# Non-linear effect of the fluctuation for the inhomogeneous chiral phase transition

PRD95, 074010 (2017)

Ryo Yoshiike (Kyoto University)

Collaborators : Tong-Gyu Lee, Toshitaka Tatsumi

ICNFP2017 8/18

# Index

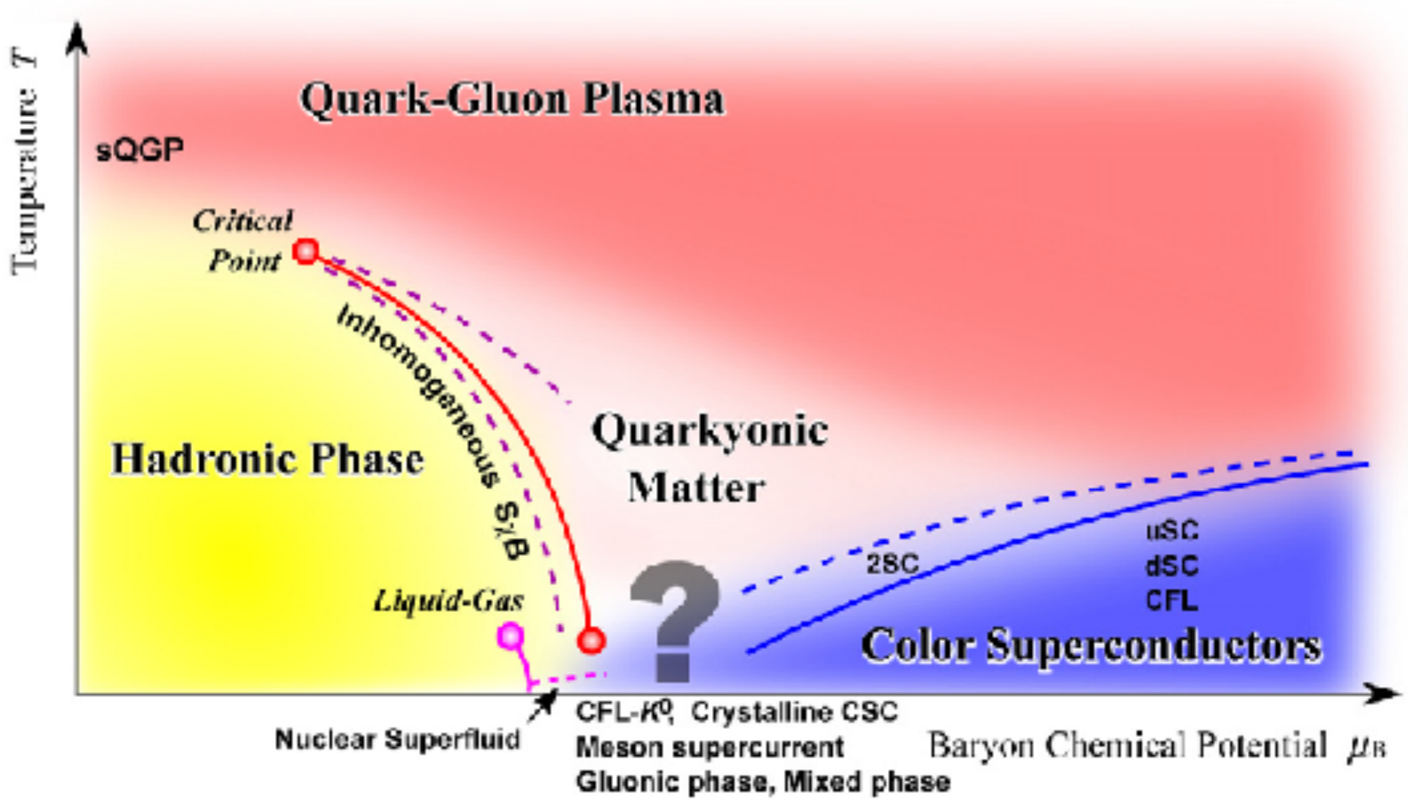
- Introduction
- Phase transition with the fluctuation
- Effect on the particle number
- Summary

# Index

- Introduction
- Phase transition with the fluctuation
- Effect on the particle number
- Summary

# QCD phase diagram

## Phase structure of quark matter




- Hadron phase
- Quark-gluon plasma
- Color superconductivity etc…
- Inhomogeneous chiral phase(iCP)

K. Fukushima, T. Hatsuda,
Rep. Prog. Phys. 74, 014001 (2011)

Chiral symmetry

※chiral limit

Restored phase

$\langle \bar{\psi}\psi \rangle = 0$

chiral condensate

SSB

Broken phase

$\langle \bar{\psi}\psi \rangle \neq 0$

spatially modulating

cf. FFLO superconductivity, CDW, SDW

# What is the inhomogeneous chiral phase?

「New phase where the quark condensates spatially modulate」

NJL model within the mean field approximation (2-flavor)　※chiral limit

$$\mathcal{L}_{\mathrm{MF}} = \bar{\psi}\left[i\partial\!\!\!/ + 2G\left(\langle\bar{\psi}\psi\rangle + i\gamma^5\tau^3\langle\bar{\psi}i\gamma^5\tau^3\psi\rangle\right)\right]\psi + G\left(\langle\bar{\psi}\psi\rangle^2 + \langle\bar{\psi}i\gamma^5\tau^3\psi\rangle^2\right)$$

Inhomogeneous chiral condensate

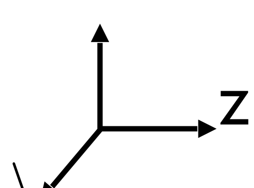
$$\Delta(\mathbf{r}) = \langle\bar{\psi}\psi\rangle + i\langle\bar{\psi}i\gamma_5\tau_3\psi\rangle$$

cf. Conventional “homogeneous” condensate : $\Delta(\mathbf{r}) = const.$

Dual chiral density wave (DCDW) type ・・・ $\Delta(\mathbf{r}) = -\frac{m}{2G}e^{iqz}$

Real kink crystal (RKC) type ・・・

$$\Delta(\mathbf{r}) = -\frac{\lambda}{2G}\left(\frac{2\sqrt{\nu}}{1+\sqrt{\nu}}\right)\mathrm{sn}\left(\frac{2\lambda z}{1+\sqrt{\nu}};\nu\right)$$



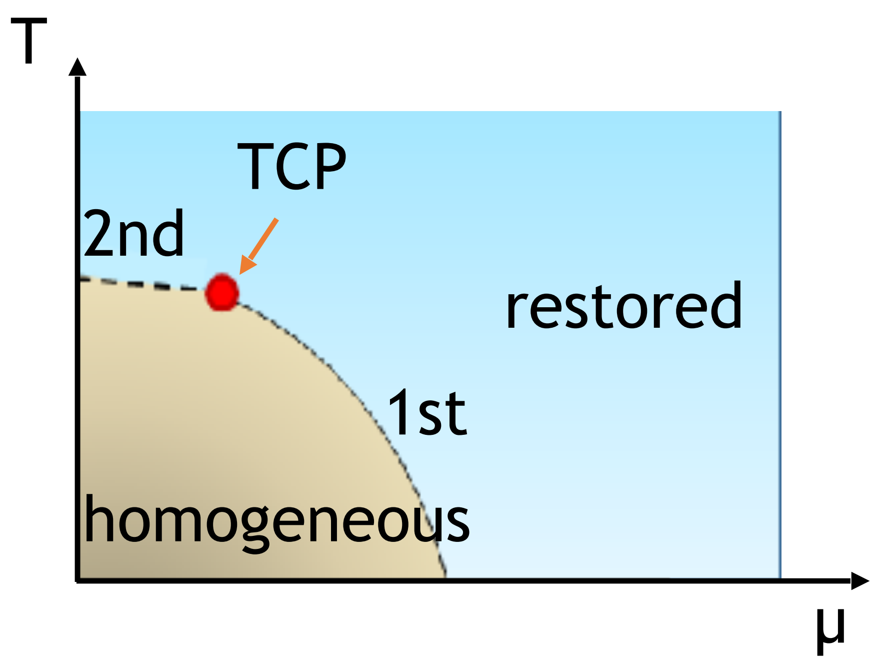

※ A self-consistent solution of the 1+1 dimensional NJL model within MFA

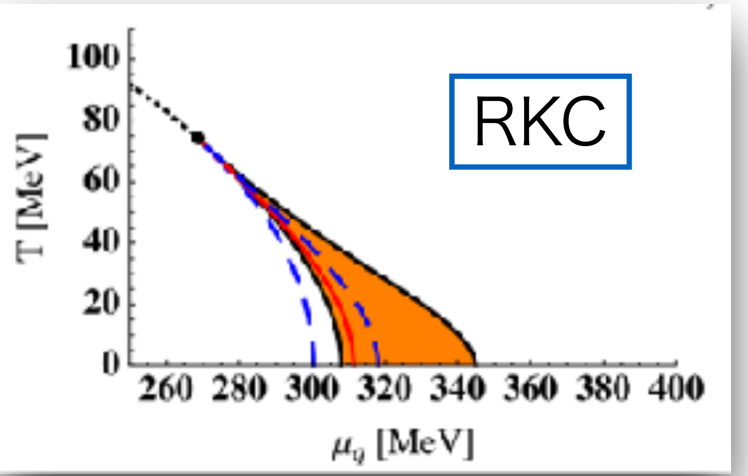
G. Basar, et al., PRD 79, 105012 (2009)

# Inhomogeneous chiral phase within MFA

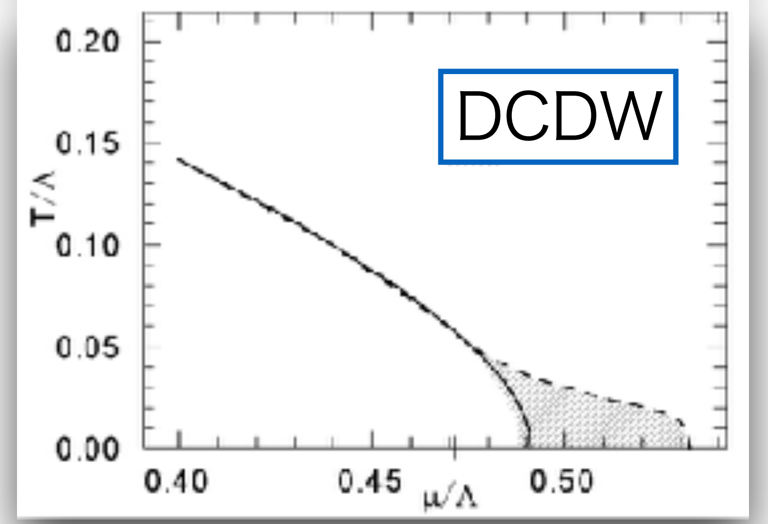
- Homogeneously broken phase
- Inhomogeneous chiral phase
- Restored phase

※chiral limit




D. Nickel,
PRD 80, 075025 (2009)

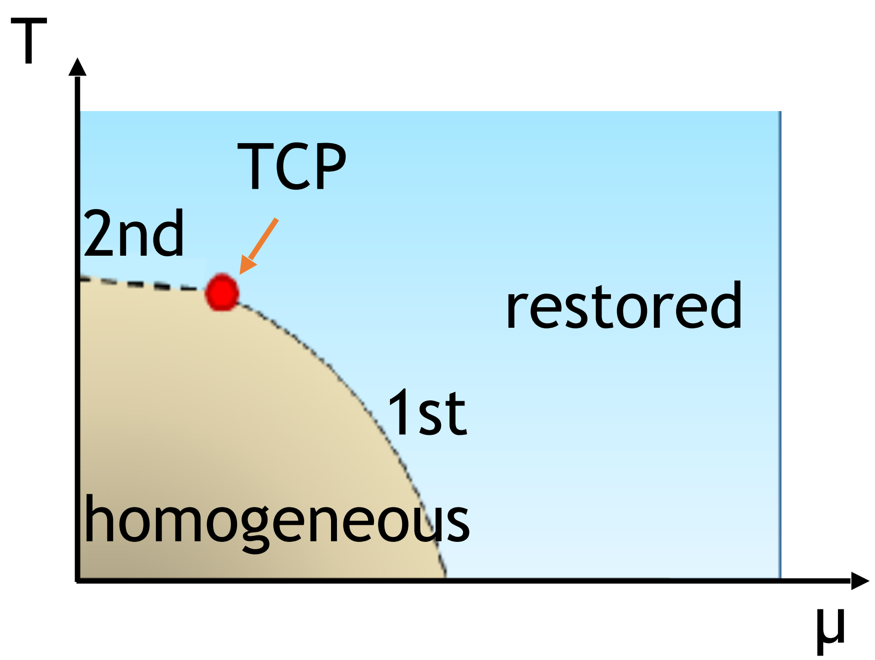


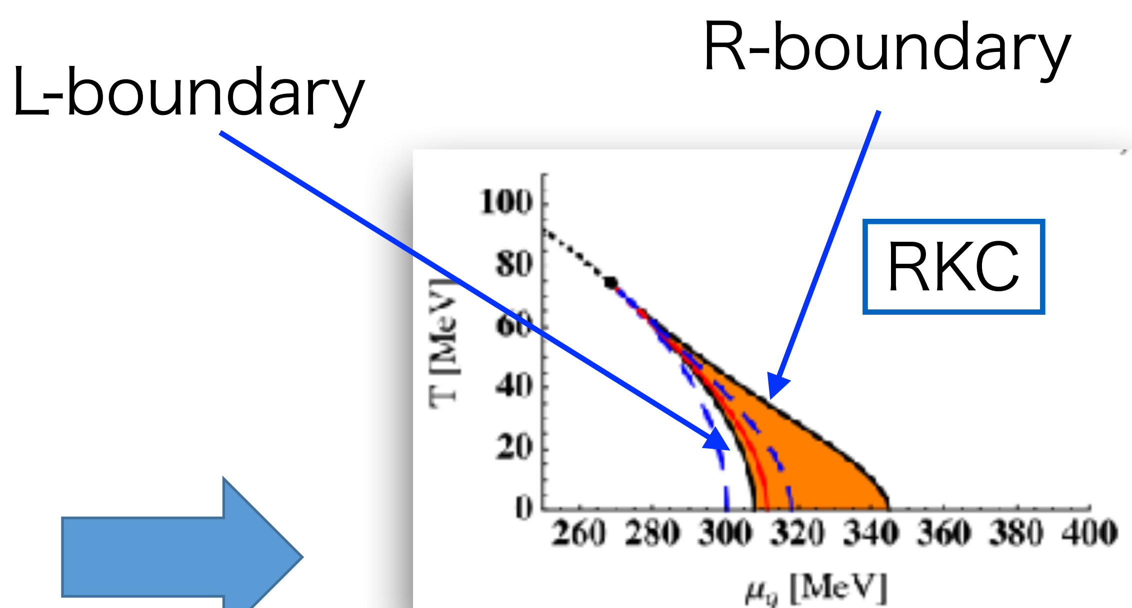

E. Nakano, T. Tatsumi,
PRD 71, 114006 (2005)

# Inhomogeneous chiral phase within MFA

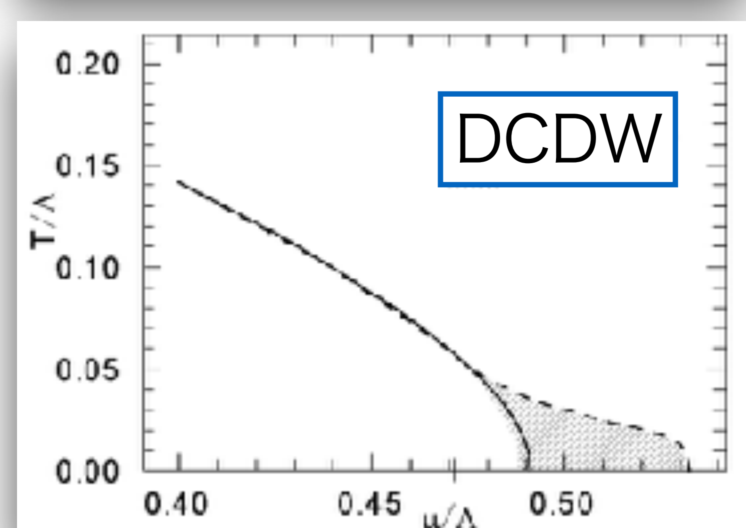
- Homogeneously broken phase
- Inhomogeneous chiral phase
- Restored phase

※chiral limit




D. Nickel,
PRD 80, 075025 (2009)

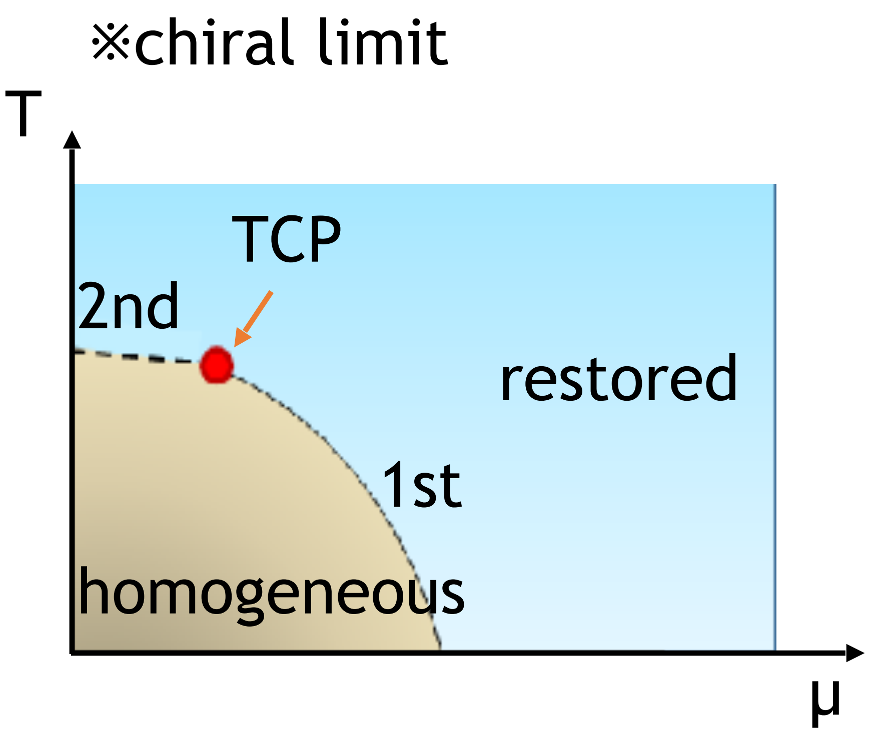


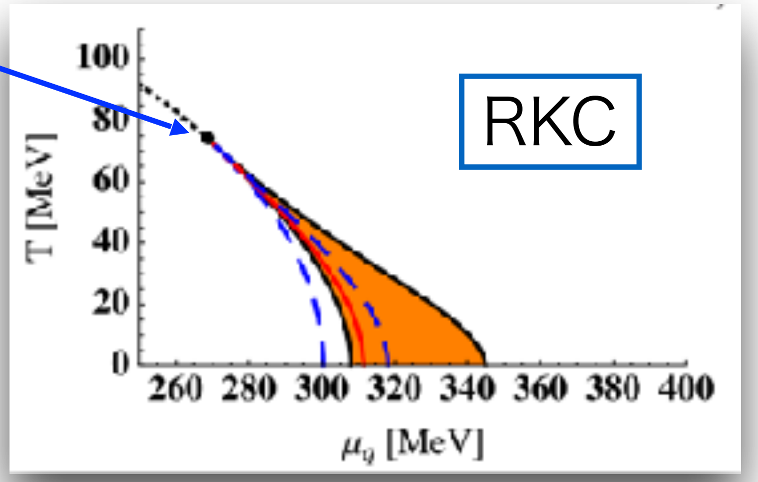

E. Nakano, T. Tatsumi,
PRD 71, 114006 (2005)

# Inhomogeneous chiral phase within MFA

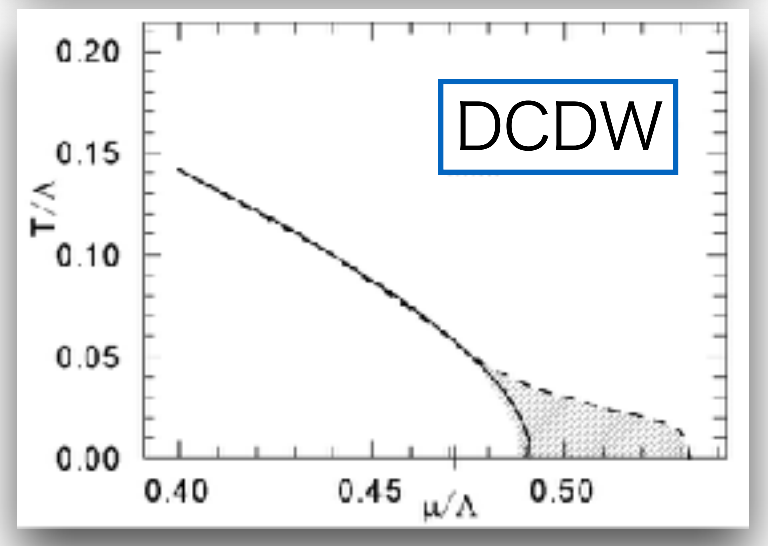
- Homogeneously broken phase
- Inhomogeneous chiral phase
- Restored phase

Lifshitz point・・・Three phases meet at this point.



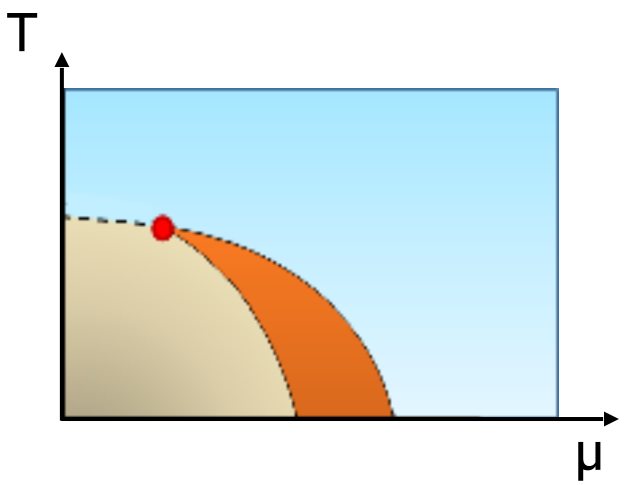

D. Nickel,
PRD 80, 075025 (2009)




E. Nakano, T. Tatsumi,
PRD 71, 114006 (2005)

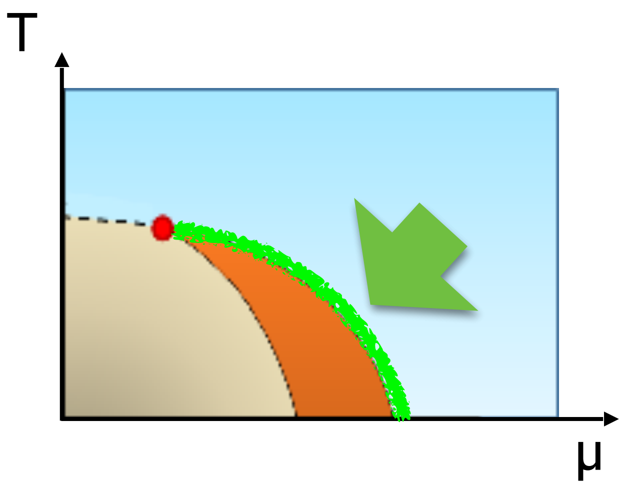
# Goal

We analyze the effect of the quantum and thermal fluctuation around R-boundary by approaching from the restored phase.



# Goal

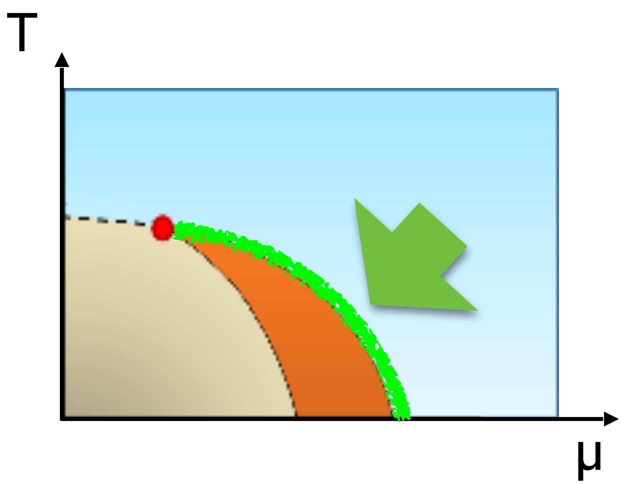
We analyze the effect of the quantum and thermal fluctuation around R-boundary by approaching from the restored phase.




R-boundary is the 2nd order phase transition(PT) line in the both case of DCDW and RKC within MFA.

# Goal

We analyze the effect of the quantum and thermal fluctuation around R-boundary by approaching from the restored phase.




R-boundary is the 2nd order phase transition(PT) line in the both case of DCDW and RKC within MFA.

・Constitute the low-energy effective Lagrangian

⬇

・Analyze the order of PT by considering the correction of the loop diagrams

cf. FFLO superconductivity [Y. Ohashi, J. Phys. Jpn. 71, 2625 (2002)]

# Index

- Introduction
- Phase transition with the fluctuation
- Effect on the particle number
- Summary

# Effective Lagrangian

Starting point is the NJL model.

$$Z \equiv \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left[\int d\tau \int d^3\mathbf{x} \left\{\bar{\psi} S_\beta^{-1} \psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma^5 \tau^a \psi)^2\right]\right\}\right]$$

$$= \int \mathcal{D}\sigma \mathcal{D}\pi^a \exp\left[-\int d\tau \int d^3\mathbf{x} \frac{1}{4G}\left(\sigma^2 + \pi_a^2\right) + \mathrm{TrLn}\left(S_\beta^{-1} - \sigma - i\gamma^5 \tau^a \pi^a\right)\right]$$

# Effective Lagrangian

Starting point is the NJL model.

$$Z \equiv \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left[\int d\tau \int d^3\mathbf{x} \left\{ \bar{\psi} S_\beta^{-1} \psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma^5 \tau^a \psi)^2\right]\right\}\right]$$

$$= \int \mathcal{D}\sigma \mathcal{D}\pi^a \exp\left[-\int d\tau \int d^3\mathbf{x} \frac{1}{4G}\left(\sigma^2 + \pi_a^2\right) + \mathrm{TrLn}\left(S_\beta^{-1} - \sigma - i\gamma^5\tau^a\pi^a\right)\right]$$

Expansion about the boson field

$$\mathcal{L}_{\mathrm{eff}}\left[\tilde{\phi}^\alpha\right] \equiv \mathcal{L}_0 + \frac{1}{2} G_{\mathrm{ps}}^{-1} \tilde{\phi}^\alpha \tilde{\phi}^\alpha + \frac{\lambda}{4!}(\tilde{\phi}^\alpha \tilde{\phi}^\alpha)^2 \qquad \phi^\alpha \equiv g_{\phi qq} \tilde{\phi}^\alpha$$

# Effective Lagrangian

**Starting point is the NJL model.**

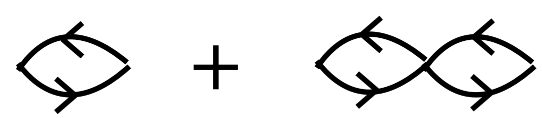
$$Z \equiv \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left[\int d\tau \int d^3\mathbf{x} \left\{ \bar{\psi} S_\beta^{-1} \psi + G\left[ (\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma^5 \tau^a \psi)^2 \right] \right\}\right]$$

$$= \int \mathcal{D}\sigma \mathcal{D}\pi^a \exp\left[ - \int d\tau \int d^3\mathbf{x} \frac{1}{4G}(\sigma^2 + \pi_a^2) + \mathrm{TrLn}\left( S_\beta^{-1} - \sigma - i\gamma^5\tau^a\pi^a \right) \right]$$

Expansion about the boson field

$$\mathcal{L}_{\mathrm{eff}}\left[\tilde{\phi}^\alpha\right] \equiv \mathcal{L}_0 + \frac{1}{2} G_{\mathrm{ps}}^{-1} \tilde{\phi}^\alpha \tilde{\phi}^\alpha + \frac{\lambda}{4!} (\tilde{\phi}^\alpha \tilde{\phi}^\alpha)^2 \qquad \phi^\alpha \equiv g_{\phi qq} \tilde{\phi}^\alpha$$

$G_{\mathrm{ps}}(\nu_n, \mathbf{q}) =$ 

$=$  $+ \cdots$

Chiral pair fluctuation

# Effective Lagrangian

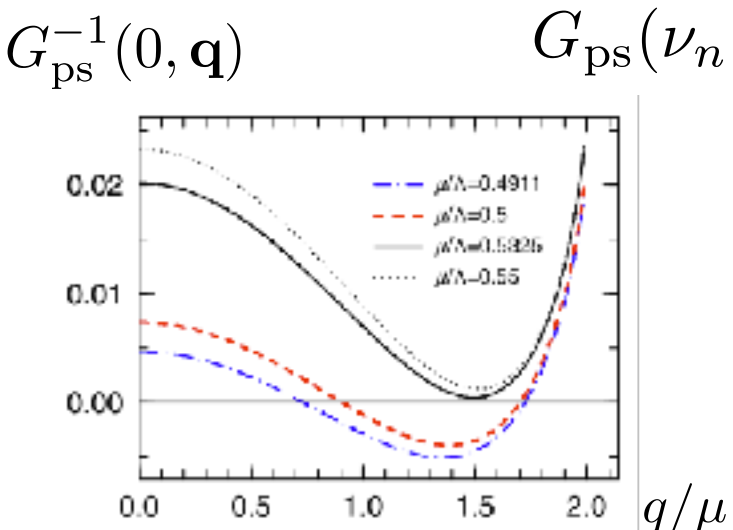
Starting point is the NJL model.

$$Z \equiv \int \mathcal{D}\psi\mathcal{D}\bar{\psi} \exp\left[\int d\tau \int d^3\mathbf{x} \left\{\bar{\psi}S_\beta^{-1}\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\tau^a\psi)^2\right]\right\}\right]$$

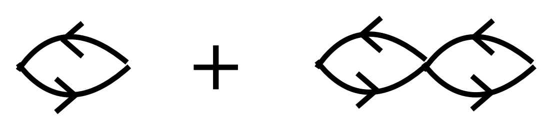
$$= \int \mathcal{D}\sigma\mathcal{D}\pi^a \exp\left[-\int d\tau \int d^3\mathbf{x} \frac{1}{4G}\left(\sigma^2 + \pi_a^2\right) + \mathrm{TrLn}\left(S_\beta^{-1} - \sigma - i\gamma^5\tau^a\pi^a\right)\right]$$

Expansion about the boson field

$$\mathcal{L}_{\mathrm{eff}}\left[\tilde{\phi}^\alpha\right] \equiv \mathcal{L}_0 + \frac{1}{2}G_{\mathrm{ps}}^{-1}\tilde{\phi}^\alpha\tilde{\phi}^\alpha + \frac{\lambda}{4!}(\tilde{\phi}^\alpha\tilde{\phi}^\alpha)^2 \qquad \phi^\alpha \equiv g_{\phi qq}\tilde{\phi}^\alpha$$




$G_{\mathrm{ps}}(\nu_n, \mathbf{q}) =$ 

$=$  $+ \cdots$

Chiral pair fluctuation

$$\simeq \left[\tau + \gamma(\mathbf{q}^2 - \mathbf{q}_c^2)^2 + \alpha|\nu_n|\right]^{-1}$$

Extremum at the finite wave number

E. Nakano, T. Tatsumi, PRD 71, 114006 (2005)

# Thermodynamic potential

$$\Omega = -\frac{T}{V}\ln\int \mathcal{D}\delta\phi^{\alpha} e^{-\int d\tau \int d^3\mathbf{x}\mathcal{L}_{\mathrm{eff}}[\phi_0+\delta\phi]}$$

$\tilde{\phi} = \phi_0 + \delta\phi$

Fluctuation

Order parameter

$$= \Omega_0 + \frac{1}{2}\int \frac{d^3\mathbf{p}_1 d^3\mathbf{p}_2}{(2\pi)^6}\Gamma^{(2)}(\mathbf{p}_1,\mathbf{p}_2)\phi_0(\mathbf{p}_1)\phi_0(\mathbf{p}_2)$$

$$+ \frac{1}{4!}\int \frac{d^3\mathbf{p}_1 d^3\mathbf{p}_2 d^3\mathbf{p}_3 d^3\mathbf{p}_4}{(2\pi)^{12}}\Gamma^{(4)}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4)\phi_0(\mathbf{p}_1)\phi_0(\mathbf{p}_2)\phi_0(\mathbf{p}_3)\phi_0(\mathbf{p}_4)$$

# Thermodynamic potential

$$\Omega = -\frac{T}{V}\ln\int \mathcal{D}\delta\phi^{\alpha} e^{-\int d\tau \int d^3\mathbf{x}\mathcal{L}_{\rm eff}[\phi_0+\delta\phi]}$$

$\tilde{\phi} = \phi_0 + \delta\phi$

Order parameter ($\phi_0$), Fluctuation ($\delta\phi$)

$$= \Omega_0 + \frac{1}{2}\int \frac{d^3\mathbf{p}_1 d^3\mathbf{p}_2}{(2\pi)^6}\Gamma^{(2)}(\mathbf{p}_1,\mathbf{p}_2)\phi_0(\mathbf{p}_1)\phi_0(\mathbf{p}_2)$$

$$+ \frac{1}{4!}\int \frac{d^3\mathbf{p}_1 d^3\mathbf{p}_2 d^3\mathbf{p}_3 d^3\mathbf{p}_4}{(2\pi)^{12}}\Gamma^{(4)}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4)\phi_0(\mathbf{p}_1)\phi_0(\mathbf{p}_2)\phi_0(\mathbf{p}_3)\phi_0(\mathbf{p}_4)$$

Analyze PT by calculating the coefficients including the fluctuations

# Thermodynamic potential

$$\Omega = -\frac{T}{V}\ln\int \mathcal{D}\delta\phi^{\alpha} e^{-\int d\tau \int d^3\mathbf{x}\mathcal{L}_{\rm eff}[\phi_0+\delta\phi]}$$

$\tilde{\phi} = \phi_0 + \delta\phi$

Order parameter ($\phi_0$) — Fluctuation ($\delta\phi$)

$$= \Omega_0 + \frac{1}{2}\int \frac{d^3\mathbf{p}_1 d^3\mathbf{p}_2}{(2\pi)^6}\Gamma^{(2)}(\mathbf{p}_1,\mathbf{p}_2)\phi_0(\mathbf{p}_1)\phi_0(\mathbf{p}_2)$$

$$+\frac{1}{4!}\int \frac{d^3\mathbf{p}_1 d^3\mathbf{p}_2 d^3\mathbf{p}_3 d^3\mathbf{p}_4}{(2\pi)^{12}}\Gamma^{(4)}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4)\phi_0(\mathbf{p}_1)\phi_0(\mathbf{p}_2)\phi_0(\mathbf{p}_3)\phi_0(\mathbf{p}_4)$$

Analyze PT by calculating the coefficients including the fluctuations

## No fluctuation case (MFA)

$$\Gamma^{(2)}(\mathbf{p}_1,\mathbf{p}_2) = G_{\rm ps}^{-1}(i\nu_n = 0,\mathbf{p}_1)\delta^3(\mathbf{p}_1+\mathbf{p}_2)$$

Static propagator

$$G_{\rm ps}^{-1}(0,\mathbf{p}_1)|_{\mathbf{p}_1=\mathbf{q}_c} = 0$$

・・・The 2nd order PT to the iCP occurs with the wave vector q_c.

Phase transition at $\tau = 0$

※No assumption about the inhomogeneous condensate

# 2nd order vertex function

$$\Gamma^{(2)}(\mathbf{q}_1,\mathbf{q}_2) \equiv (2\pi)^6 \frac{\delta^2\Omega_{\mathrm{eff}}}{\delta\phi_0(-\mathbf{q}_1)\delta\phi_0(-\mathbf{q}_2)}\bigg|_{\phi_0=0} = (2\pi)^3\delta(\mathbf{q}_1+\mathbf{q}_2)\underline{[\tau_R+\gamma(\mathbf{q}_1^2-\mathbf{q}_c^2)^2]}$$

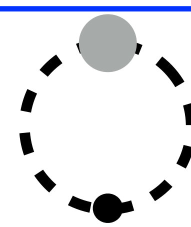
$\tau$ in MFA changes to $\tau$_R.

# 2nd order vertex function

$$\Gamma^{(2)}(\mathbf{q}_1, \mathbf{q}_2) \equiv (2\pi)^6 \frac{\delta^2 \Omega_{\text{eff}}}{\delta\phi_0(-\mathbf{q}_1)\delta\phi_0(-\mathbf{q}_2)}\bigg|_{\phi_0=0} = (2\pi)^3 \delta(\mathbf{q}_1 + \mathbf{q}_2)[\tau_R + \gamma(\mathbf{q}_1^2 - \mathbf{q}_c^2)^2]$$

$\tau$ in MFA changes to $\tau$_R.

$$\tau_R = \tau + \lambda T \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\tau_R + \gamma(\mathbf{p}^2 - \mathbf{q}_c^2)^2 + \alpha|\nu_n|}$$

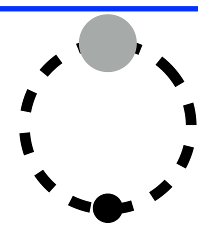
Self-energy $\Pi =$ 

# 2nd order vertex function

$$\Gamma^{(2)}(\mathbf{q}_1, \mathbf{q}_2) \equiv (2\pi)^6 \frac{\delta^2 \Omega_{\mathrm{eff}}}{\delta\phi_0(-\mathbf{q}_1)\delta\phi_0(-\mathbf{q}_2)}\bigg|_{\phi_0=0} = (2\pi)^3 \delta(\mathbf{q}_1 + \mathbf{q}_2)[\tau_R + \gamma(\mathbf{q}_1^2 - \mathbf{q}_c^2)^2]$$

$\tau$ in MFA changes to $\tau$ _R.

$$\tau_R = \tau + \lambda T \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\tau_R + \gamma(\mathbf{p}^2 - \mathbf{q}_c^2)^2 + \alpha|\nu_n|}$$

Self-energy $\Pi =$  $\sim \dfrac{\lambda T q_c}{4\pi\gamma^{1/2}} \dfrac{1}{\tau_R^{1/2}}$

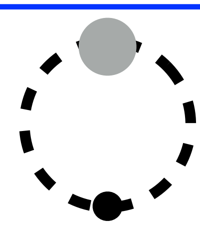
divergence at $\tau_R \to 0$

# 2nd order vertex function

$$\Gamma^{(2)}(\mathbf{q}_1,\mathbf{q}_2) \equiv (2\pi)^6 \left.\frac{\delta^2\Omega_{\rm eff}}{\delta\phi_0(-\mathbf{q}_1)\delta\phi_0(-\mathbf{q}_2)}\right|_{\phi_0=0} = (2\pi)^3\delta(\mathbf{q}_1+\mathbf{q}_2)[\tau_R+\gamma(\mathbf{q}_1^2-\mathbf{q}_c^2)^2]$$

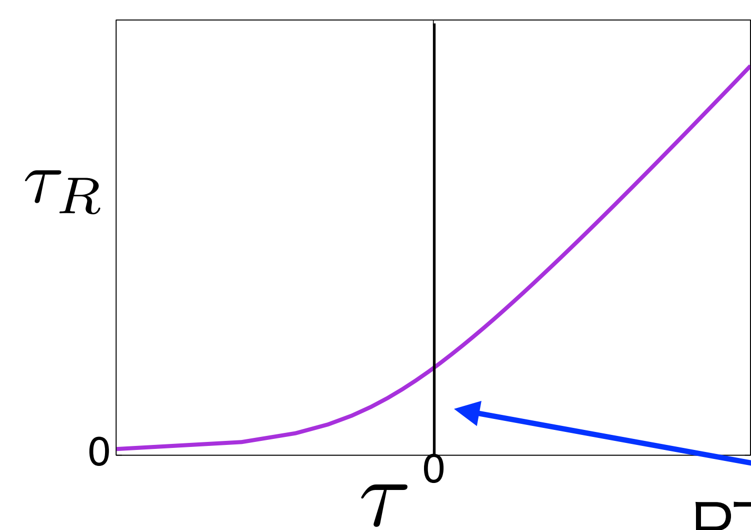
$\tau$ in MFA changes to $\tau$_R.

$$\tau_R = \tau + \lambda T \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\tau_R+\gamma(\mathbf{p}^2-\mathbf{q}_c^2)^2+\alpha|\nu_n|}$$

Self-energy $\Pi =$  $\sim \frac{\lambda T q_c}{4\pi\gamma^{1/2}}\frac{1}{\tau_R^{1/2}}$

divergence at $\tau_R \to 0$

$$\tau_R = \tau + \frac{\lambda T q_c}{4\pi\gamma^{1/2}}\frac{1}{\tau_R^{1/2}}$$




PT point within MFA

$\tau_R$ never vanishes. No 2nd order PT

# Dimensional reduction due to the temperature

The divergence vanishes at T=0

$$\tau_R = \tau + \Pi$$

Finite value even at $\tau_R = 0$

# Dimensional reduction due to the temperature

The divergence vanishes at T=0

$$\tau_R = \tau + \Pi$$

Finite value even at $\tau_R = 0$

2nd order PT at $\tau_R = 0$

# Dimensional reduction due to the temperature

The divergence vanishes at T=0

$$\tau_R = \tau + \Pi$$

Finite value even at $\tau_R = 0$

⇒ 2nd order PT at $\tau_R = 0$

Dimensional reduction

$$\Pi = \lambda T \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\tau_R + \gamma(\mathbf{p}^2 - \mathbf{q}_c^2)^2 + \alpha|\nu_n|}$$

The leading contribution to the divergence comes from n=0.

# Dimensional reduction due to the temperature

The divergence vanishes at T=0

$$\tau_R = \tau + \Pi$$

Finite value even at $\tau_R = 0$

→ 2nd order PT at $\tau_R = 0$

Dimensional reduction

$$\Pi = \lambda T \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\tau_R + \gamma(\mathbf{p}^2 - \mathbf{q}_c^2)^2 + \alpha|\nu_n|}$$

The leading contribution to the divergence comes from n=0.

n=0

→ $$\Pi \to \lambda T \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\tau_R + \gamma(\mathbf{p}^2 - \mathbf{q}_c^2)^2}$$

The number of the dimension reduce compared to T=0

→ IR divergence becomes stronger.

# Dimensional reduction due to the temperature

The divergence vanishes at T=0

$$\tau_R = \tau + \Pi$$

Finite value even at $\tau_R = 0$

➡ 2nd order PT at $\tau_R = 0$

Dimensional reduction

$$\Pi = \lambda T \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\tau_R + \gamma(\mathbf{p}^2 - \mathbf{q}_c^2)^2 + \alpha|\nu_n|}$$

The leading contribution to the divergence comes from n=0.

n=0

➡ $$\Pi \to \lambda T \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\tau_R + \gamma(\mathbf{p}^2 - \mathbf{q}_c^2)^2}$$

The number of the dimension reduce compared to T=0

➡ IR divergence becomes stronger.

cf. IR divergence of the NG mode destroys the condensate in iCP.

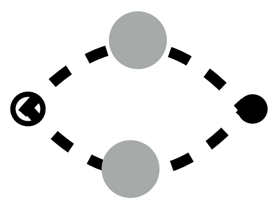
Quasi-long range order at $T \neq 0$ ➡ Stable long range order at $T = 0$

T.-G. Lee, et al., PRD92, 034024 (2015)

H. Hidaka, et al., PRD92, 034003 (2015)

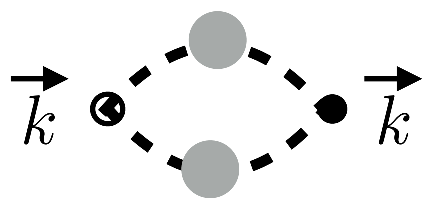
# 4th order vertex function

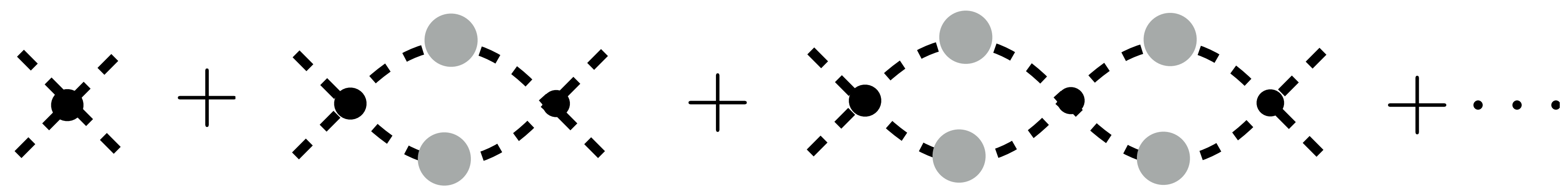
$$\Gamma^{(4)} \equiv (2\pi)^{12} \frac{\delta^4 \Omega}{\delta\phi_0(\mathbf{q}_1)\delta\phi_0(\mathbf{q}_2)\delta\phi_0(\mathbf{q}_3)\delta\phi_0(\mathbf{q}_4)}\bigg|_{\phi_0=0} = (2\pi)^3 \lambda \frac{1+\frac{2}{3}L(0)}{1-2L(0)} \delta^3(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3+\mathbf{q}_4)$$

$L(k) =$ $\vec{k}$  $\vec{k}$

# 4th order vertex function

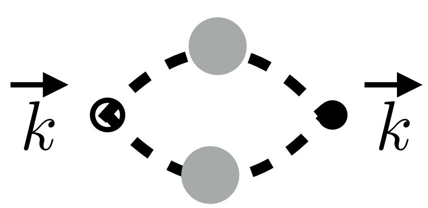
$$\Gamma^{(4)} \equiv (2\pi)^{12} \frac{\delta^4 \Omega}{\delta\phi_0(\mathbf{q}_1)\delta\phi_0(\mathbf{q}_2)\delta\phi_0(\mathbf{q}_3)\delta\phi_0(\mathbf{q}_4)}\bigg|_{\phi_0=0} = (2\pi)^3 \lambda \frac{1+\frac{2}{3}L(0)}{1-2L(0)} \delta^3(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3+\mathbf{q}_4)$$

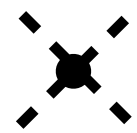
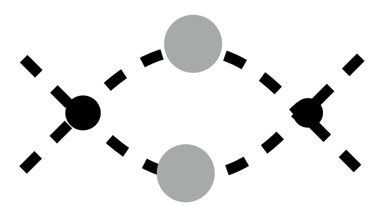
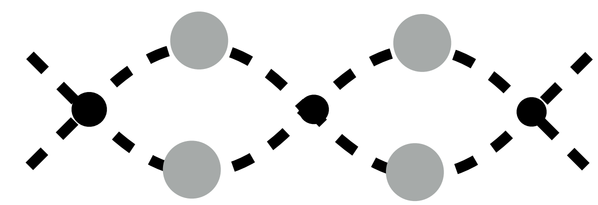
$L(k) =$ 



# 4th order vertex function

$$\Gamma^{(4)} \equiv (2\pi)^{12} \frac{\delta^4 \Omega}{\delta\phi_0(\mathbf{q}_1)\delta\phi_0(\mathbf{q}_2)\delta\phi_0(\mathbf{q}_3)\delta\phi_0(\mathbf{q}_4)}\bigg|_{\phi_0=0} = (2\pi)^3 \boxed{\lambda \frac{1+\frac{2}{3}L(0)}{1-2L(0)}} \delta^3(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3+\mathbf{q}_4)$$

$L(k) =$ 

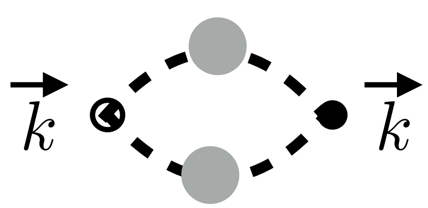
 $+$  $+$  $+ \cdots$

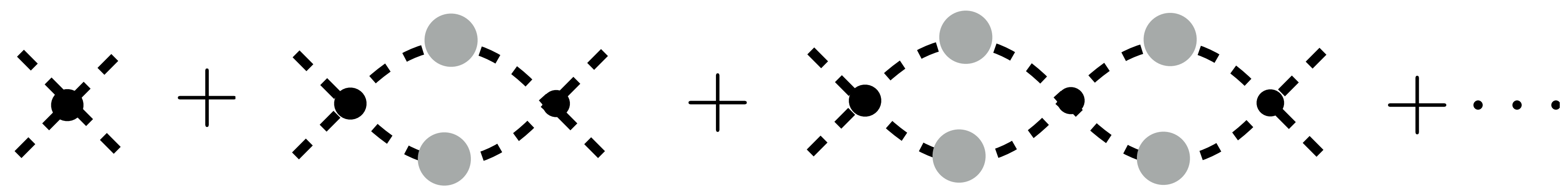
$$L(0) = -\frac{\lambda}{2} T \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{[\tau_R + \gamma(\mathbf{p}^2 - \mathbf{q}_c^2)^2 + a|\nu_n|]^2} \sim -\frac{\lambda T q_c}{16\pi\gamma^{1/2}} \frac{1}{\tau_R^{3/2}}$$

$$\boxed{L(0) \to -\infty \ (\tau_R \to 0)}$$

# 4th order vertex function

$$\Gamma^{(4)} \equiv (2\pi)^{12} \frac{\delta^4 \Omega}{\delta\phi_0(\mathbf{q}_1)\delta\phi_0(\mathbf{q}_2)\delta\phi_0(\mathbf{q}_3)\delta\phi_0(\mathbf{q}_4)}\bigg|_{\phi_0=0} = (2\pi)^3 \lambda \frac{1+\frac{2}{3}L(0)}{1-2L(0)} \delta^3(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3+\mathbf{q}_4)$$

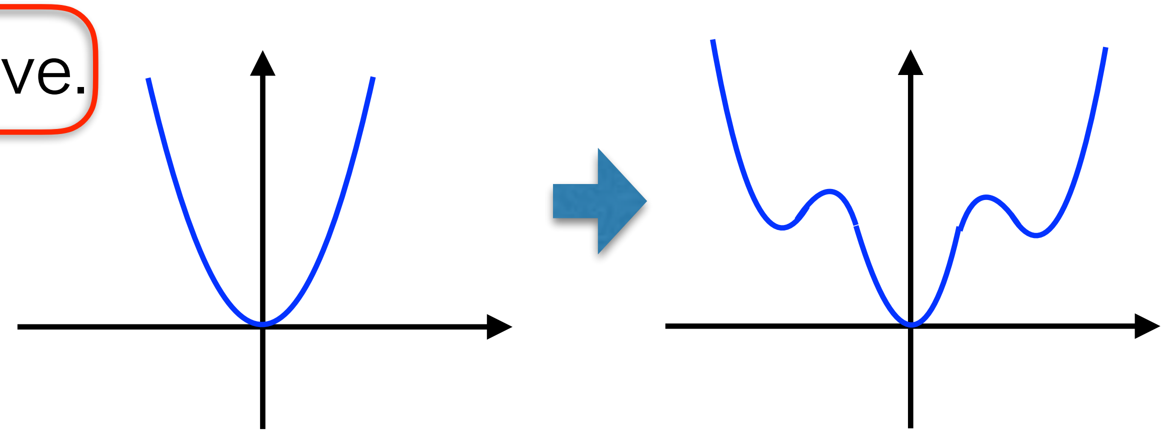
$L(k) =$ 




$$L(0) = -\frac{\lambda}{2} T \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{[\tau_R + \gamma(\mathbf{p}^2 - \mathbf{q}_c^2)^2 + a|\nu_n|]^2} \sim -\frac{\lambda T q_c}{16\pi\gamma^{1/2}} \frac{1}{\tau_R^{3/2}}$$

$$L(0) \to -\infty \quad (\tau_R \to 0)$$

$\Gamma^{(4)}$ becomes negative.



1st order PT

S. A. Brazovskii,
Zh. Eksp. Teor. Fiz. 68 (1975)

※The divergence remains even at T=0 $L(0) \sim -\tau_R^{-1/2}$

# Index

- Introduction
- Phase transition with the fluctuation
- Effect on the particle number
- Summary

# Effect on the particle number

Thermodynamic potential (Order parameter vanishes at the restored phase.)

$$\Omega = -\frac{T}{V}\ln\int \mathcal{D}\delta\phi^{\alpha}\exp\left[\mathrm{TrLn}\left(S_{\beta}^{-1}\right) - \frac{1}{2}\sum_{p} G_{\mathrm{ps}}^{-1}(p)\delta\phi^{\alpha}(p)\delta\phi^{\alpha}(-p) + \mathcal{O}(\delta\phi^{4})\right]$$

$$\sim -2N_f N_c T\sum_{m}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\ln\left[(\omega_m + i\mu)^2 + \mathbf{p}^2\right] + 2T\sum_{n}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\ln G_{\mathrm{ps}}^{-1}(i\nu_n,\mathbf{p}) + \mathrm{const.}$$

# Effect on the particle number

Thermodynamic potential (Order parameter vanishes at the restored phase.)

$$\Omega = -\frac{T}{V}\ln\int \mathcal{D}\delta\phi^\alpha \exp\left[\mathrm{TrLn}\left(S_\beta^{-1}\right) - \frac{1}{2}\sum_p G_{\mathrm{ps}}^{-1}(p)\delta\phi^\alpha(p)\delta\phi^\alpha(-p) + \mathcal{O}(\delta\phi^4)\right]$$

$$\sim -2N_f N_c T\sum_m \int\frac{d^3\mathbf{p}}{(2\pi)^3}\ln\left[(\omega_m + i\mu)^2 + \mathbf{p}^2\right] + 2T\sum_n\int\frac{d^3\mathbf{p}}{(2\pi)^3}\ln G_{\mathrm{ps}}^{-1}(i\nu_n, \mathbf{p}) + \mathrm{const.}$$

Contribution of the free quarks (MFA)

Contribution of the fluctuation

# Effect on the particle number

## Thermodynamic potential (Order parameter vanishes at the restored phase.)

$$\Omega = -\frac{T}{V}\ln\int\mathcal{D}\delta\phi^\alpha \exp\left[\mathrm{TrLn}\left(S_\beta^{-1}\right) - \frac{1}{2}\sum_p G_{\mathrm{ps}}^{-1}(p)\delta\phi^\alpha(p)\delta\phi^\alpha(-p) + \mathcal{O}(\delta\phi^4)\right]$$

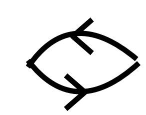
$$\sim -2N_f N_c T\sum_m\int\frac{d^3\mathbf{p}}{(2\pi)^3}\ln\left[(\omega_m + i\mu)^2 + \mathbf{p}^2\right] + 2T\sum_n\int\frac{d^3\mathbf{p}}{(2\pi)^3}\ln G_{\mathrm{ps}}^{-1}(i\nu_n,\mathbf{p}) + \mathrm{const.}$$

Contribution of the free quarks (MFA)

Contribution of the fluctuation

## Quark number density

$$G_{\mathrm{ps}}^{-1} = \frac{1}{2G}(1 - 2G\Pi_{\mathrm{ps}}^0)$$



$$n \equiv -\frac{\partial\Omega}{\partial\mu}$$

$$= N_f N_c\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left[f(|\mathbf{p}| - \mu) - f(|\mathbf{p}| + \mu)\right] + 2T\sum_n\int\frac{d^3\mathbf{p}}{(2\pi)^3}G_{\mathrm{ps}}(i\nu_n,\mathbf{p})\frac{\partial}{\partial\mu}\Pi_{\mathrm{ps}}^0(i\nu_n,\mathbf{p})$$

# Effect on the particle number

Thermodynamic potential (Order parameter vanishes at the restored phase.)

$$\Omega = -\frac{T}{V}\ln\int \mathcal{D}\delta\phi^{\alpha}\exp\left[\mathrm{TrLn}\left(S_{\beta}^{-1}\right) - \frac{1}{2}\sum_{p} G_{\mathrm{ps}}^{-1}(p)\delta\phi^{\alpha}(p)\delta\phi^{\alpha}(-p) + \mathcal{O}(\delta\phi^{4})\right]$$

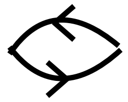
$$\sim -2N_f N_c T\sum_{m}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\ln\left[(\omega_m + i\mu)^2 + \mathbf{p}^2\right] + 2T\sum_{n}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\ln G_{\mathrm{ps}}^{-1}(i\nu_n, \mathbf{p}) + \mathrm{const.}$$

Contribution of the free quarks (MFA)

Contribution of the fluctuation

Quark number density

$$G_{\mathrm{ps}}^{-1} = \frac{1}{2G}(1 - 2G\Pi_{\mathrm{ps}}^{0})$$



$$n \equiv -\frac{\partial\Omega}{\partial\mu}$$

$$= N_f N_c\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left[f(|\mathbf{p}| - \mu) - f(|\mathbf{p}| + \mu)\right] + 2T\sum_{n}\int\frac{d^3\mathbf{p}}{(2\pi)^3} G_{\mathrm{ps}}(i\nu_n, \mathbf{p})\frac{\partial}{\partial\mu}\Pi_{\mathrm{ps}}^{0}(i\nu_n, \mathbf{p})$$

$$\equiv n^{\mathrm{flu}}$$

# Effect on the particle number

$$G^{\rm ps}(i\nu_n, \mathbf{p}) \simeq \frac{1}{\tau + \gamma\left(|\mathbf{p}|^2 - q_c^2\right)^2 + a|\nu_n|}$$

The contribution at $|\mathbf{p}| \sim q_c,\ n \sim 0$ is dominant.

$$\Rightarrow \quad \frac{\partial}{\partial\mu}\Pi^0_{\rm ps}(i\nu_n, \mathbf{p}) \sim \frac{\partial}{\partial\mu}\Pi^0_{\rm ps}(0, \mathbf{q}_c)$$

# Effect on the particle number

$$G^{\rm ps}(i\nu_n,\mathbf{p}) \simeq \frac{1}{\tau+\gamma\left(|\mathbf{p}|^2-q_c^2\right)^2+a|\nu_n|}$$

The contribution at $|\mathbf{p}|\sim q_c,\ n\sim 0$ is dominant.

$$\frac{\partial}{\partial\mu}\Pi^0_{\rm ps}(i\nu_n,\mathbf{p}) \sim \frac{\partial}{\partial\mu}\Pi^0_{\rm ps}(0,\mathbf{q}_c)$$

$$n^{\rm flu} \simeq 2\frac{\partial}{\partial\mu}\Pi^0_{\rm ps}(0,\mathbf{q}_c)T\sum_n\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\tau+\gamma\left(|\mathbf{p}|^2-q_c^2\right)^2+a|\nu_n|}$$

$\sim \tau^{-1/2}$ (divergence at $\tau\to 0$)

# Effect on the particle number

$$G^{\rm ps}(i\nu_n,\mathbf{p}) \simeq \frac{1}{\tau+\gamma\left(|\mathbf{p}|^2-q_c^2\right)^2+a|\nu_n|}$$

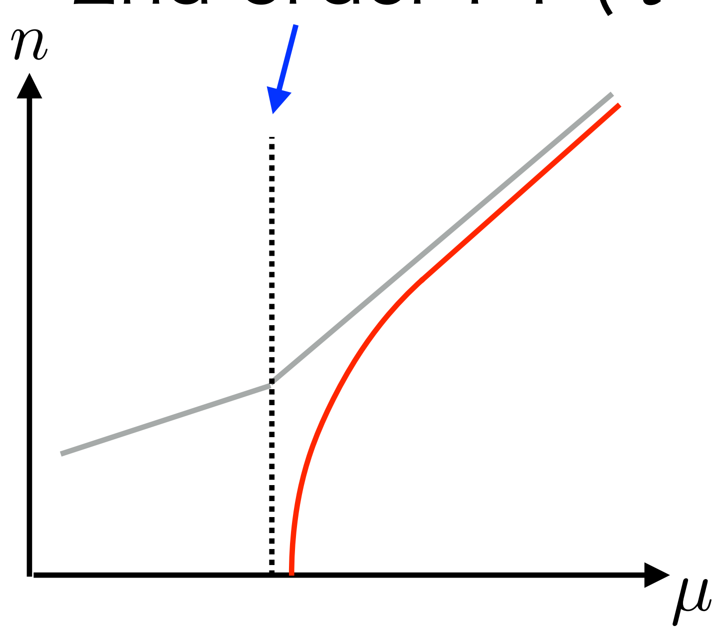
The contribution at $|\mathbf{p}|\sim q_c,\ n\sim 0$ is dominant.

$$\frac{\partial}{\partial\mu}\Pi^0_{\rm ps}(i\nu_n,\mathbf{p}) \sim \frac{\partial}{\partial\mu}\Pi^0_{\rm ps}(0,\mathbf{q}_c)$$

$$n^{\rm flu} \simeq 2\frac{\partial}{\partial\mu}\Pi^0_{\rm ps}(0,\mathbf{q}_c)\underline{T\sum_n\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{\tau+\gamma\left(|\mathbf{p}|^2-q_c^2\right)^2+a|\nu_n|}}$$

$\sim \tau^{-1/2}$ (divergence at $\tau\to 0$)

2nd order PT ($\tau$=0)



# Effect on the particle number

$$G^{\rm ps}(i\nu_n, \mathbf{p}) \simeq \frac{1}{\tau + \gamma\left(|\mathbf{p}|^2 - q_c^2\right)^2 + a|\nu_n|}$$

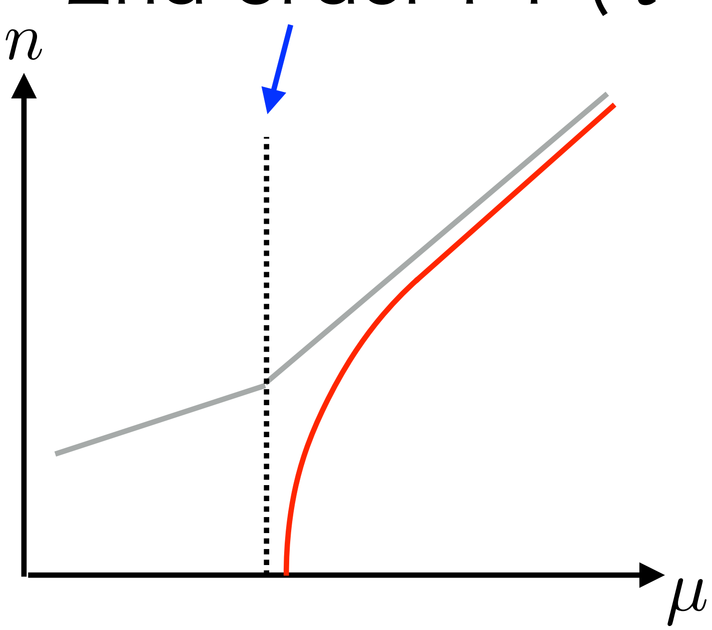
The contribution at $|\mathbf{p}| \sim q_c,\ n \sim 0$ is dominant.

$$\Rightarrow \quad \frac{\partial}{\partial \mu}\Pi^0_{\rm ps}(i\nu_n, \mathbf{p}) \sim \frac{\partial}{\partial \mu}\Pi^0_{\rm ps}(0, \mathbf{q}_c)$$

$$\Rightarrow \quad n^{\rm flu} \simeq 2\frac{\partial}{\partial \mu}\Pi^0_{\rm ps}(0, \mathbf{q}_c) \underbrace{T\sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\tau + \gamma\left(|\mathbf{p}|^2 - q_c^2\right)^2 + a|\nu_n|}}$$

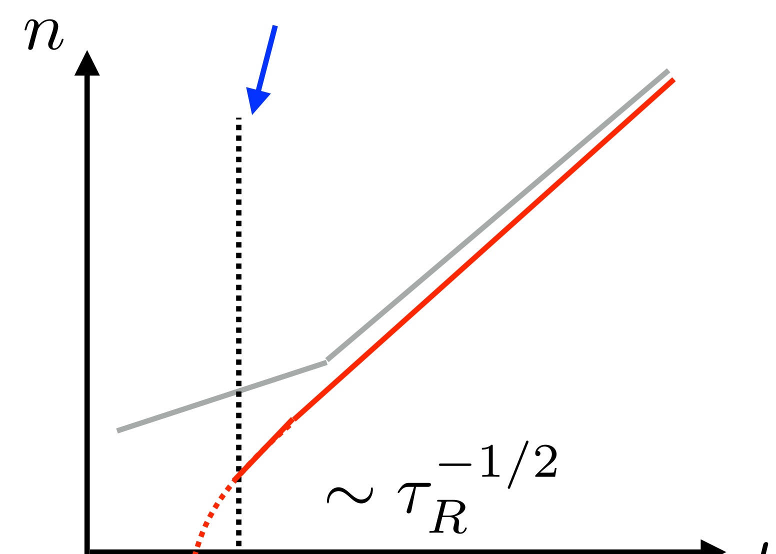
$\sim \tau^{-1/2}$ (divergence at $\tau \to 0$)

2nd order PT ($\tau$ =0)




Considering the 4th order
$\tau \to \tau_R$

1st order PT ($\tau_R \neq 0$)



# Index

- Introduction
- Phase transition with the fluctuation
- Effect on the particle number
- Summary

# Summary

・We constitute the effective theory described by chiral pair fluctuation.

・We expand the thermodynamic potential about the order parameter and analyze the change of the coefficients.

# Summary

・We constitute the effective theory described by chiral pair fluctuation.

・We expand the thermodynamic potential about the order parameter and analyze the change of the coefficients.

↓

MFA : $G_{\mathrm{ps}}^{-1}$ can become negative. $\lambda$ is always positive. 2nd order PT

# Summary

- We constitute the effective theory described by chiral pair fluctuation.
- We expand the thermodynamic potential about the order parameter and analyze the change of the coefficients.

MFA : $G_{\rm ps}^{-1}$ can become negative. $\lambda$ is always positive. 2nd order PT

| | $T = 0$ | $T \neq 0$ |
|---|---|---|
| $\Gamma^{(2)}$ | can become negative | always positive |
| $\Gamma^{(4)}$ | can become negative | can become negative |

PT may be 1st order.

PT is always 1st order.