Relativity with a preferred frame. Cosmological implications

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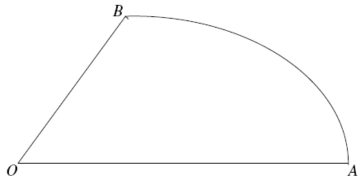
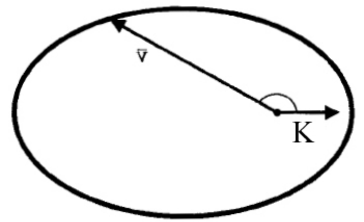
Outline

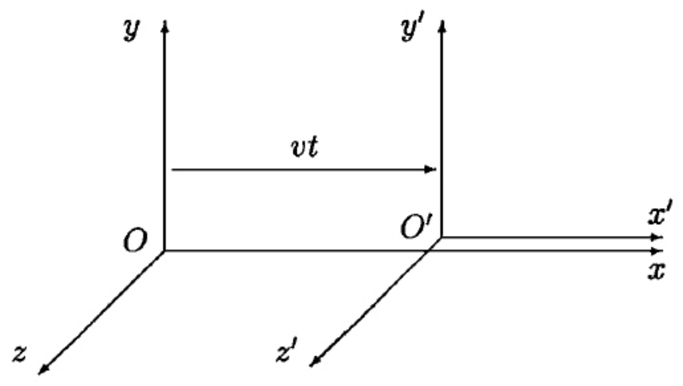
Round-trip postulate



Anisotropy in the speed of light

$$V(\mathbf{n}) = \frac{c\mathbf{n}}{1+\mathbf{kn}} \quad \text{or} \quad V(\Theta) = \frac{c}{1+k\cos\Theta}$$

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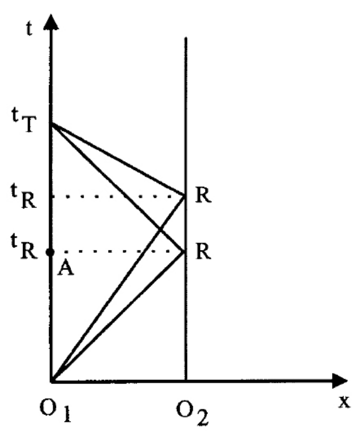

$$\textit{Einstein (standard) synchronization} \quad t_R^{(s)} = \frac{t_0 + t_A}{2} = t_0 + \frac{1}{2}(t_A - t_0)$$

$$\textit{Non-standard synchronization} \quad t_R = t_0 + \epsilon(t_A - t_0), \quad \left(\epsilon \neq \frac{1}{2}\right)$$

Group property

The transformations between inertial frames form a one-parameter group with the group parameter $a = a(v)$ (such that $v \ll 1$ corresponds to $a \ll 1$):

$$t = q(X, Y, Z, T; a), \quad x = f(X, Y, Z, T; a),$$
$$y = g(X, Y, Z, T; a), \quad z = h(X, Y, Z, T; a)$$

Lorentz transformations

$$x = \frac{X - vT}{\sqrt{1 - v^2/c^2}}, \qquad y = Y, \quad z = Z, \qquad t = \frac{T - vX/c^2}{\sqrt{1 - v^2/c^2}}$$

Lorentz group

$$x = X \cosh a - cT \sinh a, \quad t = T \cosh a - \frac{1}{c} X \sinh a$$

where a is the *group parameter*

$$a = \tanh^{-1} \frac{v}{c}$$

First principles

- **Group property**.
- **Invariance of the equation for light propagation**:

$$g_{ik}dx^i dx^k = 0; \qquad \hat{g}_{ik}dX^i dX^k = 0$$
$$(x^0 = ct,\ x^1 = x,\ x^2 = y,\ x^3 = z)$$

- **Correspondence principle**.
  In the limit of small velocities $v \ll 1$ (small values of the group parameter$a \ll 1$), the formula for transformation of the coordinate x turns into that of the Galilean transformation:

$$x = X - vT$$

- The linearity assumption is not imposed.

Procedure

- **Applying the condition of infinitesimal invariance**.
  - *Group transformations*

$$x = f(X, T; a),\ t = q(X, T; a), \quad y = g(Y, Z; a),\ z = h(Y, Z; a)$$

  - *Infinitesimal transformations*

$$x \approx X + \xi(X, T)a, \quad t \approx T + \tau(X, T)a,$$
$$y \approx Y + \eta(Y, Z)a, \quad z \approx Z + \zeta(Y, Z)a$$

  - *Determining equations*
- **Defining the group generators**:
- **Solving the Lie equations**:
- **Relating** a **to** v **by the condition:** $x = 0 \quad \text{for} \quad X = vT$

Transformations with the variable anisotropy parameter

Equations of light propagation in the frames K and K'

$$c^2dT^2 - 2Kc\ dTdX - (1-K^2)dX^2 - dY^2 - dZ^2 = 0,$$
$$c^2dt^2 - 2kc\ dtdx - \left(1-k^2\right)dx^2 - dy^2 - dz^2 = 0$$

The degree of anisotropy k becomes a variable taking part in the transformations and obeying the group property

$$k = p(K;a), \quad k \approx K + a\chi(K)$$

$$x = f(X,T,K;a),\ t = q(X,T,K;a), \quad y = g(Y,K;a),\ z = h(Z,K;a)$$

It implies that there exists a "preferred" frame in which the speed of light is isotropic: $k = 0$

Transformations with the variable anisotropy parameter

Transformations

$$x = \frac{e^{-\varphi(a)}}{\sqrt{(1-K\beta)^2-\beta^2}}\left(X - cT\beta\right),$$

$$ct = \frac{e^{-\varphi(a)}}{\sqrt{(1-K\beta)^2-\beta^2}}\left(cT\left(1-K\beta-k\beta\right) - X\left(\left(1-K^2\right)\beta + K - k\right)\right)$$

$$y = e^{-\varphi(a)}Y, \quad z = e^{-\varphi(a)}Z$$

$$\varphi(a) = \int_0^a k(\alpha)d\alpha, \quad k = k(a)$$

$$\frac{dk(\alpha)}{d\alpha} = \chi\left(k\left(\alpha\right)\right); \quad a = \frac{1}{2}\ln\frac{1+\beta-K\beta}{1-\beta-K\beta}$$

Transformations with the variable anisotropy parameter

Interval

$$dS^2 = c^2 dT^2 - 2Kc\ dTdX - (1 - K^2)dX^2 - dY^2 - dZ^2$$
$$ds^2 = c^2 dt^2 - 2kc\ dtdx - \left(1 - k^2\right) dx^2 - dy^2 - dz^2$$

Conformal invariance

$$ds^2 = e^{-2\varphi(a)} dS^2$$

$$\varphi(a) = \int_0^a k(\alpha) d\alpha$$

Special relativity with a preferred frame

The anisotropy parameter k in an arbitrary frame moving with respect to the preferred frame with velocity $\beta = \bar{v}/c$ should be given by some (universal) function $k = F\left(\bar{\beta}\right)$ of that velocity.
With accuracy up to the third order in $\bar{\beta}$, the function $k = F\left(\bar{\beta}\right)$ can be approximated by

$$k = F\left(\bar{\beta}\right) \approx \mu\bar{\beta}$$

$$k(a) = \frac{\mu\left(K\cosh a + \mu\sinh a\right)}{K\sinh a + \mu\cosh a}$$

$$\kappa(k) = \left.\frac{\partial p(k;a)}{\partial a}\right|_{a=0} = \mu - \frac{k^2}{\mu}$$

Extension to general relativity

In the special relativity with a preferred frame, the interval is not invariant but conformally modified under the transformations between inertial frames. Nevertheless, the complete apparatus of general relativity can be applied based on that there is a combination invariant under the transformations which, upon a change of variables, takes the form of the Minkowski interval while the transformations take the form of the Lorentz transformations.
The invariant combination is

$$d\tilde{s}^2 = \frac{1}{\lambda(k)^2}\left(c^2 dt^2 - 2kc\; dtdx - (1-k^2)dx^2 - dy^2 - dz^2\right)$$

where

$$\lambda(k) = \exp\left[-\int_0^k \frac{p}{\kappa(p)}dp\right]$$

with $\kappa(k)$ being the group generator for the variable k.

Extension to general relativity

Introducing the new variables

$$\tilde{t} = \frac{1}{c\lambda(k)}\left(ct - kx\right), \quad \tilde{x} = \frac{1}{\lambda(k)}x, \quad \tilde{y} = \frac{1}{\lambda(k)}y, \quad \tilde{z} = \frac{1}{\lambda(k)}z$$

converts the invariant combination into the Minkowski interval

$$d\tilde{s}^2 = c^2 d\tilde{t}^2 - d\tilde{x}^2 - d\tilde{y}^2 - d\tilde{z}^2$$

Calculating the factor $\lambda(k)$ yields

$$\lambda(k) = \left(1 - \frac{k^2}{\mu^2}\right)^{\mu/2} \quad \Rightarrow \quad B(\bar{\beta}) = \left(1 - \bar{\beta}^2\right)^{\mu/2}$$

or, with the same order of approximation, equivalently

$$B(\bar{\beta}) = 1 - \frac{\mu}{2}\bar{\beta}^2$$

Extension to general relativity

Thus, the general relativity equations in the coordinates (x^0, x^1, x^2, x^3) are valid if the locally inertial coordinates $(\xi^0, \xi^1, \xi^2, \xi^3)$ are defined as

$$\xi^0 = c\tilde{t},\ \xi^1 = \tilde{x},\ \xi^2 = \tilde{y},\ \xi^3 = \tilde{z}$$

However, in the calculation of physical effects, the 'true' time and space intervals in the 'physical' variables (t, x, y, z) are to be used.
Change of notation for physical variables

$$(t, x, y, z) \quad \Rightarrow \quad (t^*, x^*, y^*, z^*)$$

The physical coordinates (t^*, x^*, y^*, z^*) are related to the 'locally inertial' coordinates $(\xi^0, \xi^1, \xi^2, \xi^3)$ by

$$t^* = \frac{1}{c}B(\bar{\beta})\left(\xi^0 + k\xi^1\right),\ x^* = B(\bar{\beta})\xi^1,\ y^* = B(\bar{\beta})\xi^2,\ z^* = B(\bar{\beta})\xi^3; \quad B(\bar{\beta})$$

where $\bar{\beta}$ is the velocity of a locally inertial observer relative to a preferred frame.

Extension to general relativity

The relations of the 'true' ('physical') time and space intervals to the coordinates (x^0, x^1, x^2, x^3).

The invariant spacetime distance squared

$$ds^2 = g_{ik} dx^i dx^k$$

The 'physical' proper time

$$d\tau^* = \frac{1}{c} \sqrt{g_{00}} dx^0$$

The physical proper distance

$$dl^* = B(\bar{\beta}) \sqrt{\gamma_{\alpha\beta} dx^\alpha dx^\beta}, \quad \gamma_{\alpha\beta} = -g_{\alpha\beta} + \frac{g_{0\alpha} g_{0\beta}}{g_{00}}$$

In view of the fact that the time and the distance intervals are modified by the same factor $B(\bar{\beta})$, the expression for the proper velocity of a particle $v = dl^*/dt^*$ does not include that factor.

Robertson–Walker metric

Modern cosmological models are based on the assumption that the universe appear isotropic to "typical" freely falling observers, those that move with the average velocity of typical galaxies in their respective neighborhoods. The metric derived on the basis of isotropy and homogeneity (the *Robertson4–Walker metric*) has the form

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1 - K_c r^2} + r^2 d\Omega\right), \quad d\Omega = d\theta^2 + \sin^2\theta d\phi^2$$

where a co-moving reference system is used. The time coordinate $x^0 = t$ is the synchronous proper time at each point of space. The constant K_c by a suitable choice of units for r can be chosen to have the value $+1$, 0, or -1. The new radial coordinate χ is introduced by the relation $r = S(\chi)$ with

$$S(\chi) = \begin{cases} \sin\chi, & \text{if } K_c = 1; \\ \sinh\chi, & \text{if } K_c = -1; \\ \chi, & \text{if } K_c = 0; \end{cases}$$

The red shift

The *conformal time* η is introduced

$$dt = a(t)d\eta$$
$$ds^2 = a^2(\eta)\left[d\eta^2 - d\chi^2 - S^2(\chi)\, d\Omega\right]$$

A coordinate system, in which we are at the center of coordinates $\chi = 0$ and the source is at the point with a coordinate $\chi = \chi_1$, is adopted. A light ray propagating along the radial direction obeys the equation $d\eta^2 - d\chi^2 = 0$. For a light ray coming toward the origin from the source, that equation gives

$$\chi_1 = -\eta_1 + \eta_0$$

where η_1 corresponds to the moment of emission t_1 and η_0 corresponds to the moment of observation t_0. The observed frequency ν_0 is related to the frequency of the emitted light ν_1 by

$$\frac{\nu_0}{\nu_1} = \frac{a(\eta_1)\, B\left(\bar{\beta}_1\right)}{a(\eta_0)}; \quad z+1 = \frac{\nu_1}{\nu_0} = \frac{a(\eta_0)}{a(\eta_0 - \chi_1)B(\bar{\beta}_1)}; \quad B(\bar{\beta}_1) = 1 - \frac{\mu}{2}\bar{\beta}_1^2$$

The red-shift versus luminosity distance relation

$$d_L = \frac{a^2(t_0)S(\chi_1)}{a(t_1)B(\bar{\beta}_1)} = \frac{a^2(\eta_0)S(\chi_1)}{a(\eta_0 - \chi_1)B(\bar{\beta}_1)} \tag{1}$$

By eliminating $a(\eta_0 - \chi_1)$ using

$$z + 1 = \frac{a(\eta_0)}{a(\eta_0 - \chi_1)B(\bar{\beta}_1)}$$

another form of the relation for d_L is obtained, namely

$$d_L = a(\eta_0)(1 + z)S(\chi_1)$$

This relation coincides with a common form of the relation for d_L. Nevertheless, even though it does not contain the factor $B(\bar{\beta}_1)$, the dependence of d_L on z obtained by eliminating χ_1 will differ from the common one since the relation for z does contain the factor $B(\bar{\beta}_1)$.

The red-shift versus luminosity distance relation

The velocity of an object with respect to the center (with respect to a preferred frame) is

$$\bar{\beta}_1 = \left.\frac{da(t)}{dt}\right|_{t=t_0} \chi_1 = a'(t_0)\chi_1 = \frac{a'(\eta_0)}{a(\eta_0)}\chi_1$$

Then eliminating χ_1 from the relations for d_L and z

$$d_L = a(\eta_0)(1+z)S(\chi_1), \qquad z+1 = \frac{a(\eta_0)}{a(\eta_0-\chi_1)(1-\frac{\mu}{2}\bar{\beta}_1^2)}$$

yields

$$d_L = H_0^{-1}\left(z + \frac{1}{2}(1-q_0-\mu)\,z^2 + \cdots\right)$$

$$H_0 = \frac{a'(t_0)}{a(t_0)} = \frac{a'(\eta_0)}{a^2(\eta_0)}, \quad q_0 = -\frac{1}{H_0^2 a(t_0)}\left.\frac{d^2a(t)}{dt^2}\right|_{t=t_0} = 1 - \frac{a''(\eta_0)}{H_0^2 a^3(\eta_0)}$$

The red-shift versus luminosity distance relation

Compare this equation

$$d_L = H_0^{-1}\left(z + \frac{1}{2}\left(1 - q_0 - \mu\right)z^2 + \cdots\right)$$

with that obtained by solving Friedmann equations

$$d_L = H_0^{-1}\left(z + \frac{1}{2}\left(1 - q_0^{(D)}\right)z^2 + \cdots\right)$$

For a matter-dominated homogeneous cosmological model the deceleration parameter $q_0^{(D)}$ is positive for all three possible values of the parameter K_c which implies that the expansion of the universe is decelerating. However, recent observations of Type Ia supernovae, fitted into the luminosity distance versus redshift relation of that form, correspond to the deceleration parameter $q_0^{(D)} < 0$ which indicates that the expansion of the universe is accelerating. In order to explain the discrepancy within the context of General Relativity, a new component of the energy density of the universe, known as **Dark Energy** (vacuum energy) is introduced.

Baryon acoustic oscillations

Baryon acoustic oscillations (BAO) refers to a series of peaks and troughs that are present in the power spectrum of matter fluctuations due to acoustic waves which propagated in the early universe. Measurements of the length scale characteristic of these oscillations enable inferring the angular diameter distance out to galaxies probed in a survey.
In principle, the BAO scales in both transverse and line-of-sight directions can be obtained; they correspond to the quantities $\theta_s(z)$ and $\delta z_s(z)$ respectively

$$\theta_s(z) = \frac{r_s(z_d)}{d_A^{(c)}(z)}, \quad \delta z_s(z) = r_s(z_d)H(z) \tag{2}$$

where $r_s(z_d)$ is the sound horizon at the drag epoch and $d_A^{(c)}(z)$ is the comoving angular diameter distance related with the proper angular diameter distance d_A via the relation

$$d_A^{(c)}(z) = \frac{d_A}{a(t_1)} = \frac{a(t_1)S(\chi_1)}{a(t_1)} = S(\chi_1) \tag{3}$$

Baryon acoustic oscillations

However, since the current BAO observations are not sufficient to measure both scales independently, the quantity

$$r_{BAO} = \frac{r_s(z_d)}{D_V(z)} \tag{4}$$

is used where

$$D_V(z) = \left[\left(d_A^{(c)}(z)\right)^2 z/H(z)\right]^{1/3} \tag{5}$$

In the present model

$$r_{BAO} = \frac{r_s(z_d)}{B\left(\bar{\beta}\,(z)\right) D_V(z)} \tag{6}$$

The recent measurements provided the ratio

$$\frac{r_{BAO}(0.2)}{r_{BAO}(0.35)} = 1.736 \pm 0.065 \tag{7}$$

Conclusions

- The group property and the invariance of the equation of light propagation have been taken as first principles to derive transformations between inertial frames in the case when there exists an anisotropy in the light propagation.
- If an anisotropy in the light propagation is present, the strict invariance of the interval between two events is replaced by conformal invariance.
- Extension to general relativity yields corrections to cosmological models which, in principle, allow one to explain the observations, that are commonly fitted into the concordance model with a dark energy, **within a framework of a matter-dominated universe**.