Self-consistent description of particle-phonon coupling effects. Pole and non-pole (‘tadpole’) diagrams

E. E. Saperstein, Kurchatov Institute, Moscow

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The report is based on the articles:

1. E.E. Saperstein, S. Kamerdzhiev, D.S. Krepish, S.V. Tolokon-nikov, and D. Voitenkov, J. Phys. G: Nucl. Phys. 44 (2017) 065104. The first self-consistent calculation of quadrupole moments of odd semi-magic nuclei accounting for phonon-induced corrections.

2. N. V. Gnezdilov, I. N. Borzov, E. E. Saperstein, and S. V. Tolokonnikov, Phys. Rev. C 89, 034304 (2014).

Self-consistent description of single-particle levels of magic nuclei.

3. E. E. Saperstein, I. N. Borzov, and S. V. Tolokonnikov, JETP Lett., 104, 218 (2016).

On the Anomalous *A* Dependence of the Charge Radii of Heavy Calcium Isotopes.

Plan:

1. Introduction
2. Fayans energy density functional (EDF) vs Skyrme EDFs.
3. The double counting problem for particle-phonon coupling (PC) effects.
4. PC effects in single-particle energies of magic nuclei – very briefly.
5. PC effects in electromagnetic moments of odd magic (semi-magic) nuclei.
6. PC effects in charge radii – very briefly.

INTRODUCTION

Since the famous article of 1959 by S.T. Belyaev [S.T. Belyaev, Mat.-Fys. Medd. Kgl. Dan. Wid. Selsk., 31, 31 (1959)],

a crucial role of the first 2+ excitations in even-even nuclei, the quadrupole "phonons", is one of the conventional cornerstones of the low-energy nuclear theory.

In our consideration of the particle-phonon coupling (PC) effects, these 2+ states play the main role.

A consistent method to describe the PC effects within the TFFS was developed by V. A. Khodel [Sov. J. Nucl. Phys. 24, 282 (1976)]. It was based on the TFFS self-consistency relation

[S. A. Fayans and V. A. Khodel', JETP Lett. 17, 444 (1973)]

which is a consequence of the spontaneous breaking of the translation symmetry in nuclei. In a simplified form,

$$\frac{\partial U}{\partial r} = \int d^3 r' F(r, r') \frac{\partial \rho}{\partial r'}$$

where *U* is is the mean-field nuclear potential, ρ is the nucleon density distribution, *F* is the Landau--Migdal (LM) interaction amplitude.

The low-lying natural parity excitations of ee nuclei ('phonons') are interpreted as the Goldstone mode arising due to the

breaking of this symmetry. The "ghost" dipole phonon, with zero excitation energy is the head of this branch, and the corresponding vertex is

$$g_1 = \frac{\partial U}{\partial r} Y_{1M}(n)$$

the low-lying L-phonons are in many ways similar to the ghost one, their creation vertices possess a dominant surface peak

$$g_{LM} = \left(\alpha_L \frac{\partial U}{\partial r} + \chi_L(r) \right) Y_{LM}(n)$$

The second in-volume term is small

3- in 208 Pb

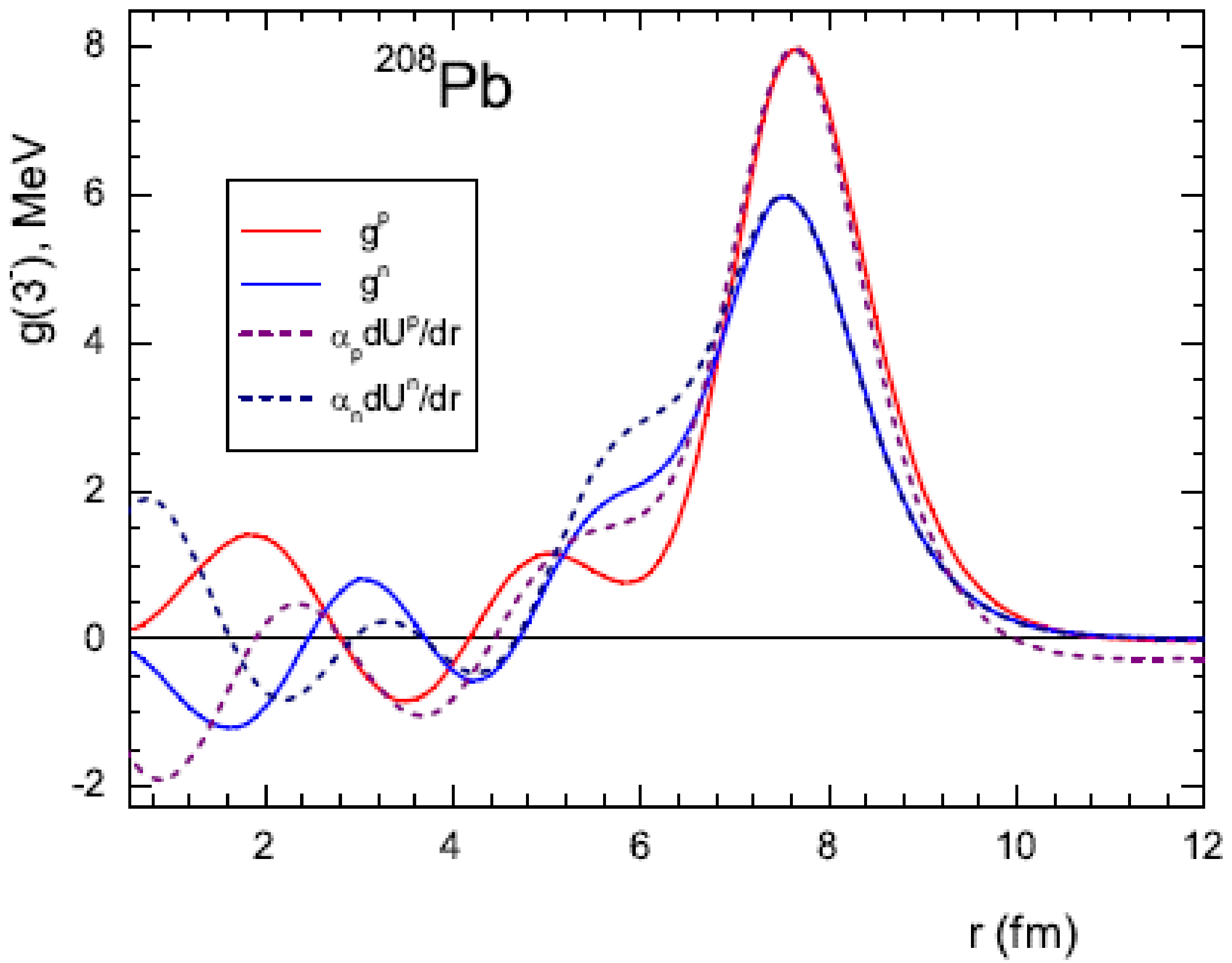
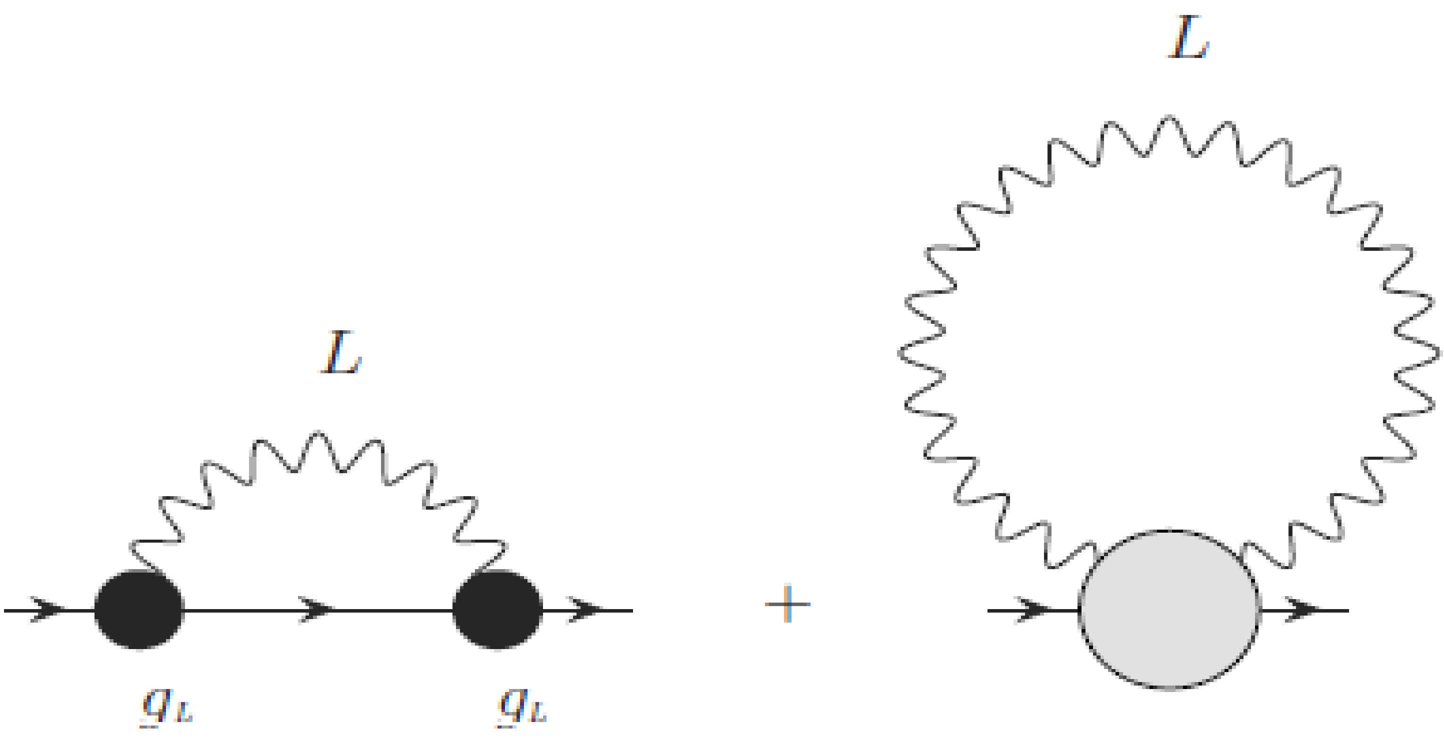


FIGURE 1. g_L vertex for the 3_1^- state in ^{208}Pb, $\alpha_L^p = 0.334$ fm, $\alpha_L^n = 0.322$ fm.

PC corrections to the mass operator



pole ***non-pole***

V.A. Khodel developed to find the non-pole ('tadpole') term within TFFS

$$g_L(\omega) = \mathcal{F} A(\omega) g_L(\omega),$$

$$\delta\Sigma^{\rm tad} = \int \frac{d\omega}{2\pi i} \delta_L g_L D_L(\omega),$$

$$\delta_L g_L = \delta_L \mathcal{F} A(\omega_L) g_L + \mathcal{F} \delta_L A(\omega_L) g_L + \mathcal{F} A(\omega_L) \delta_L g_L.$$

$$g_L(r) = \alpha_L \frac{dU}{dr} + \chi_L(r), \quad \chi_L(r)$$ ***is small***

$$\delta\Sigma_L^{\rm tad} = \frac{\alpha_L^2}{2} \frac{2L+1}{3} \Delta U(r).$$

In modern self-consistent calculations Skyrme EDFs dominate

In all our calculation we use the Fayans EDF DF3-a

[S.V. Tolokonnikov and E.E. Saperstein, Phys. At .Nucl. 73, 1684 (2010)]

a modification of the original Fayans EDF DF3

[S.A. Fayans, S.V. Tolokonnikov, E.L. Trykov, and D. Zawischa, Nucl. Phys. A 676, 49 (2000)]

Spin-orbit and tensor parameters are modified only.

For spherical nuclei, the Fayans EDF surpasses popular Skyrme EDFs in describing of

1. the charge radii,
2. the first 2+ state characteristics (excitation energies and B(E2) values) in semi-magic nuclei,
3. the single-particle energies of magic nuclei.

the Fayans EDF vs Skyrme EDFs

We use the Fayans EDF:

the main in-volume term is

$$\mathcal{E}(\rho) = \frac{a\rho^2}{2} \frac{1+\alpha\rho^\sigma}{1+\gamma\rho}. \qquad m^* = m$$

For Skyrme EDFs: $\gamma = 0, m^* \neq m$

Both peculiarities of the Fayans EDF (the ‘Fayans denominator’ and bare mass) are related to the self-consistent theory of finite Fermi systems (sc TFFS) V. A. Khodel and E. E. Saperstein, Phys. Rep. **92**, 183 (1982).

In hidden form, they describe the energy dependence effects of the sc TFFS.

S-c TFFS ***[V. A. Khodel, E. E. Saperstein, Phys. Rep. 92, 183 (1982)] starts from the quasiparticle mass operator:***

$$G_q = (\varepsilon - \varepsilon_k - \Sigma_q)^{-1}. \quad \varepsilon_k = k^2/2m$$

$$\Sigma_q(\mathrm{r}, k^2; \varepsilon) = \Sigma_0(\mathrm{r}) + \frac{1}{2m\varepsilon_{\mathrm{F}}^0}\,\mathrm{k}\,\Sigma_1(\mathrm{r})\mathrm{k} + \Sigma_2(\mathrm{r})\frac{\varepsilon}{\varepsilon_{\mathrm{F}}^0},$$

$$\varepsilon_{\mathrm{F}}^0 = (k_{\mathrm{F}}^0)^2/2m$$

k-dependence and energy dependence on equal footing

By definition,

$$\Sigma_1(\mathbf{r}) = \varepsilon_{\mathrm{F}}^0 \left.\frac{\partial\Sigma(\mathbf{r})}{\partial\varepsilon_k}\right|_0,$$

$$\Sigma_2(\mathbf{r}) = \varepsilon_{\mathrm{F}}^0 \left.\frac{\partial\Sigma(\mathbf{r})}{\partial\varepsilon}\right|_0, \quad \text{'0'}: \varepsilon = \mu, k = k_F$$

$$Z(\mathbf{r}) = \left(1 - \Sigma_2(\mathbf{r})/\varepsilon_{\mathrm{F}}^0\right)^{-1},$$

It determines the in-volume term of spectroscopic factors

$$\frac{m}{m^*(\mathbf{r})} = \frac{\left(1 + \Sigma_1(\mathbf{r})/\varepsilon_{\mathrm{F}}^0\right)}{\left(1 - \Sigma_2(\mathbf{r})/\varepsilon_{\mathrm{F}}^0\right)}.$$

In s-c TFFS, m*(r) includes ‘k-mass’ and ‘E-mass’, and the two effects strongly cancel each other, the effective mass being close to the bare one.

In the sc TFFS the EDF contains

$$Z^2(\rho), Z^3(\rho)$$

$$\mathrm{Z}(\mathbf{r}) = \frac{2}{1 + \sqrt{1 - 4C_0\lambda_{02}\rho(\mathbf{r})/\varepsilon_{\mathrm{F}}^0}},$$

Rather comlpicate density dependence of the EDF. Fayans with coauthors found that it can be approximated with the above fractional function.
In the sc TFFS, the effective mass is close to the bare one (m*(n)=0.95, m*(p)=1.05.) In the Fayans EDF the bare mass is used.
Thus, the Fayans EDF could be interpreted as an approximate version of the sc TFFS

Reasons why $m^* \approx m$

In the s-c TFFS, for nuclear matter,

$$Z_0 = 0.8,\, m^* = 0.95m$$

In any non-relativistic many-body theory of nuclear matter

$$\Sigma(\varepsilon, \varepsilon_k) = \Sigma[\hat{V}, G_0]$$

Where V is the free NN-potential and $G_0 = (\varepsilon - \varepsilon_k)^{-1}$.

V is short-range: $r_{eff} << 1/k_F$

Put it to be zero-range: $V(r_1 - r_2) = V_0 \delta(r_1 - r_2)$

All the momentum integrals diverge: cut-off at $k_{cut} >> k_F$

With accuracy $(k_F / k_{cut})^2$ ***k- and ϵ-dep. due to*** G_0

$$G_0 = (\varepsilon - \varepsilon_k)^{-1}. \qquad \frac{\partial \Sigma}{\partial \varepsilon_k} = \frac{\delta \Sigma}{\delta G_0} \frac{\partial G_0}{\partial \varepsilon_k},$$

$$\frac{\partial \Sigma}{\partial \varepsilon} = \frac{\delta \Sigma}{\delta G_0} \frac{\partial G_0}{\partial \varepsilon}.$$

$$\frac{\partial \Sigma}{\partial \varepsilon_k} = -\frac{\partial \Sigma}{\partial \varepsilon}.$$

$m^* = m$, ***with accuracy of*** $(r_{eff} k_F)^2$

The double counting problem

Such a problem exists for the study of PC corrections in any theory starting from the EDF with phenomenological parameters.

Indeed, in the EDF approach,

the PC contributions are taken into account **on average**.

If we include all the PC contributions, we must readjust these parameters anew.

Our idea is to separate **the fluctuating part** of the PC corrections that behaves in a non-regular way, depending significantly on the nucleus under consideration and the single-particle state of the odd nucleon. Their contributions to observables are, as a rule, **rather small on average but are often important to reproduce the specific experimental value in a nucleus under consideration.**

Single-particle energies of seven magic nuclei: 40,48 Ca, 56,78 Ni, 100,132 Sn, 208 Pb

N.V. Gnezdilov, I. N. Borzov, E. E. Saperstein, and S. V. Tolokonnikov, Phys. Rev. 89, 034304 (2014).

In addition to the old DF3 and DF3-a EDFs, a new one, DF3-b, is found for better description of 35 experimental spin-orbit differences!

The data from

H. Grawe, K. Langanke, and G. Mart´ınez-Pinedo, Rep. Prog.Phys. **70**, 1525 (2007) [105 levels]

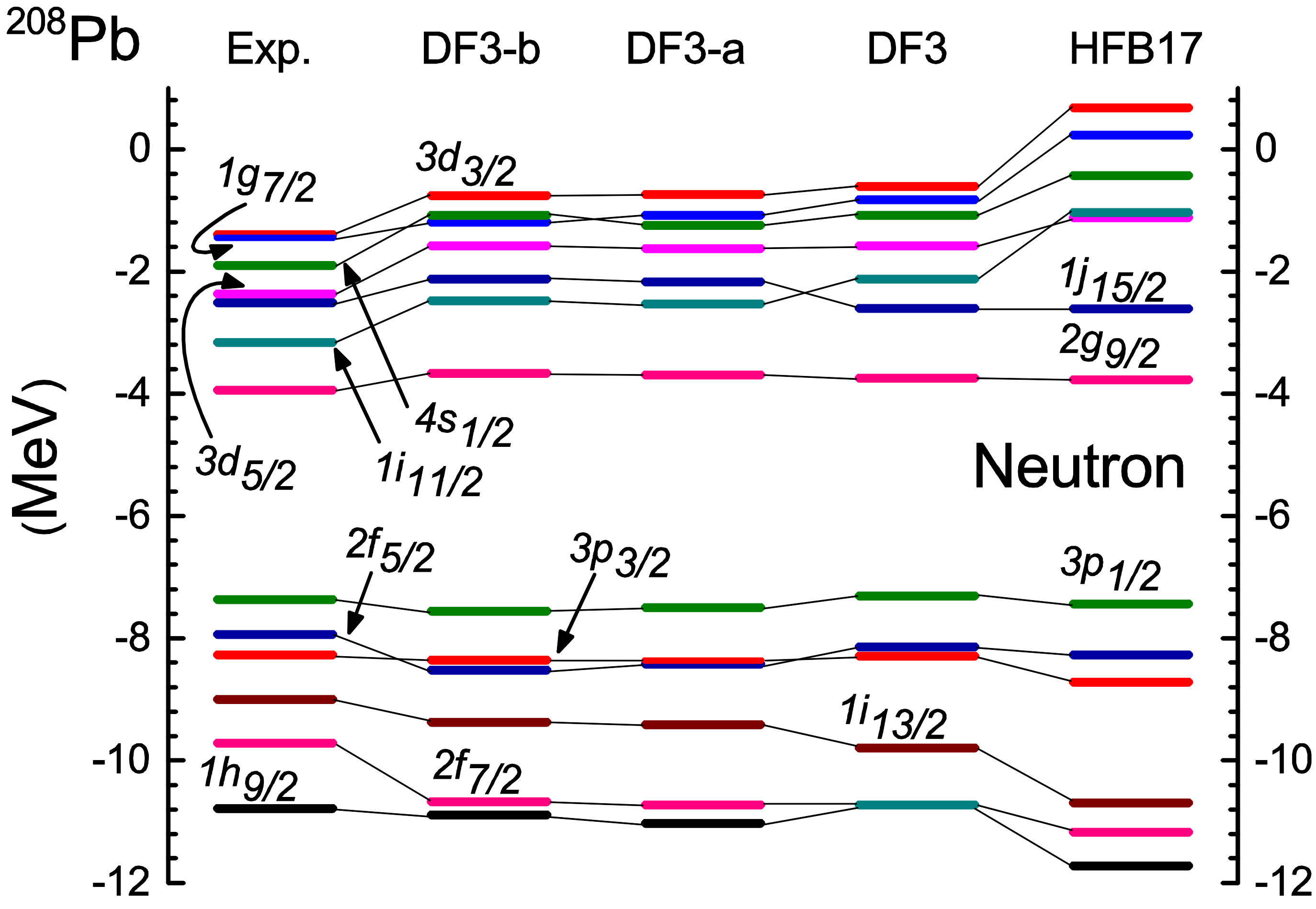
In magic nuclei, the perturbation theory (PT) in $\delta\Sigma^{PC}$

is valid. Another situation there is in semi-magic nuclei, where the Dyson equation with $\Sigma(\varepsilon)=\Sigma_0+\delta\Sigma^{PC}(\varepsilon)$

Should be solved directly, without PT

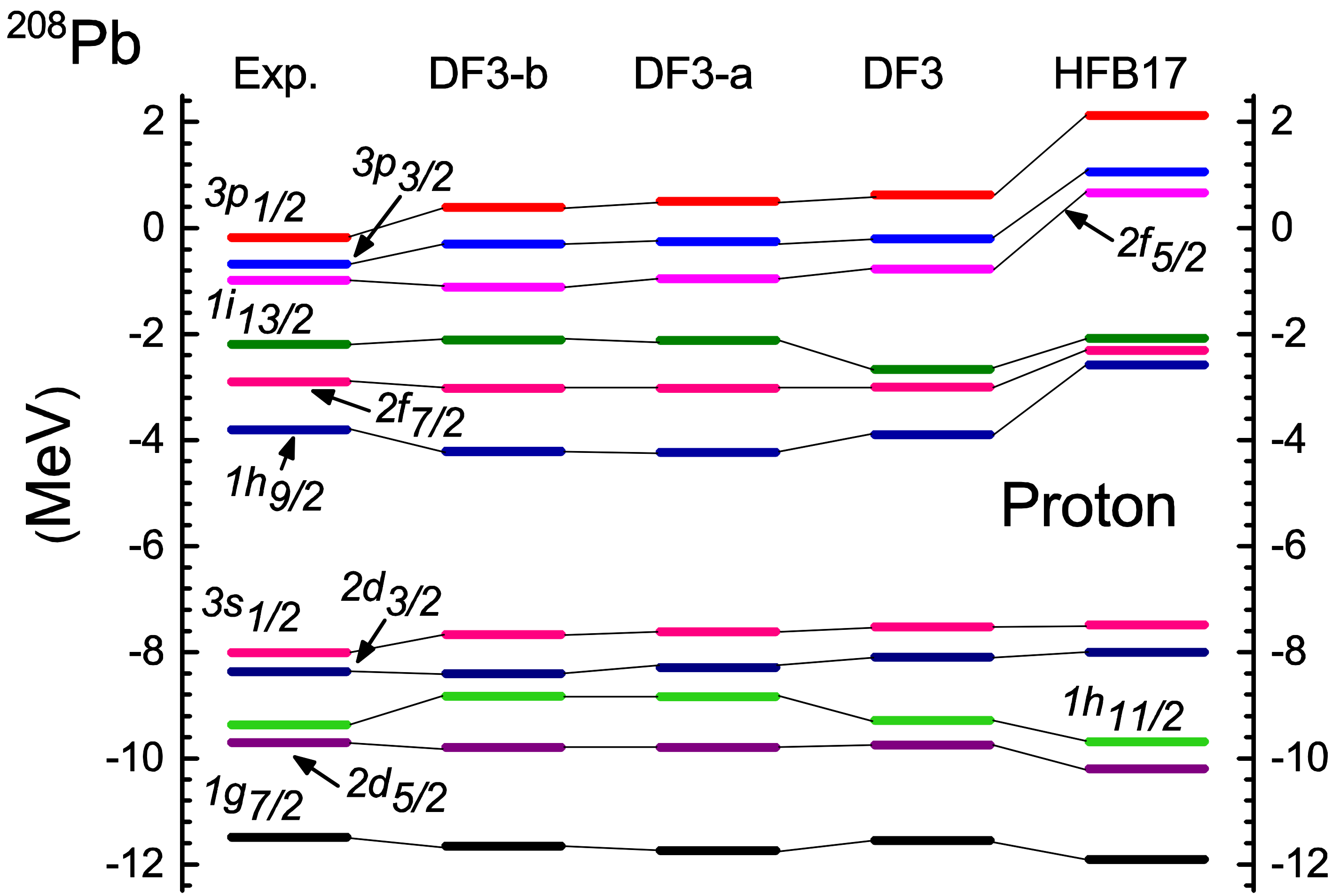
208 Pb, neutron levels without PC

N.V. Gnezdilov, I. N. Borzov, E. E. Saperstein, and S. V. Tolokonnikov, Phys. Rev. 89, 034304 (2014).



208 Pb, proton levels without PC

N.V. Gnezdilov, I. N. Borzov, E. E. Saperstein, and S. V. Tolokonnikov, Phys. Rev. 89, 034304 (2014).



Nucleus	N	DF3-b	DF3-a	DF3	HFB17
^{40}Ca	14	1.08	1.25	1.35	1.64
^{48}Ca	12	0.89	1.00	1.01	1.70
^{56}Ni	14	1.00	0.97	0.85	1.40
^{78}Ni	11	1.24	1.41	1.09	1.32
^{100}Sn	13	1.09	1.17	1.01	1.56
^{132}Sn	17	0.58	0.66	0.55	1.15
^{208}Pb	24	0.44	0.51	0.43	1.15
Total	105	0.89	0.98	0.89	1.40

Average deviations $\langle \delta\varepsilon_\lambda \rangle_{\rm rms}$ (MeV)

of predictions for SPEs without the PC corrections from the data (N is the total number of neutron and proton states). The effective mass effect !

Pole and non-pole contributions to proton SPEs in 208Pb

λ	$\delta\varepsilon_\lambda^{\text{pole}}$	$\delta\varepsilon_\lambda^{\text{tad}}$	$\delta\varepsilon_\lambda$
$3p_{1/2}$	-0.375	0.153	-0.222
$3p_{3/2}$	-0.371	0.152	-0.219
$2f_{5/2}$	-0.278	0.168	-0.110
$1i_{13/2}$	-0.534	0.266	-0.268
$2f_{7/2}$	-0.409	0.168	-0.240
$1h_{9/2}$	-0.054	0.222	0.168
$3s_{1/2}$	-0.310	0.143	-0.167
$2d_{3/2}$	-0.241	0.146	-0.095
$1h_{11/2}$	-0.017	0.246	0.229
$2d_{5/2}$	0.435	0.147	0.582
$1g_{7/2}$	-0.271	0.197	-0.074

The non-pole term is always positive,

The pole one, as a rule, negative.

Account for the pole term alone overestimates the PC correction to SPEs significantly.

For DF3-a EDF, in 208 Pb,

$\langle \delta \bar{\varepsilon}_\lambda \rangle_{\rm rms}$ = 0.51 MeV without PC,

= 0.38 MeV with PC

[RMF+PC = 0.85 MeV]

E. Litvinova and P. Ring, Phys. Rev. C **73**, 044328 (2006).

Recent calculations of PC corrections to SPEs of magic nuclei with Skyrme EDFs

1. L.-G. Cao, G. Coló, H. Sagawa, and P. F. Bortignon,

Phys. Rev. C **89**, 044314 (2014).

2. D. Tarpanov, J. Dobaczewski, J. Toivanen, and B. G. Carlsson, Phys. Rev. Lett. **113**, 252501 (2014).

In the second one, the SPEs were examined with a family of Skyrme EDFs with various m^* values, firstly, without PC and then + PC

Conclusions: without PC, the EDFs with $m^* \approx m$

are preferable, in accordance with our results, BUT

inclusion of PC corrections spoils the agreement.

I believe, it happens because of omitting the non-pole term.

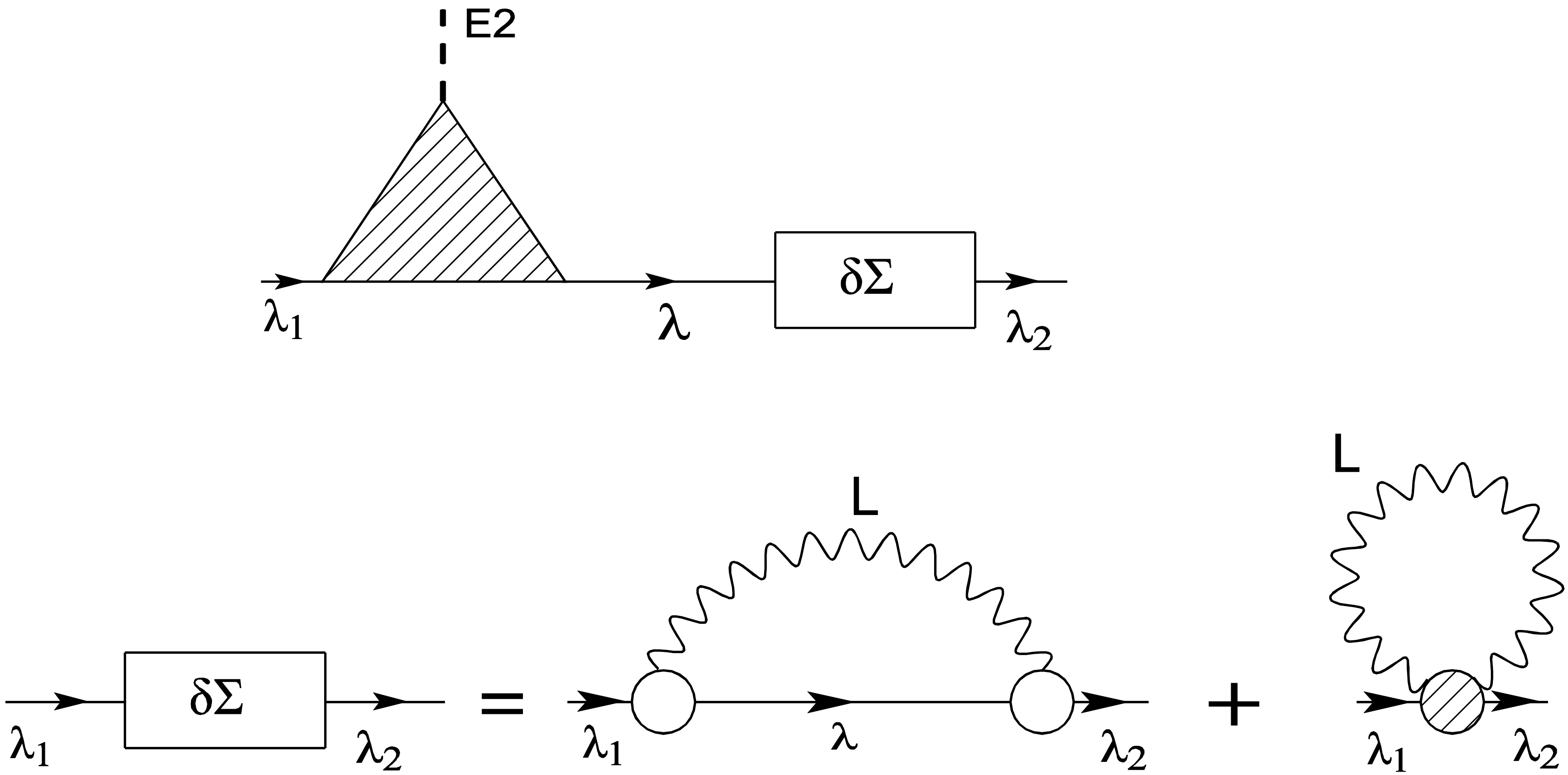
PC corrections to electromagnetic moments of odd semi-magic nuclei

Quadrupole moments of proton-odd neighbors of even Sn isotopes

$$Q_\lambda = \langle \lambda | V | \lambda \rangle_{m=j},$$

$$V = e_q V_0 + \mathcal{F} A(\omega = 0) V,$$

PC corrections to different (not all) elements of these formulas, only which behave in a non-regular way. Two main terms (due to the Z-factors and the induced interaction) are of opposite signs and cancel each other significantly leaving some room for ‘small’ corrections.



End corrections

The main ‘end correction’ is due to the diagonal term with $\lambda = \lambda_2$

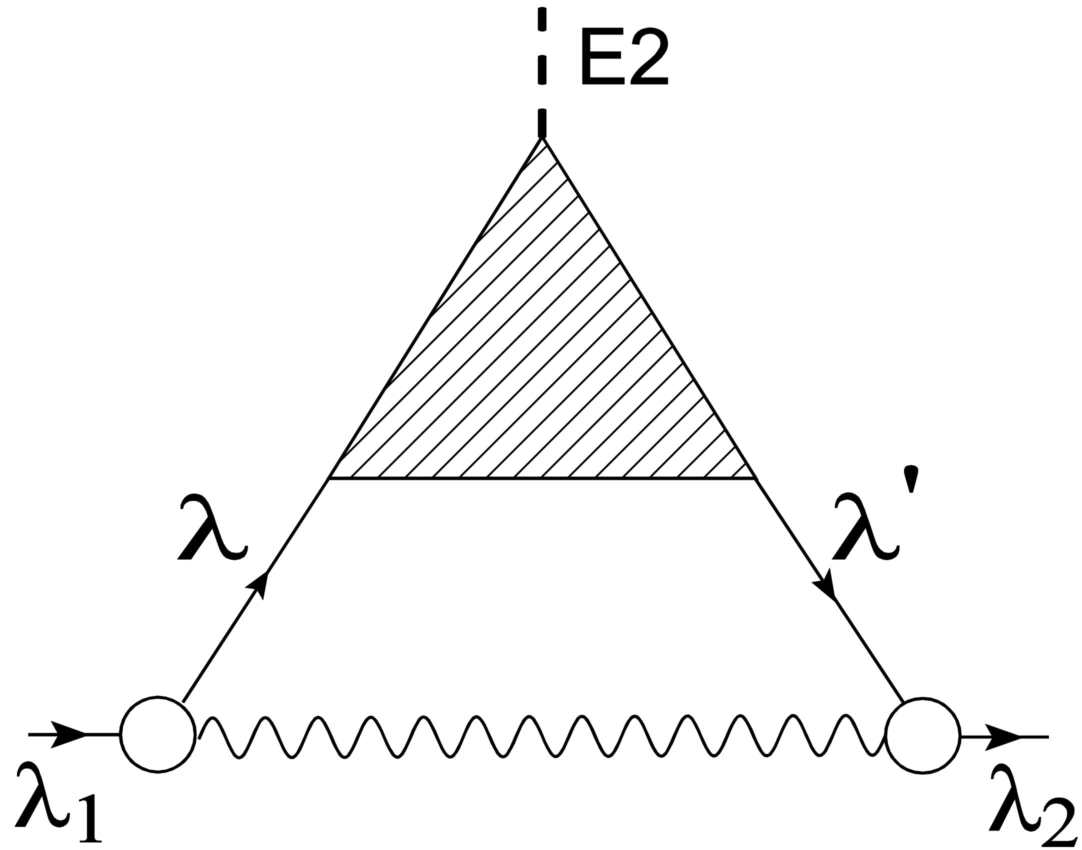
It results in a ‘renormalization’ of the ends: $|\lambda\rangle \to \sqrt{Z_\lambda}|\lambda\rangle$, where

$$Z_\lambda = \left(1 - \frac{\partial \delta \Sigma_{\lambda\lambda}(\varepsilon)}{\partial \varepsilon}\bigg|_{\varepsilon=\varepsilon_\lambda}\right)^{-1}$$

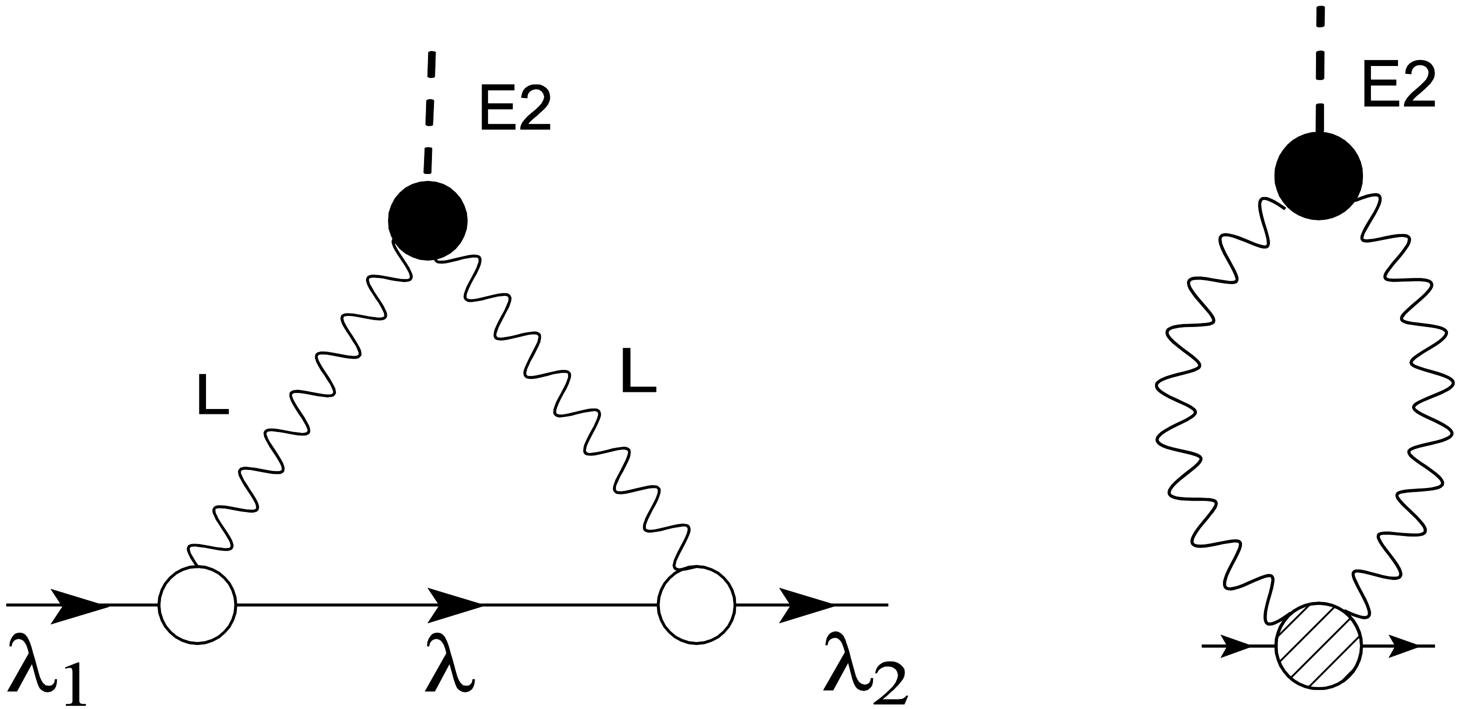
The non-diagonal term, the sum over $\lambda \neq \lambda_2$

denoted as $\delta V'_{\mathrm{end}}$

is rather small but sometimes is also important.



Phonon-induced interaction
(GGD triangle)



Diagrams of the phonon quadrupole moment, pole (GDD-triangle) and non-pole

$$\delta V_{GDD} = \delta V_{GDD}^{(1)} + \delta V_{GDD}^{(2)},$$

two terms with different behavior at $\omega_L \to 0.$
regular and singular: $1/\omega_L$

The non-pole term possesses the same singularity (of opposite sign). If the non-pole 'blob' is a constant, [the model of EPL, 103, 42001 (2013)] these two singular terms cancel each other exactly.

Final formula for the PC corrected matrix element:

$$\widetilde{V}_{\lambda\lambda} = Z_\lambda \left(V + \delta V_{GGD} + \delta V_{GDD}^{(1)} + \delta V'_{\mathrm{end}} \right)_{\lambda\lambda}.$$

p-odd neighbors of even Sn isotopes (In and Sb), Q in b

nucl.	λ	Q	Z	$\delta Q^{Z}_{\rm ptb}$	δQ_{GGD}	δQ_{GDD}	$\delta Q'_{\rm end}$	$\delta Q_{\rm ptb}$	$\delta Q_{\rm ph}$
^{105}In	$1g_{9/2}$	+0.833	0.675	-0.400	0.231	0.055	0.014	-0.100	-0.067
^{107}In	$1g_{9/2}$	+0.976	0.584	-0.692	0.404	0.094	0.021	-0.172	-0.100
^{109}In	$1g_{9/2}$	+1.113	0.573	-0.826	0.487	0.128	0.023	-0.188	-0.108
^{111}In	$1g_{9/2}$	+1.165	0.488	-1.220	0.722	0.163	0.034	-0.301	-0.147
^{113}In	$1g_{9/2}$	+1.117	0.576	-0.820	0.484	0.071	0.025	-0.240	-0.138

113 In : phonon corrections

$$\delta Q(Z) = -0.840b,\ \delta Q(ind) = 0.484b,\ \delta Q(tot) = -0.138b$$

In the final table,

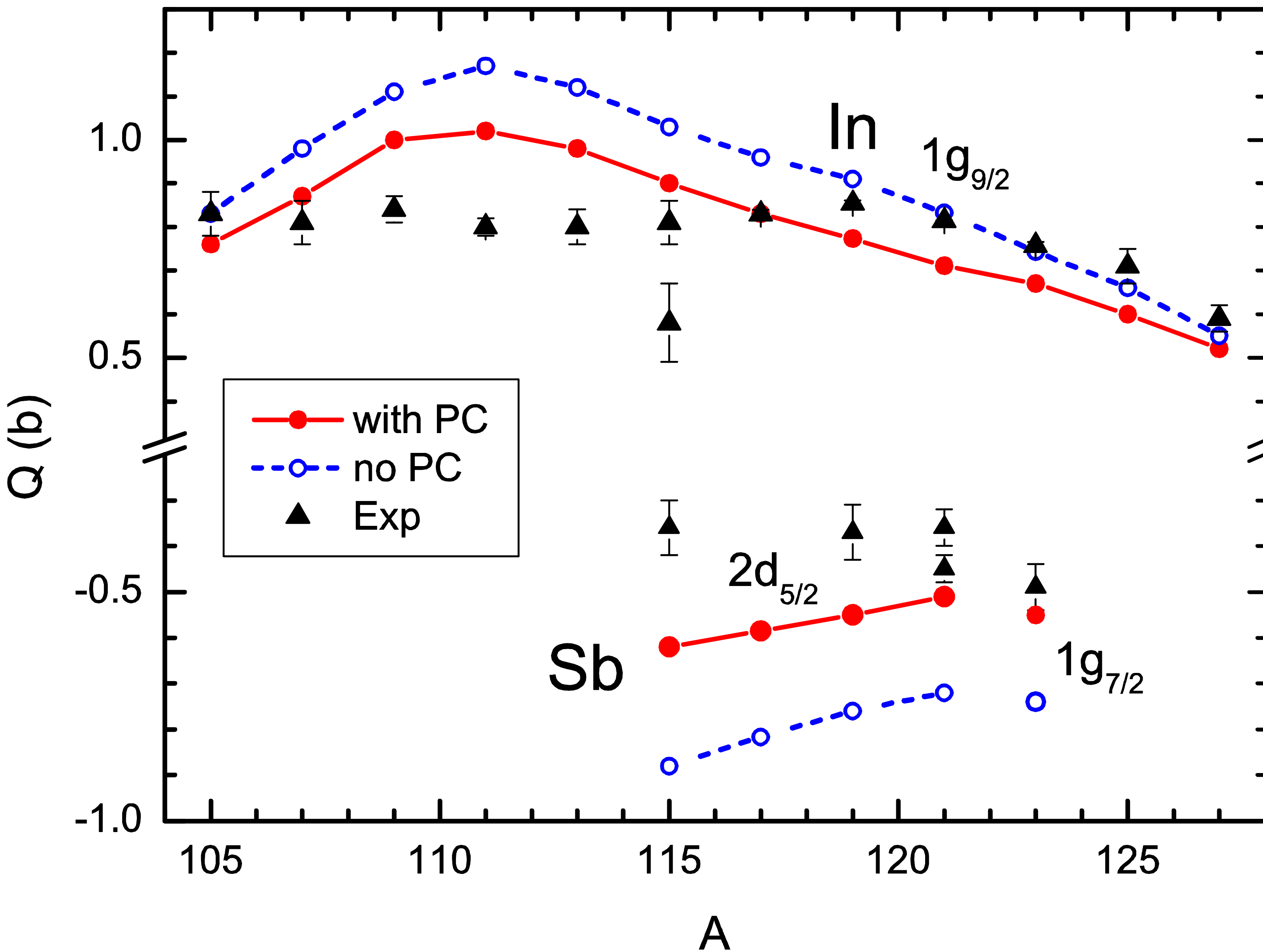
$$\delta Q = Q(no\ ph) - Q(\exp),$$

$$\delta \tilde{Q} = Q(with\ ph) - Q(\exp)$$

$< \delta Q >_{\rm rms} = 0.27$ b $\qquad < \delta \tilde{Q} >_{\rm rms} = 0.15$ b

Results:

nucl.	λ	Q_{exp}	Q_0	Q_{th}	$\tilde{Q}_{\mathrm{th}}$	δQ	$\delta\tilde{Q}$
^{105}In	$1g_{9/2}$	+0.83(5)	0.18	+0.83	0.76	0.00	-0.07
^{107}In	$1g_{9/2}$	+0.81(5)	0.18	+0.98	0.87	0.17	0.06
^{109}In	$1g_{9/2}$	+0.84(3)	0.18	+1.11	1.00	0.27	0.16
^{111}In	$1g_{9/2}$	+0.80(2)	0.19	+1.17	1.02	0.37	0.22
^{113}In	$1g_{9/2}$	+0.80(4)	0.19	+1.12	0.98	0.32	0.16
^{115}In	$1g_{9/2}$	+0.81(5)	0.19	+1.03	0.90	0.22	0.09
		0.58(9)				0.45	0.32
^{117}In	$1g_{9/2}$	+0.829(10)	0.19	+0.96	0.83	0.131	0.001
^{119}In	$1g_{9/2}$	+0.854(7)	0.19	+0.91	0.773	0.056	-0.081
^{121}In	$1g_{9/2}$	+0.814(11)	0.19	+0.833	0.711	0.019	-0.103
^{123}In	$1g_{9/2}$	+0.757(9)	0.19	+0.743	0.670	-0.014	-0.087
^{125}In	$1g_{9/2}$	+0.71(4)	0.19	+0.66	0.60	-0.05	-0.11
^{127}In	$1g_{9/2}$	+0.59(3)	0.19	+0.55	0.52	-0.04	-0.07
^{115}Sb	$2d_{5/2}$	-0.36(6)	-0.14	-0.88	-0.62	-0.52	-0.26
^{117}Sb	$2d_{5/2}$	-	-0.14	-0.817	-0.585	-	-
^{119}Sb	$2d_{5/2}$	-0.37(6)	-0.14	-0.76	-0.55	-0.39	-0.18
^{121}Sb	$2d_{5/2}$	-0.36(4)	-0.14	-0.72	-0.51	-0.36	-0.15
		-0.45(3)				-0.27	-0.06
^{123}Sb	$1g_{7/2}$	-0.49(5)	-0.17	-0.74	-0.55	-0.25	-0.06



Q (b)
A
1.0
0.5
-0.5
-1.0
105
110
115
120
125
In
1g9/2
Sb
2d5/2
1g7/2
with PC
no PC
Exp

PC corrections to charge radii

Recently, in

R. F. Garcia Ruiz, et al. (Collab.), Nature Phys. **12**, 594 (2016),

a puzzle was announced of ‘anomanolously big’ charge radii of heavy Ca isotopes. We explain it with the PC effect.

In [Nucl. Phys. A **676**, 49 (2000)] , Fayans et al. explained rather fancy (crown-like) behavior of R_{ch} in the Ca (40 – 48) chain

without PC, Fayans tried to find the best EDF without phonons.

EDF DF3: $R_{ch}(40Ca) \approx R_{ch}(48Ca)$

For a strong odd-even effect, the gradient term in paring force was added

$$\mathcal{F}^{\xi} = C_0 f^{\xi} = C_0 \left(f_{ex}^{\xi} + h^{\xi} x^{2/3} + f_{\nabla}^{\xi} r_0^2 (\nabla x)^2 \right).$$

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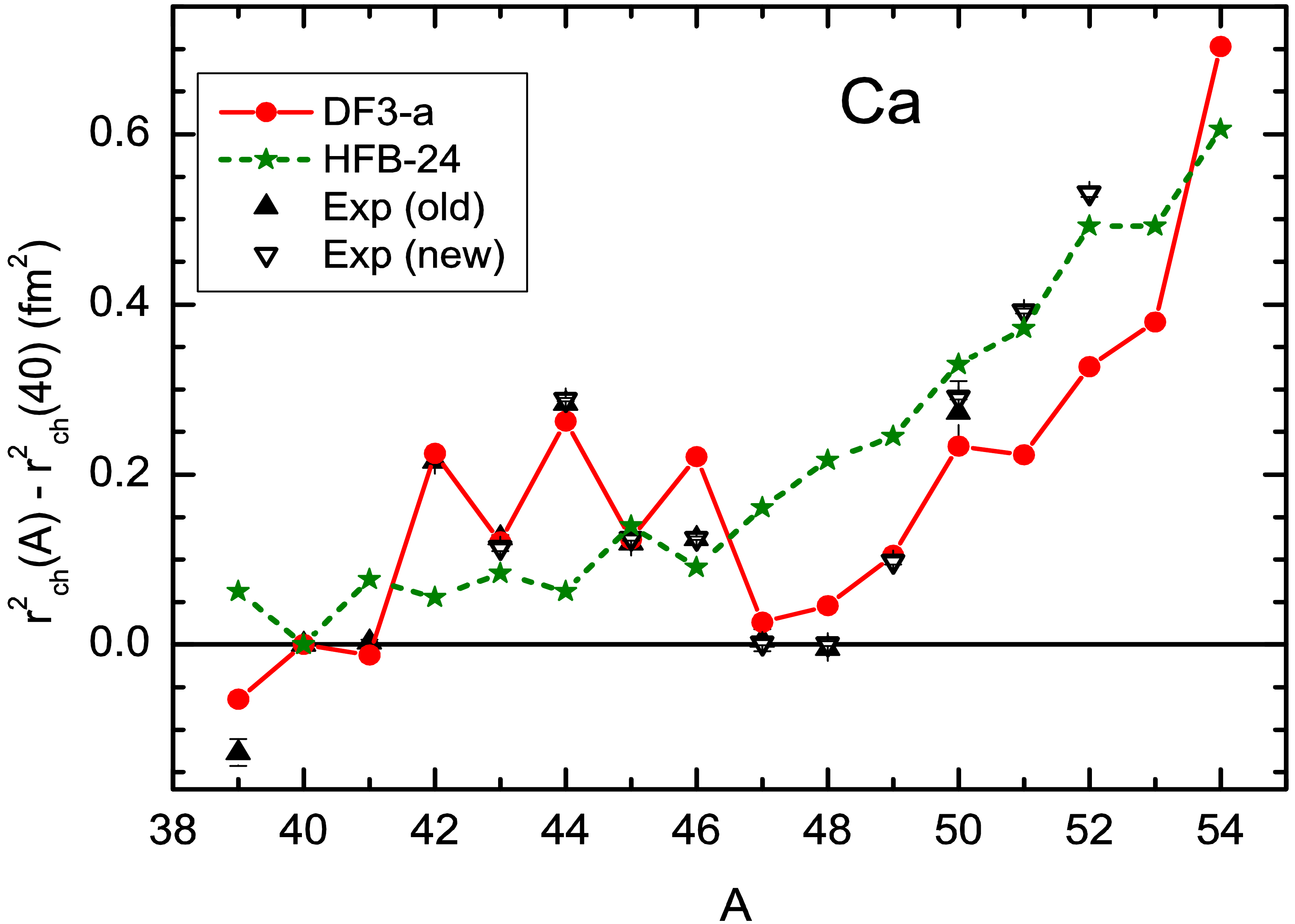
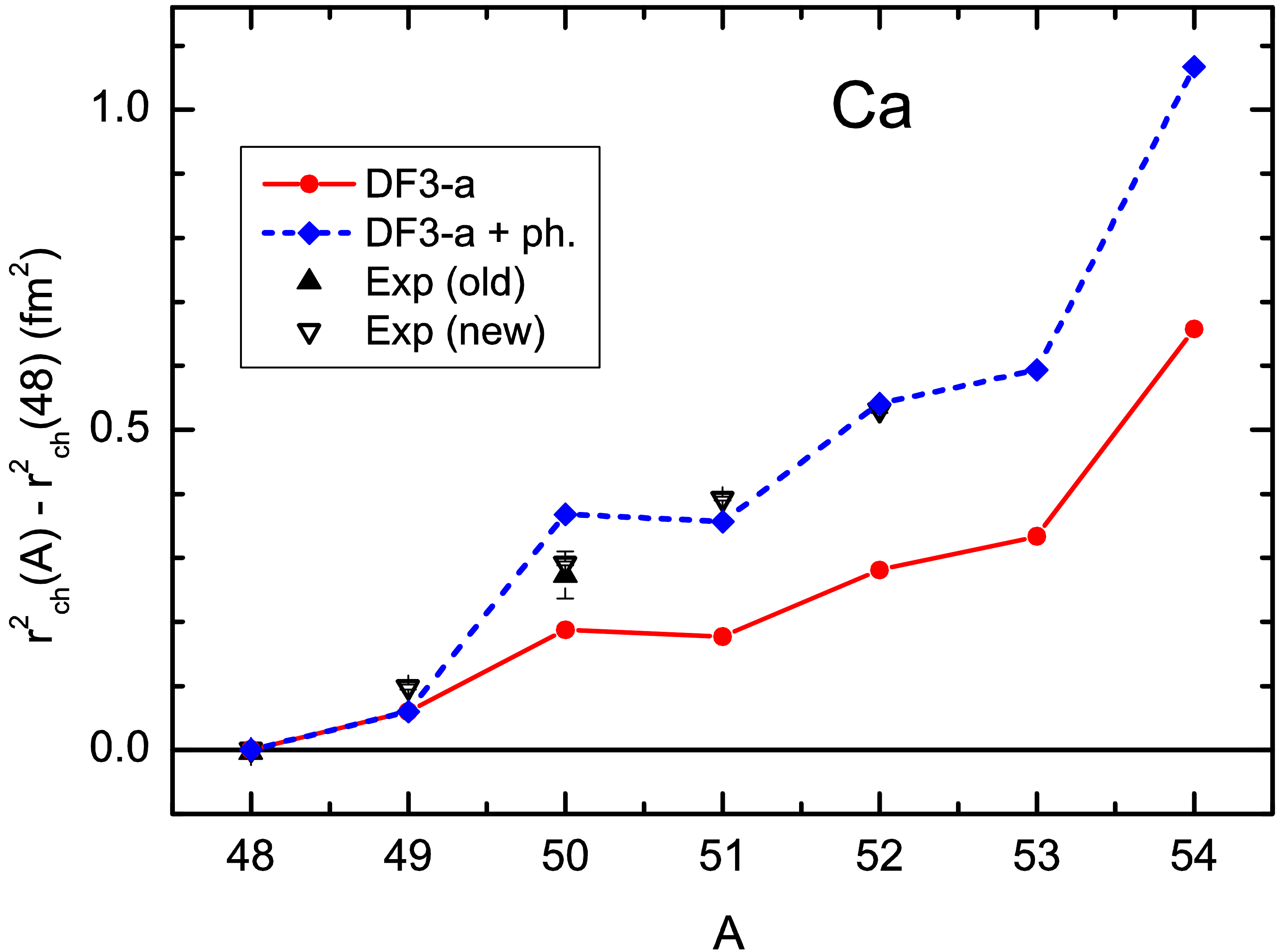


JETP Lett., 104, 218 (2016).



Ca
DF3-a
DF3-a + ph.
Exp (old)
Exp (new)
$r^2_{ch}(A) - r^2_{ch}(48)$ (fm^2)
1.0
0.5
0.0
48
49
50
51
52
53
54
A

Conclusions

For the Fayans EDF DF3-a ($m^*=m$, the 'Fayans denominator'), account for the PC corrections makes agreement with experiment better (in fact, with a record accuracy) for

1. SPEs of magic nuclei,
2. Quadrupole moments of odd semi-magic nuclei.
3. Charge radii of Ca isotopes.

Self-consistency, no adjusted parameters!

In the first two cases,

The non-pole term plays a crucial role.

Thank you