

Initial state correlations in the CGC wave function

Michael Lublinsky

Ben-Gurion University of the Negev
Israel

Alex Kovner and ML; Phys.Rev. D84 (2011) 094011, Phys.Rev. D83 (2011) 034017;
Int.J.Mod.Phys. E22 (2013) 1330001;

T. Altinoluk, N. Armesto, G. Beuf, A. Kovner and ML arXiv:1503.07126 (PLB)

T. Altinoluk, N. Armesto, G. Beuf, A. Kovner and ML arXiv:1509.03223 (PLB)

T. Altinoluk, N. Armesto, G. Beuf, A. Kovner and ML arXiv:1610.03020 (PRD)

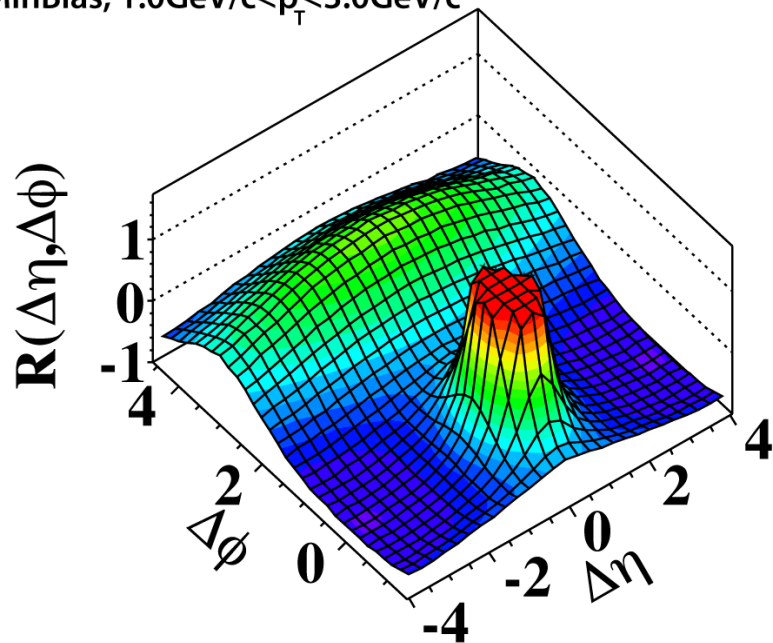
A. Kovner, ML, and V. Skokov, arXiv:1612.07790 (PRD)

A. Kovner, ML, and V. Skokov, arXiv:1706.02330

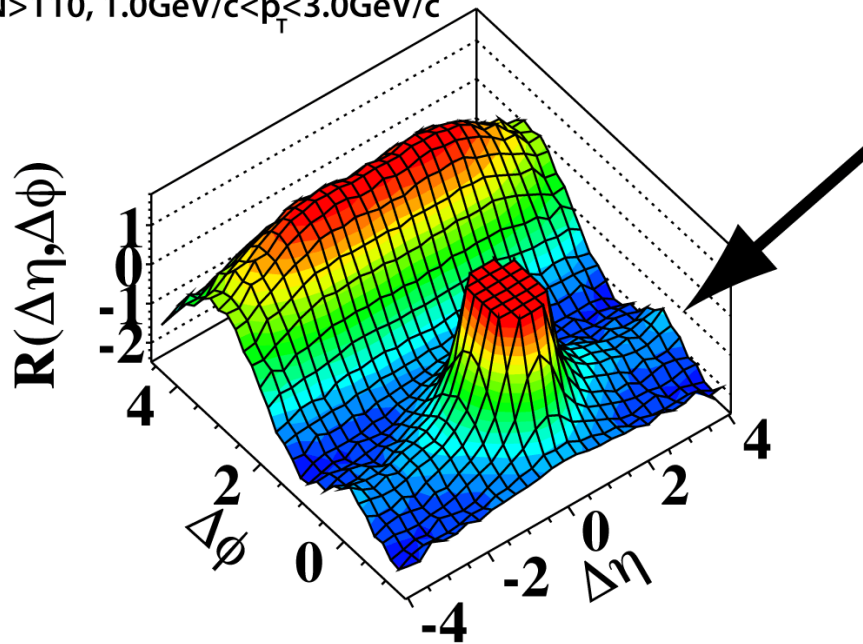
"RIDGE" - ANGULAR CORRELATIONS

Two particle correlations in $p - p$: long range in rapidity, near-side angular correlations

CMS 2010, $\sqrt{s}=7\text{TeV}$
MinBias, $1.0\text{GeV}/c < p_T < 3.0\text{GeV}/c$



$N > 110$, $1.0\text{GeV}/c < p_T < 3.0\text{GeV}/c$

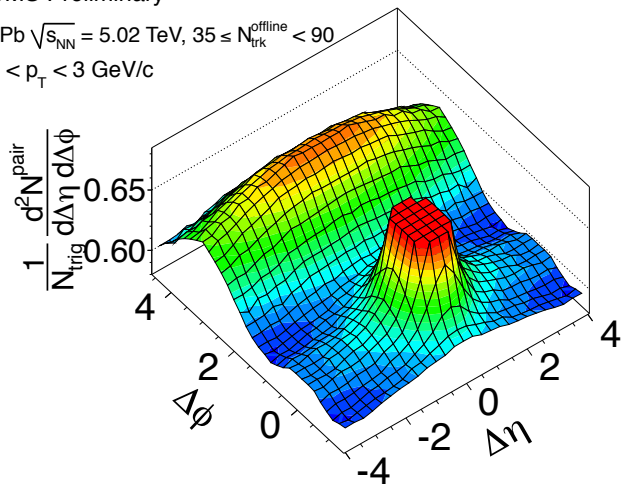


"High multiplicity" collisions with over a hundred charged particles produced

Multiplicity dependence of the ridge in pPb

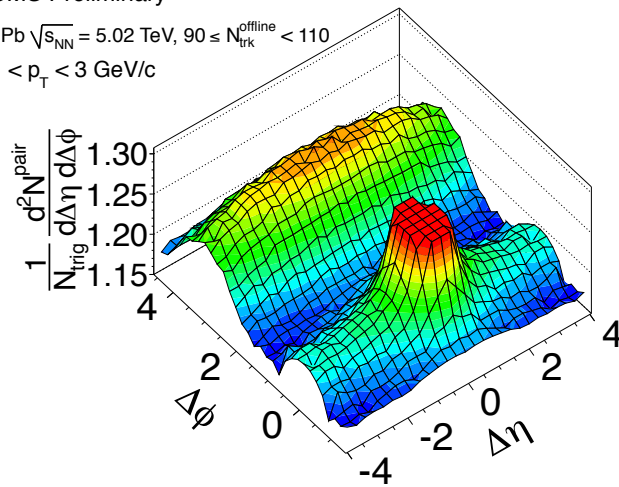
CMS Preliminary

$pPb \sqrt{s_{NN}} = 5.02 \text{ TeV}$, $35 \leq N_{\text{trk}}^{\text{offline}} < 90$
 $1 < p_T < 3 \text{ GeV}/c$



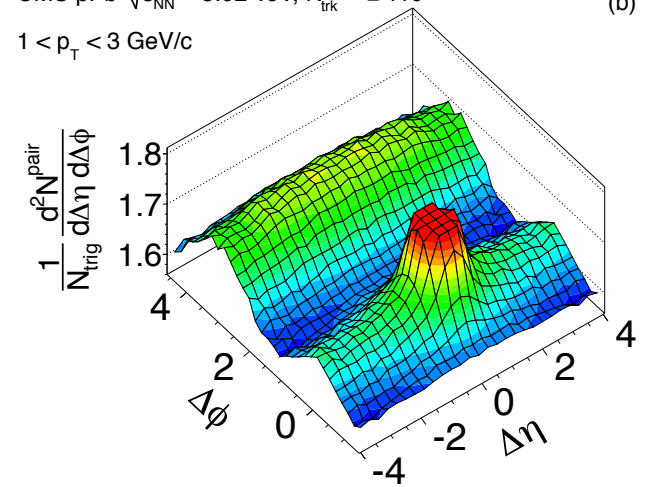
CMS Preliminary

$pPb \sqrt{s_{NN}} = 5.02 \text{ TeV}$, $90 \leq N_{\text{trk}}^{\text{offline}} < 110$
 $1 < p_T < 3 \text{ GeV}/c$



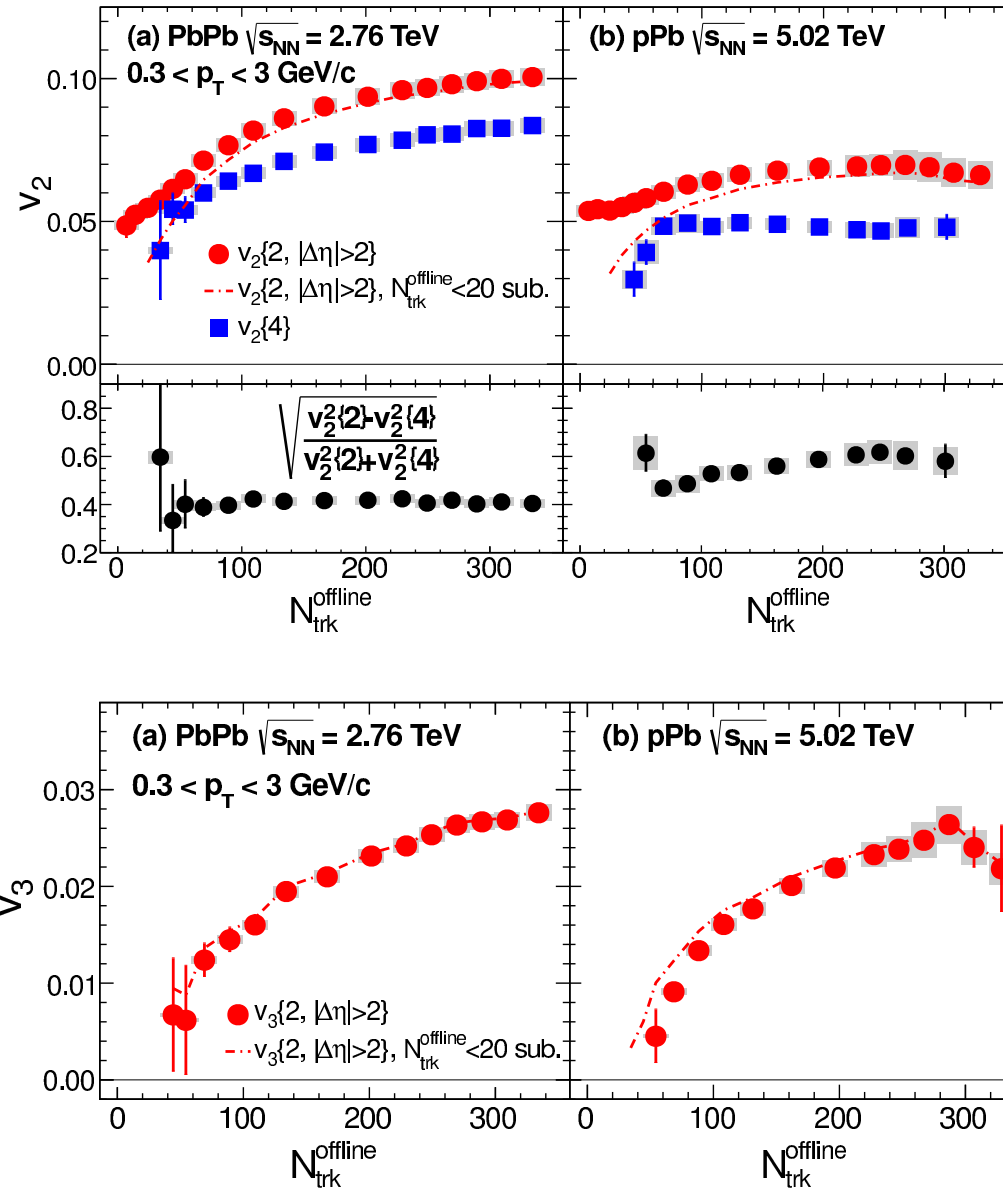
CMS $pPb \sqrt{s_{NN}} = 5.02 \text{ TeV}$, $N_{\text{trk}}^{\text{offline}} \geq 110$

$1 < p_T < 3 \text{ GeV}/c$



(b)

The correlations point to collective, or at least quasi collective behavior.



Even more exciting: recent CMS and ATLAS analysis of p-p at LHC - ridge persists even in MIN. BIAS events.

Big Questions

- **Origin of angular collimation?**

Could be many. For sure explosive "wind" from hydro would lead to some.

- **Origin of long range rapidity correlations?**

Causality: correlations exist in early stage of the collision

- **Do we see a sign of universality between $p-p$, $p-A$ and $A-A$?**

Hopefully Yes! High energy QCD implies this universality. In most of the experiments the effect emerges when high densities are involved (color glass condensate (CGC))

- **We seem to observe collective phenomena in $p-p$ and $p-A$, even up to $p_t \approx 10\text{GeV}$, and we are told that "one hydro to rule them all"**

Is it indeed just hydro and cannot be anything else?

We would like to explore other sources such as correlated structure of the initial wave function

Our Goal

To discuss some general features of gluon/quark production at high energy.

”Flow coefficients” measure correlations between the emitted particles, and are believed to encode collectivity of the final state. For double inclusive spectrum

$$\frac{d^2N}{d^2p_1 d^2p_2} \propto 1 + \sum_{n=1}^{\infty} 2V_n(\mathbf{p}_1, \mathbf{p}_2) \cos(n\Delta\phi)$$
$$v_n^2 = \frac{V_n(\mathbf{p}_T, \mathbf{p}_T^{\text{ref}})}{\sqrt{V_n(\mathbf{p}_T^{\text{ref}}, \mathbf{p}_T^{\text{ref}})}}; \quad n = 2, 3$$

Here I talk about only one source for the observed phenomena:

Correlations (Quasi-Collectivity) ”inherited” from the INITIAL CONDITIONS (CGC)

Particular questions to be addressed:

- Physics of the Glasma graphs = BEC + HBT (arXiv:1503.07126, arXiv:1509.03223)
- Beyond Glasma graphs - origin of v_3 (arXiv:1612.07790)
- Quark correlations and effect of Fermi statistics (arXiv:1610.03020)
- qqg correlation as a background for CME (arXiv:1706.02330)

Naive picture of correlated gluon production

Long range rapidity correlations come for free with boost invariance

Incoming $|\Psi_{in}\rangle$ is approximately boost invariant: exactly the same gluon distribution at Y_1 and Y_2 .

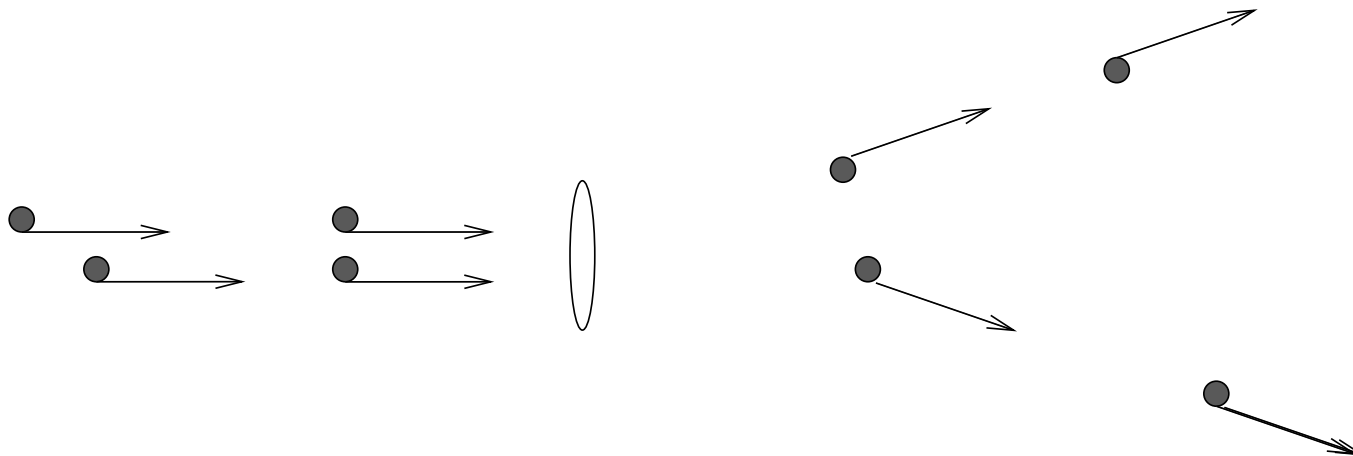
What happens at Y_1 , happens also at Y_2 : If it is probable to produce a gluon at Y_1 , it is also probable to produce a gluon at Y_2 .

But exactly by the same logic there must be angular correlations:

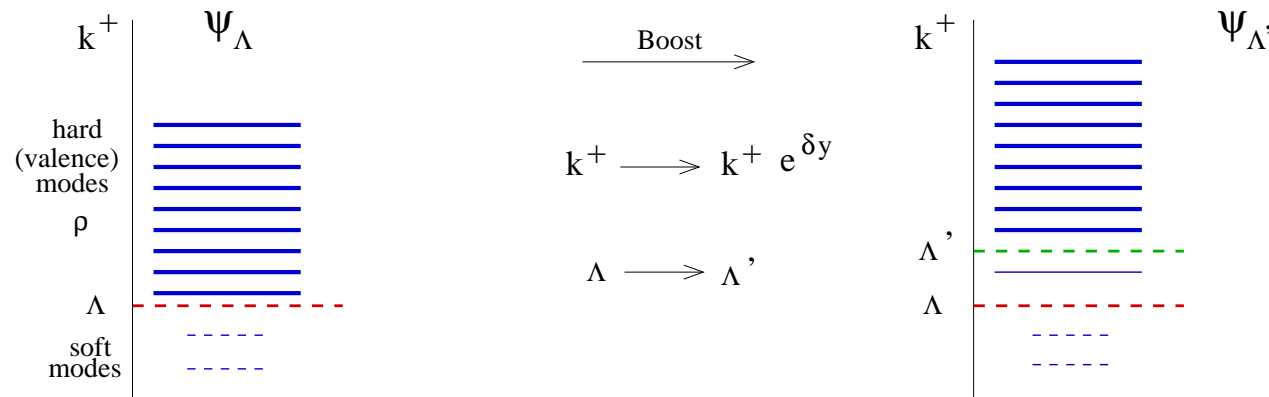
Gluons scatter on exactly the same target

If the first gluon is most likely to be scattered to the right, the second gluon at the same impact parameter will be also scattered to the right

Eikonal scattering is rapidity independent!



Light Cone Wave Function



Hard particles with $k^+ > \Lambda$ scatter off the target. In the eikonal approximation, the scattering amplitude is independent of k^+ . Hard (valence) modes are described by the valence density $\rho(x_\perp)$. **Note rapidity independence!**

Soft modes are not many. They do not contribute much to the scattering amplitude.

The boost opens a window above Λ with the width $\sim \delta y$. The window is populated by soft modes, which became hard after the boost. These newly created hard modes do scatter off the target.

In the dilute limit $\rho \sim 1$; gluon emission $\sim \alpha_s \rho$, LO = one gluon, NLO = 2 gluons

In the dense limit $\rho \sim 1/\alpha_s$, we have $\alpha_s \rho \sim 1$, and the number of gluons in the window can be very large.

Denote soft glue creation and annihilation operators as \mathbf{a} and \mathbf{a}^\dagger .

$$\mathbf{H}_{\text{QCD}} = \mathbf{H}(\rho, \mathbf{a}, \mathbf{a}^\dagger)$$

The dressed incoming hadron light cone wave function

$$|\Psi_{\text{in}}\rangle_Y = \Omega_Y(\rho, \mathbf{a}) |\rho\rangle_{\text{valence}} \otimes |\mathbf{0}_a\rangle_{\text{soft}}$$

$|\Psi_{\text{in}}\rangle_Y$ is an eigen-function of the Hamiltonian, $\mathbf{H}_{\text{QCD}} |\Psi_{\text{in}}\rangle_Y = \mathbf{E} |\Psi_{\text{in}}\rangle_Y$

The major challenge is to find Ω that diagonalises the Hamiltonian

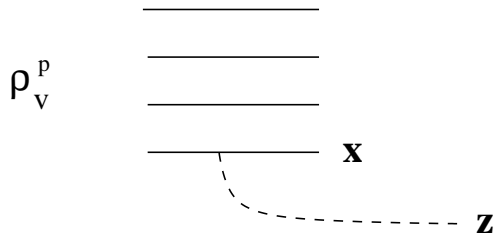
$$\Omega^\dagger \mathbf{H} \Omega = \mathbf{H}_{\text{diagonal}}$$

LCWF in Dilute Limit

Gluon coherent field operator in the dilute limit

$$\Omega_Y(\rho \rightarrow 0) \equiv \mathcal{C}_Y = \text{Exp} \left\{ i \int d^2z b_i^a(z) \int_{e^{Y_0 \Lambda}}^{e^{Y \Lambda}} \frac{d\mathbf{k}^+}{\pi^{1/2} |\mathbf{k}^+|^{1/2}} \left[a_i^a(\mathbf{k}^+, z) + a_i^{\dagger a}(\mathbf{k}^+, z) \right] \right\}$$

Linear evolution means $\delta\rho \propto \rho^p$



Emission amplitude is given by the Weizsaker-Williams field

$$b_i^a(z) = \frac{g}{2\pi} \int d^2x \frac{(\mathbf{x} - \mathbf{z})_i}{(\mathbf{x} - \mathbf{z})^2} \rho^a(\mathbf{x})$$

The operator \mathcal{C} dresses the valence charges by a cloud of the WW gluons

Density Matrix of soft modes

The wave function coming into the collision region at time $t = 0$

$$|\Psi_{\text{in}}\rangle = \Omega_Y |\rho, \mathbf{0}_a\rangle .$$

Define the reduced density matrix of soft modes

$$\hat{\rho} = \int \mathbf{D}\rho \mathbf{W}[\rho] |\Psi_{\text{in}}\rangle \langle \Psi_{\text{in}}|$$

McLerran-Venugopalan model for dense systems:

$$\mathbf{W}^{\text{MV}}[\rho] = \mathcal{N} \exp \left[- \int_{\mathbf{k}} \frac{1}{2\mu^2(\mathbf{k})} \rho(\mathbf{k}) \rho(-\mathbf{k}) \right]$$

”Dilute/Dense mix approximation”: $\Omega = \mathcal{C}$ and $W = W^{MV}$ (Gaussian),
 $\hat{\rho}$ is computable analytically

T. Altinoluk, N. Armesto, G. Beuf, A. Kovner and ML, arXiv:1503.07126

$$\hat{\rho} = \sum_n \frac{1}{n!} e^{-\frac{1}{2}\phi_i M_{ij} \phi_j} \left[\prod_{m=1}^n M_{imjm} \phi_{im} |0\rangle \langle 0| \phi_{jm} \right] e^{-\frac{1}{2}\phi_i M_{ij} \phi_j}$$

Here we have introduced compact notations:

$$\phi_i \equiv \left[\mathbf{a}_i^{\dagger a}(\mathbf{x}) + \mathbf{a}_i^a(\mathbf{x}) \right] ; \quad \mathbf{M}_{ij} \equiv \frac{\mathbf{g}^2}{4\pi^2} \int_{\mathbf{u}, \mathbf{v}} \mu^2(\mathbf{u}, \mathbf{v}) \frac{(\mathbf{x} - \mathbf{u})_i (\mathbf{y} - \mathbf{v})_j}{(\mathbf{x} - \mathbf{u})^2 (\mathbf{y} - \mathbf{v})^2} \delta^{ab}$$

M bears two polarisation, colour, and coordinate indices, collectively denoted as $\{ij\}$.

Bose Enhancement

Easy to show that correlators in this $\hat{\rho}$ Wick factorize in terms of two basic elements:

$$tr[\hat{\rho} a_a^{\dagger i}(k) a_b^j(p)] = (2\pi)^2 \delta_{ab} \delta^{(2)}(k - p) g^2 \mu^2(p) \frac{p^i p^j}{p^4}$$

$$tr[\hat{\rho} a_a^i(k) a_b^j(p)] = tr[\hat{\rho} a_a^{\dagger i}(k) a_b^{\dagger j}(p)] = -(2\pi)^2 \delta_{ab} \delta^{(2)}(k + p) g^2 \mu^2(p) \frac{p^i p^j}{p^4}$$

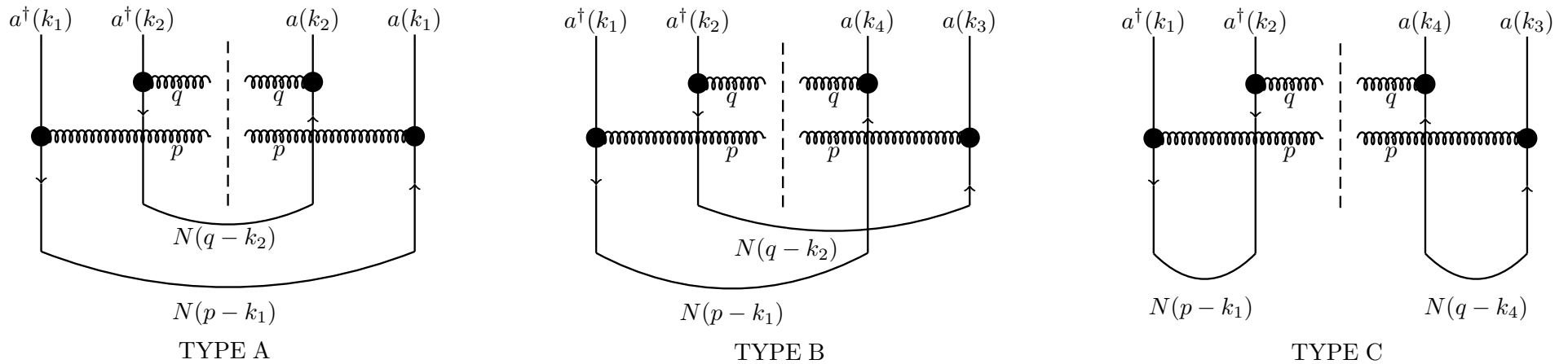
Correlator of **aaaa** enters double gluon production:

$$tr[\hat{\rho} a_a^{\dagger i}(k_1) a_b^{\dagger j}(k_2) a_a^i(k_1) a_b^j(k_2)] = S^2(N_c^2 - 1)^2 \left\{ \frac{g^4 \mu^2(k_1) \mu^2(k_2)}{k_1^2 k_2^2} + \frac{1}{S(N_c^2 - 1)} \left[\delta^{(2)}(k_1 - k_2) + \delta^{(2)}(k_1 + k_2) \right] \frac{g^4 \mu^4(k_1)}{k_1^4} \right\}$$

The first term is the “classical” square of the density.

The last term is a Bose enhancement contribution \rightarrow Glasma Graphs ($1/N_c^2$)

“Glasma graphs” A. Dumitru, F. Gelis, J. Jalilian-Marian, T. Lappi: Phys.Lett. B697 (2011) 21 (arXiv:1009.5295), followed by quite successful quantitative effort to describe all data by K. Dusling and R. Venugopalan, Phys.Rev.Lett. 108 (2012) 262001



The resulting correlations are N_c suppressed. The physics of the “Glasma graphs” is

Type A = Initial state Bose enhancement \rightarrow correlation in the final state.

Type B + Type C \rightarrow HBT correlations in the final state

$N(k) \sim - \int d^2x e^{i\vec{k}\vec{x}} \langle \frac{1}{N_c} \text{tr}[S^\dagger(x)S(0)] \rangle_{\text{Target}}$ - the (adjoint) dipole scattering probability.

(Final state correlations without final state interactions!)

$$\frac{dN}{d^2p d^2k d\eta d\xi} = \langle \sigma(\mathbf{k}) \sigma(\mathbf{p}) \rangle_{P,T}$$

Configuration by configuration

(for fixed configuration of projectile charges ρ and fixed target fields S)

$\sigma(\mathbf{k})$ is a single gluon production cross section and it is a real function of \mathbf{k} , which has a maximum at some value $\mathbf{k} = \mathbf{q}_0$. Then the two gluon production probability **configuration by configuration** has a maximum at

$$\mathbf{k} = \mathbf{p} = \mathbf{q}_0 \simeq \mathbf{Q}_s$$

The value of \mathbf{q}_0 depends on configuration, but the fact that $\mathbf{k} \simeq \mathbf{p}$ does not.

This is the near side correlation!

Is the maximum of σ_1 unique?

No, σ_1 is symmetric under $\mathbf{k} \rightarrow -\mathbf{k}$ and thus has two maxima at \mathbf{q}_0 and $-\mathbf{q}_0$

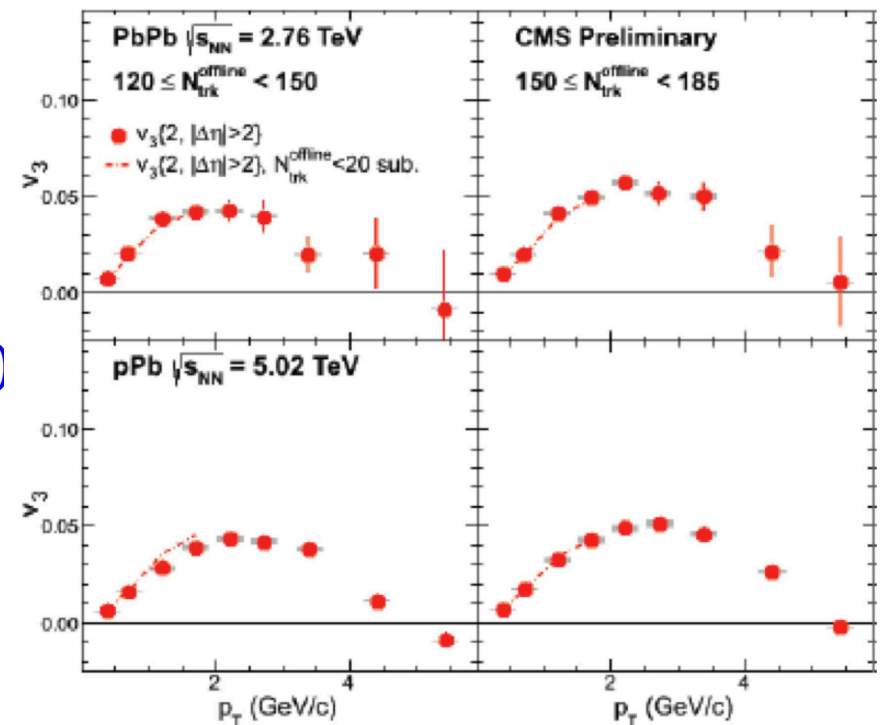
This means that σ^4 has a symmetry $\mathbf{k}, \mathbf{p} \rightarrow -\mathbf{k}, \mathbf{p}$ and therefore has maxima at relative angles $\phi = 0$ and $\phi = \pi$

It is NOT a symmetry of QCD: it is "accidental"

After all there is some asymmetry between 0 and π angles, ($v_3 \neq 0$)

Is the "dilute" CGC state (two Pomeron exchange) we are using good enough?

Better approximation to the CGC state?



LCWF in Dense Limit

Altinoluk, Kovner, ML, Peressutti, Wiedemann (2007-2009)

$$|\Psi\rangle = \Omega_Y(\rho, \mathbf{a}) |\rho\rangle_{\text{valence}} \otimes |\mathbf{0}_a\rangle_{\text{soft}}$$

In the dense regime CGC's wave-function is a squeezed state: $\Omega(\rho \sim 1/\alpha_s) = \mathcal{C} \mathcal{B}$
 \mathcal{B} is a Bogolyubov operator

$$\mathcal{B} = \exp[\mathbf{B}^{-1}[\rho] (\mathbf{a}^2 + \mathbf{a}^{\dagger 2}) + \dots]$$

\mathcal{B} defines quasiparticles above the WW background

The WW field $b_{\alpha i}$:

$$\partial_i \mathbf{b}_{\alpha i}(\mathbf{x}) = \mathbf{g} \rho_\alpha(\mathbf{x})$$

The operator B :

$$\mathbf{B} = (\mathbf{1} - \mathbf{l} - \mathbf{L})^2 = \mathbf{1} - \mathbf{l} - \mathbf{L} + [\mathbf{l}, \mathbf{L}]_+$$

where

$$\mathbf{l}_{ij}^{\alpha\beta}(\mathbf{x}, \mathbf{y}) \equiv \delta^{\alpha\beta} \frac{\partial_i \partial_j}{\partial^2}(\mathbf{x}, \mathbf{y}); \quad \mathbf{L}_{ij}^{\alpha\beta}(\mathbf{x}, \mathbf{y}) = \mathbf{U}^{\alpha\gamma}(\mathbf{x}) \frac{\partial_i \partial_j}{\partial^2}(\mathbf{x}, \mathbf{y}) \mathbf{U}^{\dagger \gamma\beta}(\mathbf{y})$$

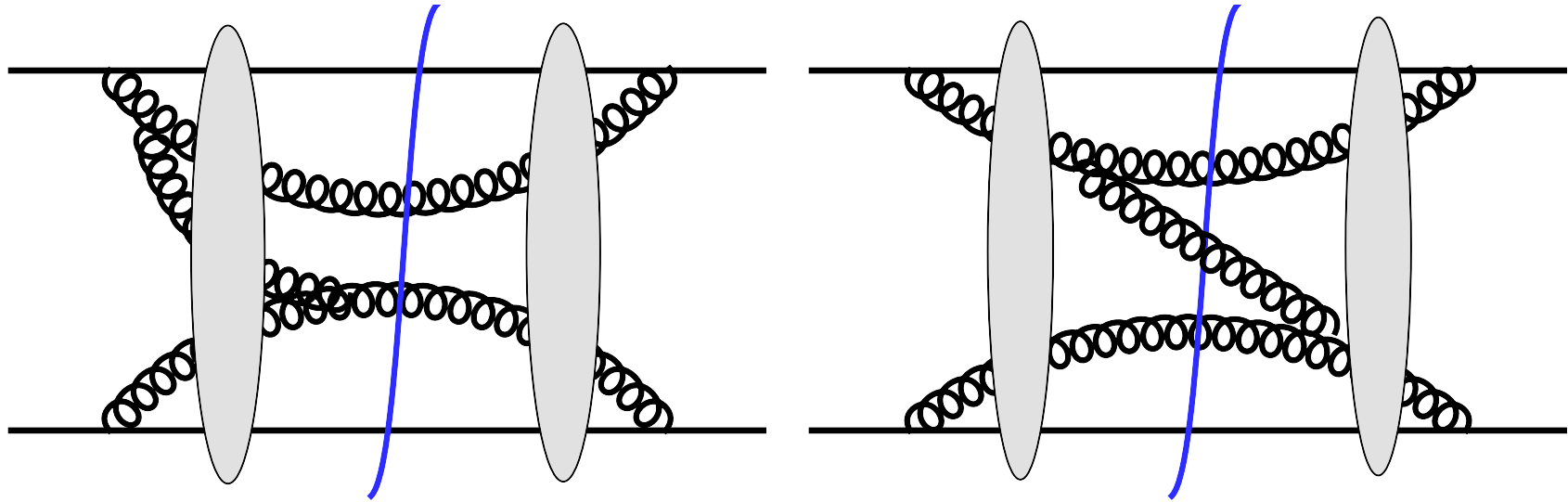


Figure 1: Schematic representation of the types of diagrams that contribute to the double inclusive production at order $g^6 \rho^4$. The horizontal lines symbolize the valence gluons constituting the valence color charge density ρ . The vertical line denotes the final state. Momenta of two final state gluons are fixed. In the case of the three gluon final state, one of the gluons is summed over inclusively. The blobs symbolize the scatterings on the target. The final state gluons can be emitted either before or after these scatterings, and all the combinations have to be accounted for.

Correlation Function

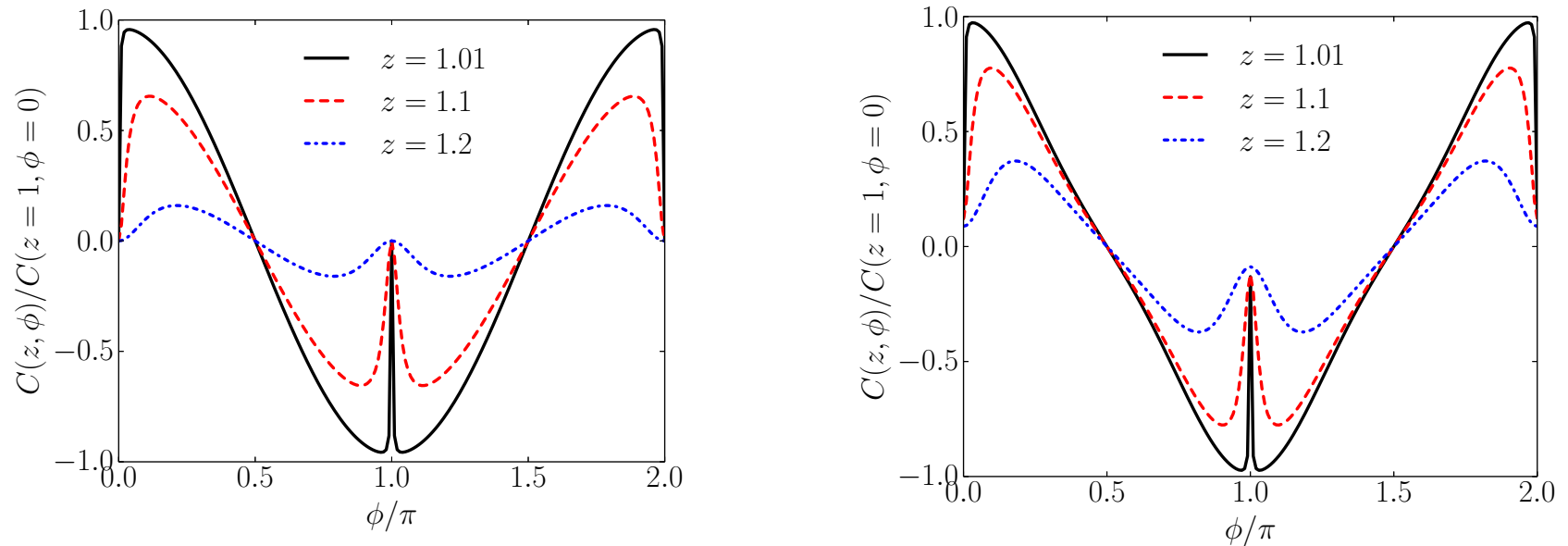


Figure 2: The correlation function as a function of the azimuthal angle, ϕ for different values of $z = p/k$. Left panel: in the projectile wave function. Right panel: double gluon inclusive production. The correlation functions are normalized by $C(z = 1, \phi = 0)$.

The odd harmonics

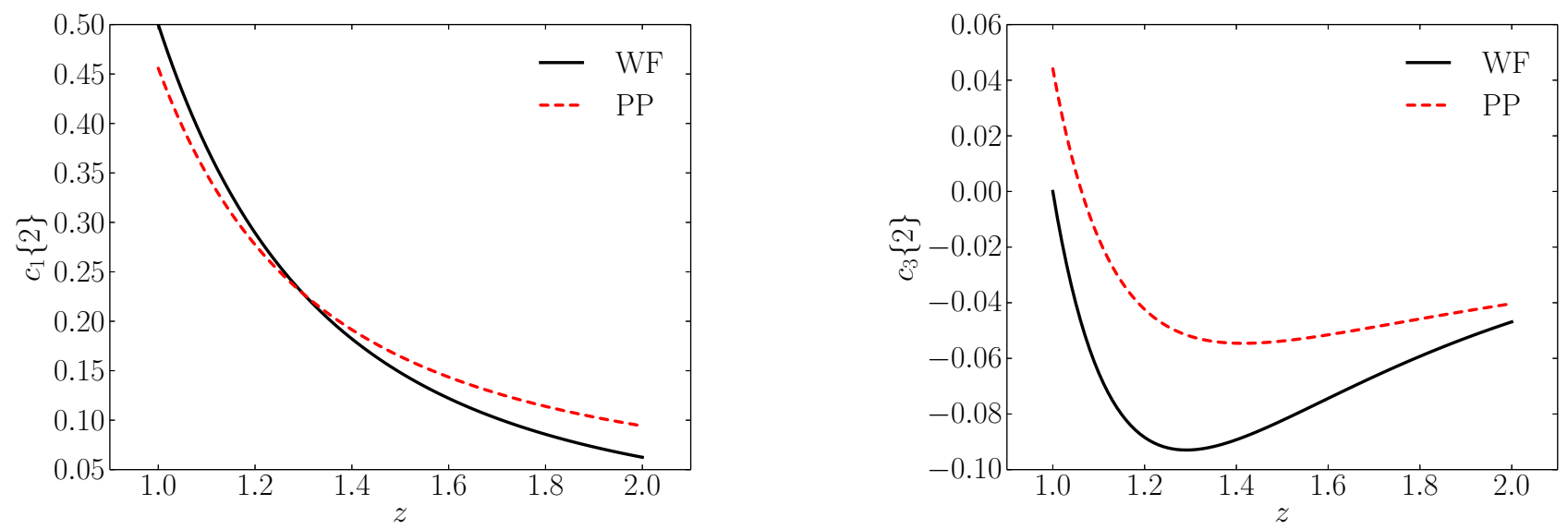


Figure 3: The first and the third cumulants as a function of $z = p/k$.

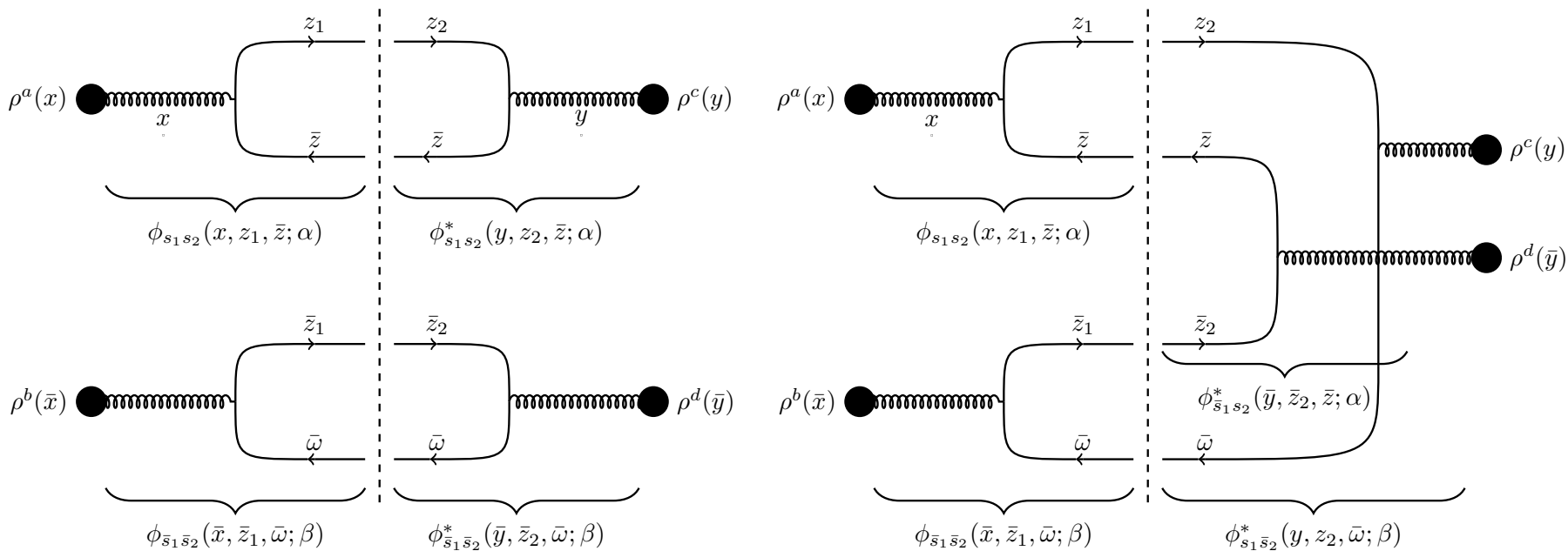
Mini-Summary of v_3

- The absence of odd harmonics in dilute-dense scattering is accidental: a denser projectile generates odd harmonics. The squeezed state has wider applicability parametrically.
- Relative to correlated piece from glasma graphs: our production cross section is $O(\alpha_s N_c)$ - so coupling suppressed but N_c enhanced.
- The sign of V_3 is only positive for $1.1 > p/k > .9$. Keep momentum of trigger fixed, increase the momentum of associated particle, the v_3 should decrease pretty fast. Some sign of this is in the data, although not clear that the momenta are large enough to trust our approximations.

Quark Correlations

Gluons are Bose enhanced: is there Pauli blocking for Quarks?

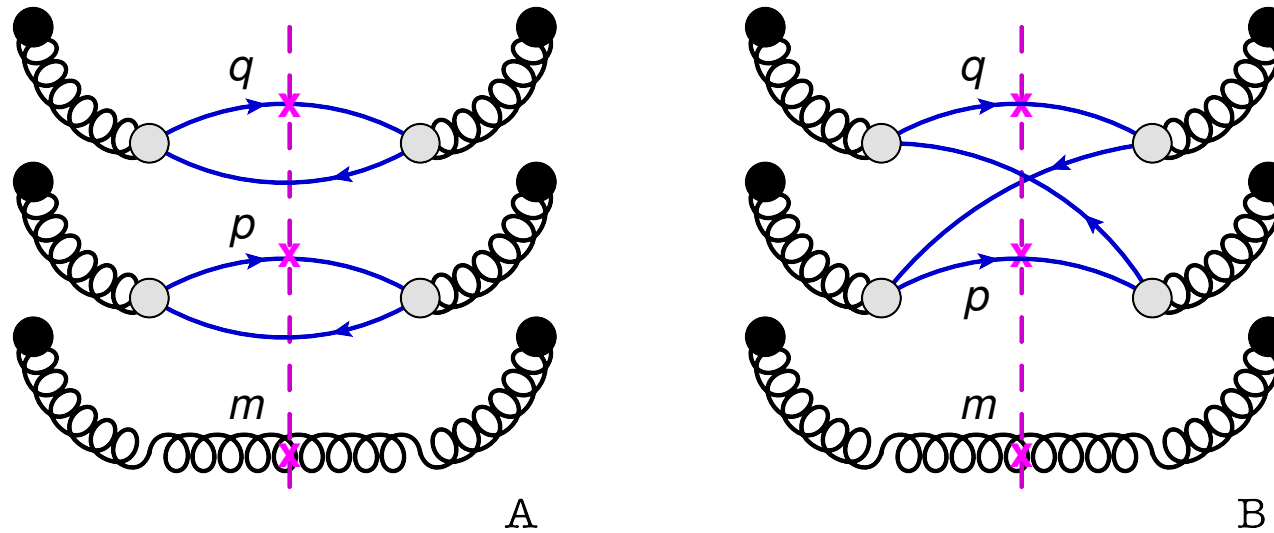
Yes, but different: for Quark-Quark production we find **short range Pauli blocking**.



Mini-Summary of quark correlations

- There is a depletion of pair production at like transverse momenta due to the Pauli blocking effect. The effect decays exponentially with the rapidity difference between the two produced quarks. This exponential decrease, however, is tempered somewhat by a factor quadratic in the rapidity difference $(\Delta\eta)^2 \exp(-\Delta\eta)$.
- The effect turns out to be parametrically $\mathcal{O}(\alpha_s^2 N_c)$ relative to gluon-gluon correlations, which for realistic values of $\alpha_s \sim 0.2$ and $N_c = 3$ results in a mild suppression factor. Thus, it is possible that the effect is big enough to be observable. Perhaps open charm-open charm correlations can be measured and analysed?
- Effects of quantum statistics are entirely encoded in a quadrupole contributions $\text{tr}[S S^\dagger S S^\dagger]$ and it is a parametrically leading contributions to the correlated particle production at large N_c .
A. Kovner and A. H. Rezaeian, arXiv:1707.06985

CME background



$$\gamma = \langle \cos(\phi_p + \phi_q - 2\phi_m) \rangle =$$

$$= \mathcal{N} \int d^2p \int d^2q \int d\phi_m \left[\frac{dN}{d\eta_1 d^2p d\eta_2 d^2q d\eta_g d^2m} \right]_{\text{corr}} \cos(\phi_q + \phi_p - 2\phi_m)$$

A - the pedestal, the rapidity-independent production, with a negative γ

B - the rapidity-dependent contribution originating from Pauli blocking, which is characterised by positive γ for small $\Delta\eta = \eta_1 - \eta_2$ and negative γ for $\Delta\eta \gg 1$.

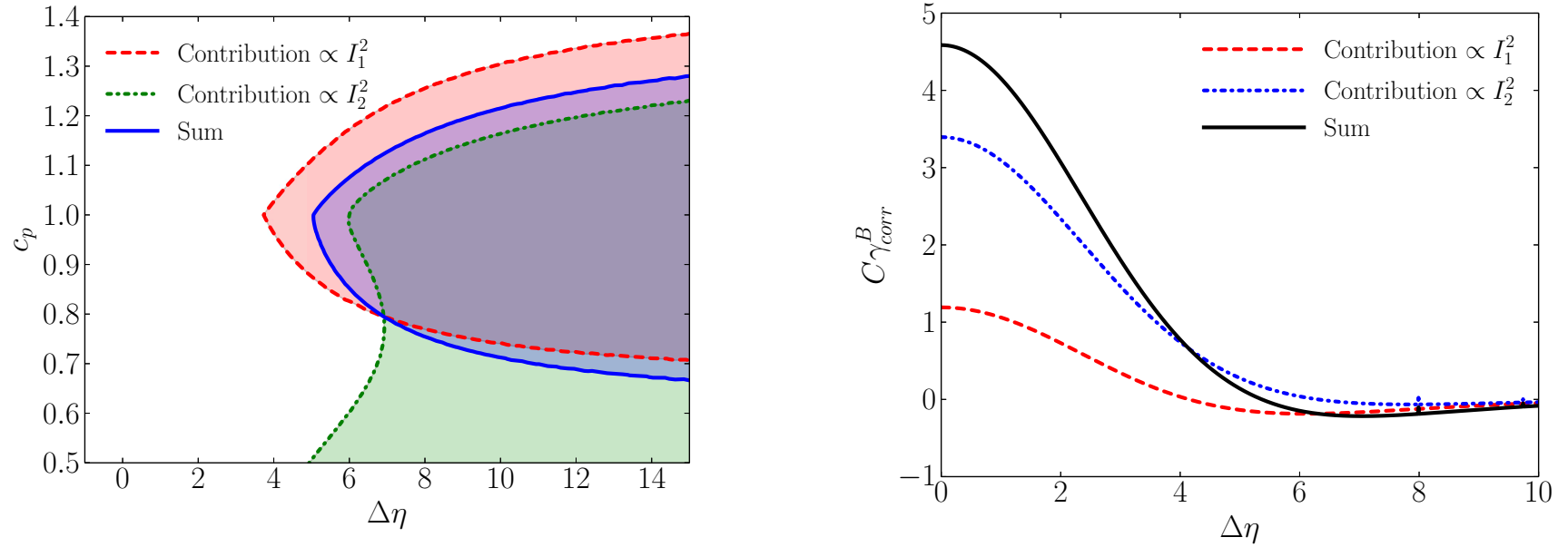


Figure 4: Left: the sign of the correlator as a function of $c_p = p/m$ and $\Delta\eta$. The shaded region show the negative values of the corresponding contributions; Right: Integrating over c_p in the range $0.9 < c_p < 1.1$.

The sign change happens at rather large values of $\Delta\eta$ which is not consistent with the experimentally observed value of $\Delta\eta \approx 1.5 - 2$. Nevertheless, qualitatively the rapidity dependence is similar to the experimentally observed one for the same charge γ .