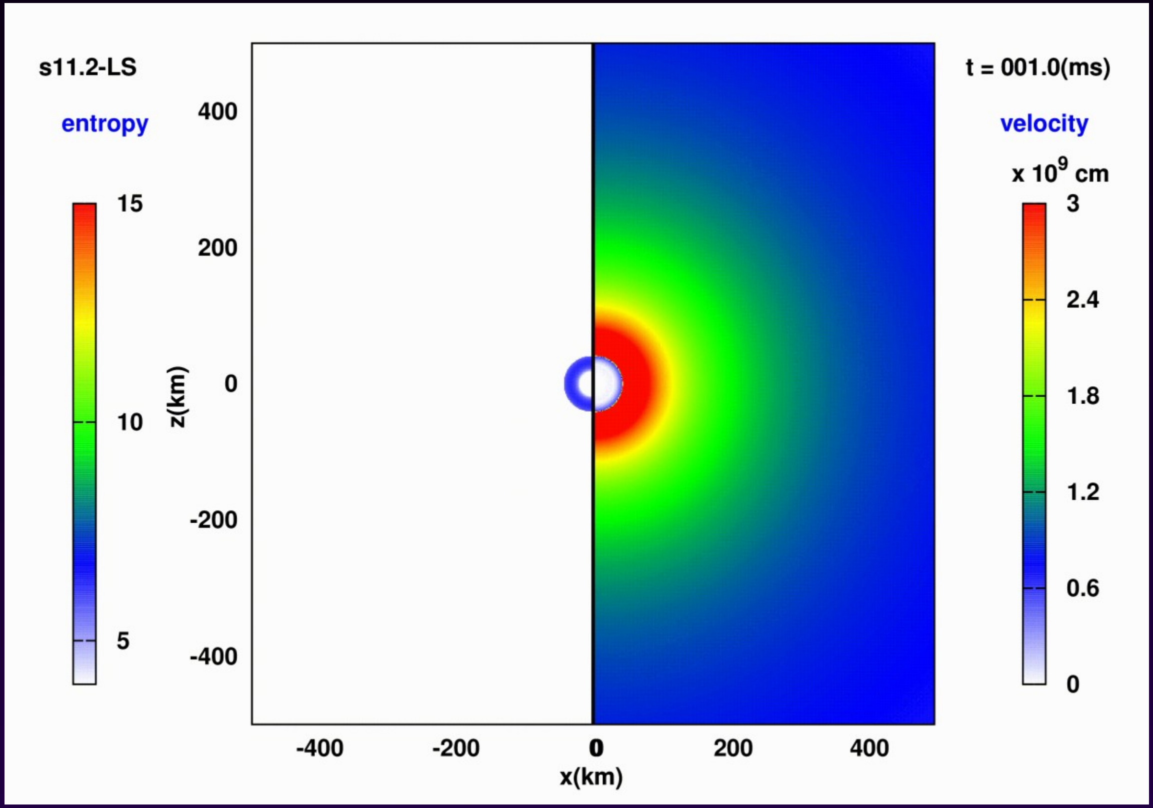
2D Full-Boltzmann Simulation
(Nagakura et al. ‘17)

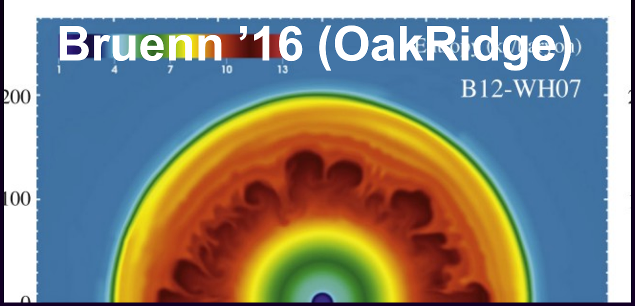


Kuroda ’16

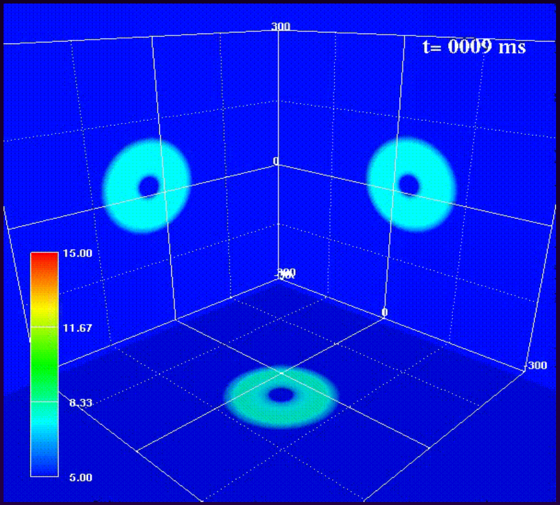


Muller ’15 (MPA)





Bruenn ’16 (OakRidge)



Takiwaki ‘12

Challenges in Supernovae

○ Prediction of observation

- Explosion (kinetic) energy ~10^{51} erg
- Ni56 Mass ~ 0.1 M
- Ejecta and/or Neutron Star Masses

- neutrinos
- Gravitational Waves

○Formation of compact stars

- Neutron Star Kick ~ 500 km/sec
- Black hole, Pulsar, Magneter

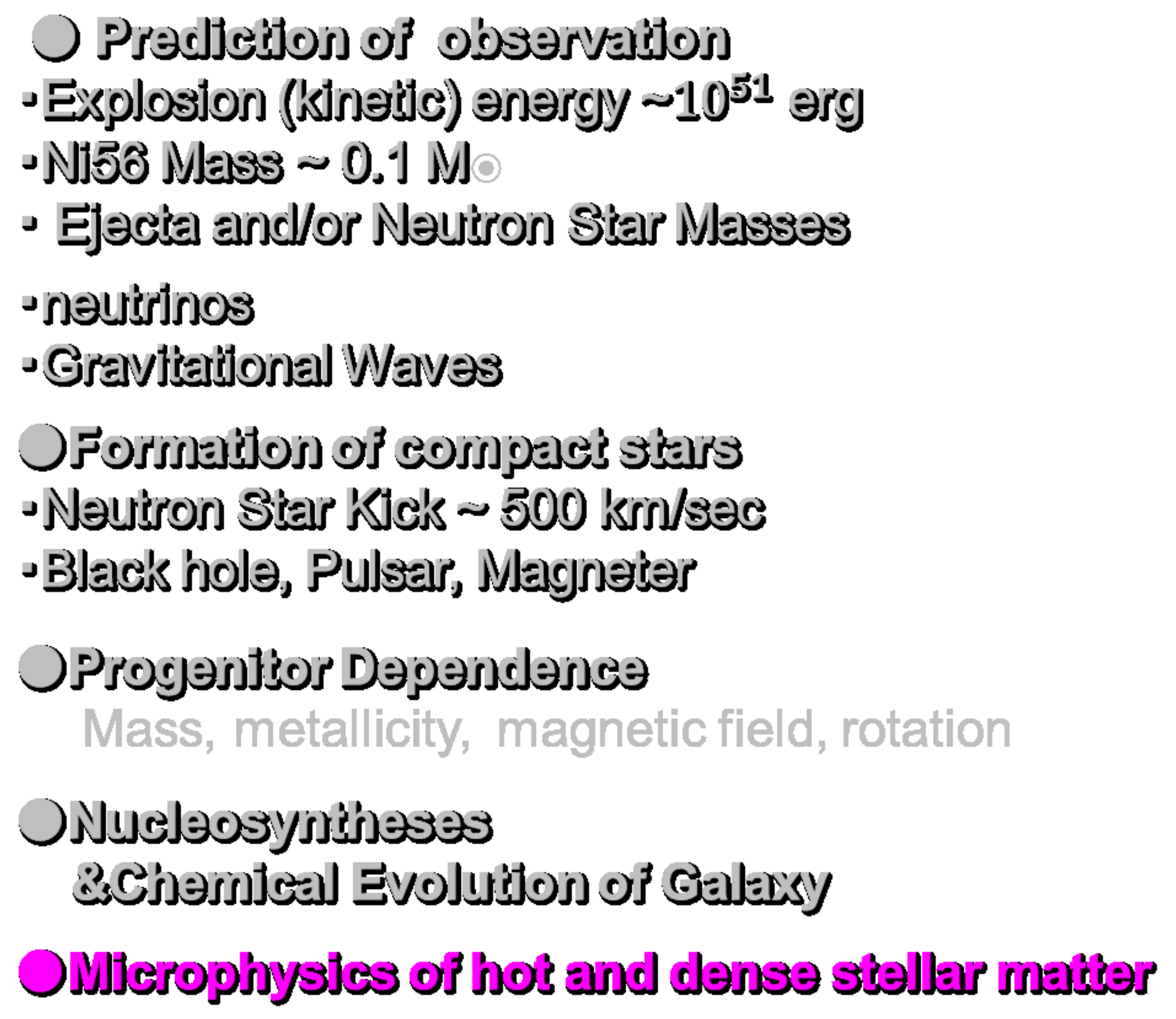
○Progenitor Dependence

Mass, metallicity, magnetic field, rotation

○Nucleosyntheses
&Chemical Evolution of Galaxy

●Microphysics of hot and dense stellar matter

Challenges in Supernovae



● **Prediction of observation**

- Explosion (kinetic) energy ~10^{51} erg
- Ni56 Mass ~ 0.1 $M_\odot$
- Ejecta and/or Neutron Star Masses

- neutrinos
- Gravitational Waves

● **Formation of compact stars**

- Neutron Star Kick ~ 500 km/sec
- Black hole, Pulsar, Magneter

● **Progenitor Dependence**

Mass, metallicity, magnetic field, rotation

● **Nucleosyntheses &Chemical Evolution of Galaxy**

● **Microphysics of hot and dense stellar matter**

enges
merical Simulation

eral relativity

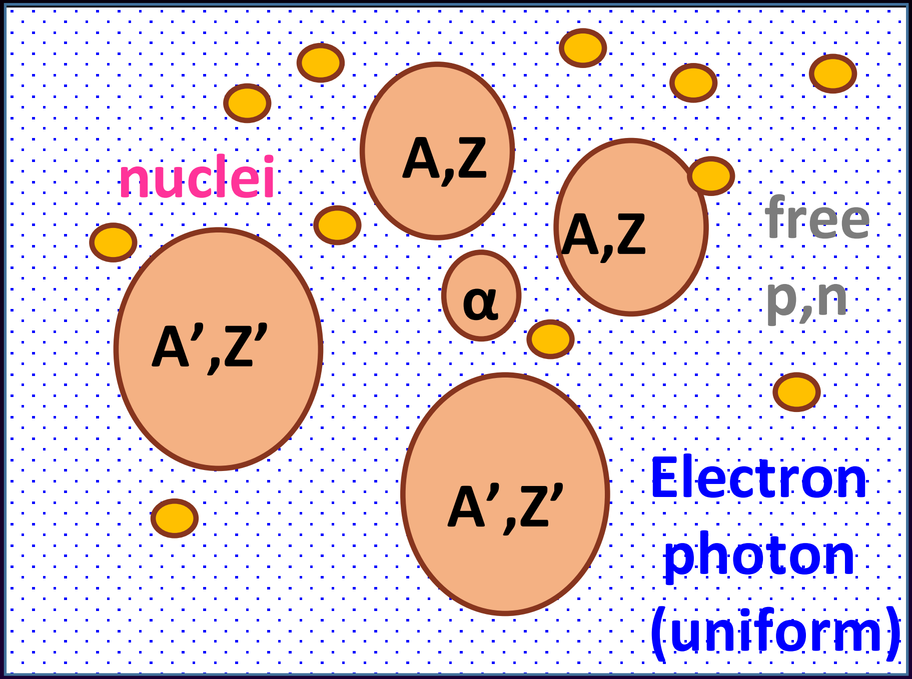
trino transport

g Term Simulation

ation of State
k Interactions

Composition of Supernova matter

Supernova matter



- neutrons and protons
- light and heavy nuclei

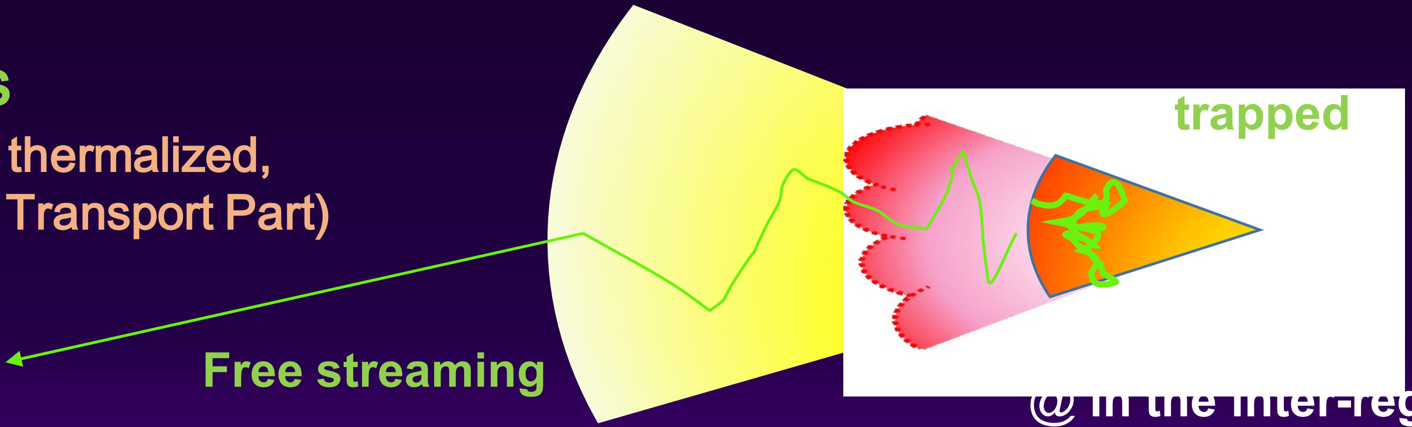
⇒ Nuclear Equation of State (EOS)

- electrons, positrons, (muons)
- photons

(Easy, ideal Fermi or Bose gasses)

- neutrinos

(not always thermalized,
Boltzmann Transport Part)



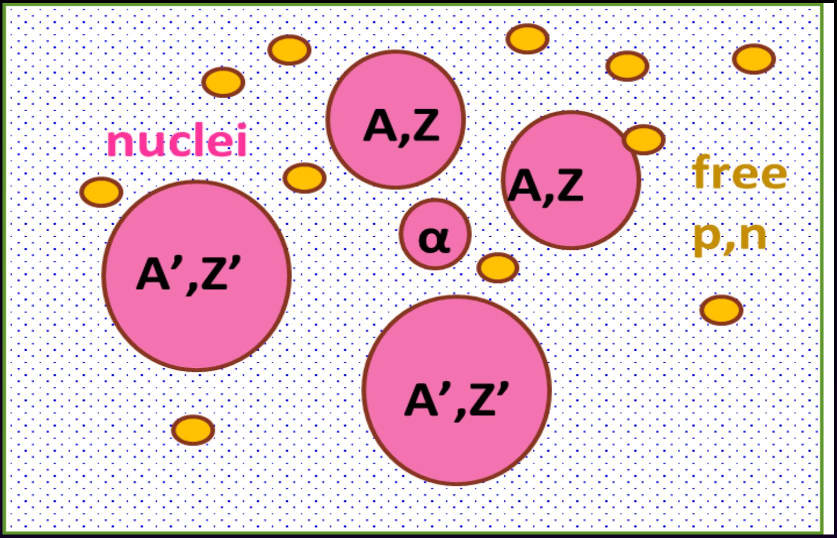
@ in the inter-region

How to calculate reliably the nuclear composition and EOS?

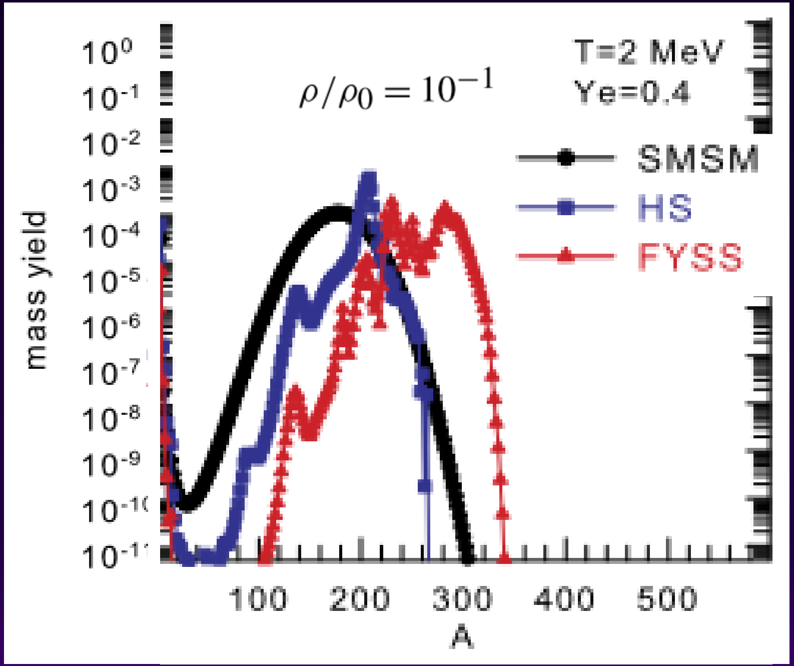
We can obtain EOS
by minimizing the free energy
at given ***T, Ye, ρ, but …***

the free energies are different in different models:

- SMSM EOS
(Botvina&Mishustin '04, '11
Buyukcizmeci et al. '13, '14)
- HS EOS
(Hempel et al. '10)
- FYSS EOS
(Furusawa et al. '11,'13,'17)
- RMF DD2 (Typel et al. '16)

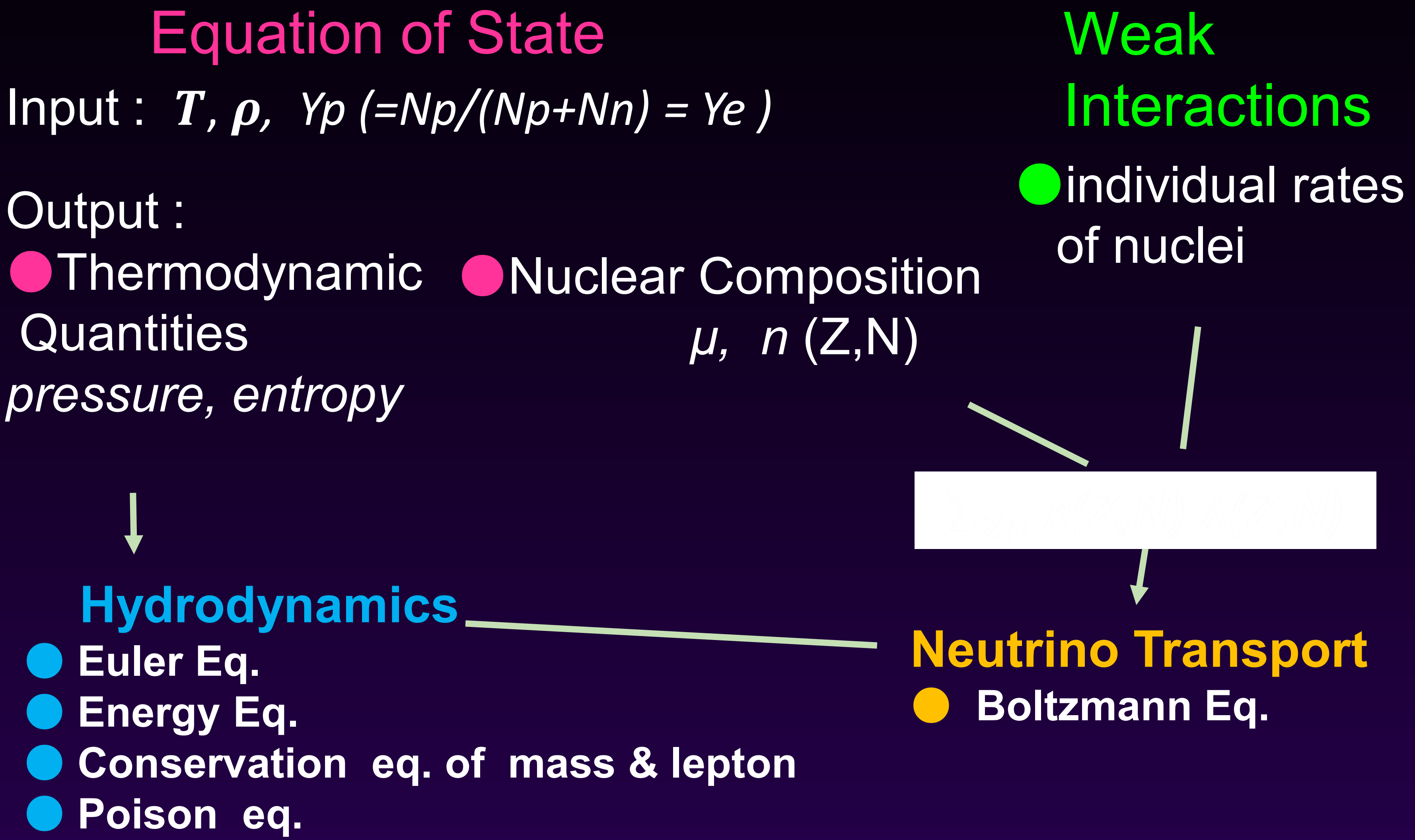


Nuclear Composition



Buyukcizmeci ('13)

Physics in Supernova Simulations



Equation of State

Input : T, ρ, *Yp (=Np/(Np+Nn) = Ye)*

Output :

- Thermodynamic Quantities *pressure, entropy*
- Nuclear Composition *μ, n* (Z,N)

Weak Interactions

- individual rates of nuclei

$$\sum_{Z,N} n(Z,N)\,\lambda(Z,N)$$

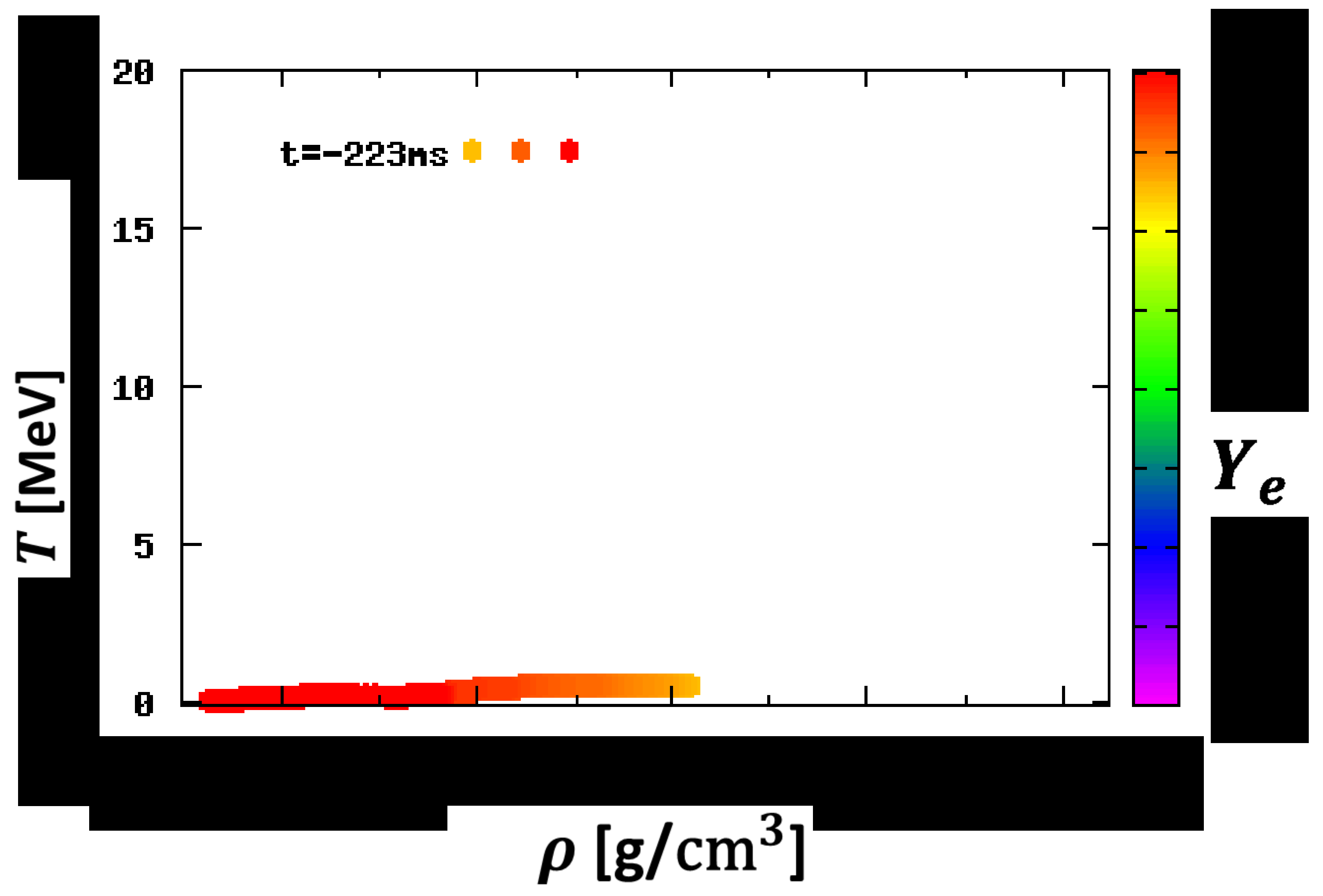
Hydrodynamics

- **Euler Eq.**
- **Energy Eq.**
- **Conservation eq. of mass & lepton**
- **Poison eq.**

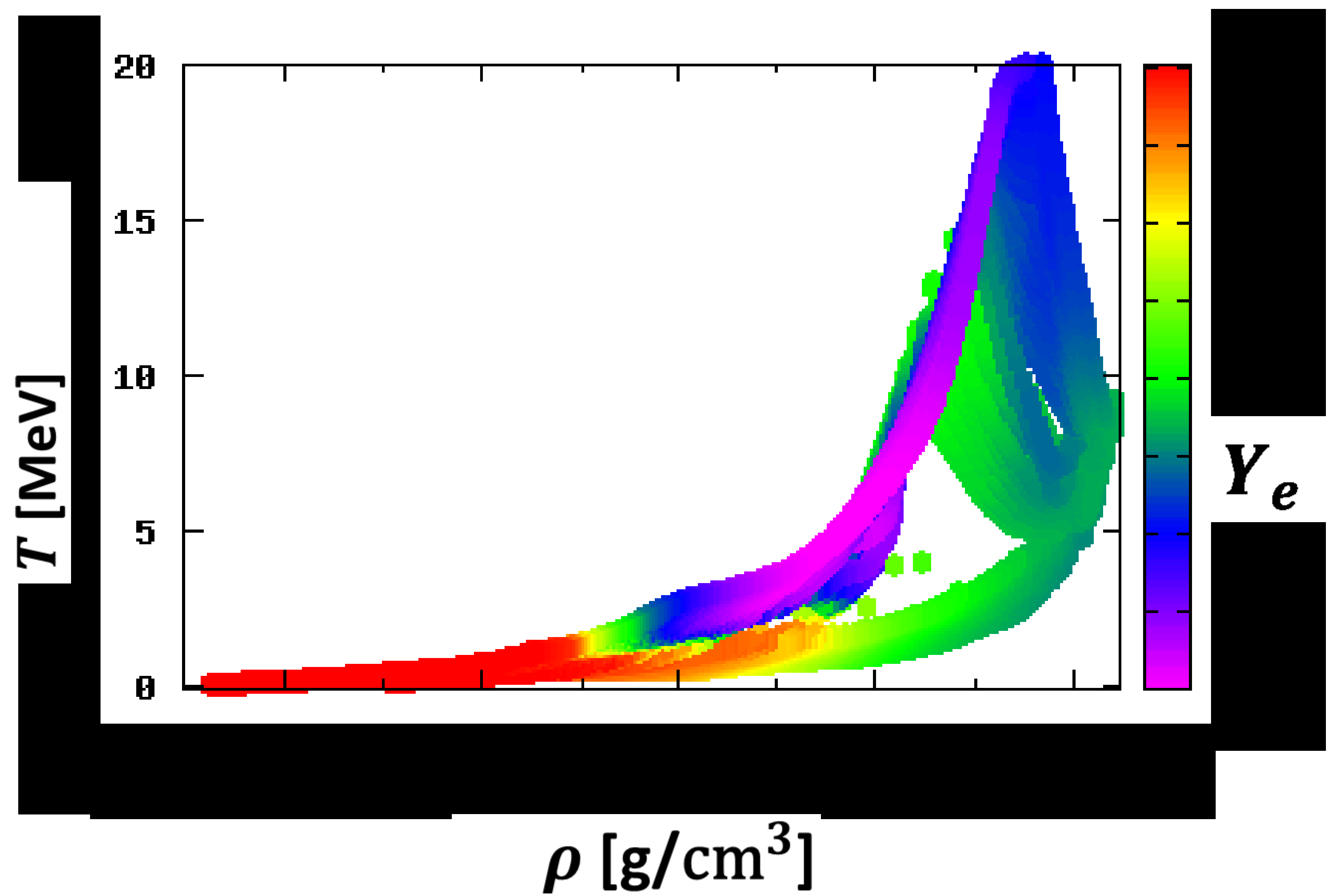
Neutrino Transport

- **Boltzmann Eq.**

Radial profiles of ρ, T, Ye

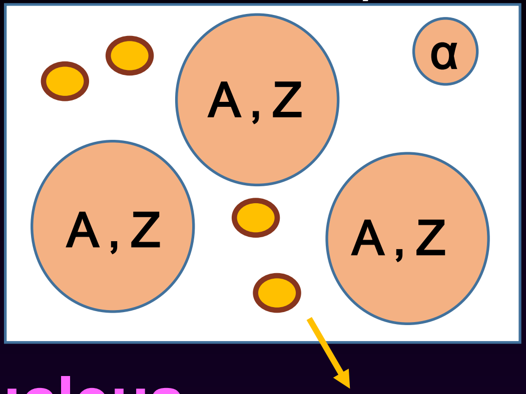


Radial profiles of ρ, T, Ye



Single Nucleus VS Multi-nucleus

● Single-Nucleus EOSs (optimize the nuclear structure)



Dripped Nucleons

- LS EOS (Lattimer and Swesty '91)
- STOS EOS (Shen et al. '98, '11)

Compressible Liquid Drop model (LDM)
or Thomas Fermi approximation

⇒Only one representative heavy nucleus
(Single Nucleus Approximation(SNA))

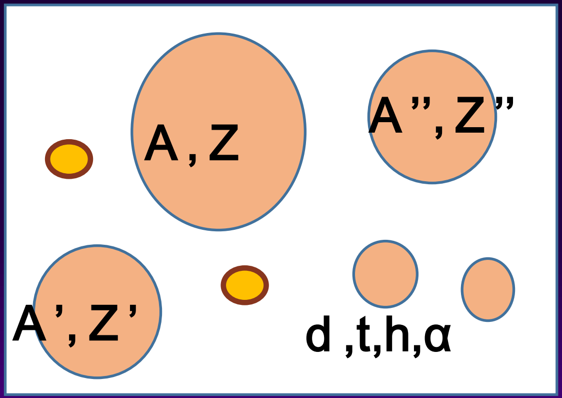
⇒Only He4 of light nuclei

● Multi-Nucleus EOSs (optimize the nuclear ensemble)

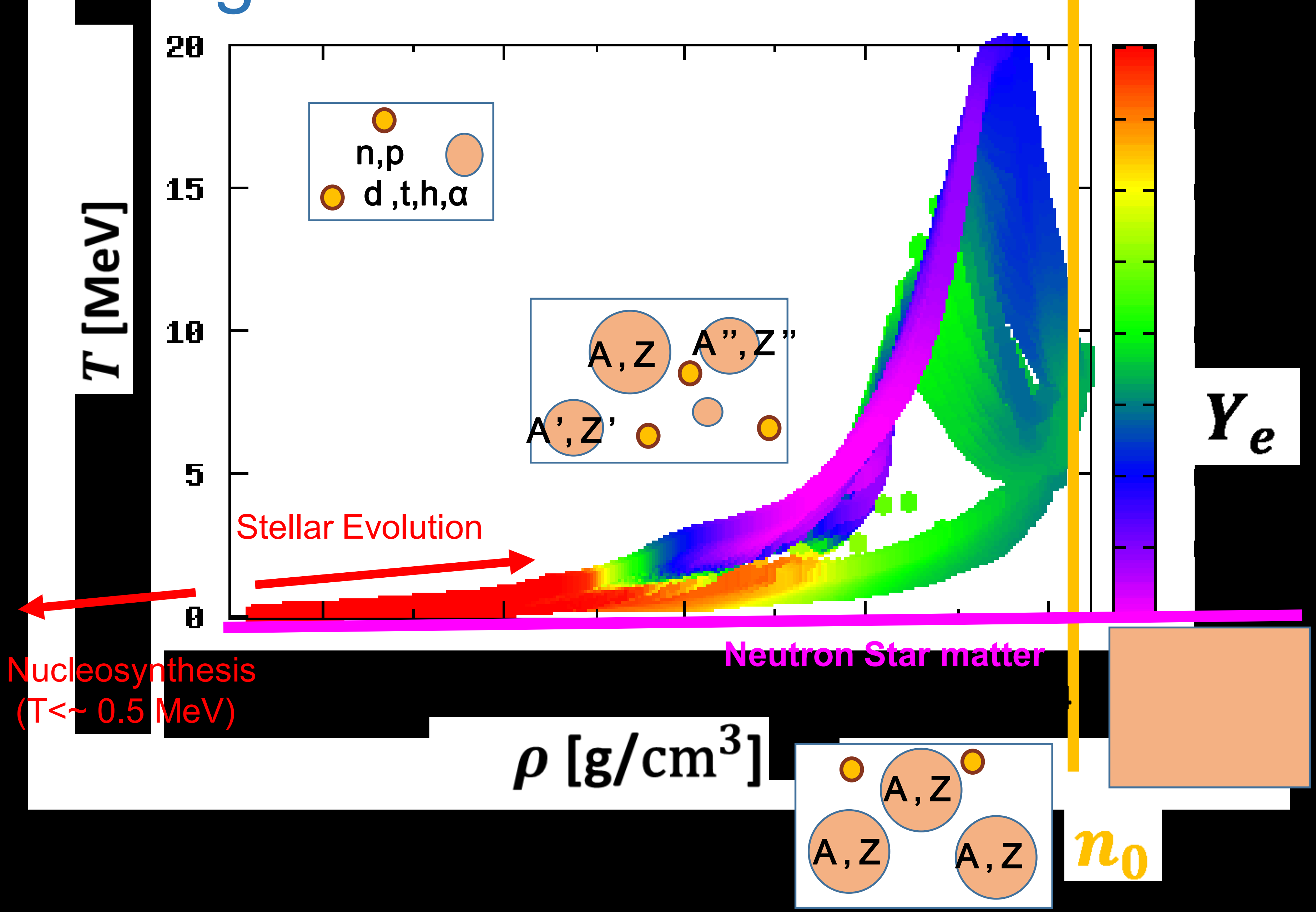
- SMSM EOS (Botvina & Mishustin '04, Buyukcizmeci et al. '10 '14)
- HS EOS (Hempel & Schaffner '10, Steiner et al. '13)
- FYSS EOS (Furusawa et al. '11,'13, '17a, '17d)

Incompressible LDM or nuclear mass data

⇒Simple evaluation of nuclear binding energies to solve full nuclear ensemble
(Multi Nucleus Approximation(MNA))



Single-Nucleus vs Multi-Nucleus

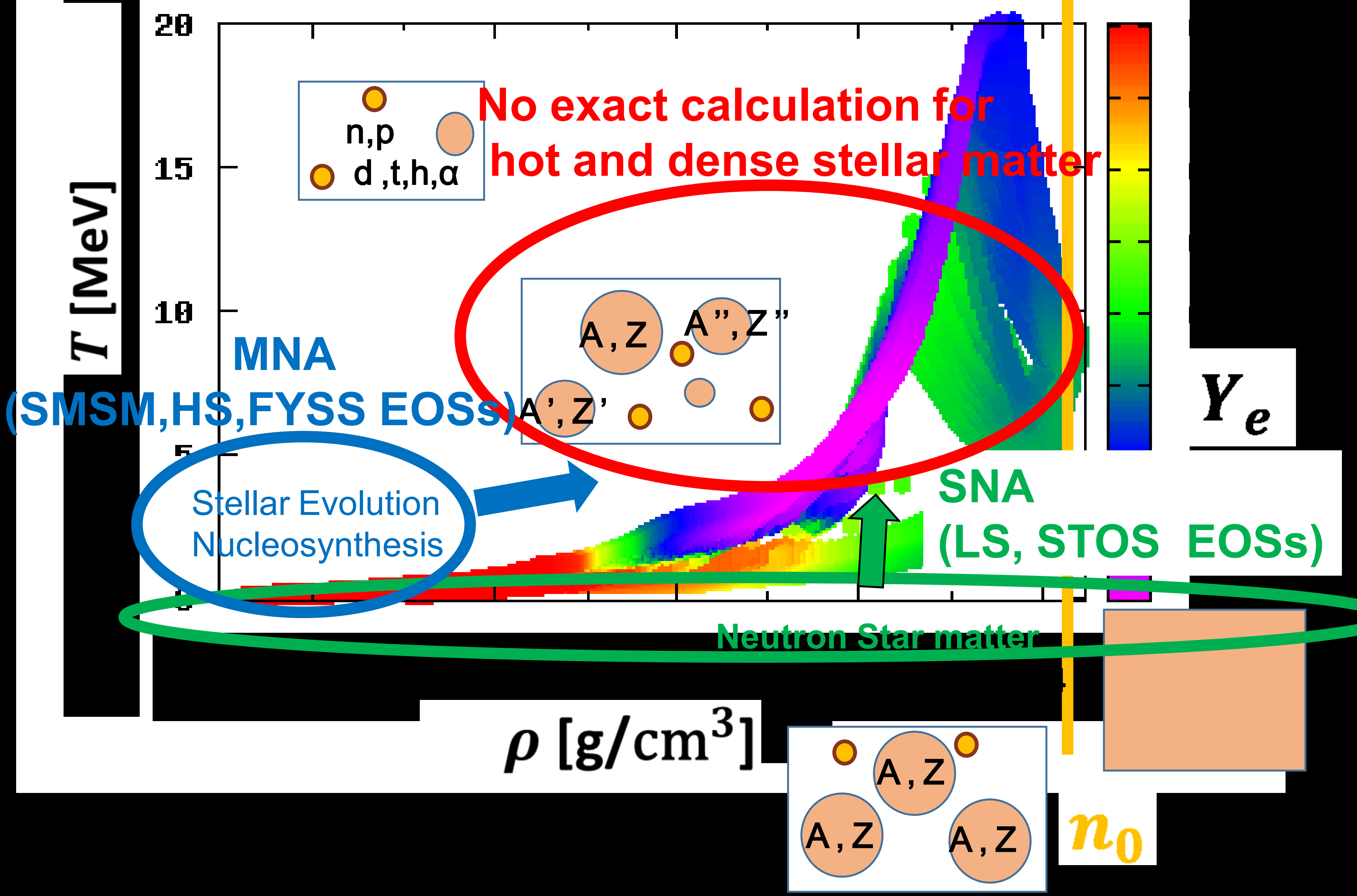


Stellar Evolution

Nucleosynthesis
(T<~ 0.5 MeV)

Neutron Star matter

Single-Nucleus vs Multi-Nucleus



No exact calculation for hot and dense stellar matter

MNA (SMSM,HS,FYSS EOSs)

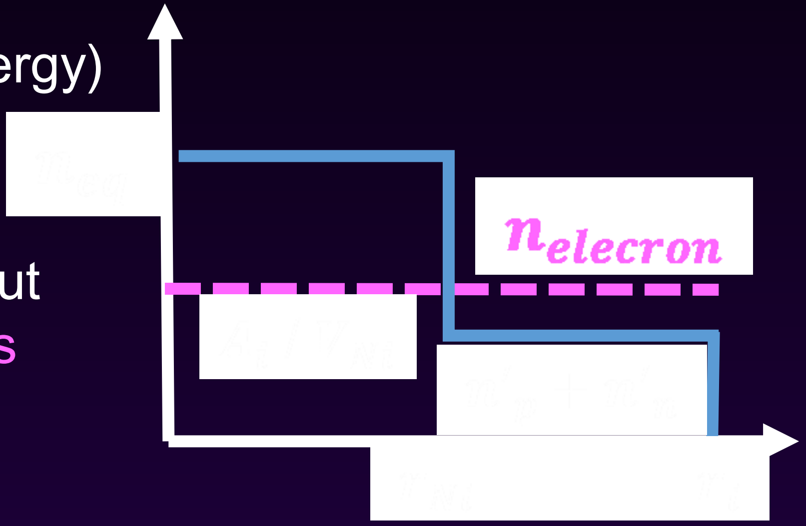
Stellar Evolution Nucleosynthesis

SNA (LS, STOS EOSs)

Neutron Star matter

Self-Consistent Calculation of nuclear ensemble and equilibrium density

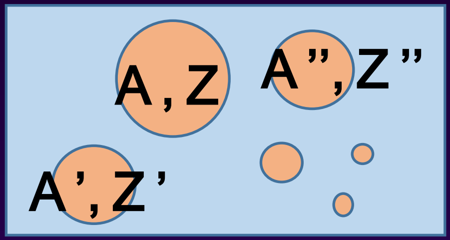
1, Solve the structure of all nuclei
(minimize the individual binding energy)



Information about
dripped nucleons
n'_p, n'_n

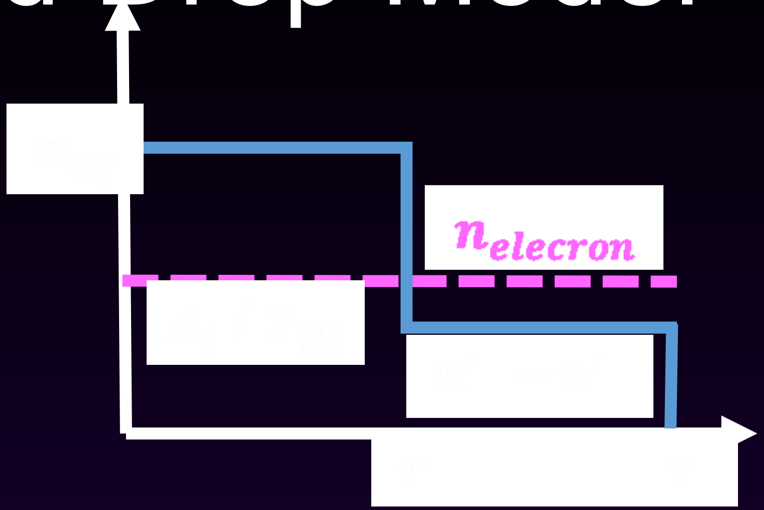
Information about
binding energies
of all nuclei

2, Solve the ensemble
(minimize the total free energy)



Convergence of all quantities
⇒ the equilibrium state at given n_B, T, Y_p

Compressible Liquid Drop Model



1, Solve the equilibrium densities of all nuclei
(minimize the individual binding energy)

$$F_i = \underline{F_i^B} + \underline{F_i^C} + \underline{F_i^S}$$

$\propto n_{eq}^{1/3}$ $\propto n_{eq-}^{-2/3}$

Bulk: Skrme Type Interaction

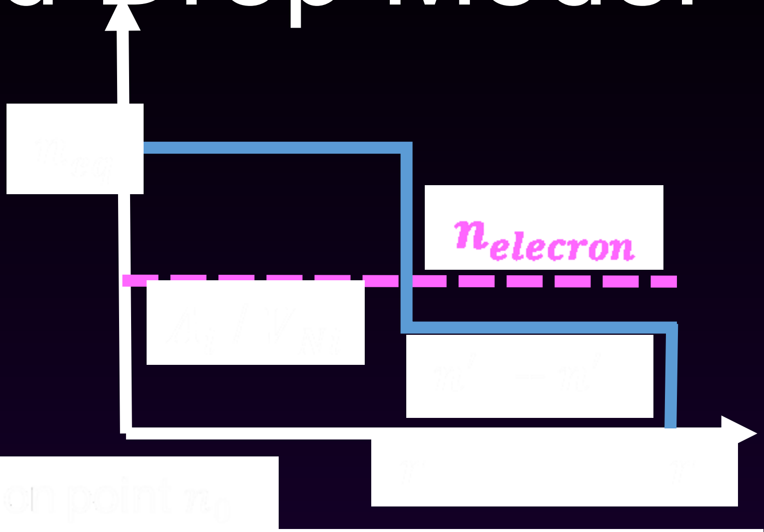
$$F_i^B(n_{eq}, T) \;=\; A_i \left[\omega_{int}(n_{eq}, Z_i/A_i, T) + \omega_{kin}(n_{eq}, Z_i/A_i)\right]$$

$$\begin{aligned}
\omega_{int}(n_B, x) &= 4x(1-x)v_s(n_B)/n_B + (1-2x)^2 v_n(n_B)/n_B, \\
v_s(n_B) &= a_1 n_B^2 + \frac{a_2 n_B^3}{1 + a_3 n_B}, \\
v_n(n_B) &= b_1 n_B^2 + \frac{b_2 n_B^3}{1 + b_3 n_B},
\end{aligned}$$

$$\omega_{kin}(n_B, x, T) = \frac{T}{2\pi^2 n_B}\left(\frac{2m_p T}{\hbar^2}\right)^{3/2} F_{3/2}(\eta_p) + \frac{T}{2\pi^2 n_B}\left(\frac{2m_n T}{\hbar^2}\right)^{3/2} F_{3/2}(\eta_n)$$

Compressible Liquid Drop Model

1, Solve the equilibrium densities of all nuclei
(minimize the individual binding energy)



$$F_i = \underbrace{F_i^B}_{} + \underbrace{F_i^C}_{\propto n_{eq}^{1/3}} + \underbrace{F_i^S}_{\propto n_{eq-}^{-2/3}}$$

Bulk: Parametric bulk property near the saturation point n_0

$$F_i^B(n_{eq},T) = A_i\left(\omega_0 + \frac{K_0}{18n_0^2}(n_{eq}-n_0)^2 + \left[S_0 + \frac{L}{3n_0}(n_{eq}-n_0)\right](1-2x)^2 - \frac{T^2}{\epsilon_0}\right)$$

Coulomb: Integration of Coulomb potential in Wigner-Seiz cell containing nucleus i

$$F_i^C(n_{eq},n'_p,n_e) = \frac{3}{5}\left(\frac{3}{4\pi}\right)^{-1/3} e^2 n_{eq}^2 \left(\frac{Z_i - n'_p V_{Ni}}{A_i}\right)^2 {V_{Ni}}^{5/3}\underline{D(u_i)}$$

Surface : Surface area times surface tension (Agwal'14) + in-medium effect

$$F_i^S(n_{eq},n'_p,n'_n,T) = 4\pi r_{Ni}^2\sigma_i \underline{\left(1-\frac{n'_p+n'_n}{n_{eq}}\right)^2}\left(\frac{T_c^2-T^2}{T_c^2+T^2}\right)^{5/4}$$

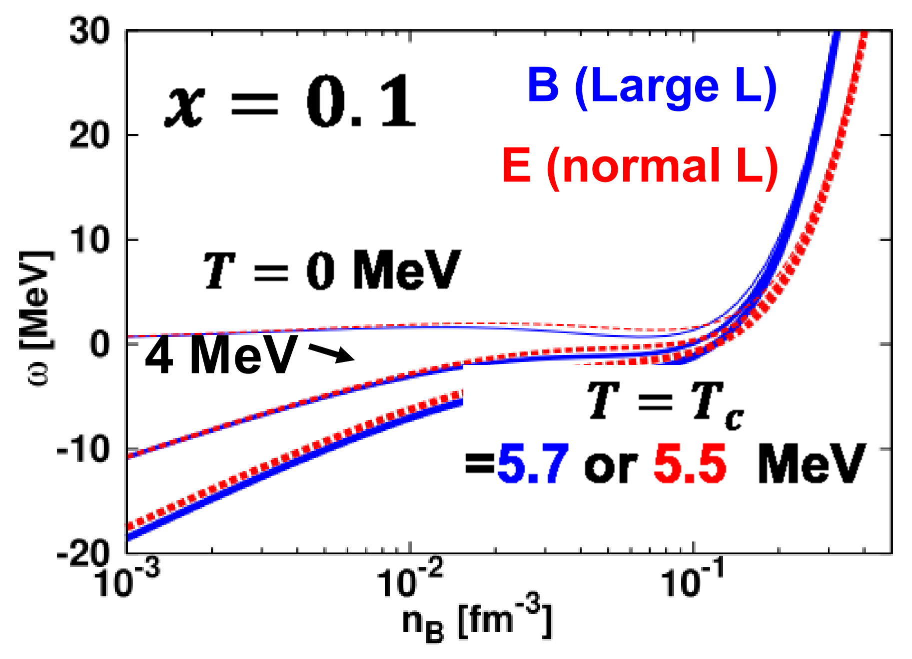
Skrme Type Interactions (Oyamatsu & Iida '03, '07)

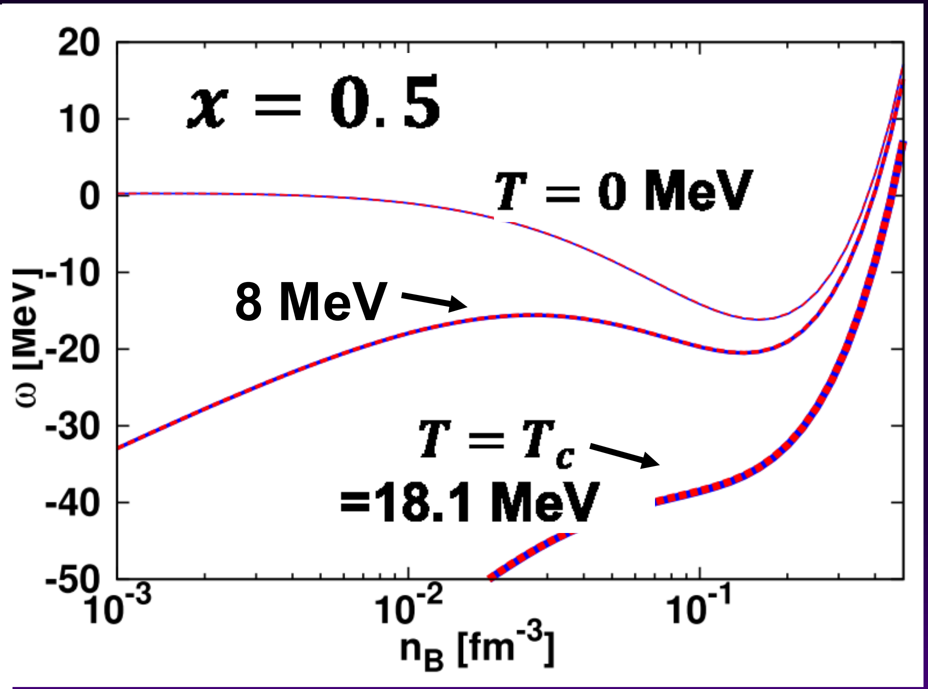
for bulk free energy (nuclei and dripped nucleon)

$$\omega(n_B, x, T) = \omega_{int}(n_B, x) + \omega_{kin}(n_B, x, T)$$

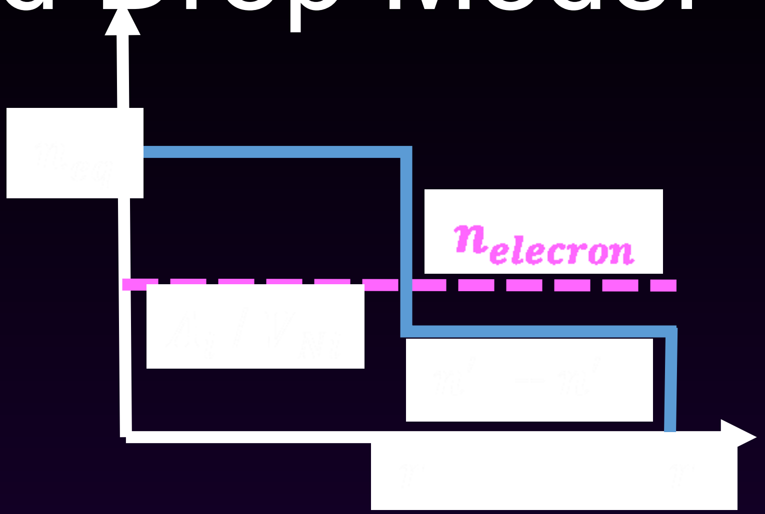
T = 0 MeV

parameter set	B	E
n_0 [fm^{-3}]	0.15969	0.15979
ω_0 [MeV]	-16.184	-16.145
K_0 [MeV]	230	230
S_0 [MeV]	33.550	31.002
L [MeV]	73.214	42.498





Compressible Liquid Drop Model



1, Solve the equilibrium densities of all nuclei
(minimize the individual binding energy)

$$F_i = \underline{F_i^B} + \underline{F_i^C} + \underline{F_i^S}$$

$\propto n_{eq}^{1/3}$ $\propto n_{eq-}^{-2/3}$

Bulk: Skrme Type Interaction

$$F_i^B(n_{eq}, T) \;=\; A_i\left[\omega_{int}(n_{eq}, Z_i/A_i, T) + \omega_{kin}(n_{eq}, Z_i/A_i)\right]$$

Coulomb: Integration of Coulomb potential in Wigner-Seiz cell containing nucleus i

$$F_i^C(n_{eq}, n_p', n_e) = \frac{3}{5}\left(\frac{3}{4\pi}\right)^{-1/3} e^2 n_{eq}^2 \left(\frac{Z_i - n_p' V_{Ni}}{A_i}\right)^2 {V_{Ni}}^{5/3} \underline{D(u_i)}$$

$$\underline{D(u_i) = 1 - \tfrac{3}{2}u_i^{1/3} + \tfrac{1}{2}u_i}$$

$T = T_c$ = 18.1 MeV

Surface : Surface area times surface tension (Agwal'14) + in-medium effect

$$F_i^S(n_{eq}, n_p', n_n', T) = 4\pi r_{Ni}^2 \sigma_i \underline{\left(1 - \frac{n_p' + n_n'}{n_{eq}}\right)^2} \left(\frac{T_c^2 - T^2}{T_c^2 + T^2}\right)^{5/4}$$

Dripped nucleons

reduce F_i^S

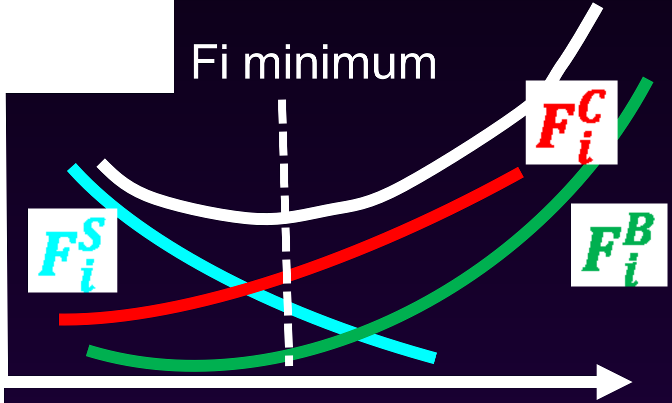
Self-Consistent Calculation of nuclear ensemble and equilibrium density

1. Solve the equilibrium densities (n_{eq}) of all nuclei (minimize the individual binding energy)

$$\underbrace{F_i^B}_{} + \underbrace{F_i^C}_{} + \underbrace{F_i^S}_{}$$

$$F_i = F_i^B + F_i^C + F_i^S$$

$\propto n_{eq}^{1/3}$ $\propto n_{eq}^{-2/3}$



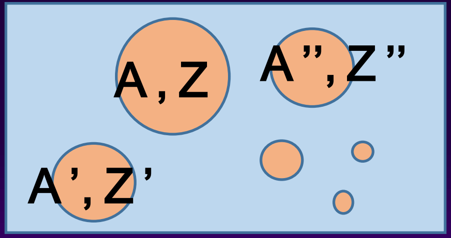
$n_{eq}(A,Z)$

2. Solve the ensemble (μ_p, μ_n, n'_p, n'_n) (minimize the total free energy)

$$n_p + n_n + 4n_\alpha + \sum_i A_i n_i = n_B$$
$$n_p + 2n_\alpha + \sum Z_i n_i = n_e$$

$$\mu = Z\mu + N\mu$$
$$n = n(\mu, T)$$

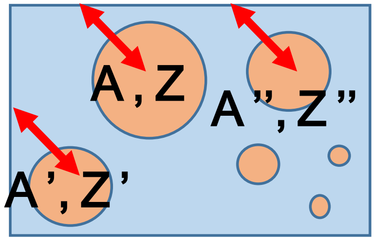
$$\mu_{p/n} = \mu'_{p/n}(n'_p, n'_n) + \sum_i n_i \frac{\partial F_i(n'_p, n'_n)}{\partial n_{p/n}}$$



Models

Model **Multi-nucleus Compressible** (MC, the new model)

solve $n_p, n_n, n_e, \{n_i\}, \{n_{eqi}\}$



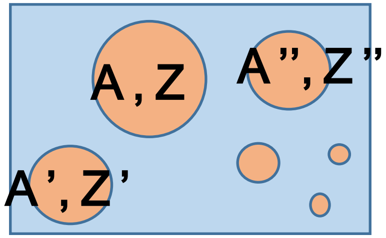
Compression or decompression

Model Multi-nucleus Incompressible (MI)

~SMSM,HS,FYSS EOSs **(MNA)**

solve $n_p, n_n, n_e, \{n_i\}$

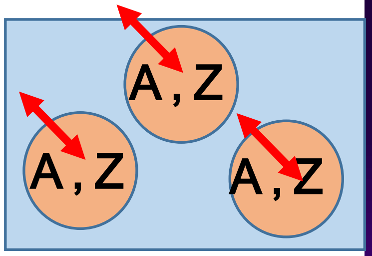
$n_{eqi} = n_{eqi}(T = 0, n'_p = 0, n'_n = 0, n_e = 0)$



Model Single-nucleus compressible (SC)

~LS, STOS EOS's **(SNA)**

solve $n_p, n_n, n_e, n_{rep}, n_{eq\,rep}, Z_{rep}, A_{rep}$



Results

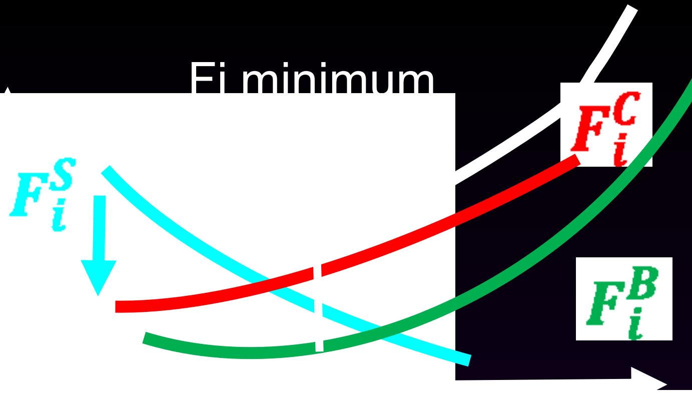
low Yp and high T

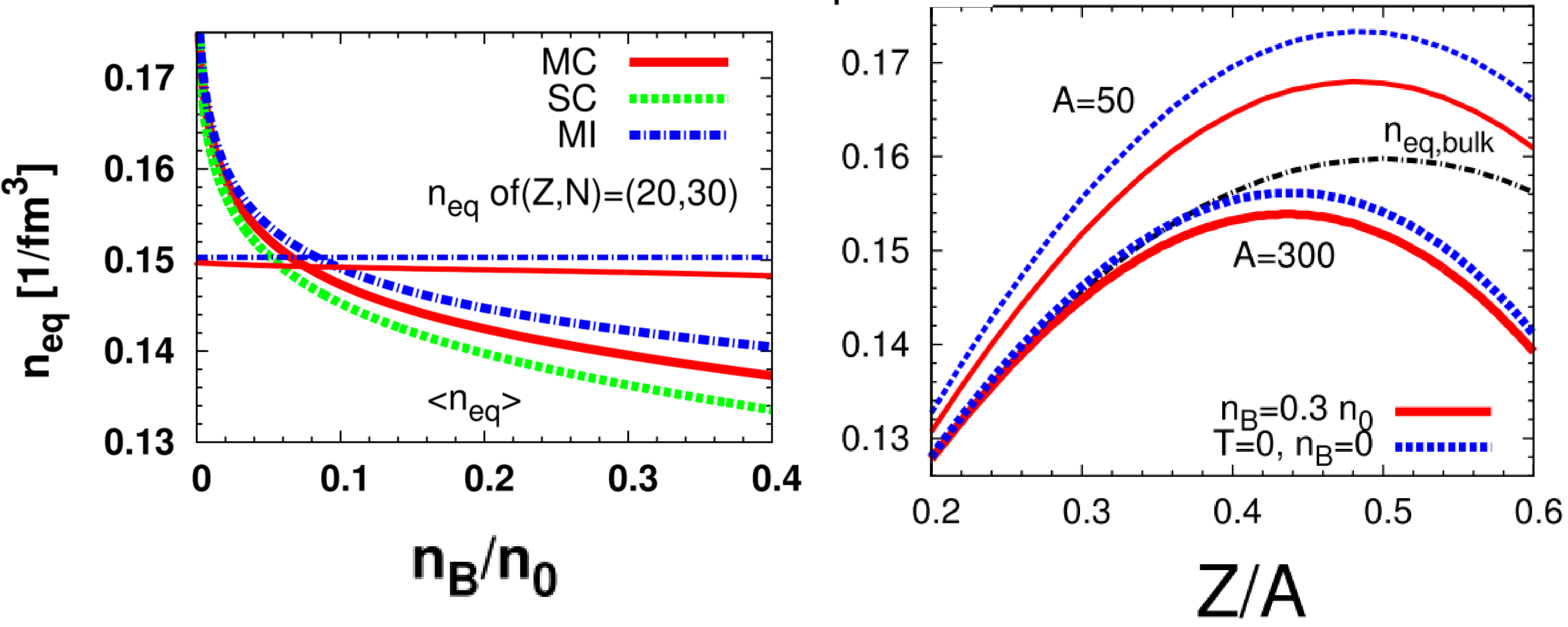
⇒ Dense dripped neutrons

⇒ Surface energy reduction

⇒ **Decompression of individual nuclei**

(Average or representative $\langle n_{eq} \rangle$ always decreases.)





Results

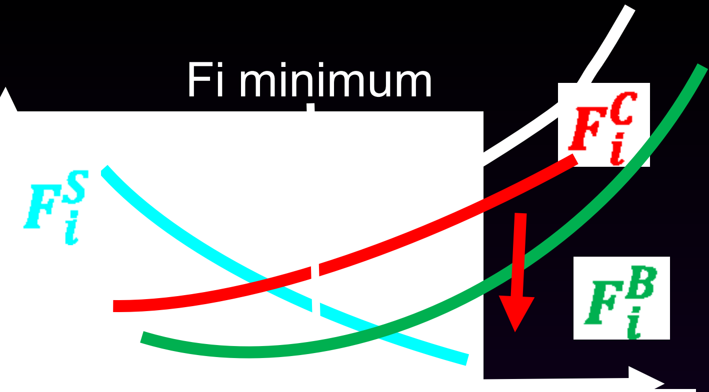
high Yp and low T

⇒ Dense electrons

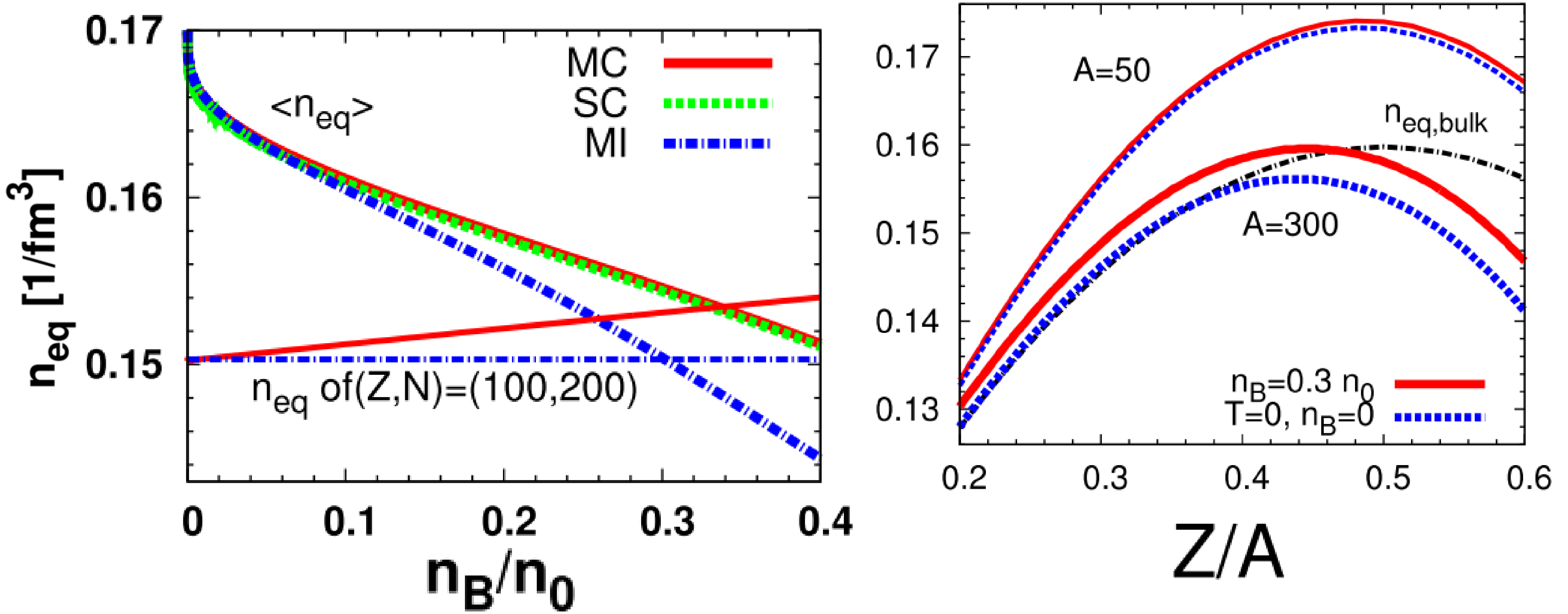
⇒ Coulomb energy reductions

⇒ **Compression of individual nuclei**

(Average or representative $<n_{eq}>$ always decreases.)

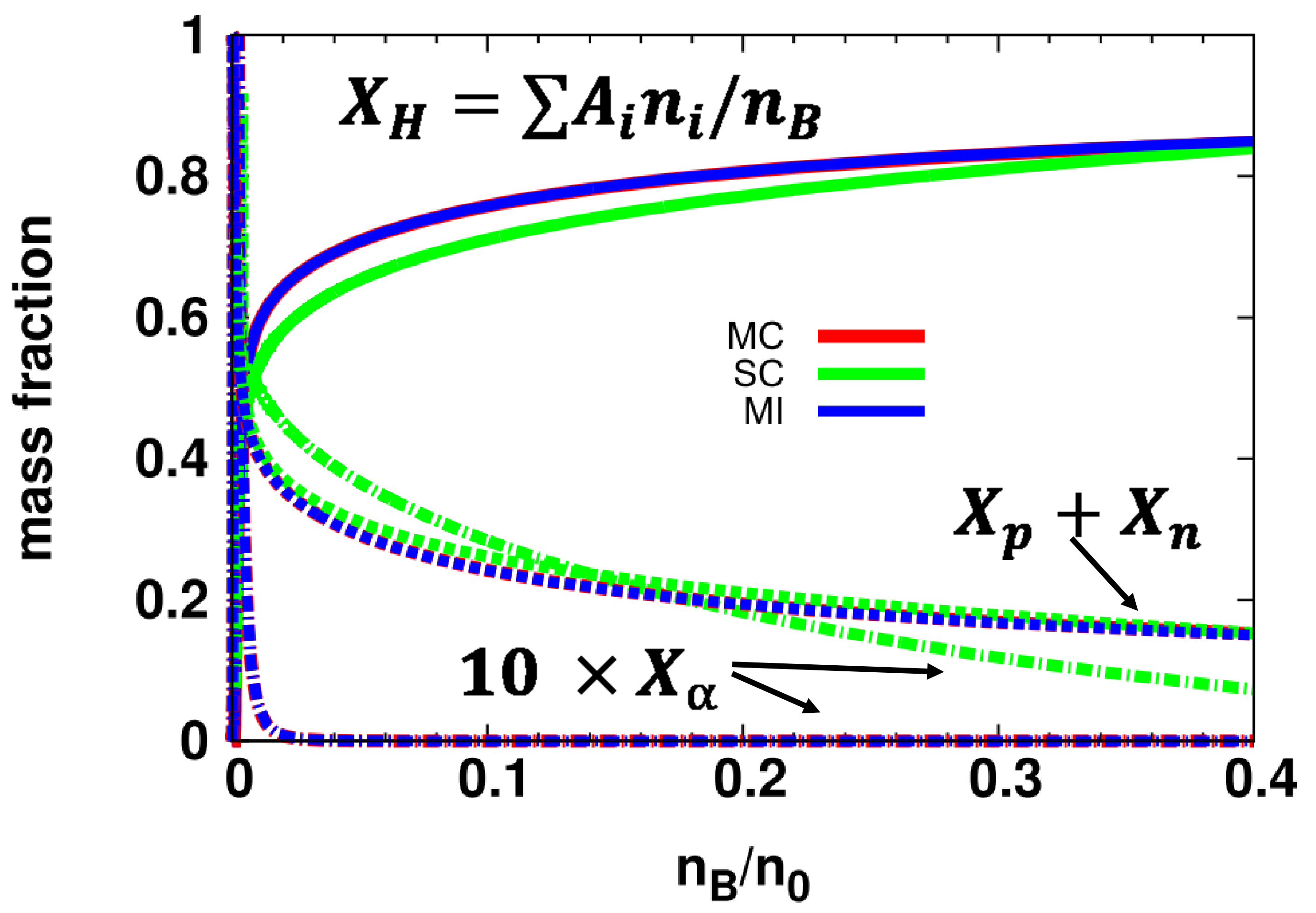


T=1 MeV Y_p=0.4



Mass Fraction

T=3 MeV Y_p=0.2



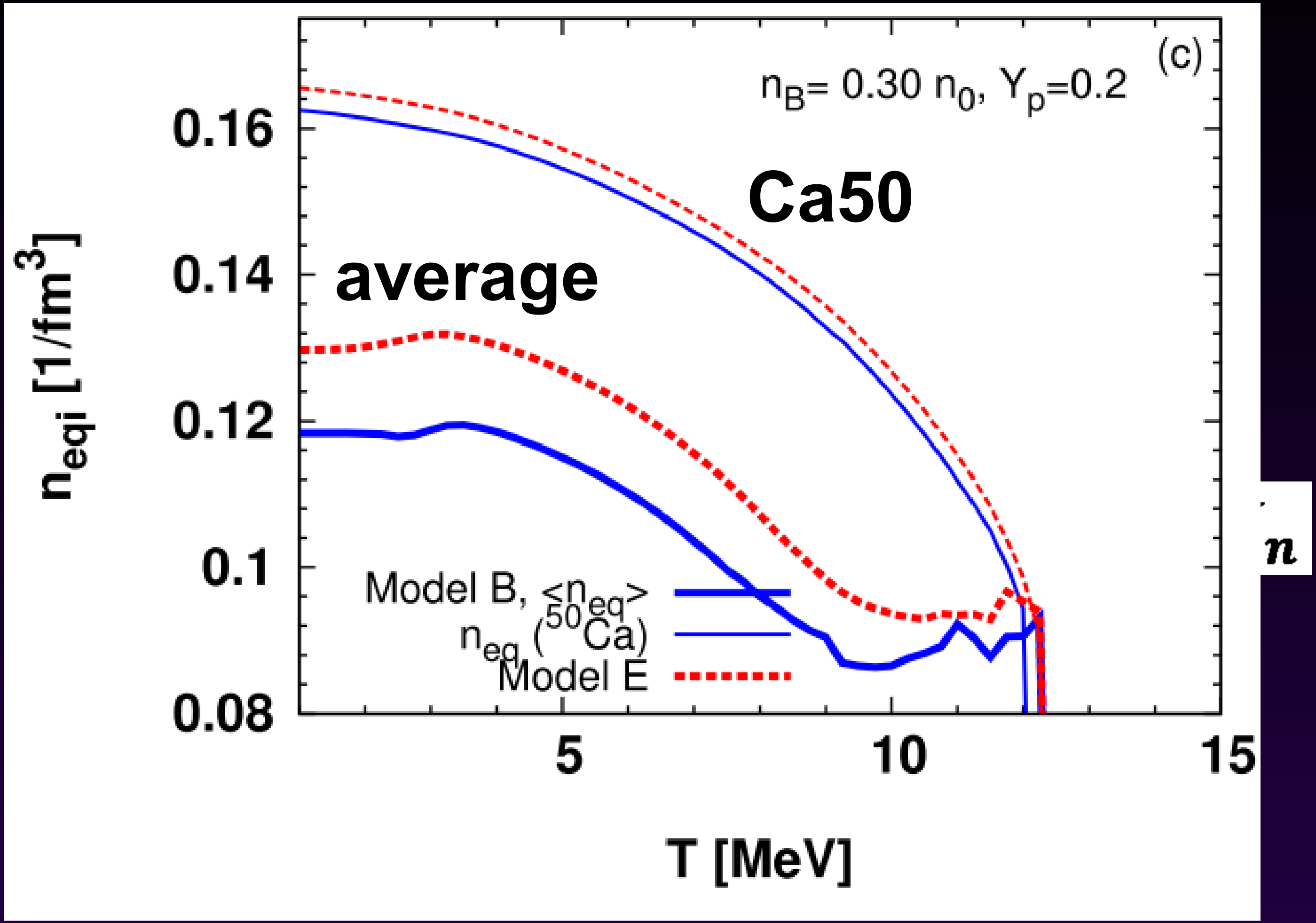
Compression or decompression hardly affects the total Mass Fraction.

(Compressible model agrees with Incompressible model)

The dripped nucleons and α particles are overestimated

in Single-Nucleus EOS

Vaporization at high Temperatures



Neutron-rich nuclei have smaller equilibrium density.
Sequential vaporizations of neutron-rich nuclei take place as temperature incerases.

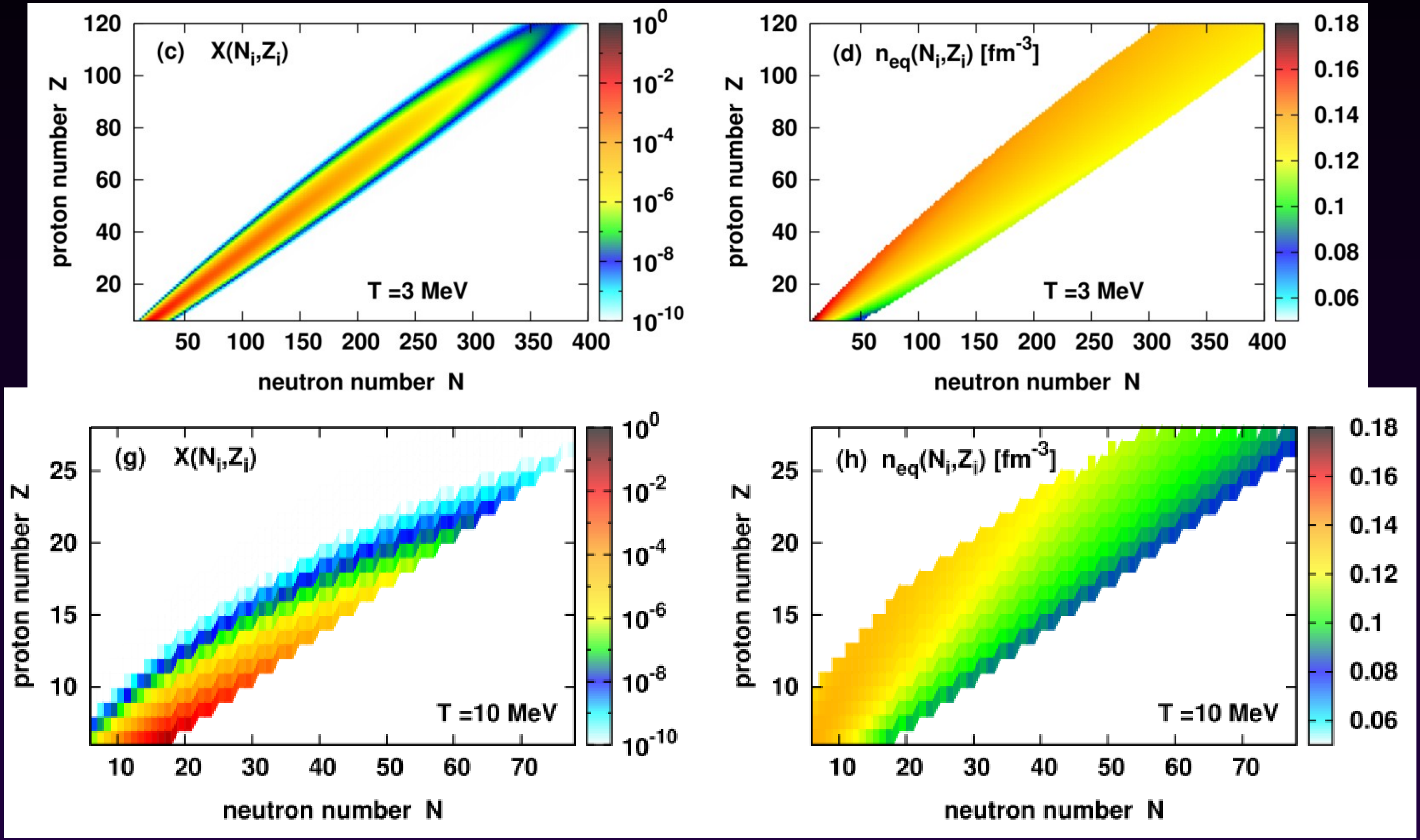
Vaporization at high Temperatures

Mass fractions

Equilibrium densities

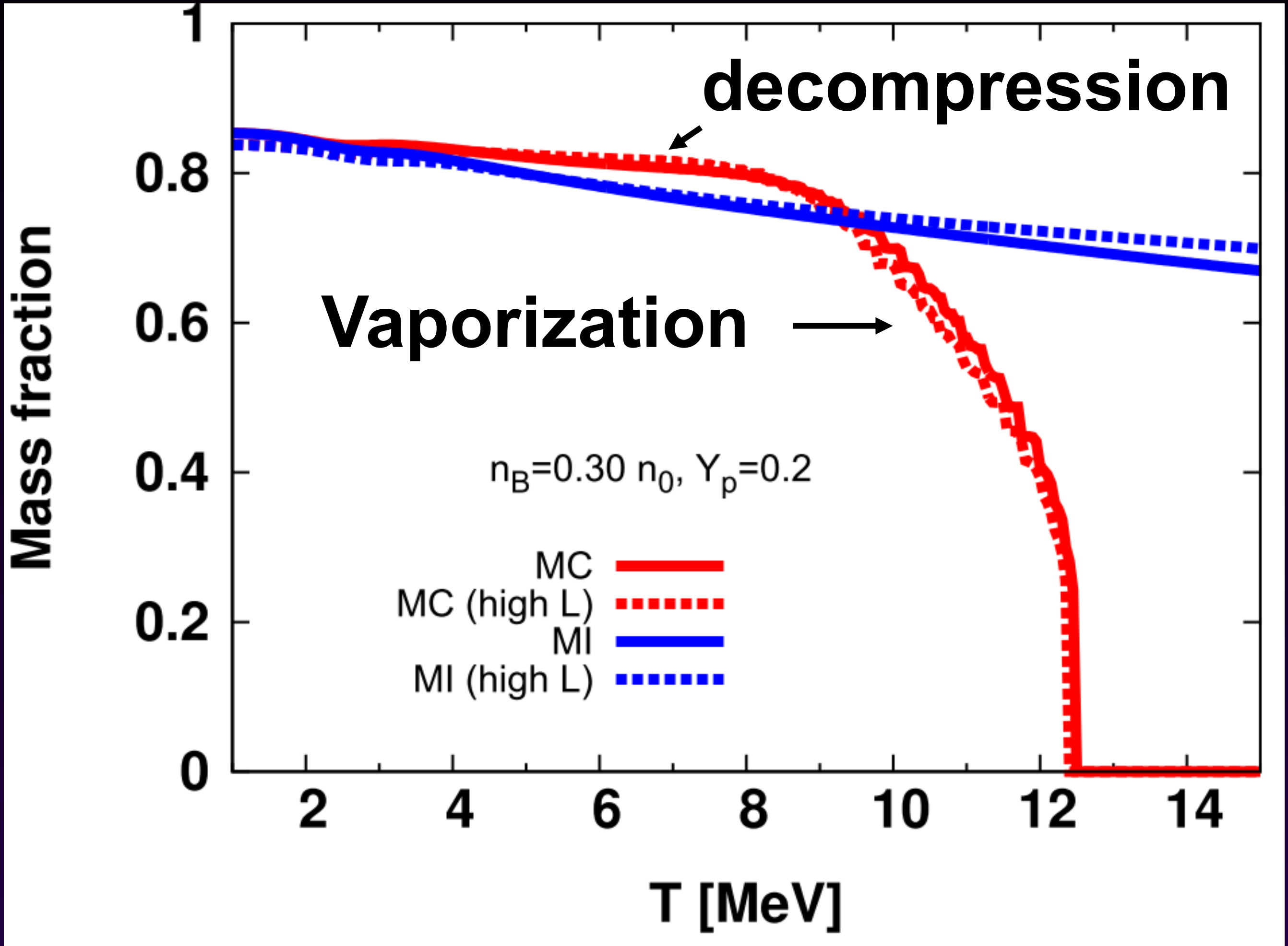
T=3 MeV

T=10 MeV



Neutron-rich nuclei have smaller equilibrium density.
Sequential vaporizations of neutron-rich nuclei take place as temperature incerases.

Mass Fraction n_B=0.30 n_0, Y_p=0.2



Incompressible model with an ordinary critical temperature T=18 MeV considerably deviates from **Compressible model** due to the lack of decompression and vaporization

Summary

○ Current SN EOS models are based on
Single Nucleus Approximation (SNA) or
simple nuclear models in Multi-nucleus ensemble (MNA).

○ We calculate the equilibration of individual nuclei
self-consistently with the full nuclear ensemble.

○ Individual nuclei are
compressed at nucleons-dripped conditions (low Yp & high T)
or decompressed at the non-drip conditions (high Yp & low T)

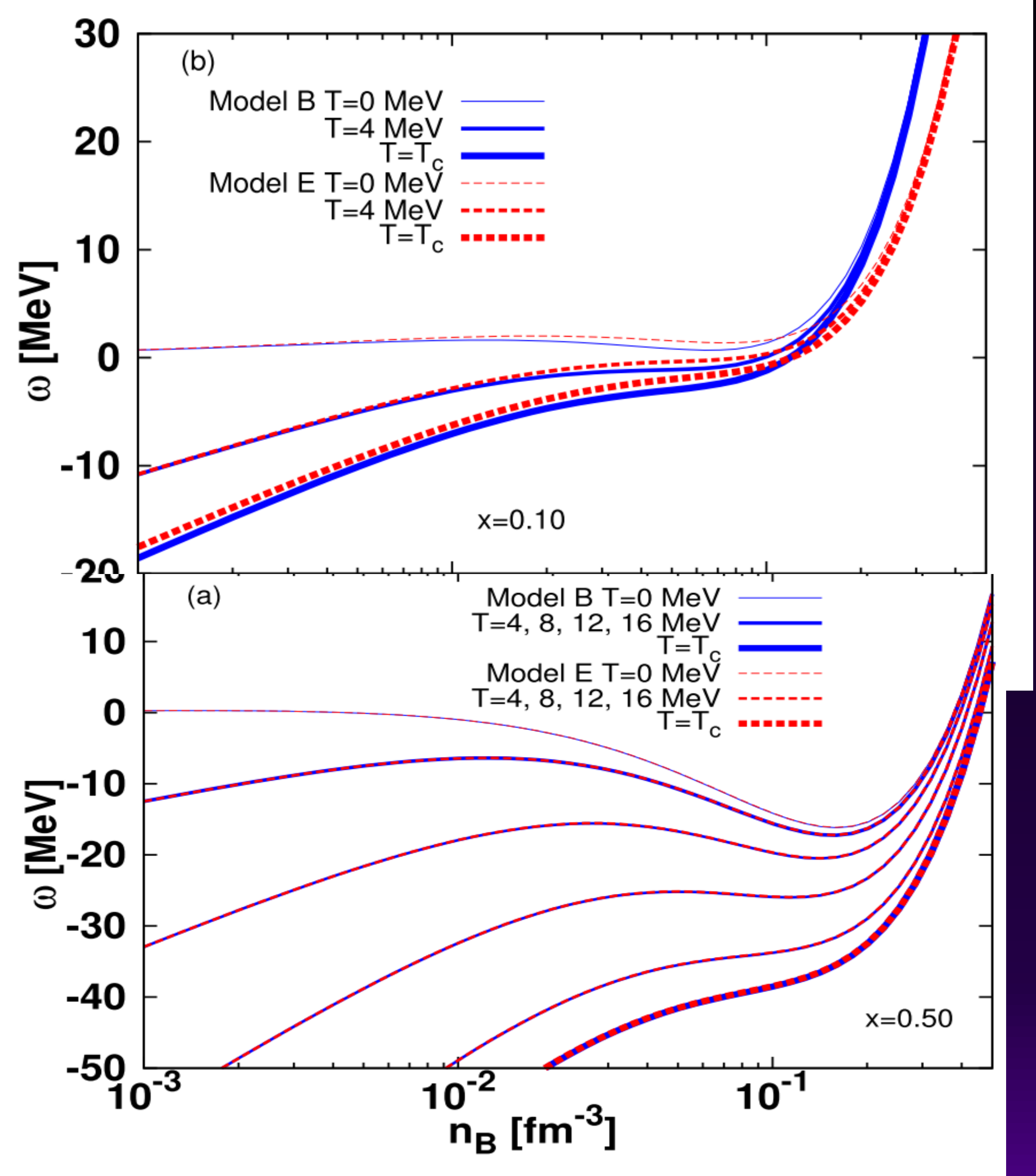
○ SNA EOS model always overestimates the fractions of dripped nucleons and light clusters and average mass number of heavy nuclei.

○ MNA EOS models are rather good at $n_B \leq 0.3\, n_0$ and $T \leq 3$ MeV
but deviate from self-consistent calculation above $T = 3$ MeV
because of the ignorance of decompression and vaporizations.

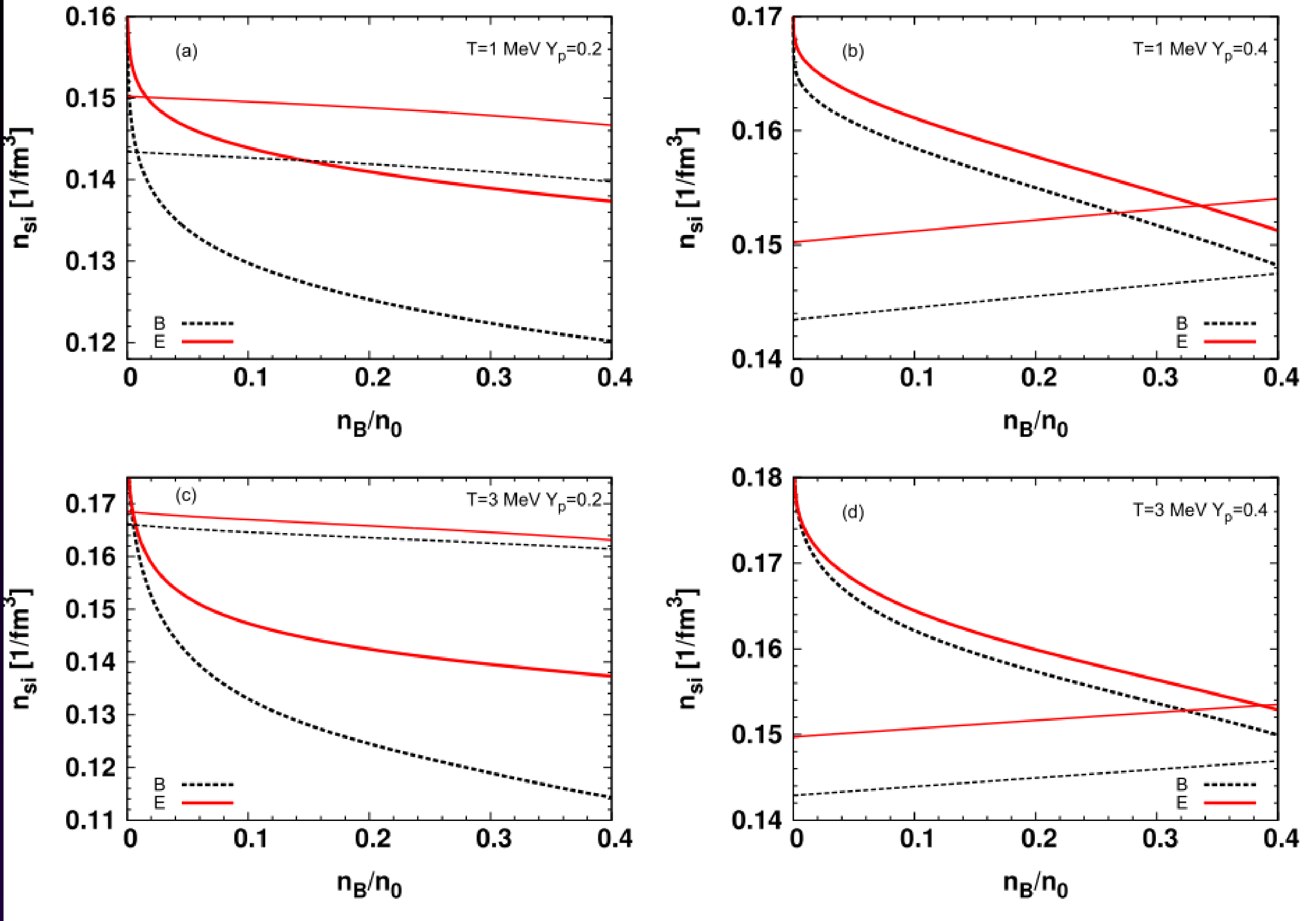
○ Multi-Nucleus EOSs are indispensable to estimate weak interaction rates.

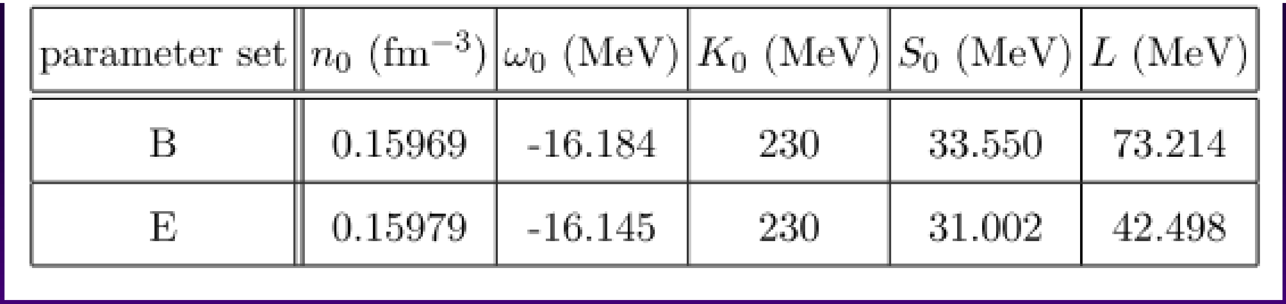
Backup

Parameter Sets for Bulk Energies



parameter set	B	E
n_0 [fm^{-3}]	0.15969	0.15979
ω_0 [MeV]	-16.184	-16.145
K_0 [MeV]	230	230
S_0 [MeV]	33.550	31.002
L [MeV]	73.214	42.498

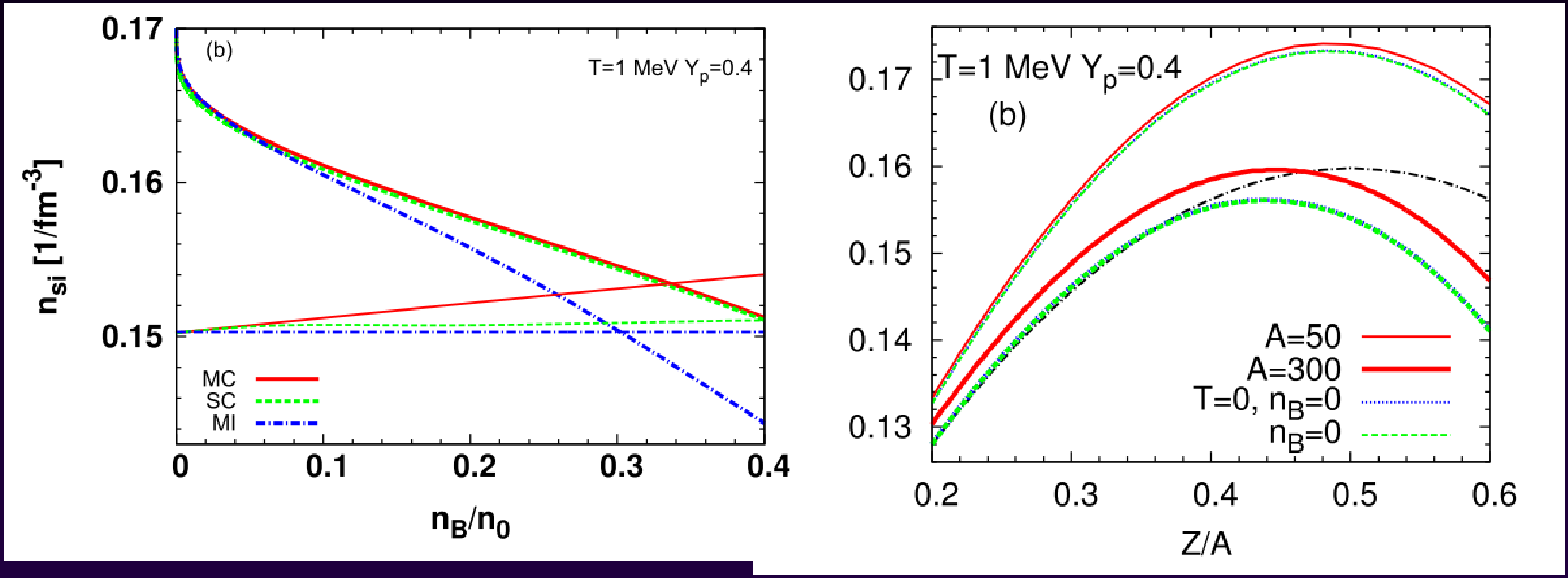




parameter set	n_0 (fm^{-3})	ω_0 (MeV)	K_0 (MeV)	S_0 (MeV)	L (MeV)
B	0.15969	-16.184	230	33.550	73.214
E	0.15979	-16.145	230	31.002	42.498

Results

At low T and high Yp conditions,



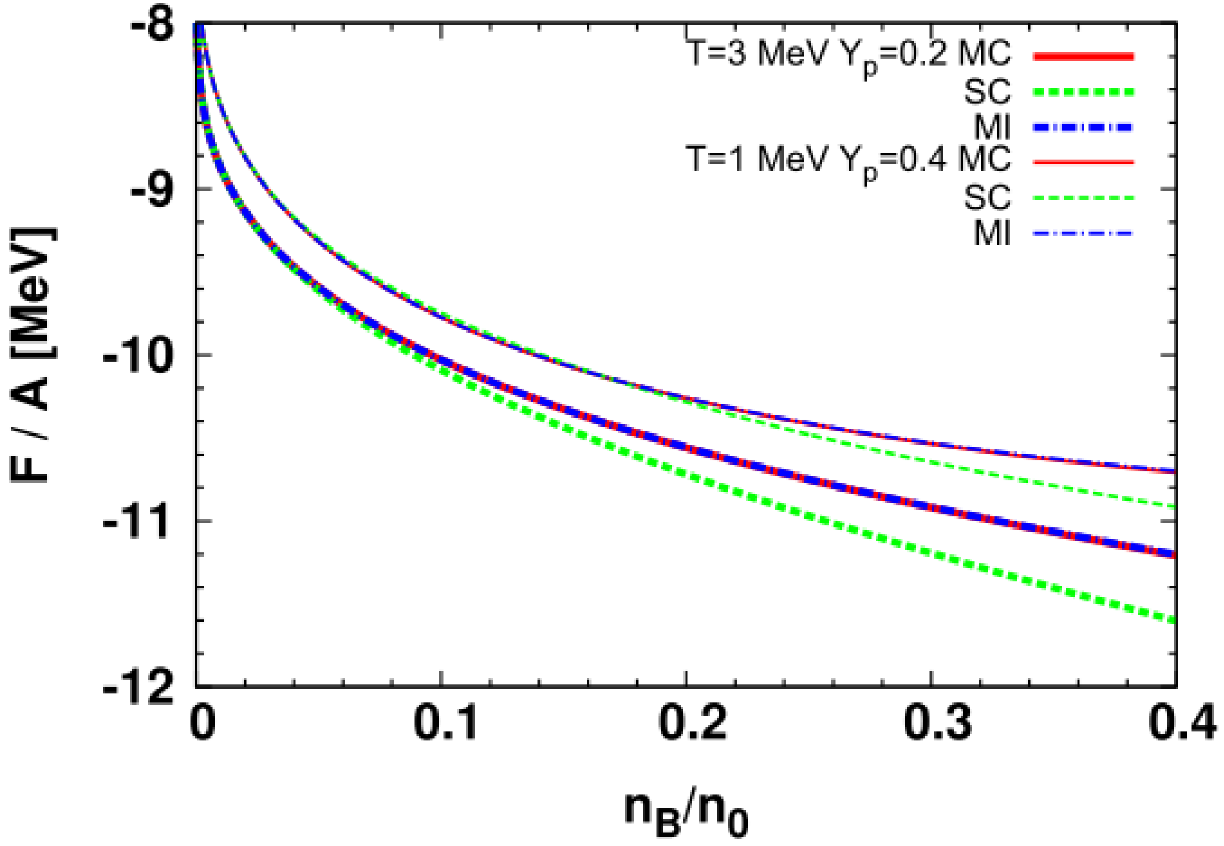


FIG. 3: Internal free energies per baryon, $(F_i^B + F_i^S + F_i^C)/A_i$, of ^{50}Ca at (T, Y_p)= (3 MeV, 0.2) (thick lines) and of ^{300}Fm at (T, Y_p)= (1 MeV, 0.4) (thin lines), as functions of n_B for Models MC (red solid lines), MI (blue dashed-dotted lines) and SC (green dashed lines).

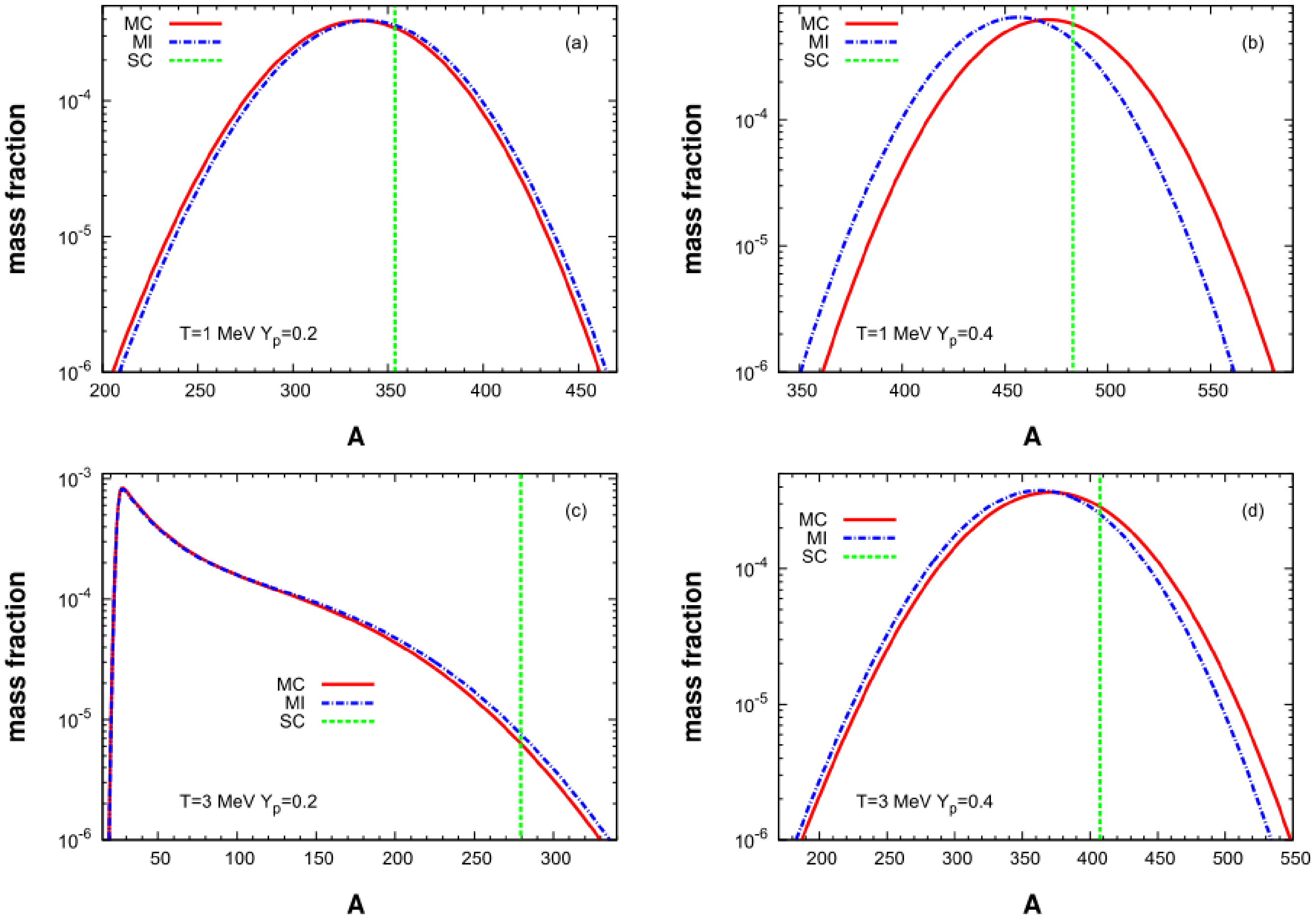
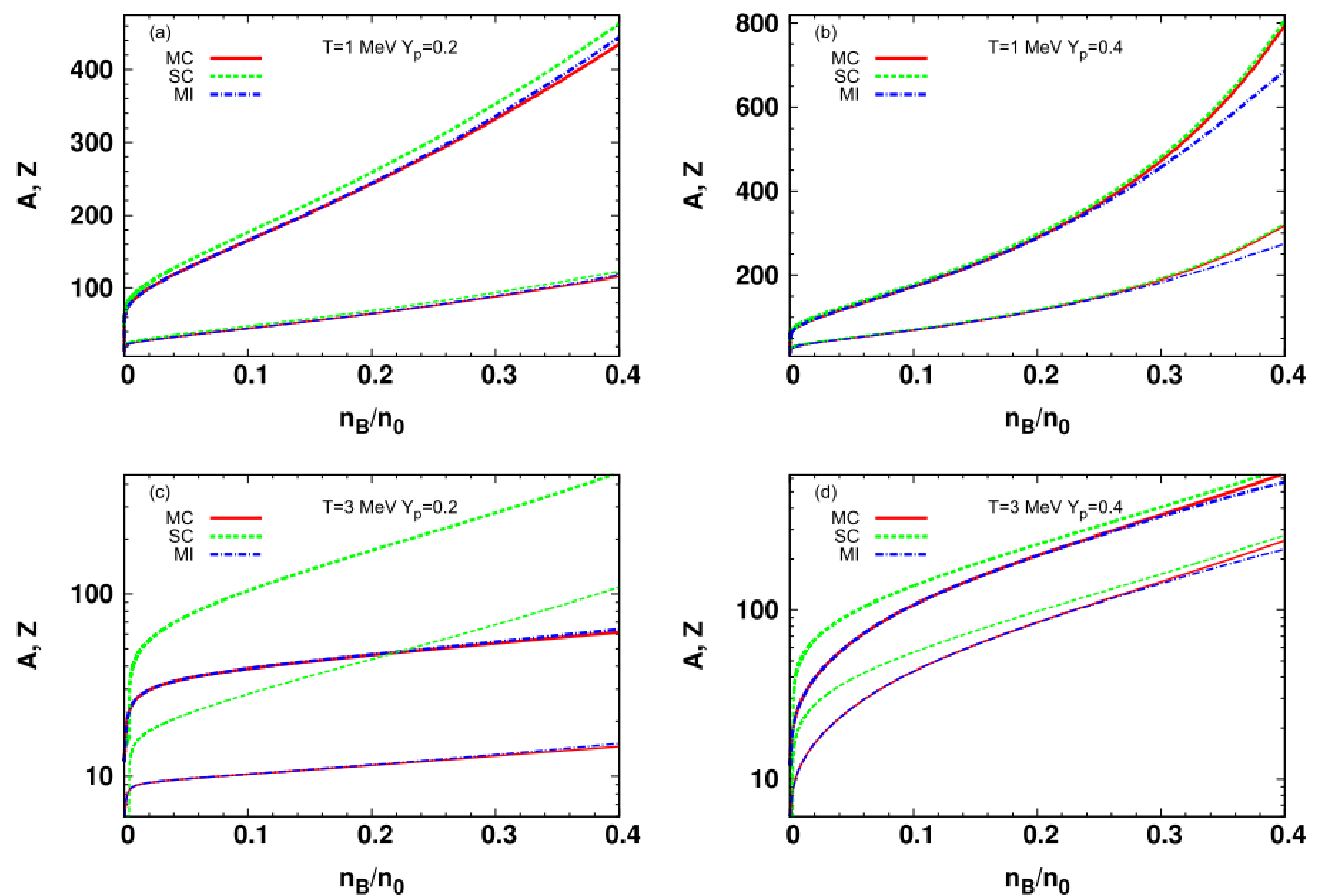
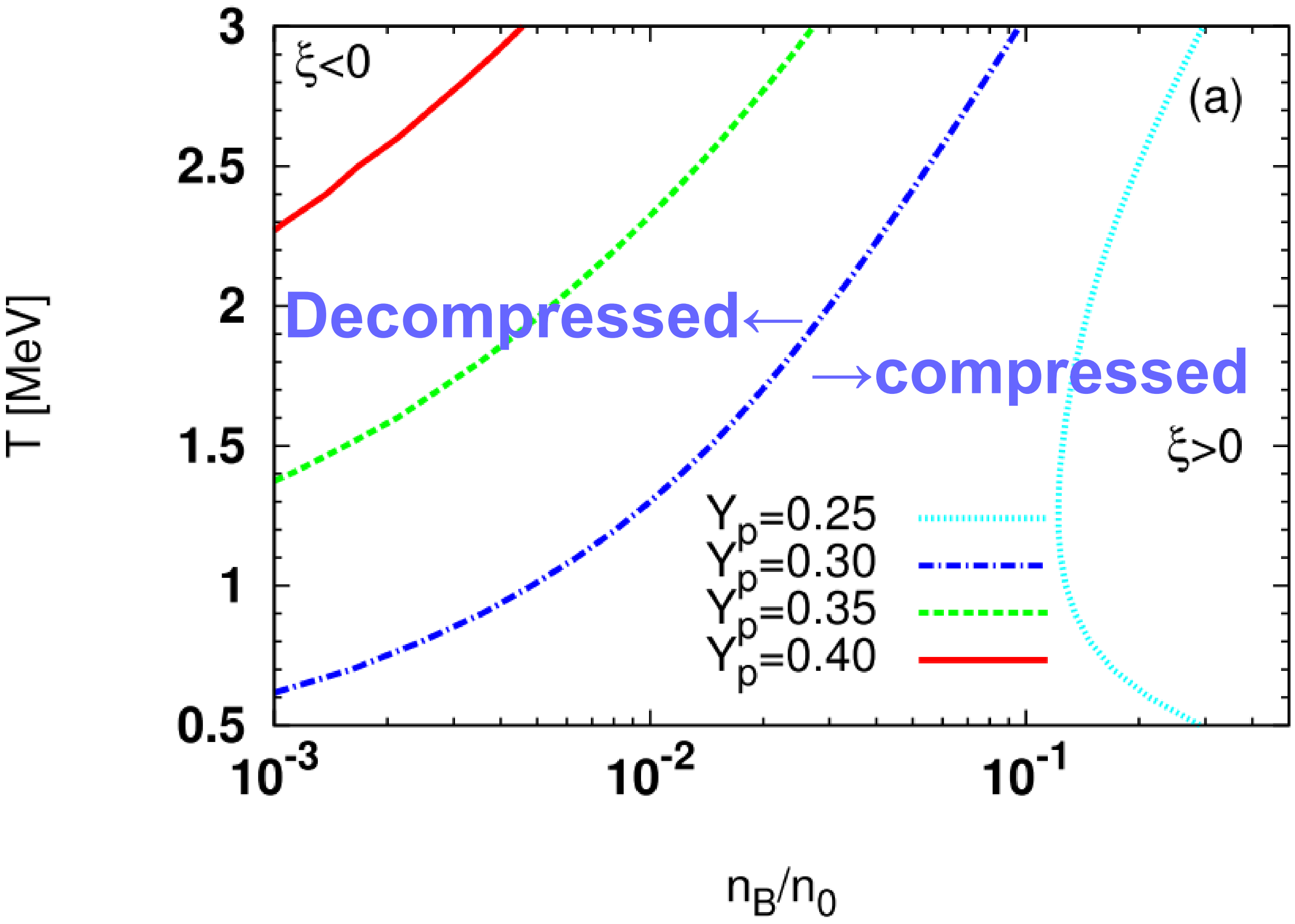


FIG. 5: Mass fractions of elements as a function of mass number for Models MC (red solid lines) and MI (blue dashed-dotted lines) in multi-nucleus description at $T = 1$ MeV (top row) and 3 MeV (bottom row) and $Y_p = 0.2$ (left column) and 0.4 (right column). Vertical green dashed lines display the mass numbers of representative nuclei in single nucleus description.

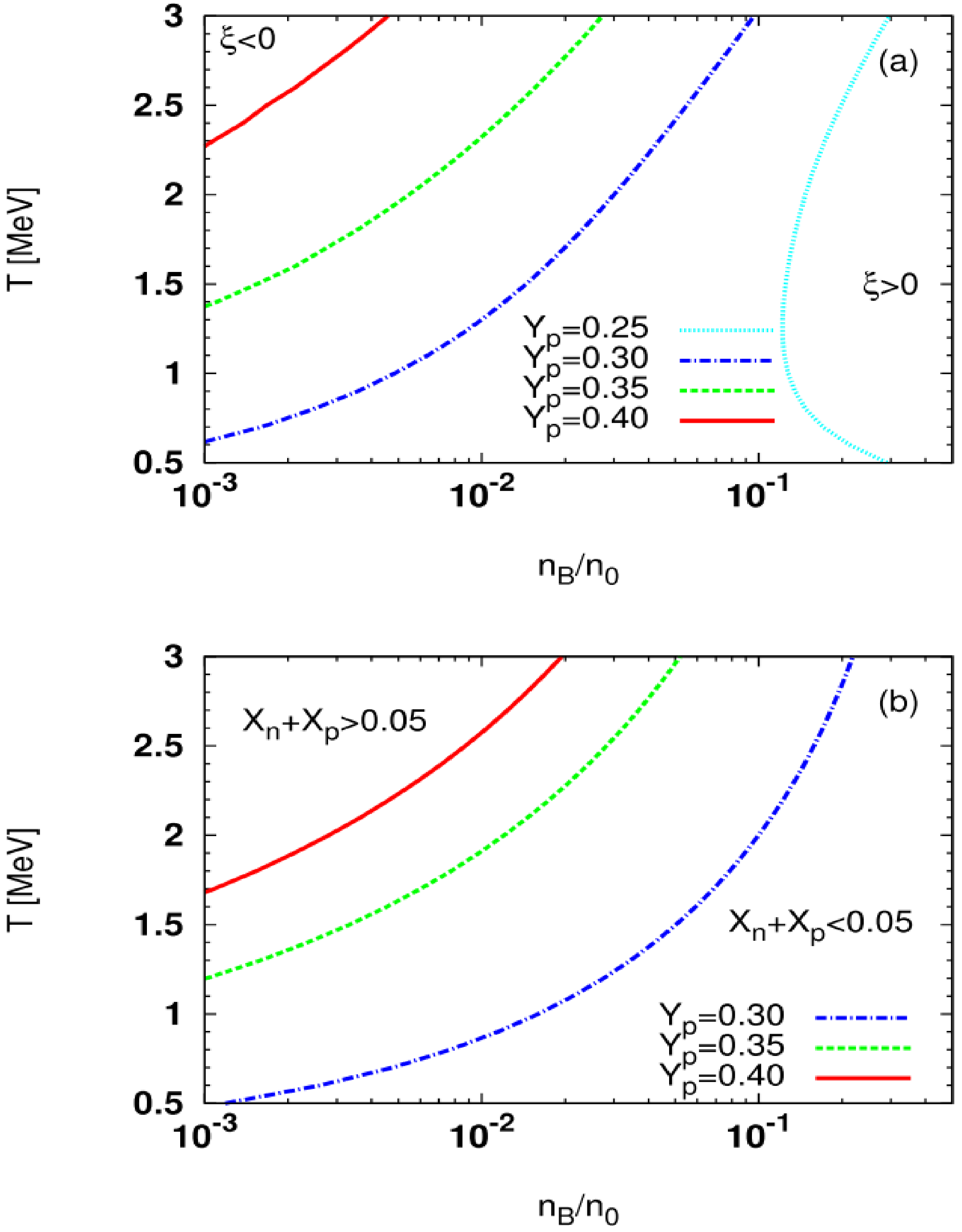

(a)
MC
SC
MI
T=1 MeV Y_p=0.2
A, Z
400
300
200
100
0
0.1
0.2
0.3
0.4
n_B/n_0
(b)
MC
SC
MI
T=1 MeV Y_p=0.4
A, Z
800
600
400
200
0
0.1
0.2
0.3
0.4
n_B/n_0
(c)
MC
SC
MI
T=3 MeV Y_p=0.2
A, Z
100
10
0
0.1
0.2
0.3
0.4
n_B/n_0
(d)
MC
SC
MI
T=3 MeV Y_p=0.4
A, Z
100
10
0
0.1
0.2
0.3
0.4
n_B/n_0

Critical Line between Compression or Decomprression

$$\xi \;=\; \sum_i n_i(\mathrm{d}n_{eq}/\mathrm{d}n_B)/(\sum_i n_i) = 0$$



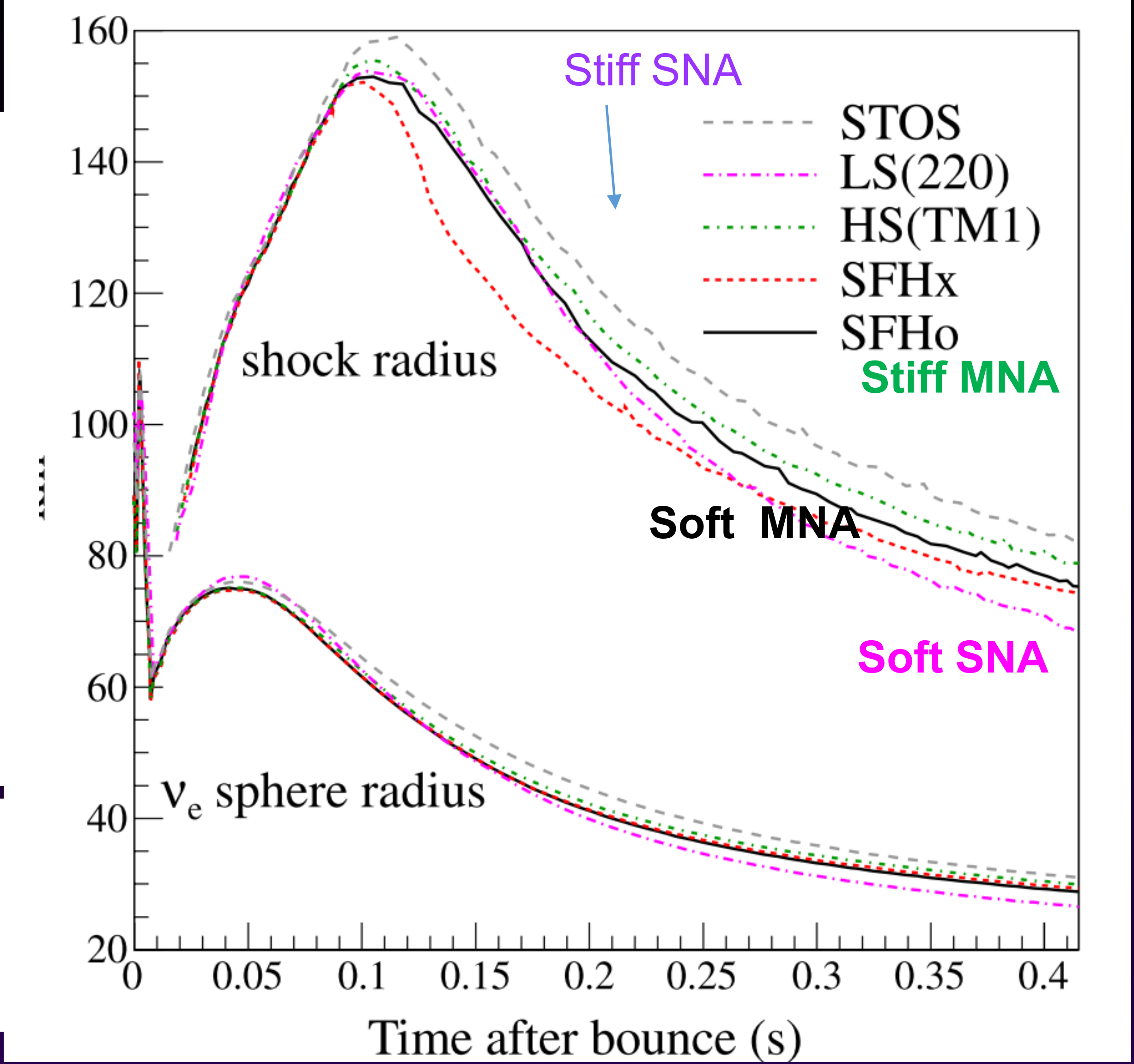
Higher T, lower nB, lower Yp ⇒ more dripped nucleons ⇒decompression


(a)
ξ<0
ξ>0
Y_p=0.25
Y_p=0.30
Y_p=0.35
Y_p=0.40
T [MeV]
3
2.5
2
1.5
1
0.5
10^{-3}
10^{-2}
10^{-1}
n_B/n_0
(b)
X_n+X_p>0.05
X_n+X_p<0.05
Y_p=0.30
Y_p=0.35
Y_p=0.40
T [MeV]
3
2.5
2
1.5
1
0.5
10^{-3}
10^{-2}
10^{-1}
n_B/n_0

Impact of EOS's on dynamics of 1D GR supernova simulations

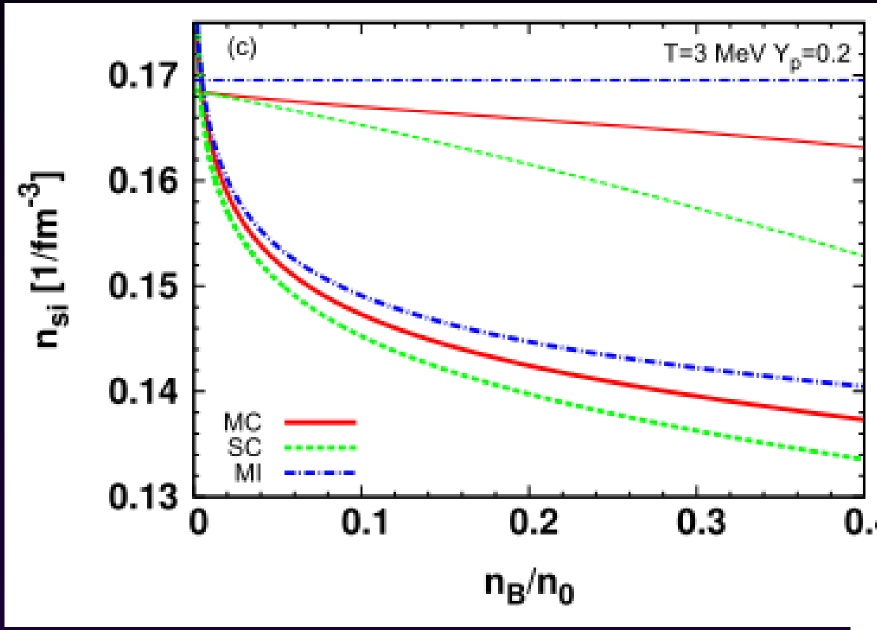
(Steiner et al. 2013)

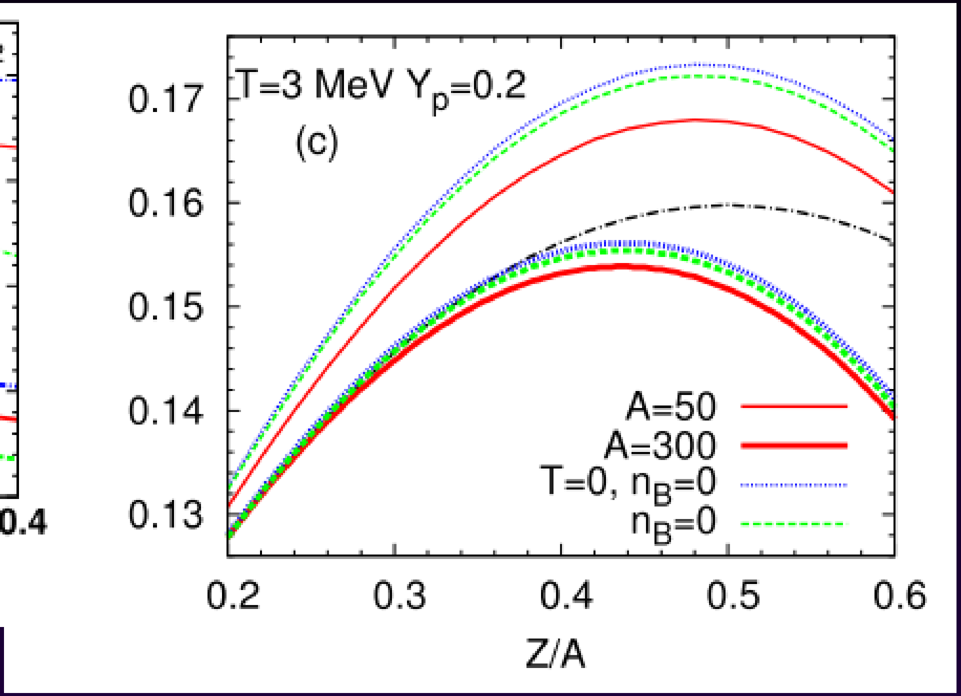
○ Impact of EOS above n_0
- Stiff EOSs give strong prompt motion at bounce
- Soft EOSs give fast contraction of proto neutron stars (and assist the neutrino heating in 2D and 3D.)



●EOSs below n_0 may have greater impact on dynamics.

Difference between stiff MNA and SNA EOS's (the same bulk propety)
>Difference between Stiff and Soft MNA EOS's (the same NSE model)


(c)
T=3 MeV Y_p=0.2
n_{si} [1/fm^{-3}]
0.17
0.16
0.15
0.14
0.13
0
0.1
0.2
0.3
0.4
n_B/n_0
MC
SC
MI


T=3 MeV Y_p=0.2
(c)
0.17
0.16
0.15
0.14
0.13
0.2
0.3
0.4
0.5
0.6
Z/A
A=50
A=300
T=0, n_B=0
n_B=0