Statistical Models vs. Mean Field Models
Model Constraints
Novel Equation of State
Applications
Conclusions

Neutron stars equation of state consistent with high-energy nuclear physics data

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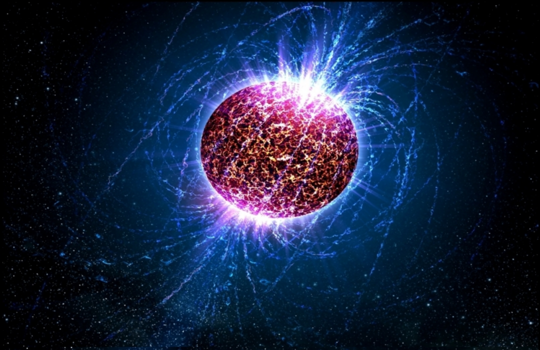

ICNFP 2017, Crete, 19.08.2017

Outline

1 Statistical Models vs. Mean Field Models

2 Model Constraints

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3 Novel Equation of State

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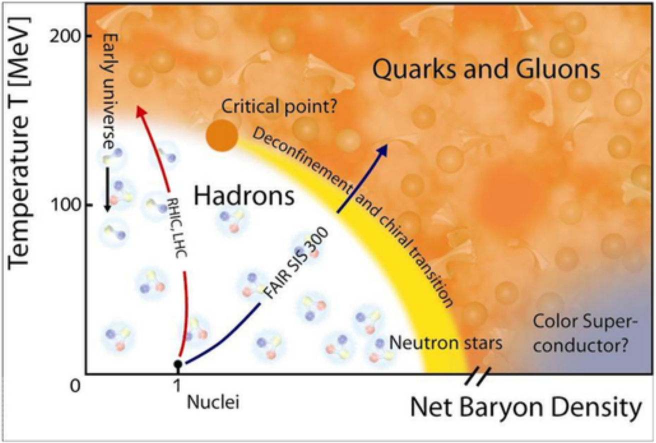
Strongly Interacting Matter Phase Diagram



Models of hadronic/nuclear matter EoS

- **Statistical Cluster models** - Fisher Droplet Model, Gas of Bags Model, Statistical Multifragmentation Model, Hadron Resonance Gas Model
  Advantage: fluctuations $\Rightarrow$ physical critical point
  Problems: Hard core repulsion violates causality

- **Relativistic Mean Field models** - Walecka Model, Chiral PerturbationTheory, etc.
  Advantage: QCD symmetries are preserved
  Problems: 1. NO fluctuations $\Rightarrow$ unrealistic critical point
  2. Only few particle species

- **Hybrid approach** - Walecka model with nonrelativistic proper volume of nucleons

$$p(T, \mu) = p_{Walecka}(T, \mu - pV_{eigen})$$

D.H. Rischke, M.I. Gorenstein, H.Stoecker and W. Greiner, Z. Phys. C 51(1991)

Why hard core repulsion?

Hadronic hard core

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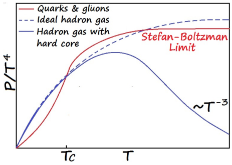
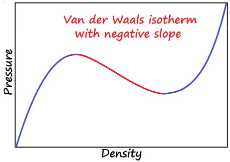
- Prevents phenomenological EoS of QCD from quark confinement at high temperatures

$$\textit{ideal gas}: \quad p \sim T^4$$

$$\textit{hard core}: \quad p \sim T$$



- Accounts for short range repulsion between the constituents (hadrons, nuclear fragments, etc.)



- Is necessary for statistical (not Van der Waals) liquid-gas phase transition in cluster models

- Important element in description of particle yields (ideal hadron gas is proven to be inadequate at high A+A collision energies)
  J. Cleymans and H. Satz, Z. Phys. C 57, 135 (1993)
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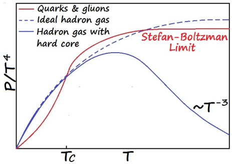

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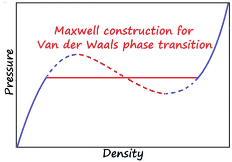

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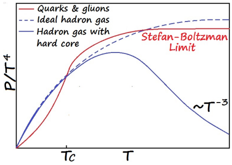

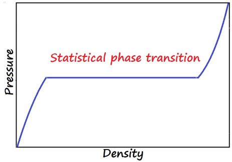

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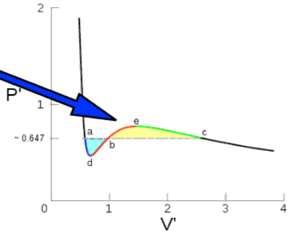
Basics of the VdWaals EOS

$$P = \underbrace{\frac{N\,T}{V - N\,b}}_{repulsion} - \underbrace{\frac{N^2 a}{V^2}}_{attraction} \quad \text{or} \quad \left(P + \frac{N^2 a}{V^2}\right)(V - N\,b) = NT$$

Although VdWaals EOS behavior contradicts the 2-nd Van Hove axiom of statistical mechanics



Van der Waal Isotherm T' = 0.90

Maxwell's rule eliminates the oscillating behavior of the isotherm in the phase transition zone by defining it as a certain isobar in that zone.

Hardcore Repulsion

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- General prescription for hard core repulsion $\Rightarrow$ **EoS of the Van der Waals type**

$$p(T,\mu) = p_{ideal}(T, \mu - pV_{eigen})$$

D.H. Rischke, et al. , Z. Phys. C 51(1991) 485

- Several hard core radii

$$p(T,\mu) = \sum_h p_{ideal}(T, \mu_h - pV^h_{eigen})$$

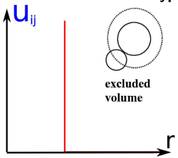
Advantage: partial account of multicomponent repulsion
Problems: NO repulsion between different particle species

- Excluded volume formalism $V^{ij}_{excl} = \frac{1}{2} \cdot \frac{4\pi}{3}(R_i + R_j)^3$

K. Bugaev, et al. , Eur. Phys. J. A 49 (2013) 30

Advantage: multicomponent repulsion
Problems: n species $\Rightarrow$ n equations – **complicated EoS**

hard-core repulsion of the Van der Waals type

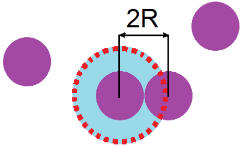


Simple multicomponent EoS is needed

Constraints on Hadronic/Nuclear Matter EoS

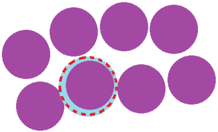
- **Multicomponent EoS** $\Rightarrow$ Grand Canonical Ensemble (GCE) is natural choice
- **Thermodynamic consistency** (in GCE pressure is function of T and μ only)
  $p = p(T, \mu, n) \Rightarrow$ contradiction with thermodynamic relation $n = \frac{\partial p}{\partial \mu}$
  L. van Hove, Physica 15, 951 (1949) and Physica 16, 137 (1950)
- **Switching between excluded and eigen volumes (per particle)**

Low densities



$V_{excl} \simeq 4 V_{eigen}$

High densities



$V_{excl} \simeq V_{eigen}$

high order virial coefficients are needed

- **Causality:** $c_{sound} \leq c_{light} = 1$, where $c_{sound}^2 = \frac{dp}{d\epsilon}\Big|_{s/n=const}$
  Existing approaches to restore causality violated by hard core are rather complicated
  K. Bugaev, Nucl. Phys. A 807 (2008)

Something more convenient for practical applications is needed

Application to Compact Astrophysical Objects

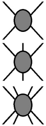
- Hadronic EoS used for modeling of the neutron star interiors **violates causality**
  H. Grigorian, COST Action @ CPOD 2016, Wroclaw

- Three, four, ... particle forces are needed - **high order virial coefficients**
  G. Baym, COST Action @ CPOD 2016, Wroclaw

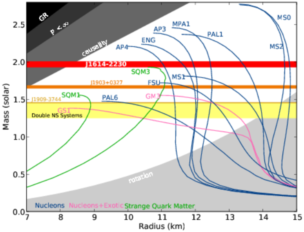
  $2 \rightarrow 2$ reactions $\Rightarrow$ **2^{nd} virial coefficient**

  $3 \rightarrow 3$ reactions $\Rightarrow$ **3^{rd} virial coefficient**

  $4 \rightarrow 4$ reactions $\Rightarrow$ **4^{th} virial coefficient**



- Observational constrains require soft/stiff EoS at low/high densities
  T. Kojo, COST Action @ CPOD 2016, Wroclaw
  hyperons $\Rightarrow$ softening of the hadronic EoS
  multicomponent EoS in GCE is needed



Objectives

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- Simple multicomponent EoS with hard core repulsion
- Thermodynamic consistency
- Correct asymptotics of excluded volume EoS at low and high densities
- Higher virial coefficients
- Causality up to densities where QGP is expected

Cluster and Virial Expansions

- GC partition for one component Boltzman system

$$Z = \exp\left[V \sum_k b_k e^{\frac{k\mu}{T}}\right] \Rightarrow \underbrace{\begin{matrix} p = T\sum_k b_k e^{\frac{k\mu}{T}} \\ n = T\sum_k k b_k e^{\frac{k\mu}{T}} \end{matrix}}_{\text{cluster expansion in GCE}}, \quad p = \underbrace{Tn\sum_k a_k n^k}_{\text{virial expansion in CE}}$$

Cluster integrals

$b_1 = \int \frac{d\Gamma}{1!V} e^{-\frac{\omega}{T}}$ - one particle thermal density

$b_2 = \int \frac{d\Gamma_1 d\Gamma_2}{2!V} e^{-\frac{\omega_1+\omega_2}{T}} f_{12}$

...

Virial coefficients

$a_1 = 1$

$a_2 = -\frac{b_2}{b_1^2}$

...

$d\Gamma = g\frac{d\tilde{r}d\tilde{k}}{(2\pi^3)}$, g - degeneracy, $\omega^2 = k^2 + m^2$, $f_{12} = e^{-\frac{U_{12}}{T}} - 1$ - Mayer function

- Multicomponent mixture of hard spheres with radii $\{R_i\}$

$$\begin{matrix} b_1^i = \phi_i, \; b_2^{ij} = -\phi_i\phi_j V_{ij}^{excl} \\ V_{ij}^{excl} = \frac{2\pi}{3}(R_i + R_j)^3 \end{matrix} \Rightarrow p = T\sum_i \phi_i e^{\frac{\mu_i}{T}} - \sum_{i,j} V_{ij}^{excl}\phi_i\phi_j e^{\frac{\mu_i+\mu_j}{T}} + \mathcal{O}(\phi^3)$$

- VdW extrapolation for one component system

$$\phi e^{\frac{\mu}{T}} \simeq \frac{p}{T} \quad \Rightarrow \quad p \simeq T\phi e^{\frac{\mu}{T}}\left(1 - \frac{bp}{T}\right) = T\phi e^{\frac{\mu - bp}{T}}$$

Extrapolation to High Densities

- Explicit expression for V_{ij}^{excl} $\Rightarrow$

$$\frac{p}{T} = \overbrace{\sum_i \phi_i e^{\frac{\mu_i}{T}}}^{\text{ideal gas}} - \sum_{i,j} \phi_j \phi_j e^{\frac{\mu_i + \mu_j}{T}} \left(\overbrace{\frac{2\pi}{3} R_i^3 + \frac{2\pi}{3} R_j^3}^{\text{the same}} + \overbrace{2\pi R_i^2 R_j + 2\pi R_i R_j^2}^{\text{the same}} \right) + \mathcal{O}(\phi^3)$$

$$= \sum_i \phi_i e^{\frac{\mu_i}{T}} \left(1 - \underbrace{\frac{4\pi}{3} R_i^3}_{\text{volume } V_i} \sum_j \phi_j e^{\frac{\mu_j}{T}} - \underbrace{4\pi R_i^2}_{\text{surface } S_i} \sum_j R_j \phi_j e^{\frac{\mu_j}{T}} \right) + \mathcal{O}(\phi^3)$$

Term proportional to circumference $L_i = 2\pi R_i^2$ is accounted implicitly

- Bulk term $\sum_j \phi_j e^{\frac{\mu_j}{T}} = \frac{p}{T} + \mathcal{O}(\phi^2)$, surface term $\sum_j R_j \phi_j e^{\frac{\mu_j}{T}} = \frac{\Sigma}{T} + \mathcal{O}(\phi^2)$
- VdW like extrapolation:

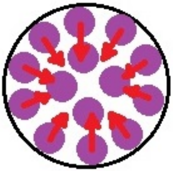
$$\frac{p}{T} = \sum_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i}{T} \right) - \textbf{ pressure}$$

$$\frac{\Sigma}{T} = \sum_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i}{T} \right) R_i - \textbf{ surface tension}$$

V. Sagun,et al., Nucl. Phys. A 924, 24 (2014)

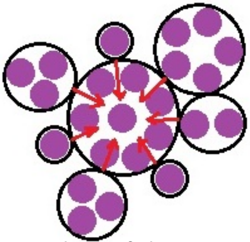
Physical Origin of the Induced Surface Tension

Vacuum



attraction of constituents
⇒ **eigen surface tension**

Medium



repulsion of clusters
⇒ **induced surface tension**

- Hard core repulsion only in part is accounted by eigen volume
- The rest corresponds to surface tension and curvature tension
  Curvature tension can be accounted explicitly or implicitly
- Physical clusters tend to have spherical (in average) shape

Extrapolation to High Densities

- Extrapolation to high densities is not unique $\Rightarrow$ equations for p and Σ can differ

$$\frac{p}{T} = \sum_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i}{T}\right)$$

$$\frac{\Sigma}{T} = \sum_i R_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i}{T}\right) \cdot \overbrace{\exp\left(\frac{(1-\alpha) S_i \Sigma}{T}\right)}^{\text{not uniqueness of extrapolation}}, \quad \alpha = const$$

- Meaning of $\alpha > 1$: one component case

$$\Sigma = pR \exp\left(\frac{(1-\alpha)S\Sigma}{T}\right)$$

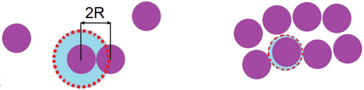
$$p = T\phi \exp\left(\frac{\mu - pV_{eff}}{T}\right) \quad \Rightarrow \quad \begin{array}{l} \textit{low densities } (\Sigma \to 0): V_{eff} = 4V \\ \textit{high densities } (\Sigma \to \infty): V_{eff} = V \end{array}$$

$$V_{eff} = V\left[1 + 3\exp\left(\frac{(1-\alpha)S\Sigma}{T}\right)\right]$$

α switches excluded and eigen volume regimes

high order virial coefficients?

- Higher virial coefficients of hard spheres



  - **Second virial coefficient** – reproduc
  - **Third virial coefficient** – reproduced for $\alpha = 1.245$ whithin 16%
  - **Fourth virial coefficient** – reproduced for $\alpha = 1.245$
- IST EoS is causal up to $\simeq 7$ normal nuclear densities where quark matter is expected

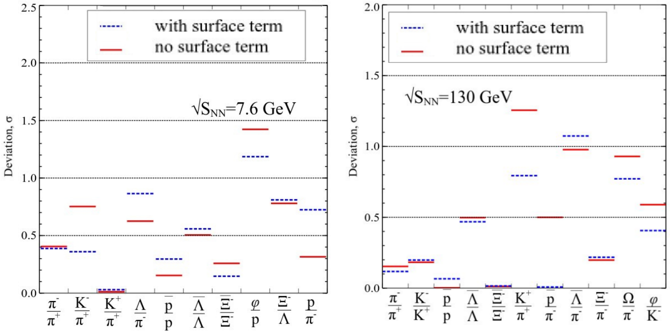
EoS with Induced Surface Tension

$$p = \sum_i p_0(T, \mu_i) \;\Rightarrow\; \begin{cases} p = \sum_i p_0(T, \mu_i - pV_i - \Sigma S_i) \\ \Sigma = \sum_i p_0(T, \mu_i - pV_i - \alpha \Sigma S_i) R_i \end{cases}$$

- Advantages compared to other EoS with hard core repulsion:
  - Multicomponent character and thermodynamic consistency
  - Correct asymptotic of excluded volume at high and low densities, higher virial coefficients
  - Wide range of causality
  - Straightforward generalization to quantum statistics and mean field mod
- Questions
  - Value of α in case of quantum statistics? Medium dependent α?
  - ...

Hadron Resonance Gas Model

- Hadrons with masses $\leq$ 2.5 GeV (widths, strong decays, zero strangeness)
- 111 independent particle ratios measured at 14 energies (from 2.7 GeV to 200 GeV)
- 14 $\times$ 4 local parameters $(T, \mu_B, \mu_{I3}, \gamma_s)$ + 5 global parameters (hard core radii)



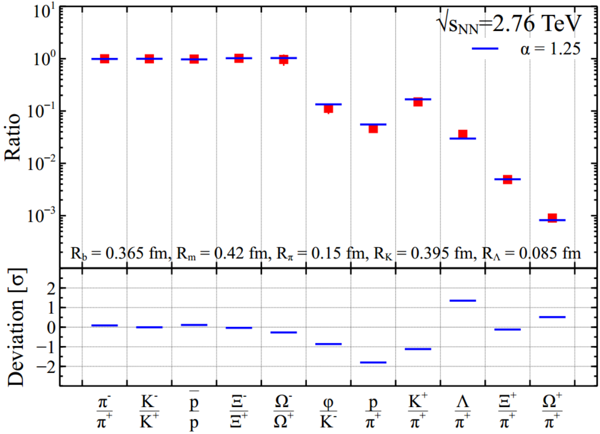
$R_b = 0.365\ fm,\ R_m = 0.42\ fm,\ R_\pi = 0.15\ fm,\ R_K = 0.395\ fm,\ R_\Lambda = 0.085\ fm$

$Overall\ \chi^2/dof \simeq 1.038$

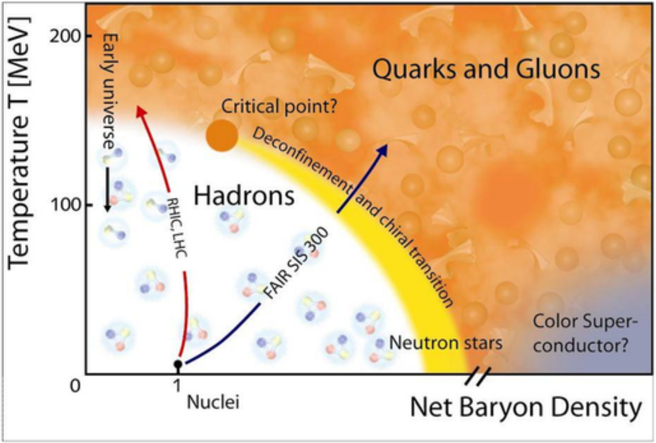
V.V. Sagun et al., arXiv:1703.00049

Hadron Resonance Gas at ALICE Energies

- 11 independent particle yields, 6 parameters (temperature + 5 hard core radii)
- Overal $\chi^2/dof \simeq 1.038$
- Freeze out temperature $T_{FO} = 154 \pm 7\ MeV$



Strongly Interacting Matter Phase Diagram



Attraction term

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$$U(n_{id}) = -C_d^2 n_{id}^{\frac{1}{3}} \equiv -Ck\,, \qquad (1)$$

where $C = C_d^2 \left(\frac{g}{6\pi^2}\right)^{\frac{1}{3}}$.
Condition of thermodynamic consistency of model with the mean-filed interaction requires the special relation between the interaction pressure and potential:

$$\frac{\partial p_{int}}{\partial n_{id}} = n_{id} \frac{\partial U(n_{id})}{\partial n_{id}} \qquad (2)$$

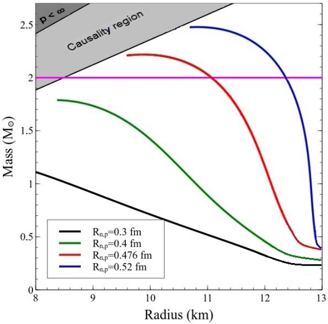
and

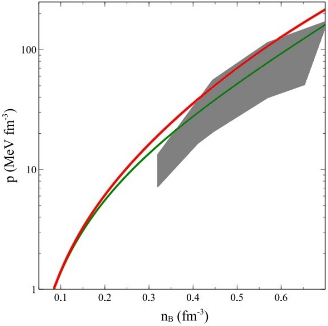
$$p_{int}(n_{id}) = n_{id} U(n_{id}) - \int_0^{n_{id}} dn\ U(n) = -\frac{gCk^4}{24\pi^2}\,. \qquad (3)$$

Such a generalization of the EoS corresponds to the substitution of the pressure with $p(m,\ \mu) \to p(m,\ \mu - U(n_{id})) + p_{int}(n_{id})$. This parametrization provides causal behaviour of the model EoS at high densities.

Mass-Radius Relation

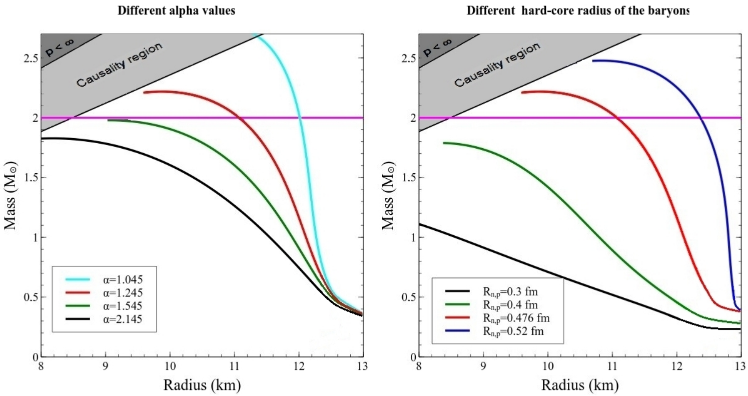
Values of the model parameters of the IST EoS found for the two limiting cases of neutron stars which satisfy all constraints and the parameters found from the description of A+A collisions.





	baryon radius (fm)	α	C	max $M_{NS}/M_{\odot}$
red curve	0.476	1.245	0.067	2.217
green curve	0.425	1.06	0.062	2.166
A+A collisions	0.355	1.245	0.067	1.544

Neutron Stars Interior



Effect of the IST

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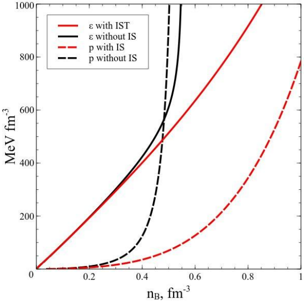

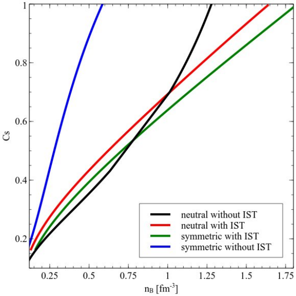

Summary

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- Using a novel a thermodynamically self-consistent IST EoS we calculated the properties of the NS at zero-temperature limit.
- It was shown that the present EoS can be successfully applied to the description of the hadron multiplicities measured in A+A collisions, to studies of the nuclear matter phase diagram and to modelling of the NS interiors.
- The found values of the hard-core radius of baryons between 0.425 fm and 0.476 fm, α between 1.06-1.245. Pressure-baryon density dependence are in good agreement with A+A collisions.
- The description of the compact stars with the IST, used to the description of the nuclear collision physics data, provide with a strong constraint on the attraction contribution in EoS at zero temperature.
- The IST EoS gives a possibility to describe the strongly interacting matter phase diagram in a wide range of its thermodynamic parameters which helps to create a solid bridge between the astrophysical and high-energy nuclear physics data.


thank you for your attention!