Hot and Dense Matter in Astrophysics

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- A massive star ($M > 8M_\odot$) forms an iron core; it cannot produce energy via fusion and contraction ensues.
- As the density increases, electron capture becomes favorable and neutrinos are produced:

$$p^+ + e^- \rightarrow n + \nu_e$$

- When the Fermi energy of neutrinos gets large enough, they become trapped and β-equilibrium is achieved.
- At this point the entropy per nucleon is $S \simeq 1$. Dripped neutrons require $S \simeq 8$ thus nuclei persist until the core contracts to $n \sim n_o$. Then nucleonic matter emerges.
- Nucleons are compressed to $n \sim 3n_o$ where the repulsive core of the strong interaction dominates their attraction and inhibits further contraction.
- The core rebounds and creates a shock wave that disrupts the star.

- Matter in β-equilibrium supported against gravitational collapse by neutron degeneracy.
- Structure determined by simultaneous solution of:
  - Interior mass,

$$m(r) = 4\pi \int_0^r \epsilon(r')r'^2 dr'$$

  - Hydrostatic equilibrium,

$$\frac{dp}{dr} = -\frac{(\epsilon + p)[Gm(r) + 4\pi G r^3 p]}{r[r - 2Gm(r)]}$$

  - EOS,

$$p = p(\epsilon)$$

Binary Mergers

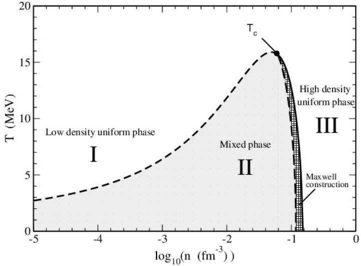
- Relativistic binaries not in equilibrium : Gravitational wave (GW) emission leads to orbital decay.
- Early stage: only gravitational interactions, GW signal contains information for the masses of the components.
- Coalescence stage: Tidal disruption of the lower-mass star, mass transfer onto the more massive one. Mass transfer rate depends on $\mathcal{C} = M_{NS}/R_{NS}$ and reflected in GW signal. Ejected matter is very neutron-rich and can lead to heavy element formation via the r-process.
- Late stage: Black hole or hypermassive neutron star formation.

	Core-collapse supernovae	**Proto-neutron stars**	**Mergers of compact binary stars**
Baryon Density(n_0)	$10^{-8} - 10$	$10^{-8} - 10$	$10^{-8} - 10$
Temperature(MeV)	$0 - 30$	$0 - 50$	$0 - 100$
Entropy(k_B)	$0.5 - 10$	$0 - 10$	$0 - 100$
Proton Fraction	$0.35 - 0.45$	$0.01 - 0.3$	$0.01 - 0.6$

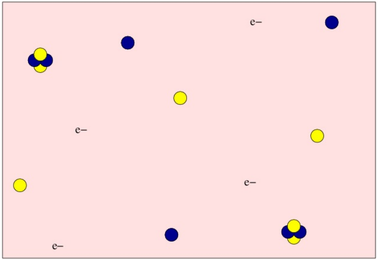
$n_0 \simeq 0.16$ fm^{-3} (equilibrium density of symmetric nuclear matter)

- Objective: Construction of EOS based on the best nuclear physics input for use in hydrodynamic simulations of core-collapse supernova explosions and binary mergers as well as the thermal evolution of proto-neutron stars.

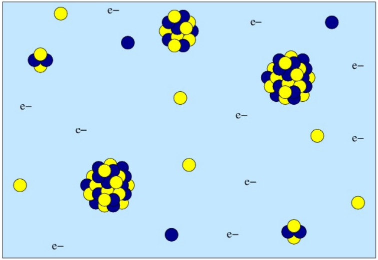
Nuclear Matter Phases



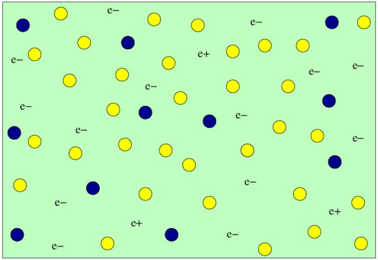
Phase I– Low Density Regime



Phase II– Mixed Phase



Phase III– $n > n_s$ Dissociated



Matter near the Equilibrium Point ($n = n_0, \alpha = 0$) of SNM

- About $\alpha = 0$,
  $E(n,\alpha) \simeq E_0(n) + S_2(n)\alpha^2 + \mathcal{O}(\alpha^4)$
- About $n = n_0$,
  $E_0(n) \simeq \mathcal{E}_0 + \frac{1}{2}K_0\left(\frac{n-n_0}{3n_0}\right)^2 + \ldots$
  $S_2(n) \simeq S_v + L\left(\frac{n-n_0}{3n_0}\right) + \ldots$
- $P(n,\alpha) = n^2\frac{\partial E}{\partial n} \simeq n^2\left[\frac{K_0}{3n_0}\left(\frac{n-n_0}{3n_0}\right) + \frac{L}{3n_0}\alpha^2\right] + \ldots$
- $\hat{\mu} = \mu_n - \mu_p = 2\frac{\partial E}{\partial \alpha} \simeq 4\alpha S_2(n)$

EOS Laboratory Constraints

- Saturation density, $n_0 = 0.16 \pm 0.01 \text{ fm}^{-3}$
  High-energy electron scattering: $r_0 \propto \pi/qR$, $n_0 = \left(\frac{4}{3}\pi r_0^3\right)^{-1}$
- Energy per particle, $\mathcal{E}_0 = -16 \pm 1$ MeV
  Fits to masses of atomic nuclei :
  $B(N,Z) = \mathcal{E}_o A - b_{surf} A^{2/3} - S_v \frac{(N-Z)^2}{A} - b_{Coul} Z^2 A^{-1/3}$
- Symmetry energy, $S_v = 30 - 35$ MeV
  (fits to masses of atomic nuclei)
- Slope of S_2, $L = 40 - 70\text{MeV}$
  (variety of experiments)
- Compression modulus, $K_0 = 240 \pm 30$ MeV
  Giant monopole resonances : $E_{GMR} = \left(\frac{K_A}{m<r^2>}\right)^{1/2}$
  $K_A = K_0 + K_{surf} A^{-1/3} + K_\tau \frac{(N-Z)^2}{A^2} + K_{Coul} \frac{Z^2}{A^{4/3}}$
- Effective mass, $M^*/M = 0.8 \pm 1$
  Neutron evaporation spectra : $N(E_n) \propto a$, $a_{FermiGas} = \frac{\pi^2 m^*}{2k_F^2}$.

- Largest observed mass, $M = 2.01 M_\odot$
  (binaries)
- Largest observed frequency, $\Omega = 114$ rad/s
  (pulsars)
- Inferred radius range, 9 km $\leq R \leq$ 15 km
  (photospheric emission, thermal spectra)
- ...

Solution of Tolman-Oppenheimer-Volkoff (TOV) equations and EOS predicts M_{max}, R_{max}, I_{max}, Ω_{max}, etc.

(Future) Information from Supernovae and Binary Mergers

Supernovae:

- EOS controls electron capture rates (via $\hat{\mu}$) and therefore the neutrino signal.
- GW amplitude related to PNS compactness and thus the high-density properties of the EOS.

Binary mergers:

- Tidal disruption of NS during coalesence of BH-NS binary depends on the stiffness of the EOS. GW frequency sensitive to orbital frequency at disruption.
- Short gamma-ray bursts may be produced in BH-NS mergers. The luminocity and the lifetime of the (metastable) remnant are related to the thermal and high-density properties of the EOS respectively.
- r-process production rates and final abundances depend on the composition of the ejecta and thus the EOS.
- NS radii: tidal deformity, $\lambda \propto R_{NS}^5$.

Approaches to the Nucleon-Nucleon Interaction

- High-precision interactions fitted to NN scattering data
  - meson-exchange models
    e.g. Nijmegen, Paris, Juelich-Bonn
  - sums of local operators
    e.g. Urbana, Argonne

- Interactions from chiral EFT

- RG-evolved potentials

Extension of the above to bulk matter by a variety of techniques: SCGF, BHF, variational, etc.

- Phenomenological approaches: Skyrme, Gogny, RMFT
  Typically treated at the HF level, fitted to extracted quantities.

- $\hat{V}_{NN} = \sum_{i<j} \hat{V}_{ij} + \sum_{i<j<k} \hat{V}_{ijk}$, zero-range
- Evaluating $\hat{H} = \hat{T} + \hat{V}_{NN}$ in the HF approximation gives

$$
\begin{aligned}
\mathcal{H}_{Skyrme} \quad = \quad & \frac{\hbar^2}{2m_n}\tau_n + \frac{\hbar^2}{2m_p}\tau_p \\
& + n(\tau_n + \tau_p)\left[\frac{t_1}{4}\left(1 + \frac{x_1}{2}\right) + \frac{t_2}{4}\left(1 + \frac{x_2}{2}\right)\right] \\
& + (\tau_n n_n + \tau_p n_p)\left[\frac{t_2}{4}\left(\frac{1}{2} + x_2\right) - \frac{t_1}{4}\left(\frac{1}{2} + x_1\right)\right] \\
& + \frac{t_o}{2}\left(1 + \frac{x_o}{2}\right) n^2 - \frac{t_o}{2}\left(\frac{1}{2} + x_o\right)(n_n^2 + n_p^2) \\
& \left[\frac{t_3}{12}\left(1 + \frac{x_3}{2}\right) n^2 - \frac{t_3}{12}\left(\frac{1}{2} + x_3\right)(n_n^2 + n_p^2)\right] n^{\epsilon}
\end{aligned}
$$

Thermodynamics

- Single-particle energy spectrum:

$$\varepsilon_i = k_i^2 \frac{\partial \mathcal{H}}{\partial \tau_i} + \frac{\partial \mathcal{H}}{\partial n_i} \equiv \varepsilon_{k_i} + V_i$$

- $$n_i = \tfrac{1}{2\pi^2} \left(\tfrac{2m_i^* T}{\hbar^2} \right)^{3/2} F_{1/2i}$$

$$\tau_i = \frac{1}{2\pi^2} \left(\frac{2m_i^* T}{\hbar^2} \right)^{5/2} F_{3/2i}$$

$$F_{\alpha i} = \int_0^\infty \frac{x_i^\alpha}{e^{-\psi_i} e^{x_i} + 1} dx_i$$

$$x_i = \tfrac{\varepsilon_{ki}}{T}, \quad \psi_i = \tfrac{\mu_i - V_i}{T} = \tfrac{\nu_i}{T}$$

- Rest of state variables :
  - Energy density $\varepsilon = \frac{\hbar^2}{2m_n^*} \tau_n + \frac{\hbar^2}{2m_p^*} \tau_p + U(n)$
  - Chemical potentials $\mu_i = T\psi_i + V_i$
  - Entropy density $s_i = \frac{1}{T} \left[\frac{5}{3} \frac{\hbar^2}{2m_i^*} \tau + n_i (V_i - \mu_i) \right]$
  - Pressure $P = T(s_n + s_p) + \mu_n n_n + \mu_p n_p - \varepsilon$
  - Free energy density $\mathcal{F} = \varepsilon - Ts$

- Nucleons, Ψ, coupled to σ, ω, and $\vec{\rho}$ mesons:

$$\begin{aligned}\mathcal{L} &= \bar{\Psi}\left[\gamma_\mu\left(i\partial^\mu - g_\omega\omega^\mu - \frac{g_\rho}{2}\vec{\rho}^\mu.\vec{\tau}\right) - (M - g_\sigma\sigma)\right]\Psi \\ &+\frac{1}{2}\left[\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2 - \frac{\kappa}{3}(g_\sigma\sigma)^3 - \frac{\lambda}{12}(g_\sigma\sigma)^4\right] \\ &+\frac{1}{2}\left[-\frac{1}{2}f_{\mu\nu}f^{\mu\nu} + m_\omega^2\omega^\mu\omega_\mu\right] \\ &+\frac{1}{2}\left[-\frac{1}{2}\vec{B}_{\mu\nu}\vec{B}^{\mu\nu} + m_\rho^2\vec{\rho}^\mu\vec{\rho}_\mu\right]\end{aligned}$$

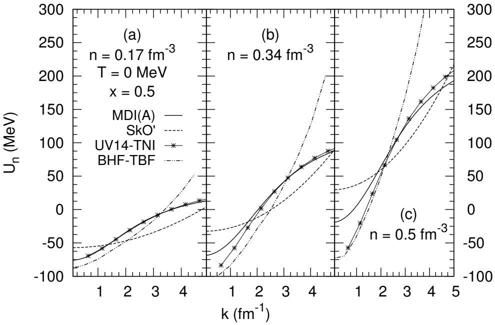
- Extensions: Hyperons, scalar-isovector δ-meson, density-dependent couplings, derivative couplings, nucleon form factors, etc.

- Negligible meson-field fluctuations, uniform and static system $\Rightarrow$

$$
\begin{aligned}
\sigma_0 &= \frac{g_\sigma}{m_\sigma^2} < \bar{\Psi}\Psi > - \frac{1}{m_\sigma^2}\left(\frac{\kappa}{2} g_\sigma^3 \sigma_0^2 + \frac{\lambda}{6} g_\sigma^4 \sigma_0^3\right) \\
&= \frac{g_\sigma}{m_\sigma^2} n_s - \frac{1}{m_\sigma^2}\left(\frac{\kappa}{2} g_\sigma^3 \sigma_0^2 + \frac{\lambda}{6} g_\sigma^4 \sigma_0^3\right) \\
\omega_0 &= \frac{g_\omega}{m_\omega^2} < \bar{\Psi}\gamma^0\Psi > = \frac{g_\omega}{m_\omega^2} n \\
\rho_0 &= \frac{g_\rho}{2m_\rho^2} < \bar{\Psi}\gamma^0\tau_3\Psi > = -\frac{g_\rho}{2m_\rho^2} n(1-2x)
\end{aligned}
$$

- Spectrum: $\epsilon_{i\pm} = \pm(p_i^2 + M^{*2})^{1/2} + \frac{g_\omega^2}{m_\omega^2} n + \frac{g_\rho^2}{4m_\rho^2}(n_i - n_j)$
- Diagonal elements of the stress-energy tensor $T_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu q_i)}\partial_\nu q_i - g_{\mu\nu}\mathcal{L}$, give:
  - Energy density, $\varepsilon = < T_{00} >$
  - Pressure, $P = \frac{1}{3} < T_{ii} >$
- Dirac mass, M^*, derived from the requirement $\frac{\delta \Omega}{\delta \sigma} = 0$

- Single particle potential, $U(\rho, p)$ in both Skyrme and MFT models grows monotonically with momentum;
  inconsistent with optical model fits to nucleon-nucleus reaction data.
- Microscopic calculations (RBHF, variational calculations, etc.) show distinctly different behaviors in their momentum dependence, consistent with optical model fits.
- The above features were found necessary to account for heavy-ion data on transverse momentum and energy flow (in conjuction with $K \sim 240$ MeV).



- SkO': P. G. Reinhard *et al.*, PRC **60**, 014316 (1999)
- BHF-TBF: W. Zuo *et al.* PRC **74**, 014317 (2006)
- UV14-TNI: R. B. Wiringa PRC **38**, 2967 (1988)

Landau Fermi Liquid Theory

- Interaction switched-on adiabatically
- Entropy density and number density maintain their free Fermi-gas forms:

$$s_i = \frac{1}{V}\sum_{k_i}\left[f_{k_i}\ln f_{k_i} + (1-f_{k_i})\ln(1-f_{k_i})\right]$$

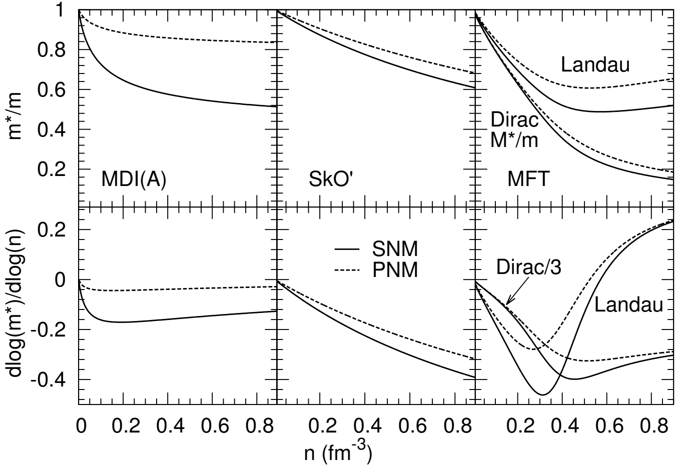
$$n_i = \frac{1}{V}\sum_{k} f_{k_i}(T)$$

- $\int d\varepsilon \frac{\delta s}{\delta T} \quad \Rightarrow \quad s_i = 2a_i n_i T$

$$a_i = \frac{\pi^2}{2}\frac{m_i^*}{k_{Fi}^2} \qquad \text{level density parameter}$$

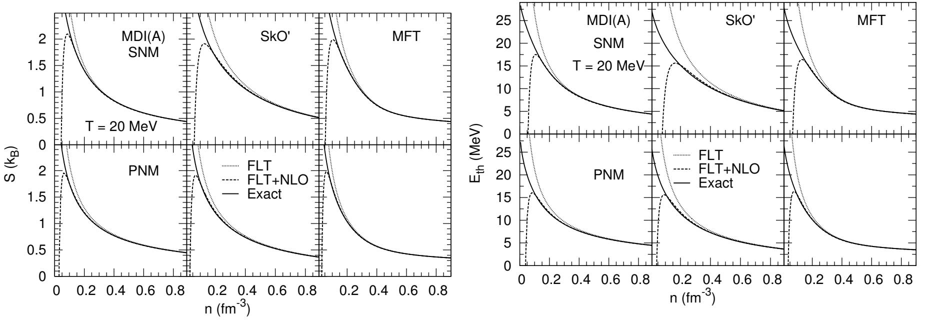
- Other thermodynamics via Maxwell's relations: $\frac{d\varepsilon}{ds} = T$, $\frac{dp}{dT} = -n^2\frac{d(s/n)}{dn}$, $\frac{d\mu}{dT} = -\frac{ds}{dn}$, $\frac{d\mathcal{F}}{dT} = -s$, etc.

Effective masses



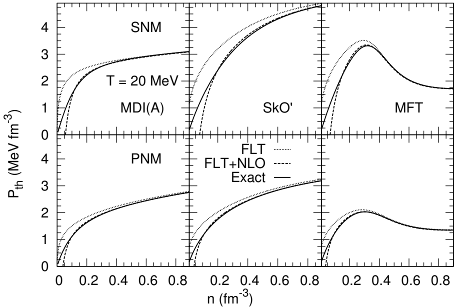
- For n large, $\frac{dm^*}{dn} \simeq 0$
- $m^* = \frac{m}{1+\beta(x)n}$
- $m^* = E_F^* = (p_F^2 + M^{*2})^{1/2}$
- Minimum at n s.t. $\frac{p_F}{M^*} + \frac{dM^*}{dp_F} = 0$

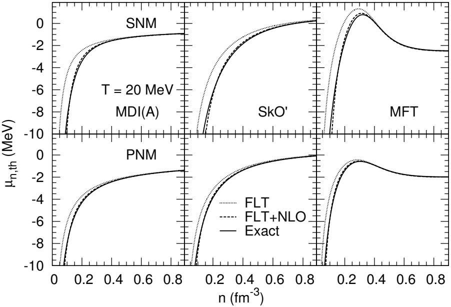
Results: S and E_{th}



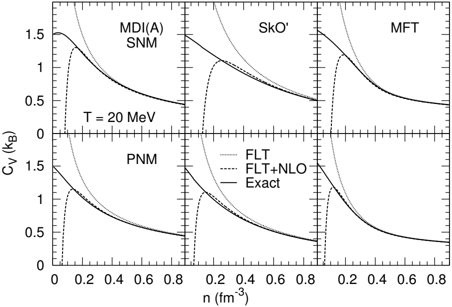
- The three models produce quantitatively similar results.
- Agreement with exact results extended to $n \simeq 0.1$ fm^{-3}.
- Better agreement for PNM than for SNM.

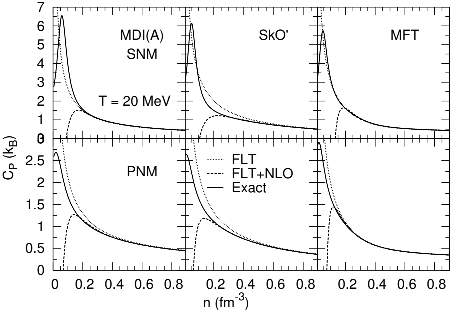
Results: P_{th} and μ_{th}





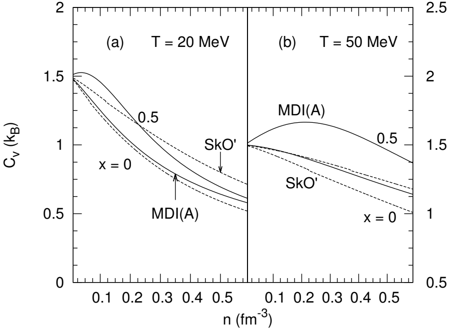
- Model dependence is evident- due to $\frac{dm^*}{dn}$.
- Agreement with exact results extended to $n \simeq 0.1$ fm^{-3}.
- Better agreement for PNM than for SNM.

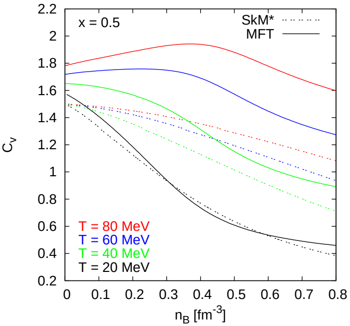


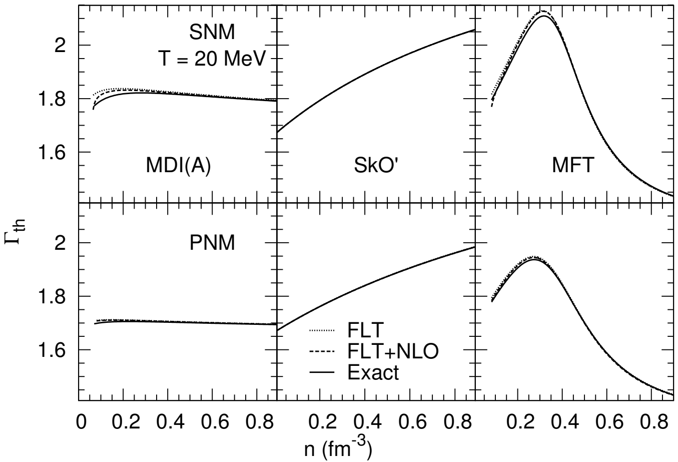


- The MDI and MFT C_V exceed the classical value of 1.5 in the nondegenerate limit. In this regime the T-dependence of the spectrum becomes important.
- The peaks in C_P are due to the proximity to the nuclear liquid-gas phase transition.

Results: Specific Heats

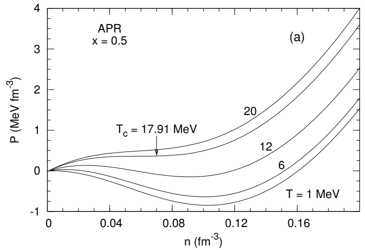




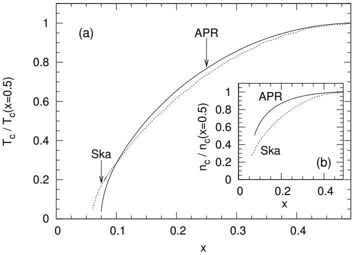


- $\Gamma_{th} = 1 + \frac{P_{th}}{\varepsilon_{th}}$
- Weak composition and temperature dependence
- Finite-range effects suppress density dependence

- The critical temperature and density of the transition for a given composition are obtained by the condition $\left.\frac{dP}{dn}\right|_{n_c,T_c} = \left.\frac{d^2P}{dn^2}\right|_{n_c,T_c} = 0.$



- (n_c, T_c) is the termination point of the phase boundary separating the homogeneous and the inhomogeneous phases of nuclear matter.

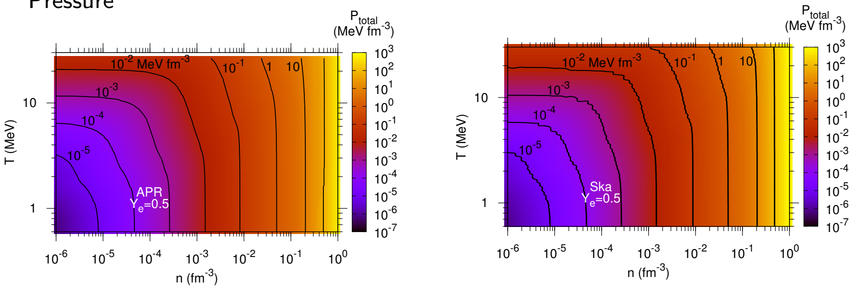


Approaches to the Subnuclear Region

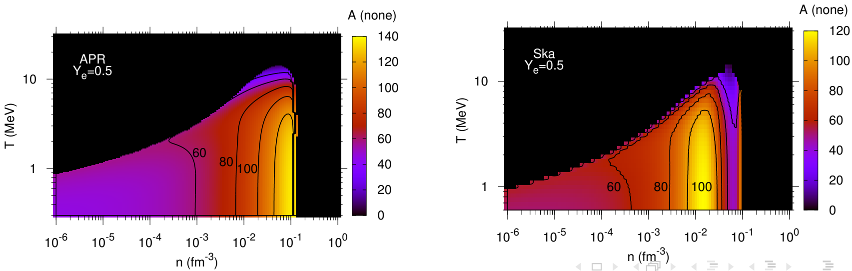
- Nuclear statistical equilibrium
  - Statistical ensemble of nucleons and nuclei in thermodynamic equilibrium
  - Chemical potentials of nuclei, $\mu_a = N_a\mu_n + Z_a\mu_p$
  - Maxwell-Boltzmann statistics
  - Abundances determined by the Saha equation
  - Requires nuclear binding energies as input
- Lattimer-Swesty
  - Single representative species of nucleus described by the liquid-drop model
  - Light nuclei represented by α-particles
  - Nucleons treated by the same model used in the homogeneous phase
  - Hard-sphere interactions
  - Equilibrium obtained by minimizing the free energy of the system wrt its internal variables.
- Virial expansion
  - Nondegenerate limit expansion of the grand potential in small fugacity: $z = \exp[(\mu - m)/T] \ll 1$
  - Coefficients depend on scattering phase shifts corresponding to the interaction
  - Supplements NSE
- Nucleons-in-cell: Molecular dynamics, Thomas-Fermi, Hartree-Fock ...

Results using the Lattimer-Swesty approach

Pressure



Nuclear size



Conclusions

- Matter for astrophysical applications covers a wide range in (n, x, T)-space; much of which cannot be accessed by terrestrial experiments.
- Upcoming astrophysical applications and laboratory experiments involving heavy ions/rare isotopes will probe higher densities and temperatures allowing a tighter grip on the EOS.
- The structure of the interaction, and thus m^*, is crucial in the determination of thermal effects. These, in turn, are important in neutrino and gravitational radiation emission.
- Open questions:
  - Neutron star M_{max}
  - Emergence of non-nucleonic DoF.
  - Treatment of the inhomogeneous phase.
  - Phase transitions