DAMPING OF SIMPLE MODES OF HIGH-ENERGY NUCLEAR EXCITATIONS: DISPERSIVE OPTICAL MODELS AND THEIR IMPLEMENTATIONS

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I. Introduction

The resonance-like structures, corresponding to simple modes of nuclear excitations (single-quasiparticle- and particle-hole-type) are observed in nuclear reactions at high excitation energies (up to a few tens of MeV). Therefore, for these energies the mean-field concept does work, or (that is the same) nuclei are “grey” (not “black”) for the mentioned degrees of freedom.

To describe coupling of these high-energy modes to many-quasiparticle (chaotic) states (the spreading effect) only a phenomenological way seems to be realistic and related to the corresponding optical models. Microscopically-based transition to these models is based on many-body Green function method introduced in nuclear physics by Migdal [1].

The main topic of this presentation is the description of the recently developed particle-hole dispersive optical model (PHDOM) and first implementations of this model. The “traditional” single-quasiparticle dispersive optical model (SQDOM) is also discussed together with a description of deep-hole states.

II. PHDOM

1. General description (“in words”)

1.1. The particle–hole dispersive optical model is developed recently [2] to describe in a semimicroscopic way the main properties of a great variety of high-energy (p-h)-type nuclear excitations (including giant resonances) in “hard” medium-heavy mass spherical nuclei.

1.2. Within the model, the main relaxation modes of the above-mentioned excitations are commonly taken into account.

These modes are:

(i) distribution of the p-h strength, or Landau damping ,~ the result of shell structure of nuclei;

(ii) coupling of (p-h)-type states to the s.p. continuum ~ nuclei are the open Fermi-systems;

(iii) coupling of (p-h)-type states to many quasi-particle (chaotic) configurations, or the spreading effect ~ high excitation energies.

1.3. Within the PHDOM, which is a semi-microscopic model, Landau damping and coupling to the s.p. continuum are described microscopically (in terms of a mean field and p-h interaction), while the spreading effect is treated phenomenologically and in average over the energy (in terms of the specific p-h interaction, or a p-h self-energy term).

1.4. Microscopically based transition to the PHDOM (as well as to the single-quasiparticle DOM) is performed with the use of the many-body Green function method. Actually, the PHDOM is an extension of the standard and non-standard continuum-RPA (cRPA) versions on phenomenological account for the spreading effect. The imaginary part of the strength of the specific energy-averaged p-h self-energy term determines also the corresponding real part via a proper dispersive relationship, which follows from the spectral expansion for the 2p-2h Green function (2p-2h configurations are doorway-states for the spreading effect).

1.5. The unique feature of the PHDOM is its ability to describe:

(i) the energy-averaged double p-h transition density and, therefore, various strength functions at arbitrary (but high-enough) excitation energies, including giant resonances;

(ii) direct-nucleon-decay properties of the (p-h)-type states, including the so-called direct+semidirect (DSD) reactions induced by a s.p. external field;

(iii) a spreading (dispersive) shift of the energy of (p-h)-type resonance structures.

1.6. Ingredients of the model used in first implementations:

(i) Landau-Migdal p-h interaction and a phenomenological partially self-consistent mean field;

(ii) The energy-dependent imaginary part of the strength of the energy-averaged p-h self-energy term responsible for the spreading effect.

In conclusion, PHDOM can be considered as the model of interacting and damping quasiparticles.

2. Basic relationships (schematically)

2.1. PHDOM is finally formulated in terms of energy-averaged quantities. The main one is the p-h Green function (effective p-h propagator) $A(x, x', \omega)$, which satisfy the Beth-Goldstone-type integral equation:

$$A(\omega) = A_0(\omega) + A_0(\omega)FA(\omega).$$

Here, ω is the excitation energy, $F(x_1, x_2)$ – p-h interaction (in implementations, taken as the Landau-Migdal forces); $A_0(x_1, x_2, \omega)$ – "free" p-h propagator, which corresponds to the model of non-interacting damping quasiparticles. The expression for this key quantity is derived within the discrete PHDOM version with the use of a statistical assumption: after energy averaging the different p-h configurations are "decaying" into chaotic states independently of one another.

2.2. Let $H_0(x)$ be a s.p. Hamiltonian that determines a set of s.p. energies $\{\varepsilon_\lambda\}$ and wave functions $\{\varphi_\lambda\}$. Then the expression for $A_0(x,x',\omega)$ derived within the PHDOM in terms of $\{\varphi_\lambda\}$ ("λ-representation") is a direct extension of the discrete-RPA version:

$$A_0(x,x',\omega) = \sum_{\lambda\mu} \varphi^*_\mu(x)\varphi_\lambda(x)\varphi^*_\lambda(x')\varphi_\mu(x')A_{\lambda\mu}(\omega),$$

$$A_{\lambda\mu} = \frac{n_\lambda - n_\mu}{\varepsilon_\lambda - \varepsilon_\mu - \omega + (n_\lambda - n_\mu)[iW(\omega) - P(\omega)]f_\lambda f_\mu}.$$

Here, $n_{\mu,\lambda}$ are the occupation numbers, $[-iW(\omega) + P(\omega)]$ is the intensity of the properly parameterized p-h self-energy term responsible for the spreading effect, $f_\lambda = \int f(x)|\varphi_\lambda(x)|^2 dx$ with $f(x)$ being the Woods-Saxon function. The imaginary part of the mentioned intensity, $W(\omega)$, determines the real part via a proper dispersive relationship [2,3].

2.3. The PHDOM continuum version follows from the approximate transformation of the above-given expression for $A_0(x,x',\omega)$ to the form, which contains also the optical-model Green functions $g(x,x',\varepsilon_\mu \pm \omega)$. The non-homogeneous equation for these functions contains the optical-model-like addition to the mean field: $[-iW(\omega)+P(\omega)]f_\mu f(x)$. The homogeneous equation for the continuum-state w.f's. $\varphi^{(+)}_{\varepsilon>0}$ contains the same addition. In fact, we have a deal with the effective optical-model potential with $W(\omega)$ being noticeably less than the imaginary part used for the description of nucleon-nucleus scattering due to a destructive interference of particles and holes spreading.

2.4 The simplest version of the dispersive relationship

$$P(\omega) = \frac{2}{\pi} P.V. \int_0^{\infty} W(\omega') \left\{ \frac{\omega'}{\omega^2 - \omega'^2} + \frac{1}{\omega'} \right\} d\omega$$

is adopted to satisfy the condition $P(\omega \to 0) \to 0$.

The simplest parametrization of $W(\omega)$ leads to:

$$W(\omega) = \alpha_W \frac{\omega^2}{1+\omega^2/B^2}, P(\omega) = \frac{\alpha_W}{\pi} \frac{\omega^2}{1+\omega^2/B^2} \ln \frac{\omega}{B}.$$

Here, the intensity parameter α_W and "saturation" parameter B are the adjustable parameters (saturation-like energy dependence of $W(\omega)$ follows from an analysis of the observable total width of various giant resonances).

In implementations of the PHDOM we use more sophisticated parametrization of $W(\omega)$, which contains also an adjustable "gap parameter" Δ. (It is supposed, that the spreading effect "is started" from a finite excitation energy $\omega_{min} = \Delta$). The dispersive relationship is properly adopted [3].

2.5. Expressions for main PHDOM quantities are derived starting from the energy-averaged p-h Green function.

(i) Double transition density and strength function related to a s.p. external field (probing operator) $V_0(x)$:

$$\rho(x,x',\omega) = -\frac{1}{\pi}\,\mathrm{Im}\,A(x,x',\omega);\; S_{V_0}(\omega) = -\frac{1}{\pi}\,\mathrm{Im}(V_0^{+}AV_0).$$

- These quantities can be evaluated at arbitrary (but high-enough excitation energy).
- The energy-averaged double tr. density can't be factorized (in other words, one-body tr. density can't be defined).
- Since existing computer codes for calculation of inelastic hadron-nucleus scattering accompanied by excitation of a given giant resonance (GR) exploits only one-body tr. density, we suggest to use the "projected" one-body tr. density

$$\rho_g(x,\omega) = \int \rho(x,x',\omega)V_{0,g}(x')dx' \,/S_{V_{0,g}}^{1/2};\; S_{V_{0,g}} = \left|(\rho_g(\omega)V_{0,g})\right|^2.$$

Here, $V_{0,g}$ - a specific for excitation of the given GR external field.

(ii) The effective field $V(x,\omega)$, defined by $(AV_0) = (A_0V)$, satisfies the equation

$$V(\omega) = V_0 + A_0(\omega)FV(\omega)$$

and determines the strength function $S_{V_0}(\omega) = -\frac{1}{\pi}\mathrm{Im}(V_0^+ A_0 V)$.

(iii) The squared amplitude of the one-nucleon direct+semidirect (DSD) reaction induced by an external field $V_0(x)$ and accompanied by excitation of an one-hole (μ^{-1}) state of the product nucleus is expressed in terms of the effective field:

$$\left|M^{DSD}_{V_0,c}(\omega)\right|^2 = n_\mu \left|\left(\psi^{(-)*}_{\epsilon=\epsilon_\mu+\omega} V(\omega)\psi_\mu\right)\left(\psi^*_\mu V^*(\omega)\psi^{(+)}_{\epsilon=\epsilon_\mu+\omega}\right)\right|,$$

$$b_c(\delta) = \int_{(\delta)}\left|M^{DSD}_{V_0,c}(\omega)\right|^2 d\omega \Big/ \int_{(\delta)} S_{V_0}(\omega)d\omega.$$

Here, c is a set of the reaction-channel quantum numbers that includes the quantum numbers of μ^{-1} state and (p-h)-type nuclear excitation; $b_c(\delta)$ is the partial branching ratio for direct one-nucleon decay of the mentioned excitation from an energy interval δ. Within the cRPA, the total branching ratio is equal to unity independently of the interval δ.

2.4. Weak violations of model unitarity caused by the use of:

(i) an energy dependence of $[-iW(\omega) + P(\omega)]$ (takes place for any types of high-energy p-h excitations);

(ii) approximate spectral expansion of optical-model Green functions $g(x, x', \varepsilon)$ (takes place only for isoscalar monopole (ISM) excitations);

can be eliminated [4].

(i) The expansion element $A_{0,\lambda\mu}(\omega)$ of the "free" p-h propagator can be represented as the sum of resonant (direct) and non-resonant (backward) terms:

$$A_{0,\lambda\mu}(\omega) = A^{r}_{0,\lambda\mu}(\omega) + A^{nr}_{0,\lambda\mu}(\omega)$$

Within the realistic approximation $\left|\frac{d^2W}{d\omega^2}\right| \ll \left|\frac{dP}{d\omega}\right|$, unitarity can be restored, provided the modified propagator is used:

$$A^{m}_{0,\lambda\mu}(\omega) =$$

$$\left(1 - f_\lambda f_\mu \frac{dP(\omega)}{d\omega}\right) A^{r}_{0,\lambda\mu}(\omega) + \left(1 + f_\lambda f_\mu \frac{dP(\omega)}{d\omega}\right) A^{nr}_{0,\lambda\mu}(\omega).$$

As a result, the p-h strength $S^{m}_{\lambda\mu} = -\frac{1}{\pi}\int \mathrm{Im}A^{m}_{0,\lambda\mu}(\omega)d\omega \simeq (1 - n_\lambda)n_\mu$ is restored and, also, $\mathrm{A}^{m}_{0,\lambda\mu}(\omega) = \mathrm{A}^{m}_{0,\mu\lambda}(-\omega)$.

(ii) To avoid within the model of ISM “spurious” excitations generated by the unit external field, in description of ISM strength functions related to the external field $V_0(\mathbf{r}) = V_0(r)Y_{00}$, the radial part $V_0(r)$ should be modified:

$$V_0 \rightarrow V_0(r) - < V_0 >,$$

Where averaging is performed on the ground-state matter density.

This modification allows one, in particular, to exclude small negative values of ISM strength functions, calculated within the PHDOM initial version.

In description of ISM excitations, both above modifications should be used to get an unitary version of the PHDOM.

3. First implementations of PHDOM

3.1. Investigations of isoscalar monopole (ISM) excitations allow to get info about nuclear matter incompressibility coefficient.

First, we study within the PHDOM the ISM relative energy-weighted strength functions

$$y_g(\omega) = \omega S_{V_{0,g}}(\omega) / EWSR_{V_{0,g}},$$

corresponding to the ISM external fields $V_{0,g}(x)$

$V_{0,1} = r^2 Y_{00}$ and $V_{0,2} = r^2(r^2 - \eta)Y_{00}$

(η is an adjustable parameter), which lead to excitation of the isoscalar monopole giant resonance (ISGMR) and its overtone (ISGMR2) [4,5].

The strength functions calculated within the PHDOM versions for ^{208}Pb are shown in the Figs. 1,2.

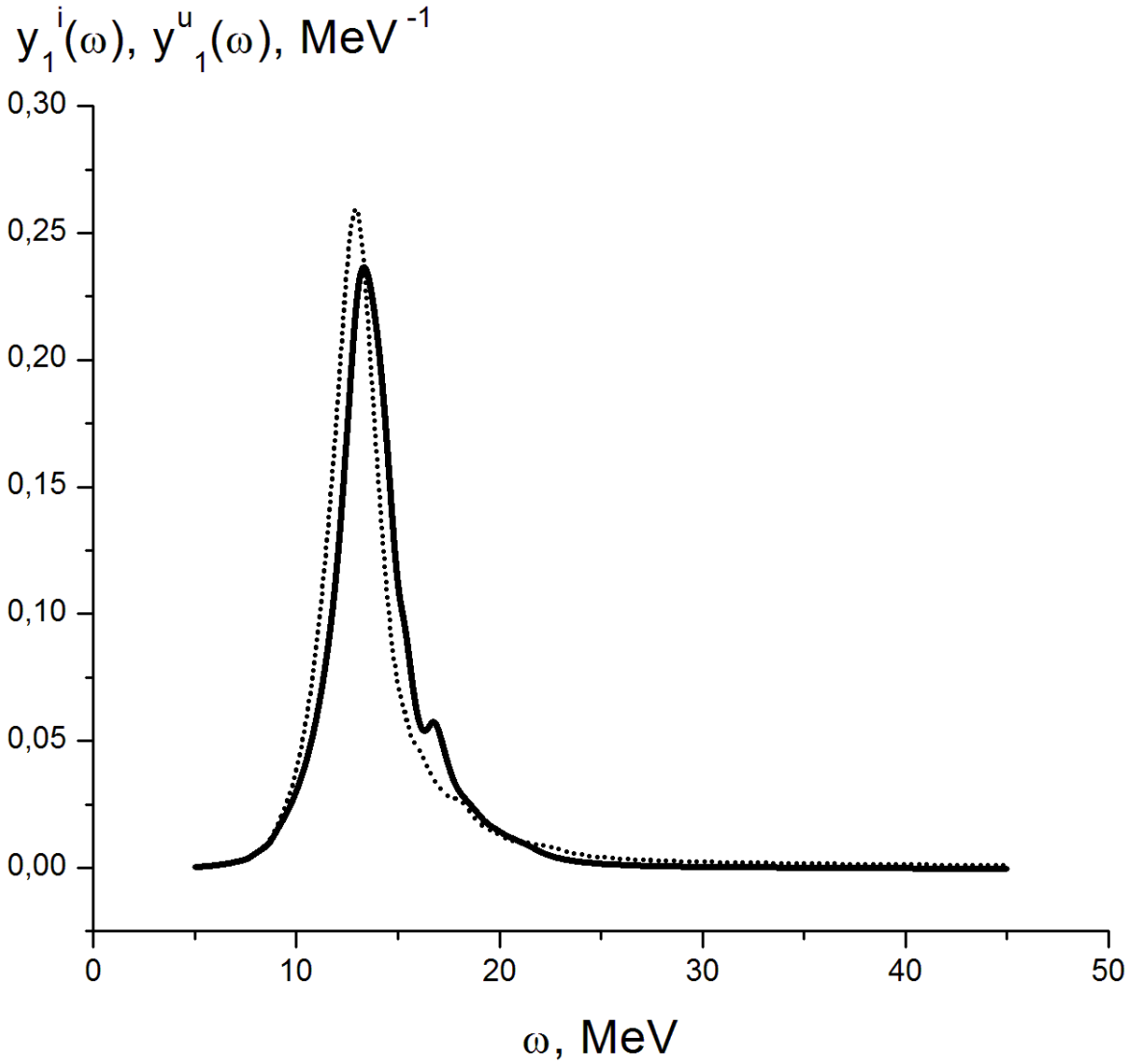

Fig. 1. The relative energy-weighted strength functions calculated for the ISGMR in ^{208}Pb within the initial (thick line) and unitary (dotted line) versions of the PHDOM.

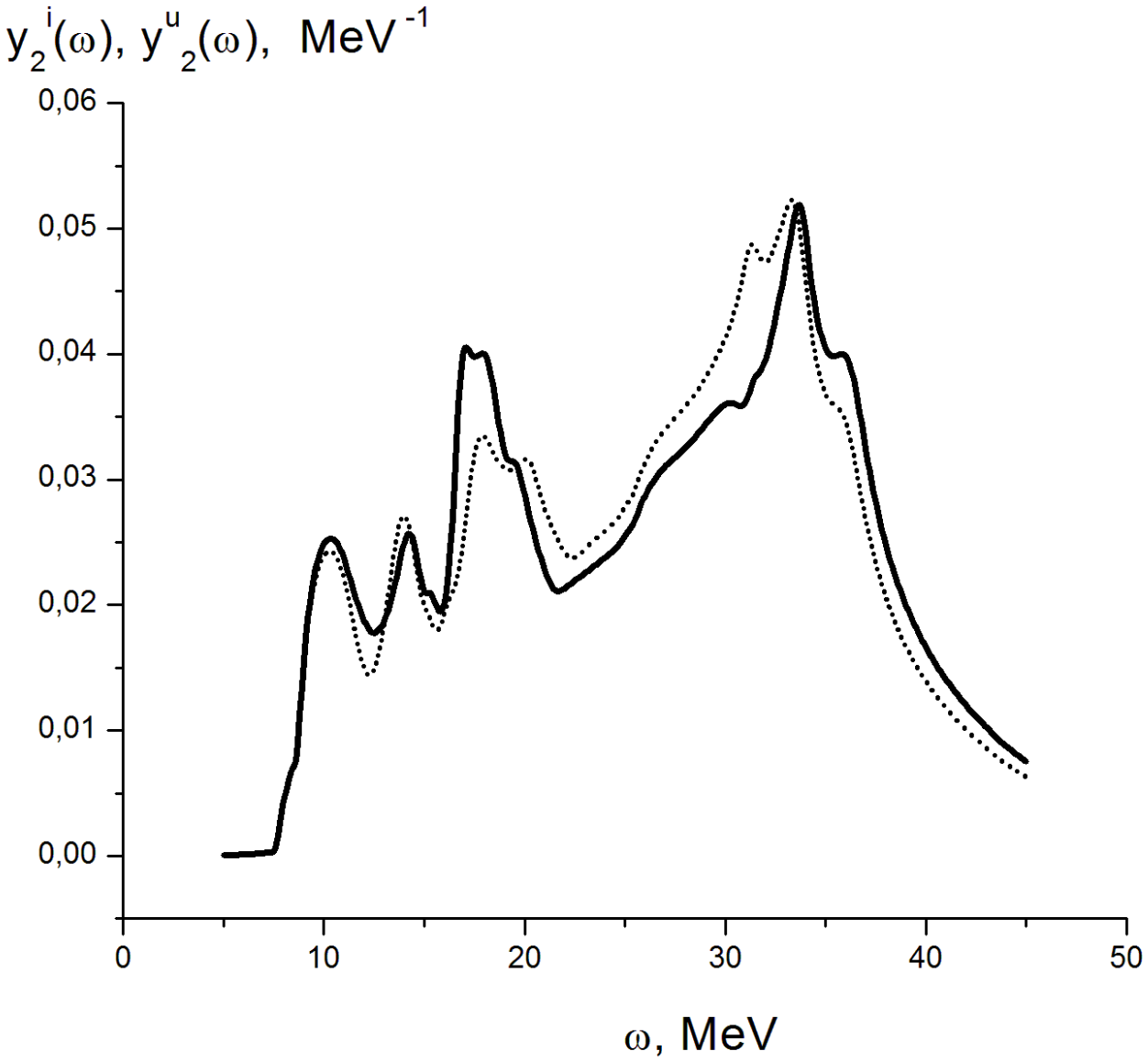

Fig. 2. Same as in Fig. 1 but for the ISGMR2.

To deduce the ISM strength distribution from the inelastic (α, α')-scattering cross sections at small angles, it is necessary to know the ISM energy-averaged (two-dimensional) double transition density:

$$\rho(r, r', \omega) = 4\pi\,(rr')^2 \rho(\mathbf{r}, \mathbf{r}', \omega)$$

at arbitrary energies.

In Fig.3 we show evaluated for ^{208}Pb the "diagonal" quantity $\rho(r, r, \omega)$ (the solid thick line) in a comparison with the squared projected and properly normalized semi-classical collective-model one-body transition densities, $\rho_1^{pr}(r, \omega)$ and $\rho_1^{cl}(r, \omega)$ (the solid thin and dashed lines, respectively) in a vicinity of the ISGMR [5]. Differences at ISGMR "tails" are clearly seen (existing computational codes "accept" only one-body transitional density).

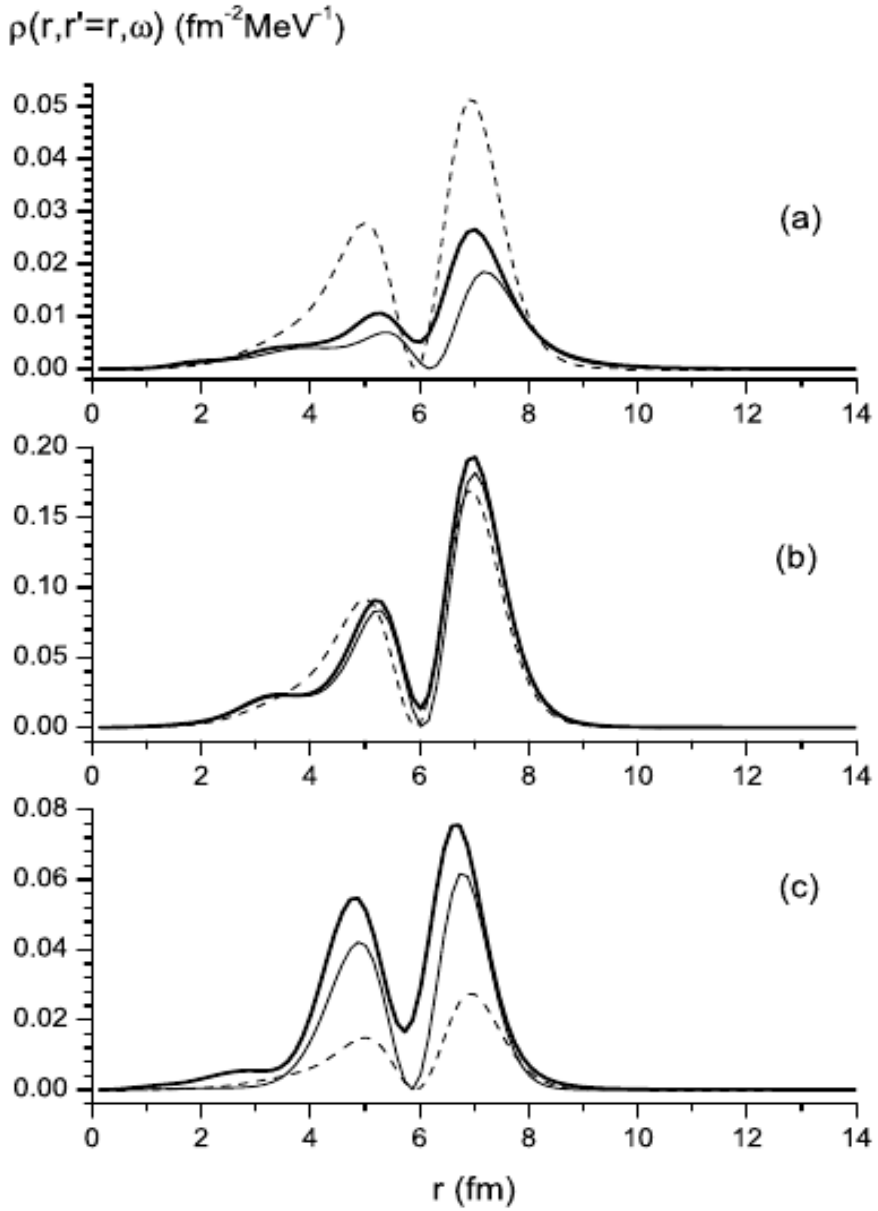

Fig. 3. The ISM double transition density calculated at different excitation energies in the vicinity of ISGMR: 10.8 MeV (a), 13.8 MeV (b), 16.8 MeV (c)

- The partial branching ratios for direct neutron decay of the ISGMR in ^{208}Pb calculated within the PHDOM initial and unitary versions (b_μ^i and b_μ^u respectively) for the excitation energy interval $\delta = 12.5 \div 15.5$ MeV in a comparison with the corresponding experimental data [4] are shown in Table 1.
- In calculations, the pure single-hole structure of μ^{-1} states of ^{207}Pb is supposed.

Table 1

μ^{-1}	$b_\mu^i, \%$	$b_\mu^u, \%$	$b_\mu^{exp}, \%$
$3p_{1/2}$	1.65	2.85 %	0.75 ± 1.1
$1i_{13/2}$	0.36	0.39	2.9 ±1.3
$2f_{5/2}$	11.6	11.7	2.6 ± 1.7
$3p_{3/2}$	2.90	3.18	5.3 ± 1.8
$2f_{7/2}$	9.76	10.5	11.4 ± 1.3
Incl. $(2d_{5/2})(1g_{7/2})$	0.0	0.0	

3.2. Simplest photonuclear reactions [6].

The isovector giant dipole and quadrupole resonances (IVGDR and IVGQR) are systematically studied by means of photonuclear reactions. The simplest reactions are photoabsorption and DSD photoneutron and inverse reactions.

To describe these reactions within the PHDOM we use the corresponding external fields as follows ($Q_{LM} = r^L Y_{LM}$):

IVGDR$\rightarrow V_0(x) = -\frac{1}{2}\tau^{(3)} Q_{1M}$; ISGQR+IVGQR $\rightarrow \frac{1}{2}\left(1 - \tau^{(3)}\right) Q_{2M}$.

Within the accuracy $1 \ll (N - Z) \ll A$, the equations for isovector ($T = 1$) and isoscalar ($T = 0$) effective fields are decoupled. These fields $V_{LM}^{T=1}\left(V_{0,LM}^{T=1} = Q_{LM}\right)$ and $V_{2M}^{T=0}\left(V_{0,2M}^{T=0} = Q_{2M}\right)$ determine, in particular, the neutron effective fields:

$$V_{1M}^{(n)} = -\frac{1}{2} V_{1M}^{T=1}; \quad V_{2M}^{(n)} = \frac{1}{2}\left(V_{2M}^{T=0} - V_{2M}^{T=1}\right).$$

The latters determine the amplitudes of the DSD photoneutron and inverse reactions. The excitation of the IVGQR (and ISGQR) in these reactions is possible only due to a p-h interaction.

Photoabsorption cross section

$$\sigma_{a,E1}(\omega) \quad (\sigma_{a,E1} + \sigma_{a,E2})$$

The adjustable parameters obtained to describe $\sigma_a^{exp}(\omega)$ (Table 2)

Nucleus	^{89}Y	^{140}Ce	^{208}Pb
α, MeV^{-1}	0.125	0.10	0.08
k_1'	0.15	0.13	0.17

^{208}Pb

$\delta = 7.5 - 37.5$ MeV

$\sigma_{int}^{calc} = 3633$ mbMeV

$\sigma_{int}^{exp} = 3583$ mbMeV

A consistency of the model

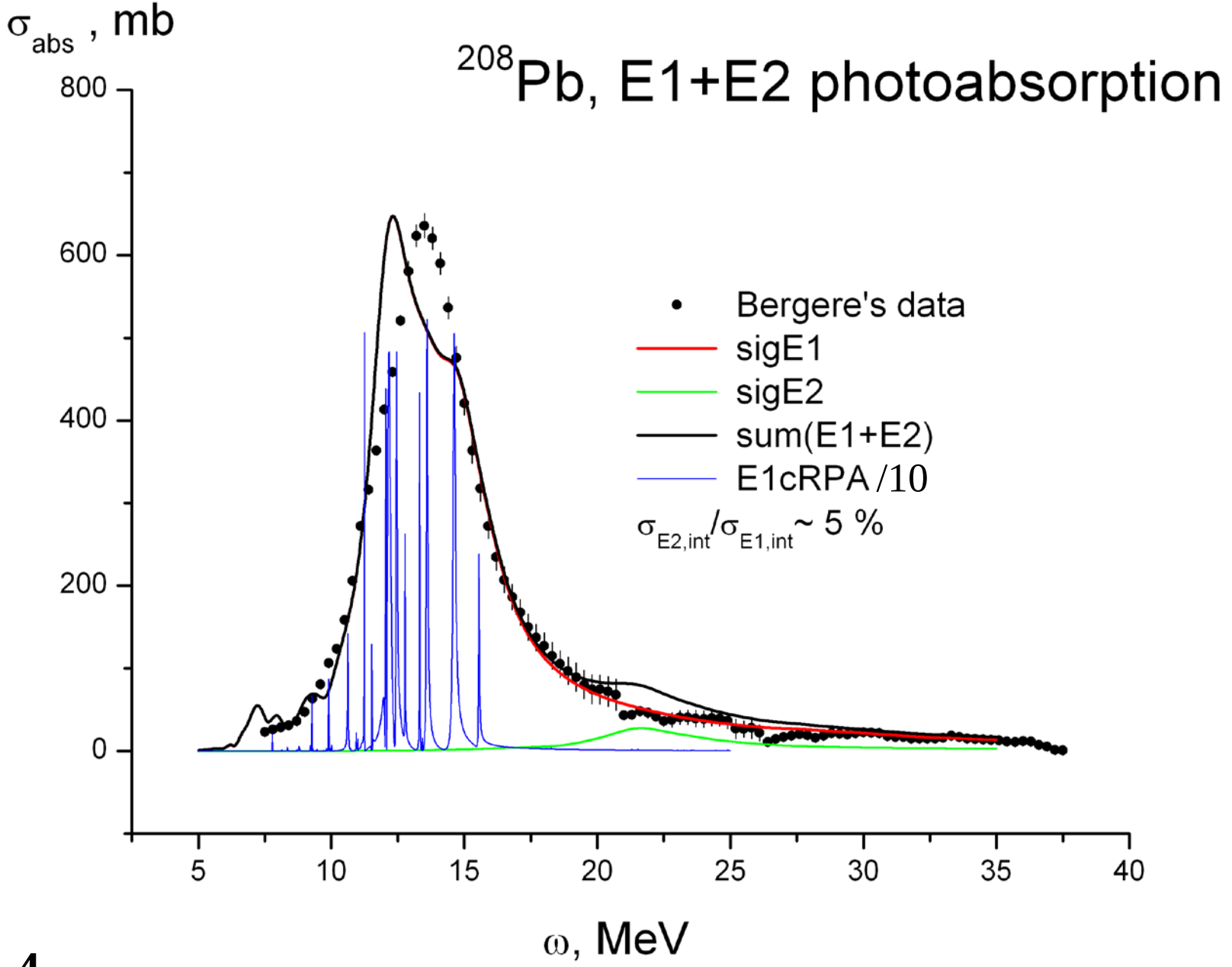
σabs , mb
208Pb, E1+E2 photoabsorption
800
600
400
200
0
5
10
15
20
25
30
35
40
ω, MeV
Bergere's data
sigE1
sigE2
sum(E1+E2)
E1cRPA /10
σE2,int/σE1,int~ 5 %

Fig. 4.

DSD neutron radiative capture

No free parameters!

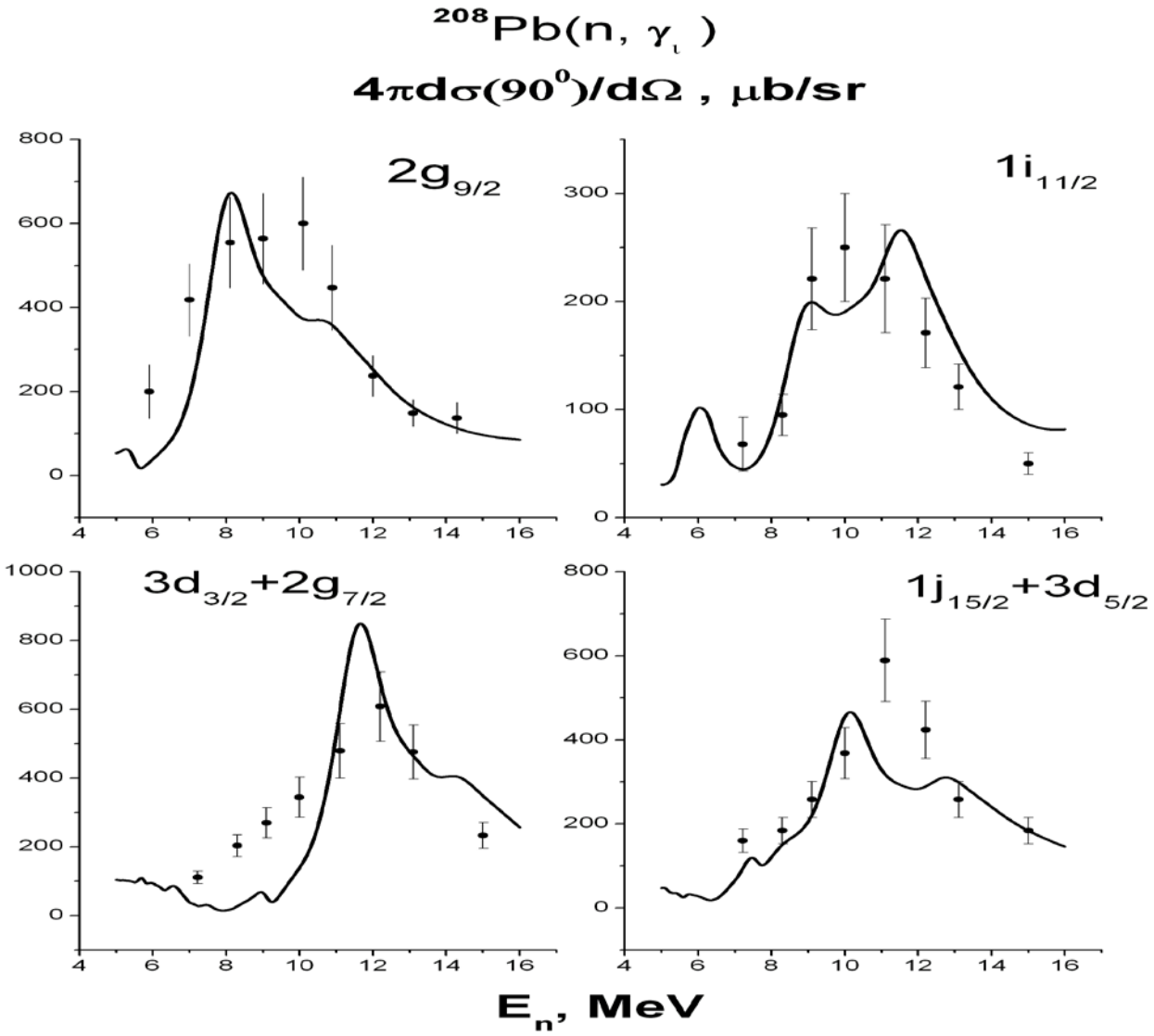

Fig. 5.

Partial DSD ^{208}Pb(γ,n) reaction cross sections (predictions).

Partial branching ratios for IVGDR direct neutron decay

Table 3

μ	$3p_{1/2}$	$2f_{5/2}$	$3p_{3/2}$	$1i_{13/2}$	$1h_{9/2}$	$1f_{7/2}$
b_μ	1.79	3.61	3.10	1.37	2.15	0.53

$$b_{tot} = 12.55\%$$

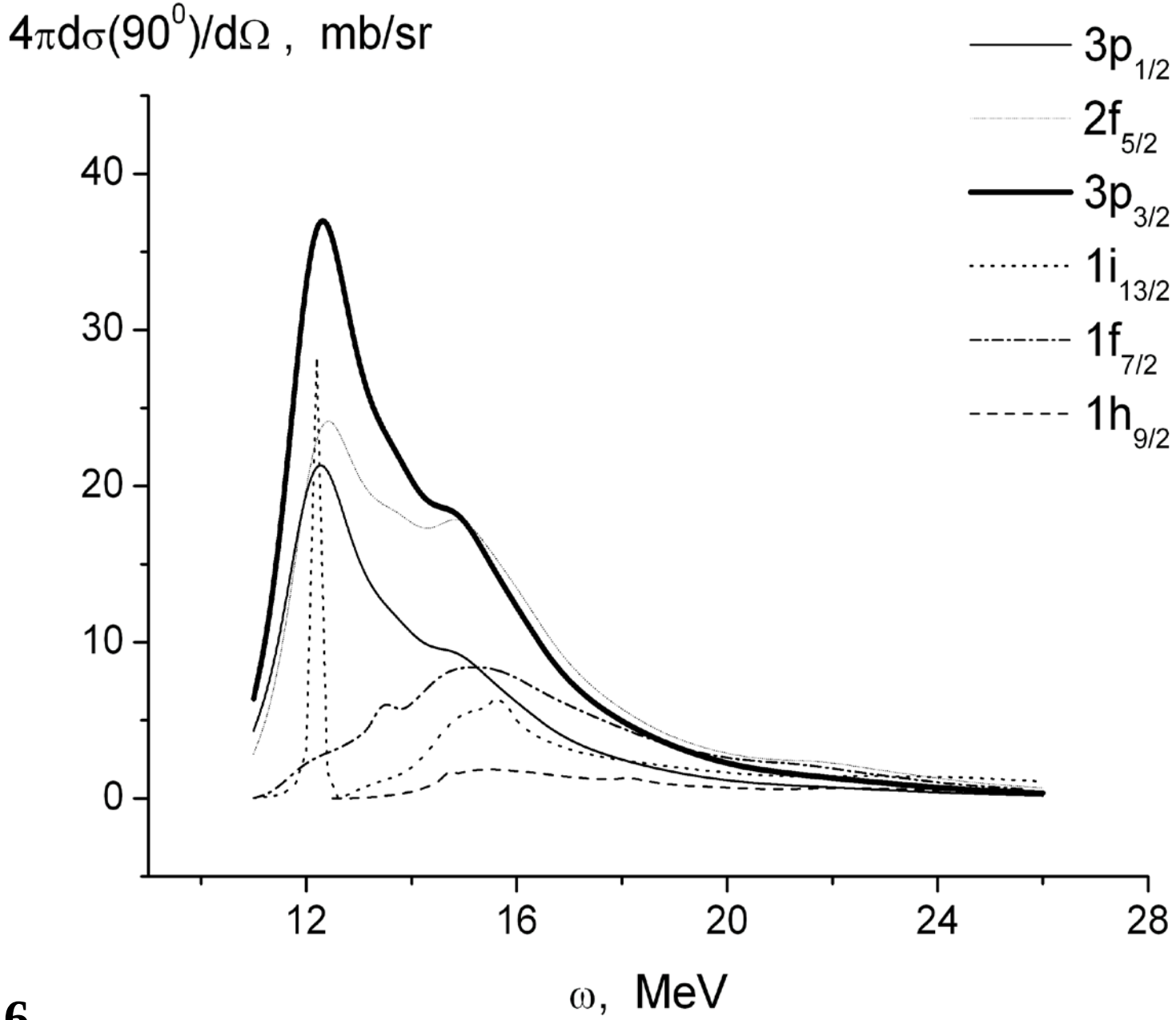
4πdσ(90⁰)/dΩ , mb/sr
3p1/2
2f5/2
3p3/2
1i13/2
1f7/2
1h9/2
40
30
20
10
0
12
16
20
24
28
ω, MeV

Fig. 6.

IVGDR+IVGQR

The asymmetry of the DSD partial differential (γ,n) and inverse reaction cross sections is linear on the E2-reaction amplitude and, therefore, is the appropriate subject for study of the IVGQR in photonuclear reactions.

$$\alpha_\mu = \frac{d\sigma_\mu^{(-)}(\omega,\theta_1)}{d\sigma_\mu^{(+)}(\omega,\theta_1)};$$

$$\frac{d\sigma_\mu^{(\mp)}(\omega,\theta_1)}{d\Omega} = \frac{d\sigma_\mu(\omega,\theta_1)}{d\Omega} \mp \frac{d\sigma_\mu(\omega,\pi-\theta_1)}{d\Omega};$$

$$\theta_1 = 55^\circ$$

The adjustable “velocity” parameter $k_2' = 0.1$

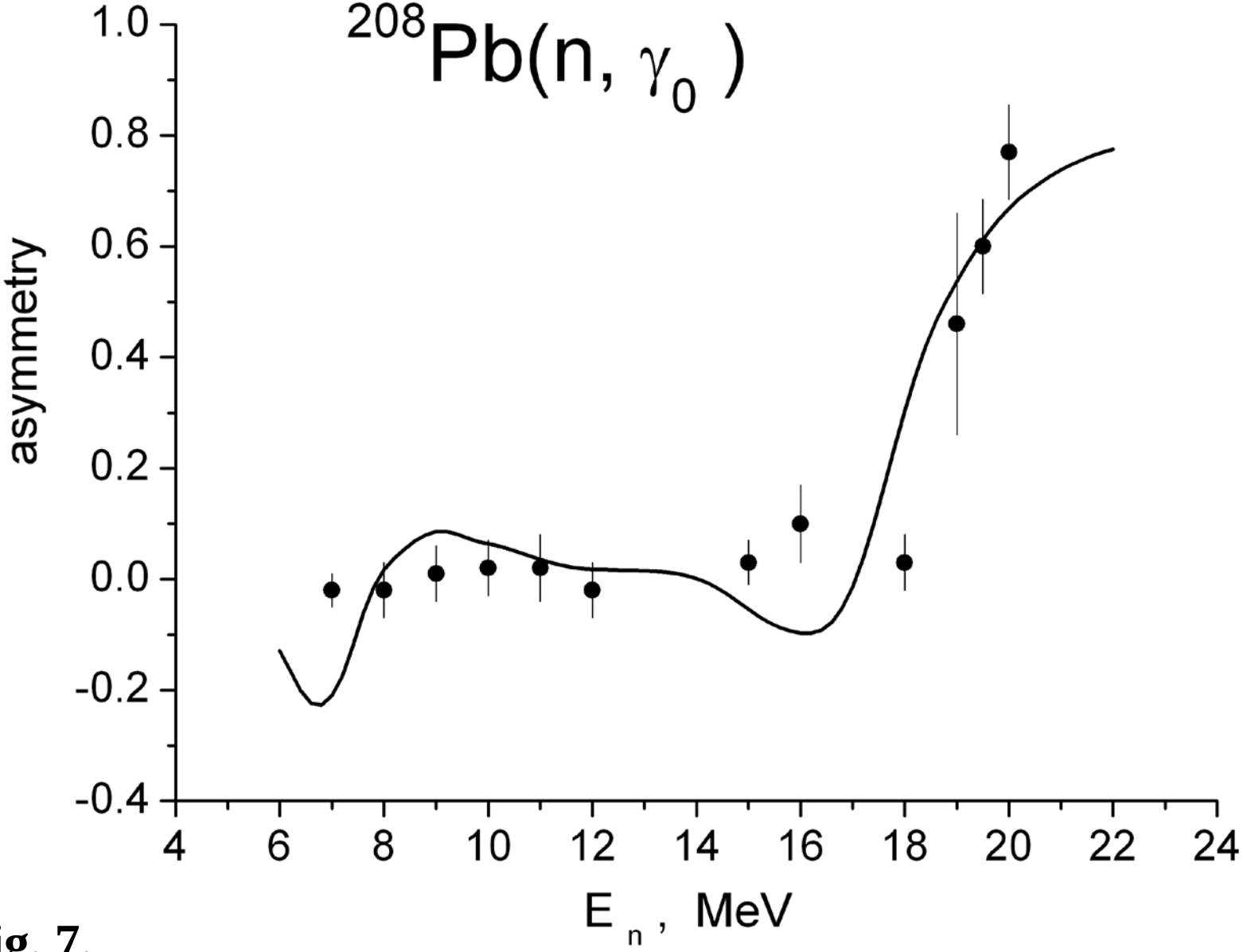
^{208}Pb(n, γ_0)
asymmetry
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
4
6
8
10
12
14
16
18
20
22
24
E n , MeV

Fig. 7.

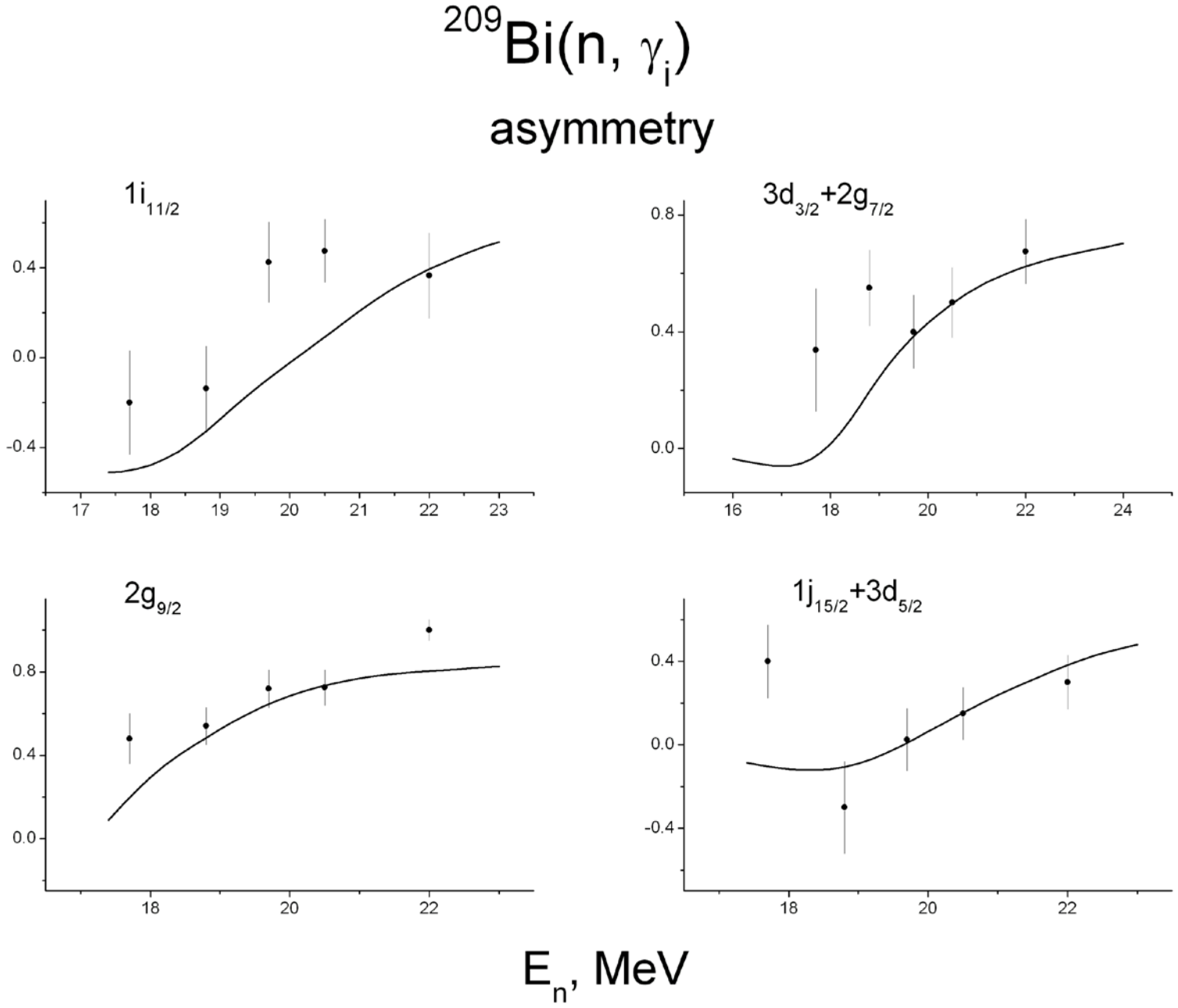
^{209}Bi(n, γ_i)
asymmetry
$1i_{11/2}$
$3d_{3/2}+2g_{7/2}$
$2g_{9/2}$
$1j_{15/2}+3d_{5/2}$
E_n, MeV

Fig. 8.

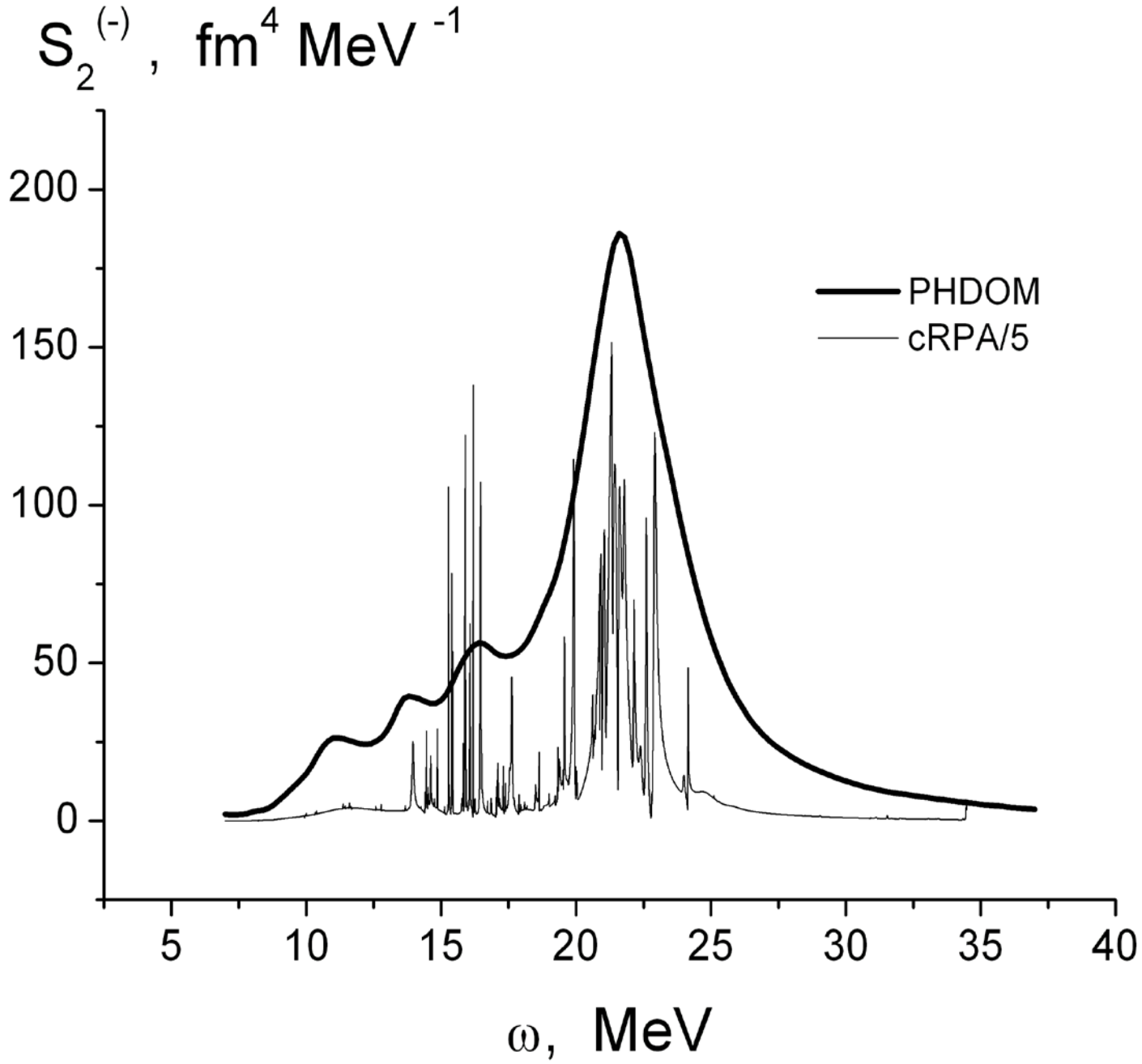
S2(-), fm4 MeV-1
200
150
100
50
0
5
10
15
20
25
30
35
40
ω, MeV
PHDOM
cRPA/5

Fig. 9.

3.3. IAR spreading width – the challenge for nuclear theory

- A small value (several tens keV) of the spreading width of the Isobaric Analog Resonances (IARs), $\Gamma_A^{\downarrow}$, is the impressive manifestation of the approximate isospin-symmetry conservation in medium-heavy mass nuclei. For the IARs, the spreading effect is strongly suppressed and realized only due to isospin mixing. In mentioned nuclei, the mean mixing mechanism consists in IAR coupling to its overtone (IVGMR(-)) via a variable part $V_C(r)$ of the mean Coulomb field $U_C(r)$.
- To get a quantitative estimation of $\Gamma_A^{\downarrow}$, we use the "Coulomb description" of IAR properties [7], which allows, in particular, to express the IAR total width Γ_A in terms of the energy-averaged strength function related to the external field $V_C^{(-)} = \left(U_C(r) - \omega_A + \frac{i}{2}\Gamma_A\right)\tau^{(-)}$:

$$\Gamma_A = \frac{2\pi}{S_A} S_{V_C^{(-)}}(\omega = \omega_A)$$

Here, $S_A \leq N - Z$ is the IAR Fermi strength, ω_A is the IAR excitation energy counted off the parent-nucleus ground-state energy.

Since the IAR is located at the distant "tail" of the IVGMR$^{(-)}$, the PHDOM is quite appropriate for evaluation of the Coulomb strength function and also the squared Coulomb DSD-reaction amplitudes. The latter determine the IAR partial and total proton escape widths:

$$\Gamma_A^{\uparrow} = \sum_{\nu} \Gamma_{A,\nu}^{\uparrow} = \frac{2\pi}{S_A} \sum_{\nu} \left| M^{DSD}_{V_C^{(-)},\nu}(\omega = \omega_A) \right|^2 .$$

Then, the IAR spreading width can be found as $\Gamma_A^{\downarrow} = \Gamma_A - \Gamma_A^{\uparrow}$.

Only one specific phenomenological parameter, α_W, determines within the model the $\Gamma_A^{\downarrow}$ value. (Other model parameters are found from independent data). We adjust α_W to describe the observable total width of the IVGMR(-).

In Fig. 10 we show the Coulomb strength function calculated for the ^{208}Pb parent nucleus in a wide energy interval that includes the IVGMR(-). As expected, no any resonance structure has been found at the IAR energy ω_A.

The preliminary results allows us to hope that the old theoretical problem – quantitative estimation of the IAR spreading width – might be solved within the PHDOM.

Table 4

Nucleus	$\Gamma_A^{\downarrow,calc}, MeV$	$\Gamma_A^{\downarrow,exp}, MeV$
^{208}Pb	80	78 ± 8
^{209}Pb	87	75 ± 7

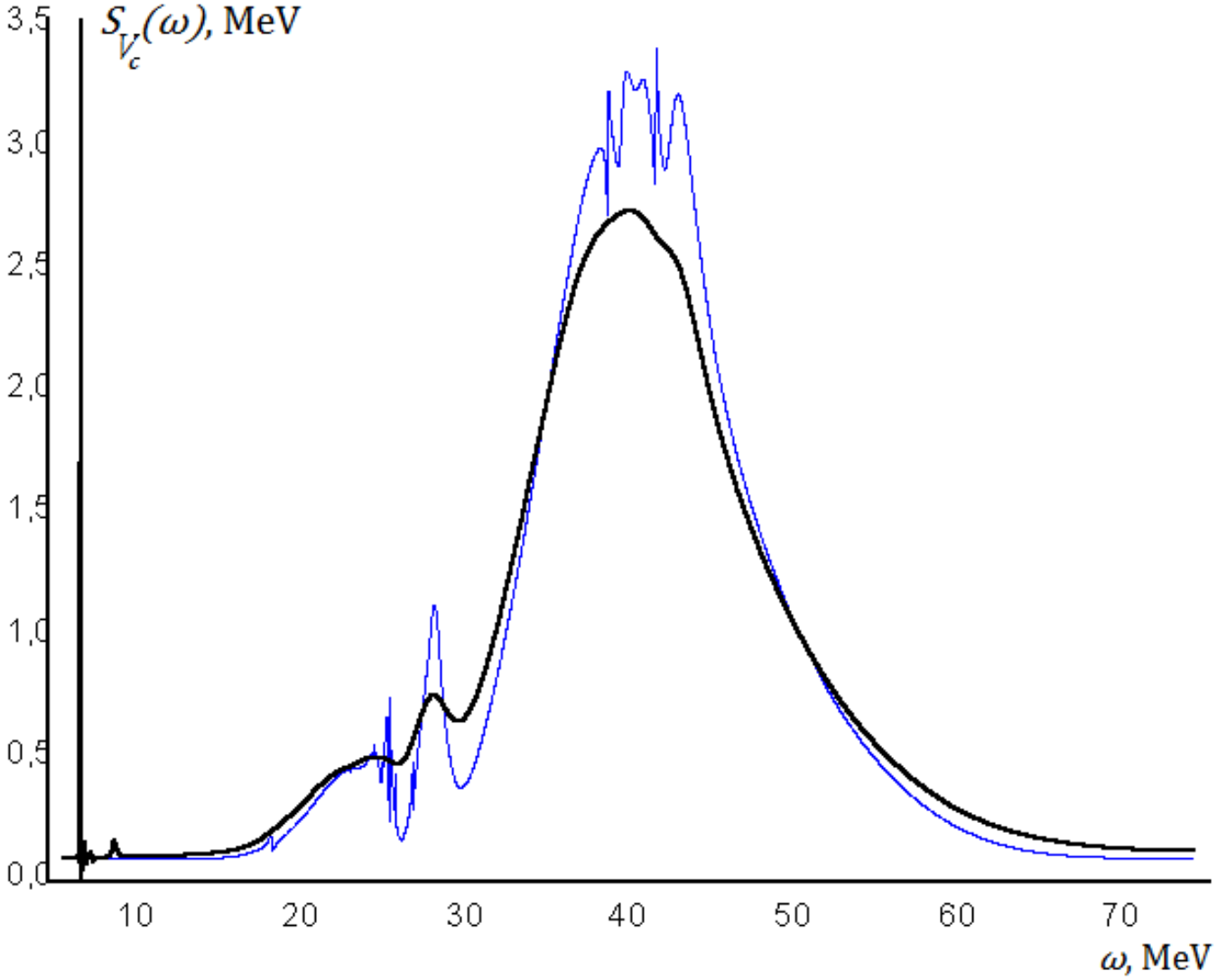

Fig. 10. The Coulomb strength functions calculated within the PHDOM (bold line) and cRPA(thin line) for the ^{208}Pb parent nucleus.

III. An unitary version of the single-quasiparticle dispersive optical model (SQDOM)

1. Introductory words

- Being the oldest nuclear model, the OM was first formulated to describe nucleon-nucleus scattering in terms of a phenomenological energy-dependent complex-valued potential. Up to now this model is widely used by experimentalists.
- An OM dispersive version has been proposed in an attempt to understand structure of the mentioned potential [8].
- In applying to description of damping of deep-hole states we suggest an unitary version of the SQDOM [9]

2. Basic relationships of the SQDOM

- The microscopacally-based transition to an OM is performed by energy averaging the Dyson equation for the single-quasiparticle Green function (GF). The free term in this equation describes “potential” motion, while the energy-averaged mass operator is responsible for the spreading effect and can be properly parameterized with taking analytical properties of this operator into account.
- The observable energies of deep-hole states can be finally described supposing the existence of an energy-dependent part of the nuclear mean field: $\Delta^p(r,\varepsilon) = \Delta^p(\varepsilon)f(r)$ (below we consider this part as an addition to the mean field used in implementations of the PHDOM).

Let $h_{jl}(r,\varepsilon) = h_{jl}(r) + \Delta^p(r,\varepsilon)$ be the radial part of the s-p Hamiltonian. Then the Green function of the corresponding Schrödinger equation ("potential" GF), satisfying the equation

$$\left\{h_{jl}(r,\varepsilon) - \varepsilon\right\} g^p_{jl}(r,r',\varepsilon) = -\delta(r-r'),$$

has non-unit residues S^p_λ at their poles (spectroscopic factors) related to bound states λ:

$$g^p_{jl}(r,r',\varepsilon \to \varepsilon_\lambda) = g^p_\lambda(\varepsilon)\chi_\lambda(r)\chi_\lambda(r');\ g^p_\lambda(\varepsilon) = \frac{S^p_\lambda}{\varepsilon - \varepsilon_\lambda};$$

$$S^p_\lambda = 1/q^p_{\lambda\lambda}(\varepsilon = \varepsilon_\lambda);\ q^p(r,\varepsilon) = 1 - \frac{d\Delta^p(\varepsilon)}{d\varepsilon} f(r).$$

This unitarity violation can be eliminated by the use of the modified "potential" GF

$$g^{p,m}_{jl}(r,r',\varepsilon) = \left(q^p(r,\varepsilon)\right)^{1/2} g^p_{jl}(r,r',\varepsilon) \left(q^p(r',\varepsilon)\right)^{1/2},$$

whose pole representation leads to the unit spectroscopic factors $S^{p,m}_\lambda \simeq 1$.

- The first step in formulation of the SQDOM is the use of the modified "potential" GF as the free term in the energy averaged Dyson equation ($\varepsilon < \mu, \mu$ is the chemical potential):

$$g_{jl}(r, r', \varepsilon) = g_{jl}^{p,m}(r, r', \varepsilon) + \int g_{jl}^{p,m}(r, r_1, \varepsilon)[iW(r_1, \varepsilon) + \Delta^d(r_1, \varepsilon)]\, g_{jl}(r_1, r', \varepsilon) dr_1.$$

Here, $W(r, \varepsilon) = W(\varepsilon) f(r)$ and $\Delta^d(r, \varepsilon) = \Delta^d(\varepsilon) f(r)$ are, respectively, the imaginary and the real parts of the strength of the properly parameterized energy-averaged mass operator.

- The simplest version of the dispersive relationship (the latter follows from the spectral expansion for the 3-quasiparticle GF) is the following ($E = \varepsilon - \mu$):

$$\Delta^d(\varepsilon) = \frac{2E}{\pi}\, P.V. \int_0^\infty \frac{W(E')}{E^2 - E'^2} dE'.$$

- The basic integral equation can be transformed into the differential equation for an auxiliary GF:

$$g_{jl}(r,r',\varepsilon) = \left(q^p(r,\varepsilon)\right)^{1/2} g_{jl}^{aux}(r,r',\varepsilon)\left(q^p(r',\varepsilon)\right)^{1/2},$$

$$\left\{h_{jl}(r,\varepsilon) + \Delta^{eff}(r,\varepsilon) + iW^{eff}(r,\varepsilon) - \varepsilon\right\} g_{jl}^{aux}(r,r',\varepsilon) = -\delta(r-r'),$$

where $\Delta^{eff}(r,\varepsilon) = \Delta^p(r,\varepsilon) + \Delta^d(r,\varepsilon)\, q^p(r,\varepsilon)$ and $W^{eff}(r,\varepsilon) = W(r,\varepsilon)q^p(r,\varepsilon)$.

Thus, the quantities $W(r,\varepsilon)$ and $\Delta^d(r,\varepsilon)$ are renormalized due to unitarity restoration on "potential" level.

- Since the observable single-hole resonance structures are non overlapped (i.e., the W value is not too large), the above-given equation can be solved in the pole approximation. As a result, the single-hole strength function

$$S_\lambda(\varepsilon) = -\frac{1}{\pi}\,\mathrm{Im}\int g_{jl}(r,r',\varepsilon \to \varepsilon_\lambda)dr$$

can be found.

- In some simplifying (but realistic) approximations the strength function can be described by a Lorentzian, whose width Γ_λ and integral strength S_λ are:

$$\Gamma_\lambda = 2W_{\lambda\lambda}(\varepsilon_\lambda)S_\lambda,\ S_\lambda = \left(q^d(\varepsilon_\lambda)\right)^{-1}_{\lambda\lambda},$$

$$q^d(r,\varepsilon) = 1 - \frac{d\Delta^d(\varepsilon)}{d\varepsilon} f(r).$$

One can see that one more violation of model unitarity is due to an energy dependence of the real (dispersive) part of energy-averaged mass operator.

- This violation can be eliminated by the use of the modified optical-model GF

$$g^m_{jl}(r,r',\varepsilon) = \left(q^d(r,\varepsilon)\right)^{1/2} g_{jl}(r,r',\varepsilon) \left(q^d(r',\varepsilon)\right)^{1/2}.$$

In this case the modified single-hole strength S^m_λ is close to unity, but the width $\Gamma^m_\lambda = \Gamma_\lambda$ is not changed.

3 First implementations

- Detailed info concerned with two sets of exp. data $\{\varepsilon_\lambda\}$ and $\{\Gamma_\lambda\}$ is available for neutron and proton one-hole states in the ^{90}Zr and ^{208}Pb parent nuclei.
- These data together with the dispersive relationship are used to deduce within the model the quantities $W(\varepsilon), \Delta^p(\varepsilon), \Delta^d(\varepsilon)$ and to establish the contribution of the dispersive part in the full energy-dependent OM potential:

$$\Delta^{eff} = \Delta^p + \Delta^d \left(1 - \frac{d\Delta^p}{d\varepsilon}\right).$$

An example is given in Fig.11. Within the unitary version of the model this contribution is suppressed.

- Properties of deep-hole states in the ^{132}Sn parent nucleus have been predicted.
- The continuum version of the SQDOM is under consideration.

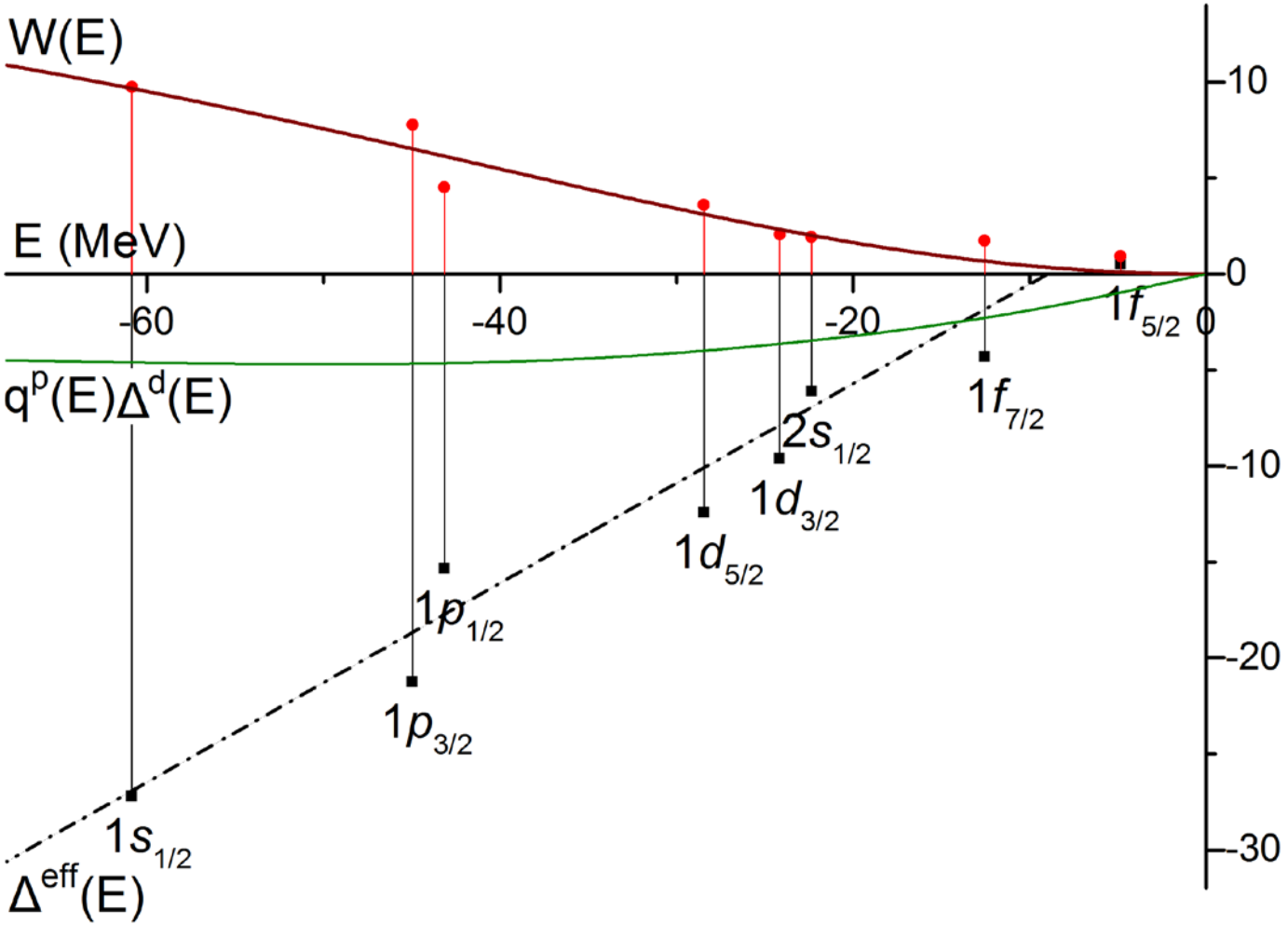

Fig. 11. The thick solid, thin solid, and dash-dotted curves represents calculated functions $W, q^p\Delta^d$ and Δ^{eff} respectively for the neutron subsystem of the ^{90}Zr parent nucleus.

IV. Conclusive remarks

- We proposed the new semimicroscopic model (PHDOM), that takes commonly into account the main relaxation modes of high-energy (p-h)-type nuclear excitations in medium-heavy mass “hard” spherical nuclei.
- The unique feature of the model is its ability to describe the energy-averaged double transition density (and, therefore, strength functions) at arbitrary (but high enough) excitation energies, and also direct-nucleon-decay properties of mentioned excitations.
- Considering the ^{208}Pb parent nucleus as an example, we apply a simple version of the model to the description of isoscalar monopole excitations, simplest photo-nuclear reactions, the spreading width of the IAR.
- In applying to the description of deep-hole states, we propose an unitary version of the single-quasiparticle dispersive optical model. This version is realized for the ^{90}Zr and ^{208}Pb parent nuclei.

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