



# Coherent diffusive photonics

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**Quantum Optics &  
Quantum Information**



Thermal light – incoherent, diagonal in energy basis

Thermodynamic light – flows like heat and retains coherence  
off-diagonal elements do not decay

Entangled state from uncorrelated modes

Optical equaliser

Reservoir: Non-Landauer erasure

Passive linear structure, dissipative coupling

## Quantumness by dissipation (some examples)

Interaction with a common environment can lead to the creation of an entangled state from an initial separable state

*F. Benatti and R. Floreanini, J. Phys. A: Math. Gen. 39, 2689 (2006);  
D. Mogilevtsev, T. Tyc, and N. Korolkova, Phys. Rev. A 79, 053832 (2009)*

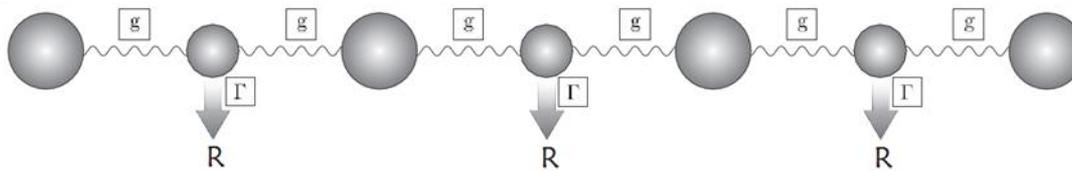
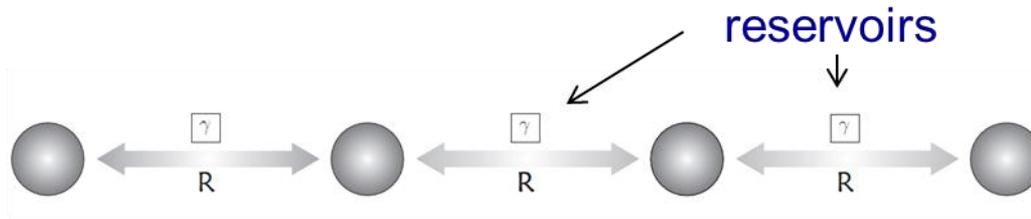
Quantum computation, quantum state engineering, and quantum phase transitions driven by dissipation

*F. Verstraete, M. M. Wolf, and J. I. Cirac, Nature Physics 5, 633 (2009)*

Dissipatively driven entanglement of two macroscopic atomic ensembles

*C. A. Muschik, E. S. Polzik, and J. I. Cirac, Phys. Rev. A 83, 052312 (2011)*

# System: chain of dissipatively coupled modes

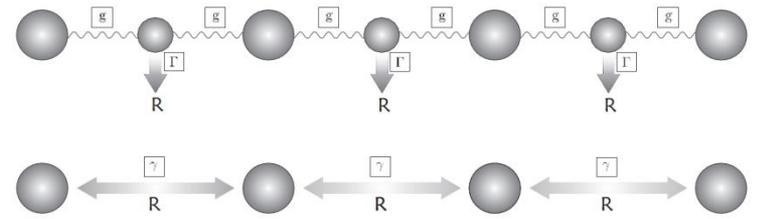


$$\frac{d}{dt}\rho = \sum_{j=1}^N \gamma_j \left( 2A_j \rho A_j^\dagger - \rho A_j^\dagger A_j - A_j^\dagger A_j \rho \right)$$

$A_j = a_j - a_{j+1}$  - Lindblad operators       $\gamma_j$  - relaxation rates into  $j$ -reservoir

$$\gamma_j = \begin{cases} \gamma, & 1 \leq j \leq N, \\ 0, & j \leq 0, j \geq N + 1 \end{cases} \quad \text{- finite size homogeneous chain}$$

## Coupled tight-binding chain of harmonic oscillators



$$\rho(t) = \int d^2 \vec{\alpha} P(\vec{\alpha}, \vec{\alpha}^*; t) |\vec{\alpha}\rangle \langle \vec{\alpha}|, \quad |\vec{\alpha}\rangle = \prod_j |\alpha_j\rangle$$

Lindblad  $\rightarrow$  Fokker-Planck for P-function  $\rightarrow$  Dynamics for coherent amplitudes

$$\frac{d}{dt} \alpha_k = -(\gamma_k + \gamma_{k-1}) \alpha_k + \gamma_k \alpha_{k+1} + \gamma_{k-1} \alpha_{k-1}$$

**- same equation as time-dependent classical random walk in 1D**

*For dissipatively coupled chain of two-level systems (“fermionic chain”) see:  
D Mogilevtsev, G Ya Slepyan, E Garusov, Ya Kilin and N Korolkova: Quantum tight-binding chains with dissipative coupling, New J. Phys. 17, 043065 (2015).*

$$\frac{d}{dt}\alpha_k = -(\gamma_k + \gamma_{k-1})\alpha_k + \gamma_k\alpha_{k+1} + \gamma_{k-1}\alpha_{k-1}$$

- time-dependent classical random walk in 1D

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## Continuous limit – heat transport Fourier equation

$\langle a_j(t) \rangle$  - 1D heat transport equation for  $\langle a(x; t) \rangle$ ,  $x - j$ -s mode

$$\frac{d}{dt}\langle a(x; t) \rangle \approx a^2\gamma \frac{\partial^2 \langle a(x; t) \rangle}{\partial x^2}$$

$\langle a_j^\dagger(t)a_k(t) \rangle$  - 2D heat transport equation; etc

"heat-like" flow of quantum correlations btw different modes in the chain (can be even entangled); "effective temperature"; heat conductivity - etc

**collective phenomenon**

## collective phenomenon

$\alpha_j$  - complex, no classical probabilities

collective symmetrical superposition of all modes:  $A_{sum} = \sum_{j=1}^{N+1} a_j / \sqrt{N+1}$

**Conserved:** average of any function of  $A_{sum}, A_{sum}^\dagger$

$$W_1 = \sum_{k=1}^{N+1} \langle a_k(t) \rangle = \sum_{k=1}^{N+1} \langle a_k(0) \rangle$$

## **Optical equalizer:**

Multi-mode quantum state is symmetrised over all modes

## Coherent symmetrisation: output – not a statistical mixture but a pure state

$$\sum_{k=1}^N \langle \hat{a}_k(t) \rangle = \sum_{k=1}^N \langle \hat{a}_k(0) \rangle \quad \text{- preserved all time}$$

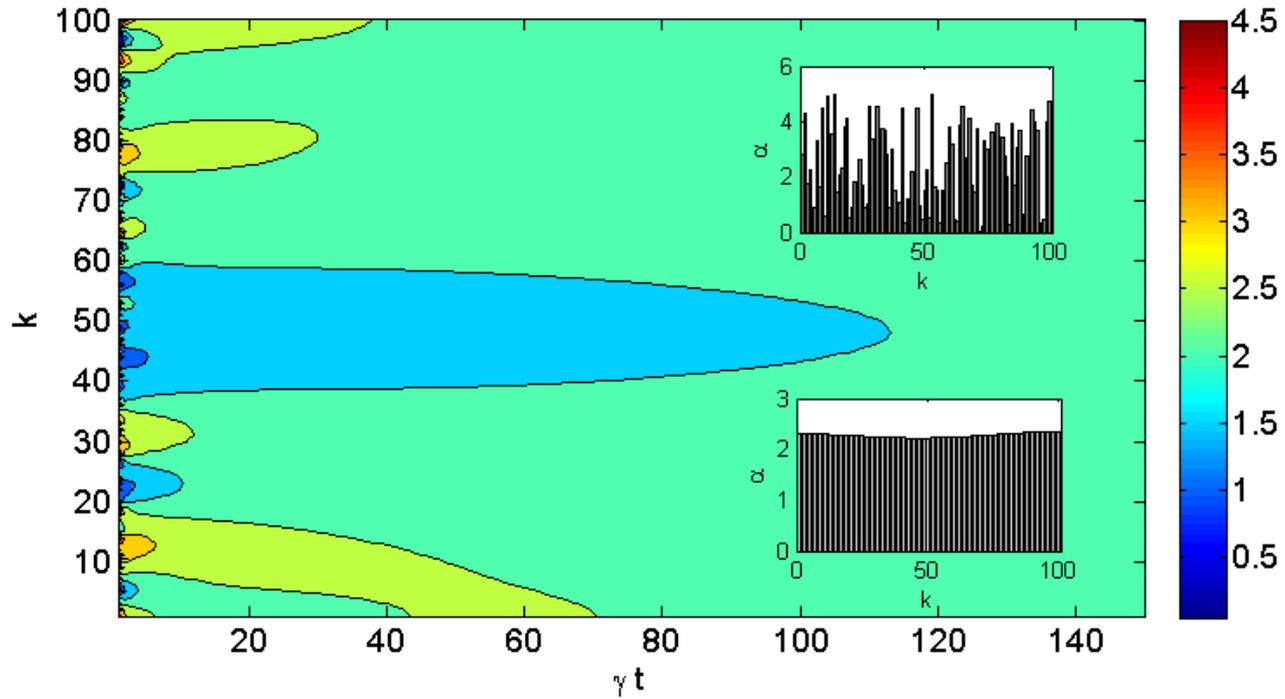
**Input coherent state:**  $|\Psi_{coh}\rangle = \prod_{j=1}^N |\alpha_j\rangle_j$

**Output:**  $|\Psi_{purecoh}\rangle = \prod_{j=1}^N |\alpha_s\rangle_j = |\Phi_{st}\rangle, \quad \alpha_s = \frac{1}{N} \sum_{j=1}^N \alpha_j$

Can eliminate light: for same amplitudes & random phase output tends to zero;

Can suppress fluctuations: zero-mean random fluctuations will be smoothed out:

$$\alpha_j = \bar{\alpha} + \delta\alpha, \quad \langle \delta\alpha \rangle = 0, \quad N \gg 0 \quad \text{yields a set of coherent states each with } \bar{\alpha}$$

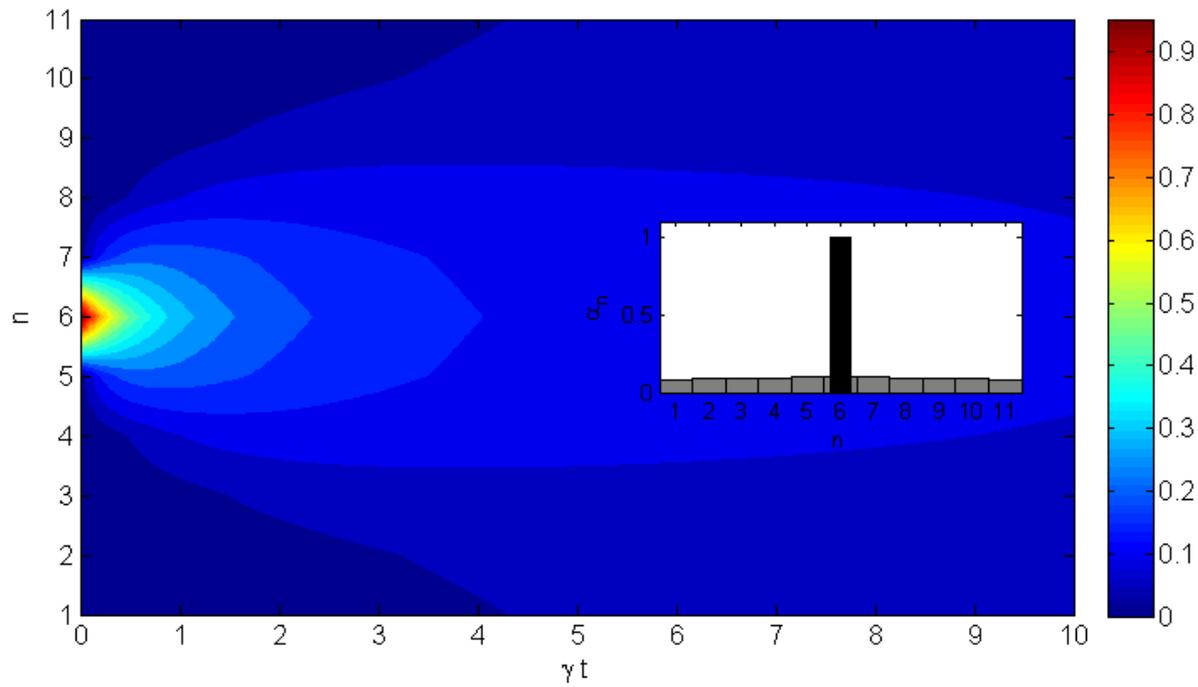


Amplitudes of the coherent states in the bosonic chain of 100 modes.

Top insert: Initial distribution of real coherent amplitudes.

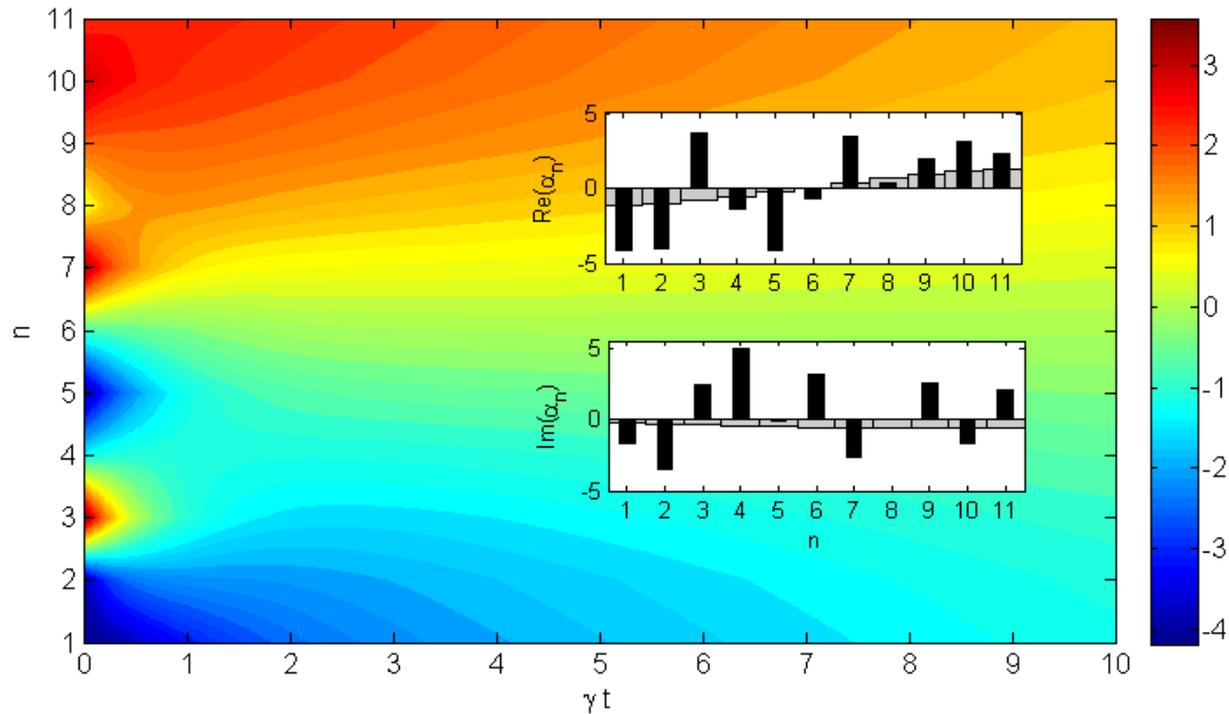
Bottom insert: Amplitude distribution at  $\gamma t = 120$

$$\frac{d}{dt} \langle a_k \rangle = -(\gamma_k + \gamma_{k-1}) \langle a_k \rangle + \gamma_k \langle a_{k+1} \rangle + \gamma_{k-1} \langle a_{k-1} \rangle$$



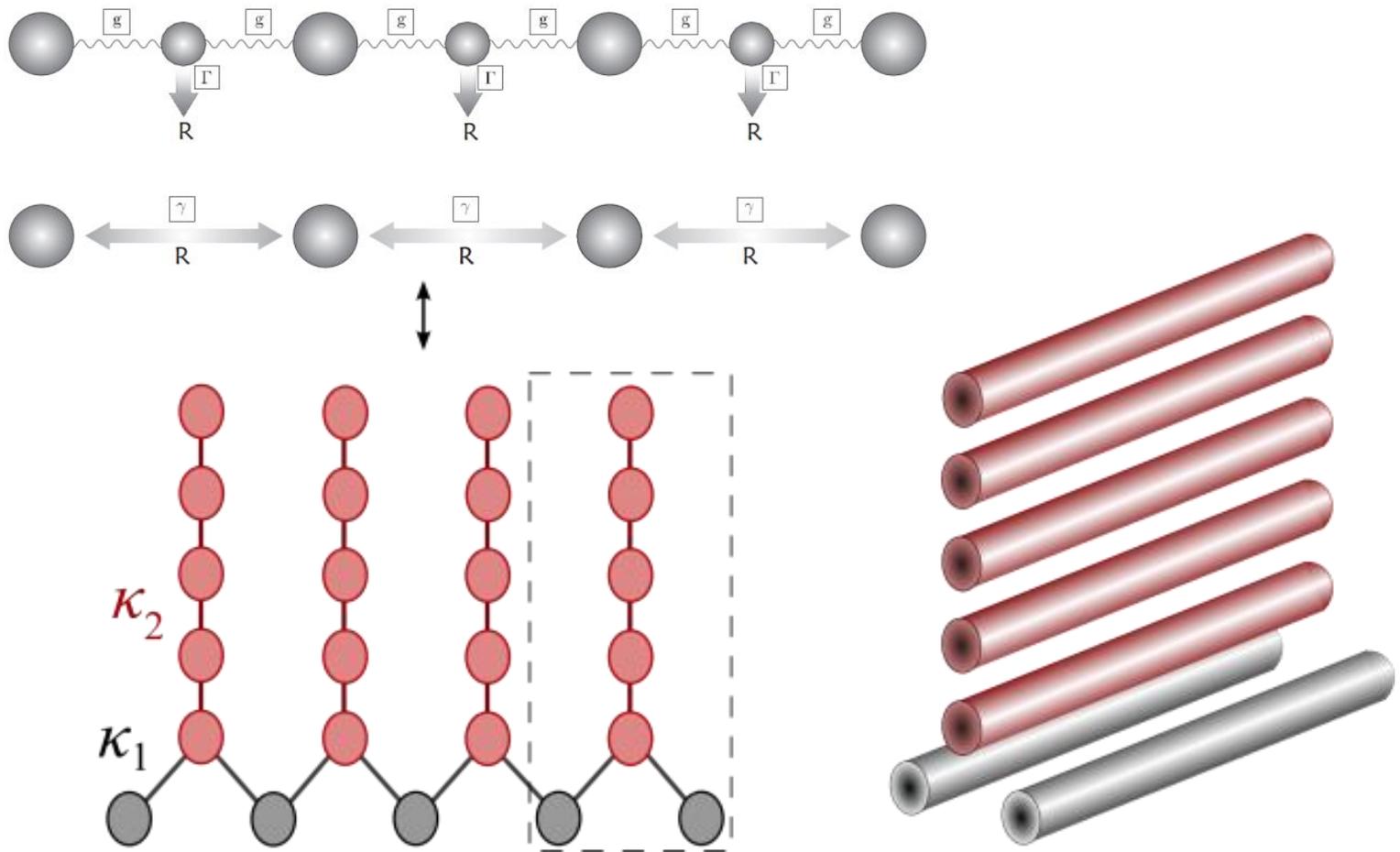
Amplitudes of the coherent states propagating through the dissipatively coupled chain of 11 modes. Insert: initial (black bar) and final (grey bars) distributions of coherent amplitudes at  $\gamma t = 10$

The states are always coherent with amplitudes  $a_n$



Amplitudes of the coherent states propagating through the dissipatively coupled chain of 11 modes. Insert: initial (black bars) and final (grey bars) distributions of real and imaginary parts of coherent amplitudes at  $\gamma t = 10$

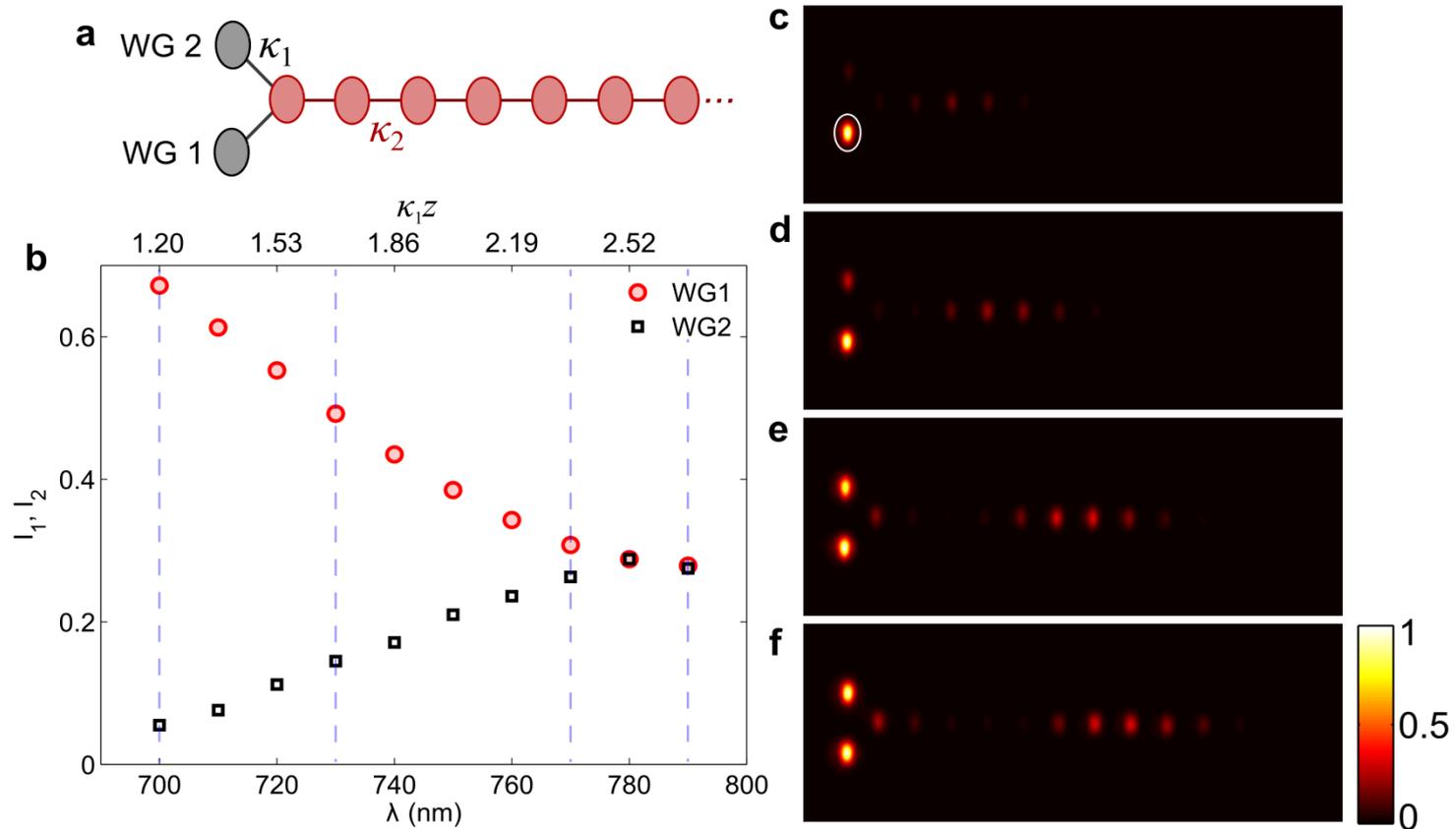
The states are always coherent with amplitudes  $\alpha_n$



## Implementation

*Experiment: Sebabrata Mukherjee and Robert Thomson,  
Photonic Instrumentation Group, Heriot Watt Univ, UK*

# Experimental results for the simplest element



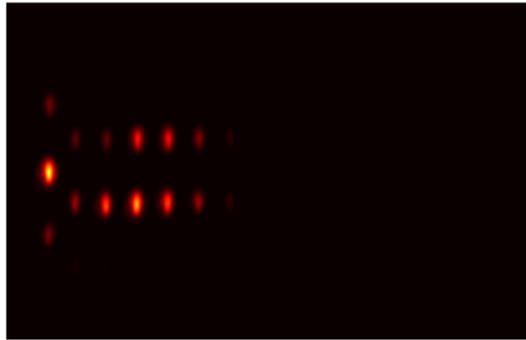
*Experiment/simulation: Sebabrata Mukherjee and Robert Thomson,  
Photonic Instrumentation Group, Heriot Watt Univ, UK*

# Experimental/theoretical results for the 5 waveguide chain

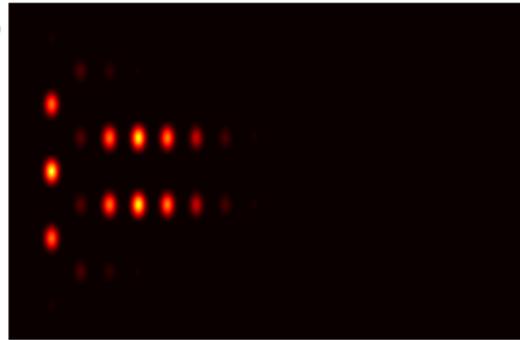
Experiment

Theory

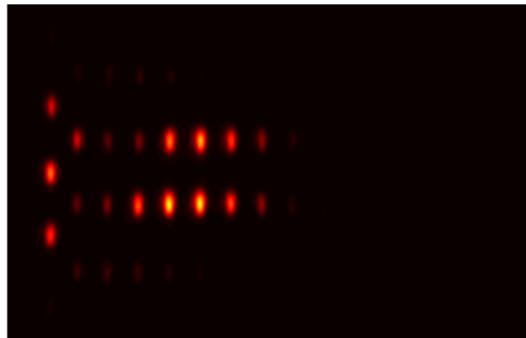
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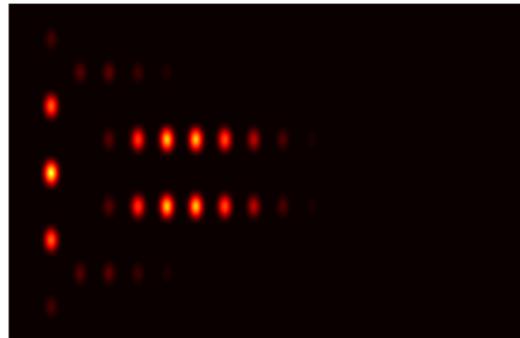
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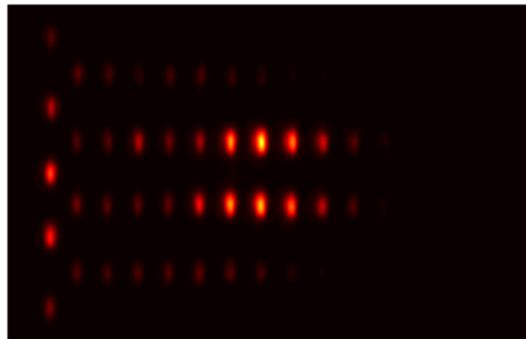
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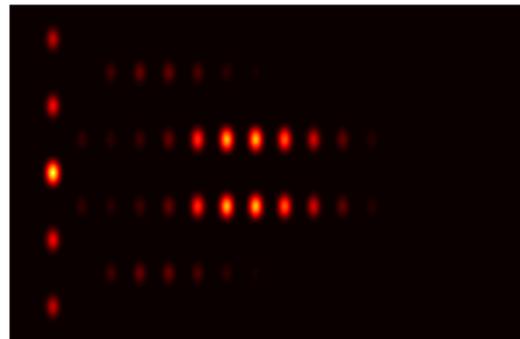
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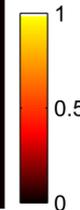


f



Intensity distributions at the output of the 30-mm-long photonic lattice;

effectively - 5 coupled modes.



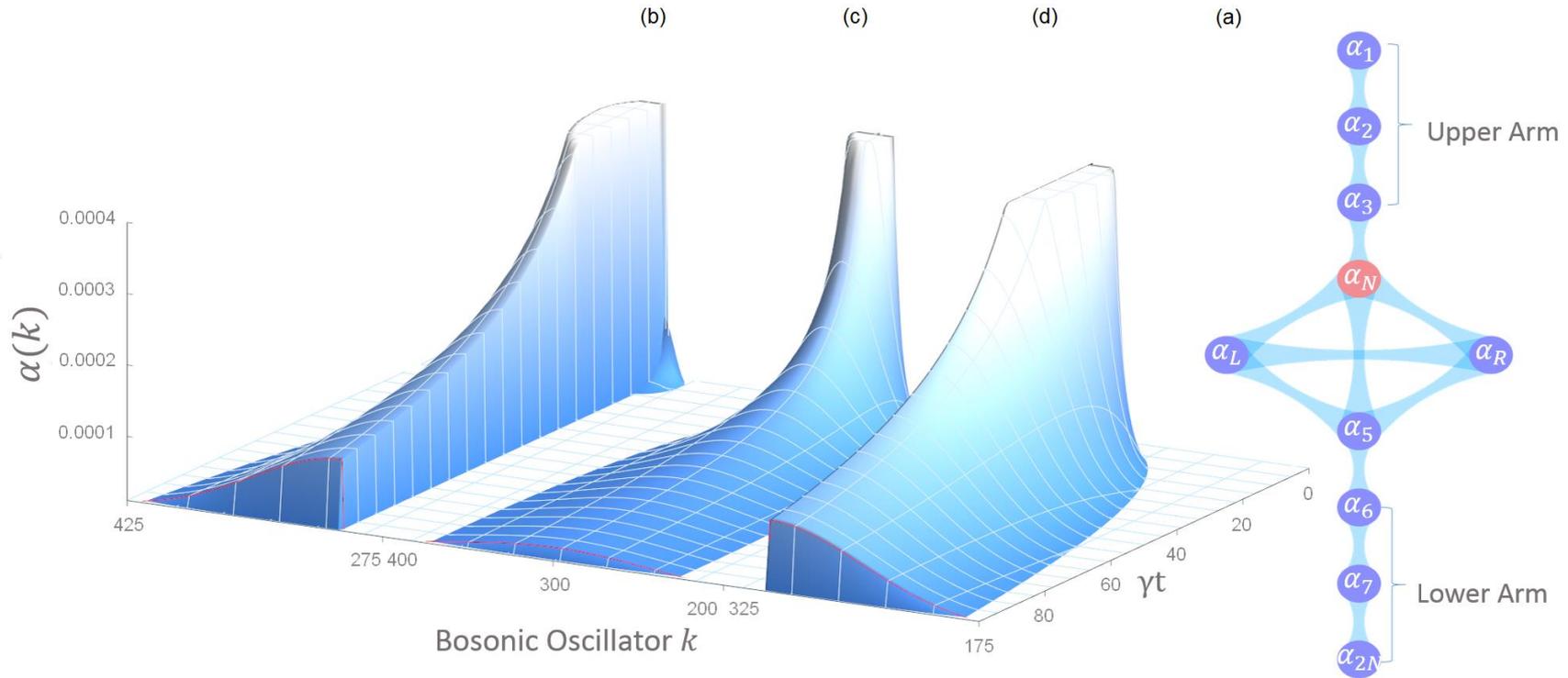
*Experiment/simulation: Sebabrata Mukherjee and Robert Thomson, Photonic Instrumentation Group, Heriot Watt Univ, UK*

Beyond the equalisation:

diffusive dissipative distribution and localization

# Diffusive light distribution:

$$L_{central} = a_N + a_S - a_L - a_R$$



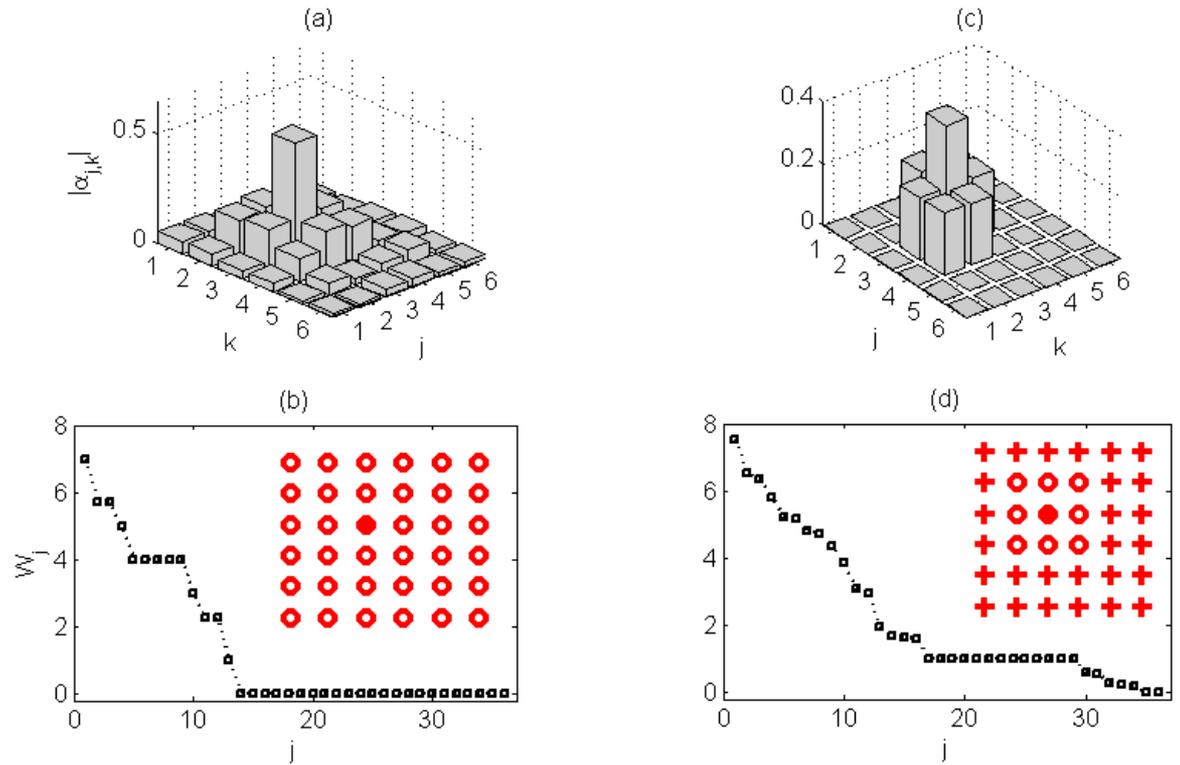
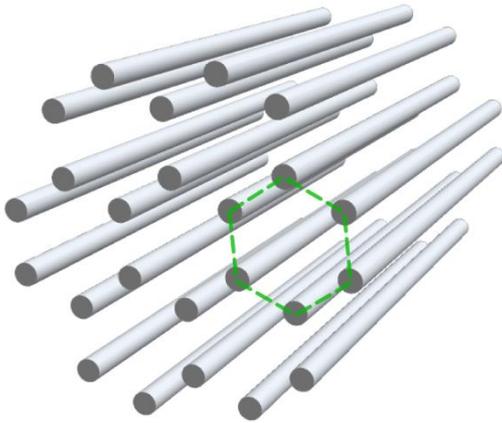
**(a) The simplest dissipative distributing structure with two arms.  $N = 600$ .**

(b) Both control modes R and L are excited equally (or if both control modes are left in the vacuum state). Light is directed into the upper arm only.

(c) When the control mode L is excited initially, the excitation spreads equally into both arms.

(d) When the control modes are excited with opposite phases, light is guided to the lower arm.

# Dissipative localization in a perfect lattice



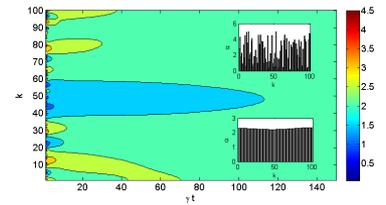
Stationary distributions of absolute values of mode amplitudes for 6x6 square lattice in absence of additional losses (a) and for additional losses (c) in sites of the lattice denoted by crosses in the inset of the panel (d).

Panels (b, d) show eigenvalues of the system without additional losses (b) and with additional losses (d). Filled dots in the insets on (b, d) show a position of the initial excitation.

Stationary states:

Correlated; entangled; Gibbs state, ... etc

$$\frac{d}{dt} \langle a_k \rangle = -(\gamma_k + \gamma_{k-1}) \langle a_k \rangle + \gamma_k \langle a_{k+1} \rangle + \gamma_{k-1} \langle a_{k-1} \rangle$$



Equaliser: renders output state completely symmetrical with respect to all modes

## Other input states?

Represent as a sum of coherent state projectors:  $\rho_0 = \sum_{\forall j} P_{jl} \prod_{l=1}^N |\alpha_{jl}\rangle \langle \alpha_{jl}|_l$

Output stationary state:  $\rho_{st} = \sum_{\forall j} \bar{P}_j \prod_{l=1}^N |\bar{\alpha}_j\rangle \langle \bar{\alpha}_j|_l$

$$\bar{\alpha}_j = \frac{1}{N} \sum_{l=1}^N \alpha_{jl},$$

$$\bar{P}_j = \sum_{l=1}^N P_{jl}$$

Coherent states input – uncorrelated output  
(but “equalised”)

$$\rho_0^{term} = \prod_{l=1}^N \int d^2\alpha_l \frac{1}{\pi n_l} \exp\{-|\alpha_l|^2/n_l\} |\alpha_l\rangle \langle \alpha_l|_l \quad - \text{thermal state}$$

### Thermal states – correlated output:

$$\rho_{st}^{term} = \frac{1}{\pi n_0} \int d^2\alpha \exp\{-|\alpha|^2/n_0\} \prod_{l=1}^N |\alpha\rangle \langle \alpha|_l$$

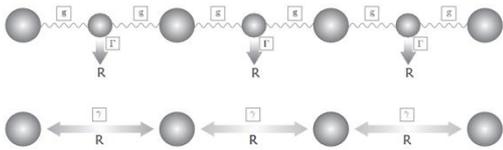
$$n_0 = \frac{\sum_{l=1}^N n_l}{N^2},$$

$$\rho_{st}^{term} = \frac{n_0^{A^\dagger A}}{(n_0 + 1)^{A^\dagger A}},$$

$$A = \frac{1}{N} \sum_{l=1}^N a_l$$

$$\langle a_k^\dagger a_l \rangle = n_0$$

## Other interesting stationary states



Gibbs state (max. entropy for the given  $\sum_{k,l=1}^{N+1} \langle a_k^\dagger a_l \rangle$ ):

$$\rho_{st} = \exp\{-\beta A_{sum}^\dagger A_{sum}\} / \text{Tr}\{\exp\{-\beta A_{sum}^\dagger A_{sum}\}\}$$

Entangled state, e. g. for 1 photon in the chain:

$$|\Phi_{st}\rangle = A_{sum}^\dagger \prod_{\forall j} |0\rangle_j$$

Dissipatively coupled chain of bosonic modes  
as a reservoir:

non-equilibrium reservoir, non-Landauer erasure

## Ideal reservoir:

- the reservoir is some entity able to **drive the system towards some state** independently of the initial state of the system
- the states before and after the interaction should belong to **the same class of macro-states** characterized by a few common parameters (i.e., T for the Gibbs state)
- the ideal reservoir should **asymptotically disentangle** itself from the system

**Cyclic process:** one cannot use the same reservoir - its state will be changed by the interaction process. Need:

Either:

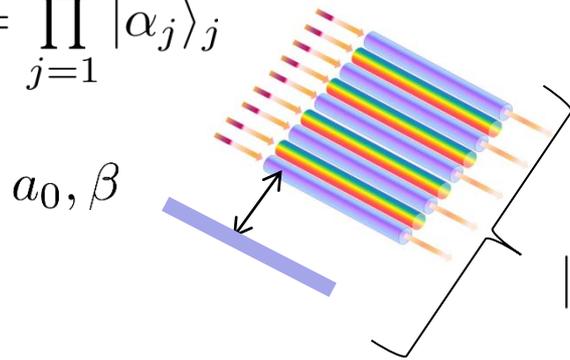
**an infinite and cost-less supply of fresh reservoirs** in the same state at each stage  
(a common assumption in quantum thermodynamics)

Or: mechanism **to return the reservoir to the original class** of macro-states at each step

Dissipatively coupled bosonic modes:

reservoir with an intrinsic mechanism for reverting to the original class of macro-states

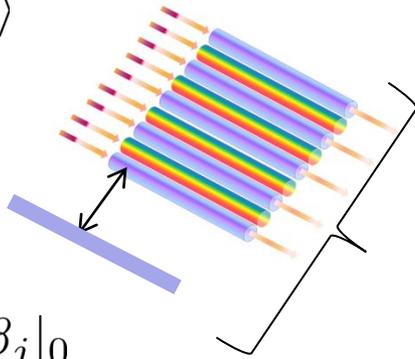
$$|\Psi_{coh}\rangle = \prod_{j=1}^N |\alpha_j\rangle_j$$



$$|\Phi_{st}\rangle = \prod_{j=0}^{N+1} |\bar{\alpha}\rangle_j, \quad \bar{\alpha} = \frac{1}{N+2}(\beta + (N+1)\alpha)$$

$N + 1 \gg 1$   $a_0, \beta$  becomes arbitrarily close to the coherent state of amplitude  $\alpha$

$$|\Psi_{coh}\rangle = \prod_{j=1}^N |\alpha_j\rangle$$



$$\rho_{a0} = \sum_j p_j |\beta_j\rangle \langle \beta_j|_0$$

$$\rho_{st} = \sum_j p_j \prod_{k=0}^{N+1} |\bar{\alpha}_j\rangle \langle \bar{\alpha}_j|_k$$

$$\bar{\alpha} = \frac{1}{N+2} (\beta + (N+1)\alpha)$$

**Fidelity**  $F = \langle \Phi | \rho_{st} | \Phi \rangle = \sum_j p_j \exp\left\{-\frac{|\beta_j - \alpha|^2}{N+2}\right\} \rightarrow 1, \quad N \gg 1$

turns any state into the coherent state with the amplitude practically equal to amplitudes of the other oscillators the chain

reservoir washing away any information about the initial state

Energy cost:

$$\Delta E = \frac{N + 1}{N + 2} \sum_j p_j |\beta_j - \alpha|^2$$

- energy difference between the initial and asymptotic states of the chain plus the mode

For large N, energy difference:

- is independent of the chain size
- equals to the energy of the coherently shifted initial state of the mode

chain + mode system loses energy driving the mode toward equilibrium with reservoir

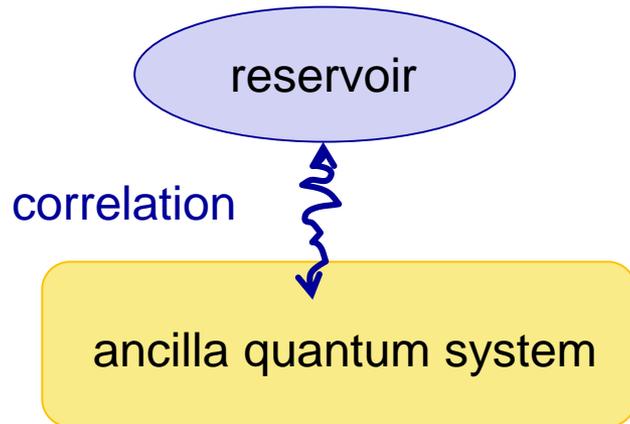
**Landauer principle:** erasure of 1 bit of info increases the entropy by  $k \ln 2$

Information is physical

to erase information irreversibly by action of the environment, energy transfer into the environment should occur.

Quantum equivalent?

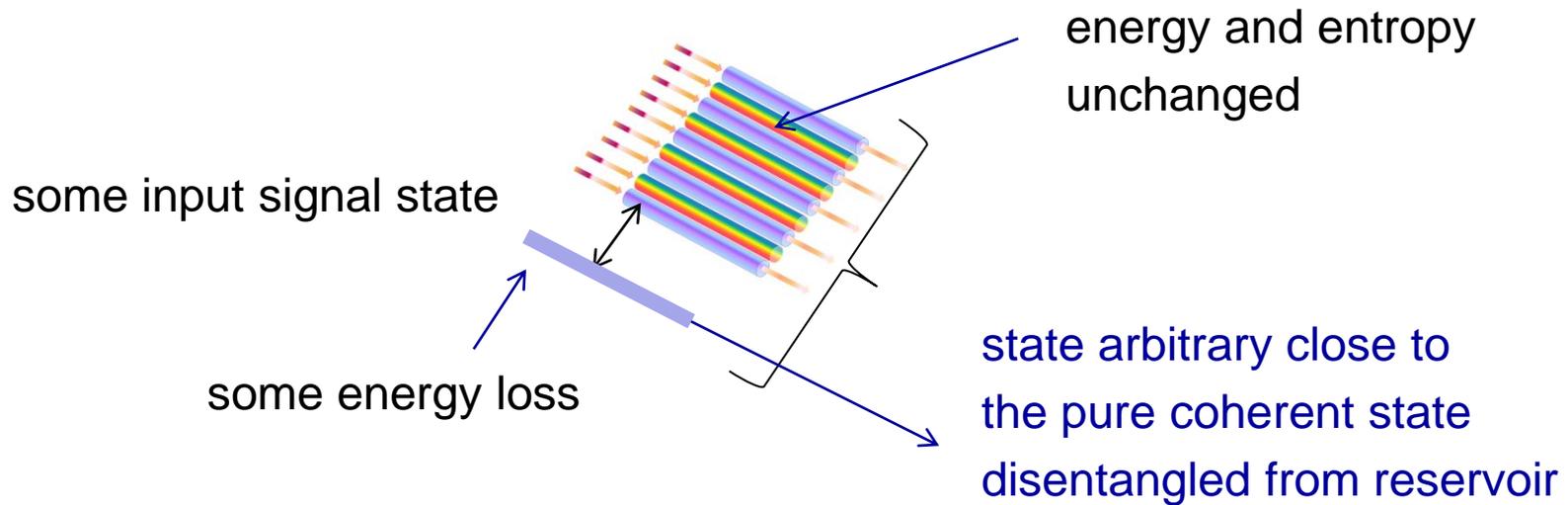
The quantum Landauer's principle holds if the reservoir is a closed system in the Gibbs state and is completely uncorrelated with the signal system to be erased.



the state of the signal can be erased  
*without*  
entropy increase of the reservoir

*For quantum principle see e.g: D. Reeb and M. M. Wolf, New J. Phys. 16, 103011 (2014)*

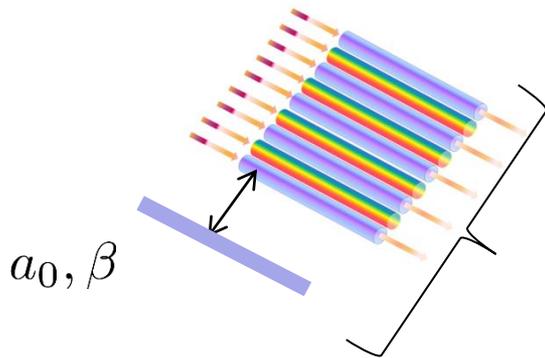
our chain of bosonic modes as reservoir transforms the signal catalytically  
**- coherence is retained**



$$\Delta E = \frac{N+1}{N+2} \sum_j p_j |\beta_j - \alpha|^2$$

- energy difference between the initial and asymptotic states of the chain plus the mode

$$\Delta E = \frac{N+1}{N+2} \sum_j p_j |\beta_j - \alpha|^2$$



the energy balance of the signal mode is a difference between energies of its initial and asymptotic states:

$$\Delta E_0 \approx \sum_j p_j |\beta_j|^2 - |\alpha|^2$$

if positive:

$$\Delta E = \Delta E_0 \Rightarrow |\alpha| = \sum_j p_j |\beta_j| \cos\{\arg(\beta_j) - \arg(\alpha)\}$$

for such signal states erasure can be done without energy loss from the reservoir

**non-Landauer erasure**

= the equality of coherences of the initial and the asymptotic state

**catalytic coherence**

Combination of seemingly classical dynamics with distinctly quantum results

Random walk equation for coherent amplitudes

Multi-mode light flows like heat remaining coherent and even entangled

Optical equaliser – output symmetric over all states

Coherence flows like heat and behaves like heat

As reservoir: retains coherence and transforms the signal catalytically

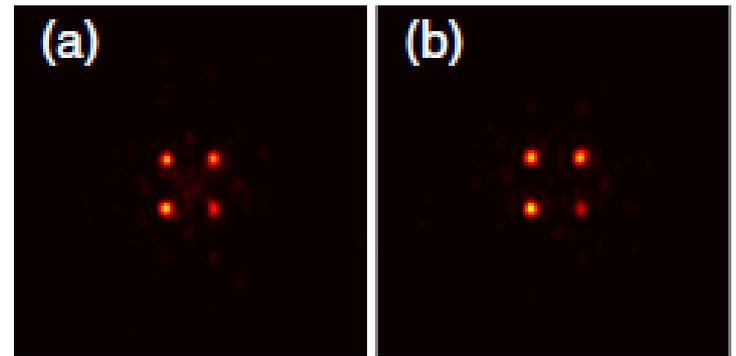
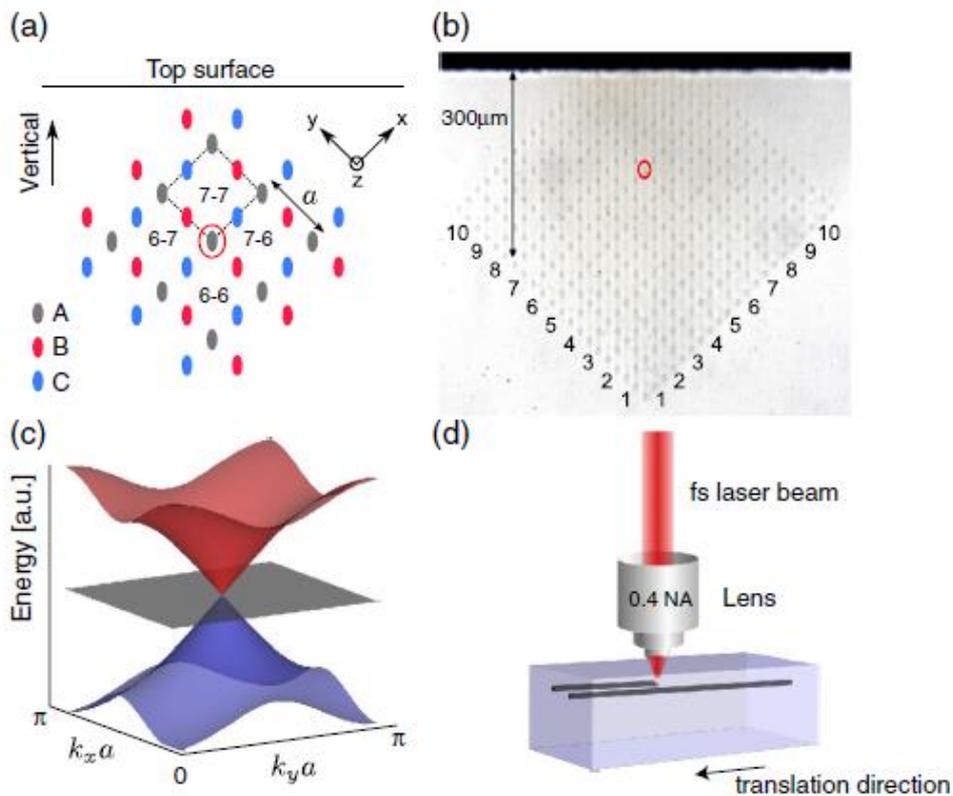
(washes out any info about the signal without losing energy and without losing coherence)

2<sup>nd</sup> type phase transition from positive to negative temperatures

Energy density of stationary states for bosonic modes is given by Fermi-Dirac

## **Applications:**

Quantum simulation, topological effects,  
modelling condensed matter systems etc



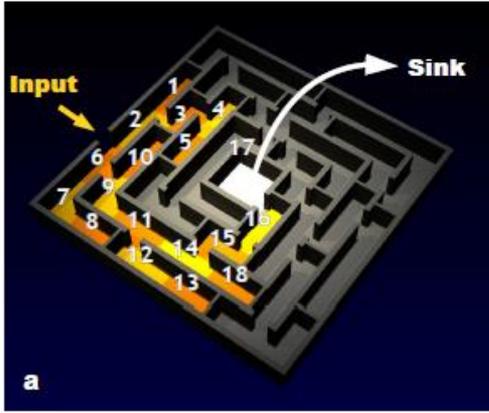
(a)–(d) The nondiffracting states

FIG. 1 (color online). (a) Edge-centered square (Lieb) lattice.

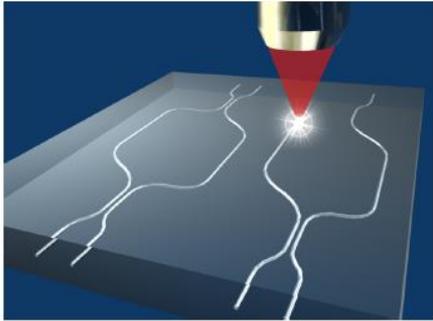
*S. Mukherjee et al, Observation of a localized flat-band state in a photonic Lieb lattice, Phys. Rev. Lett. 114, 245504 (2015);*

*Modulation-assist. tunnelling in laser fabr. photonic Wannier-Stark ladders, arXiv:1505.05217;*

*Observation of localized flat-band modes in a quasi-one-dimensional photonic rhombic lattice, Opt. Lett. 40, 5443 (2015)*



*F. Caruso, A. Crespi,  
A. G. Ciriolo, F. Sciarrino,  
B. R. Osellame,  
Fast escape of a quantum  
walker from an integrated  
photonic maze.  
Nature Comm. 7, 11682 (2016)*



*F. Flamini et al, Thermally-Reconfigurable Quantum Photonic Circuits at Telecom Wavelength by Femtosecond Laser Micromachining. Light: Science & Applications (2015) 4, e354*

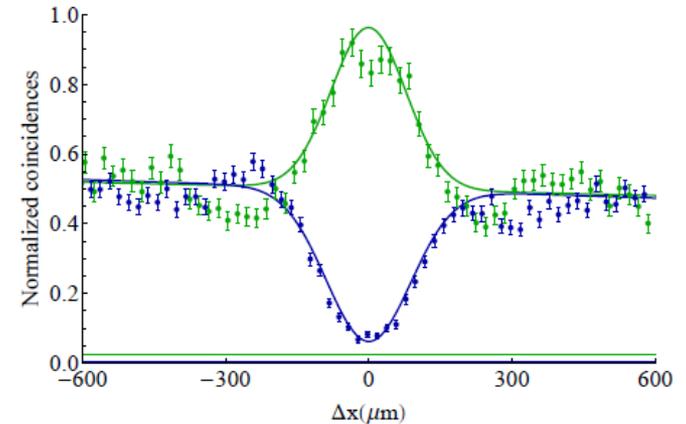
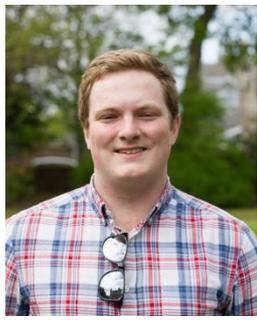


Figure 5. Hong-Ou-Mandel interference experiment for input state  $|1, 1\rangle$  at  $\Delta V \simeq 1.97$  V ( $\phi \sim \pi/2$ ). Blue Points: coincidences for output (1, 1). Green points: coincidences of output (0, 2). Solid lines: best fit of the measured data. Horizontal

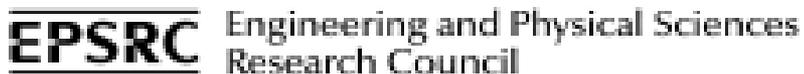


[www.st-andrews.ac.uk/~qoi](http://www.st-andrews.ac.uk/~qoi)

Photonic Instrumentation  
Group, Heriot Watt Uni, UK



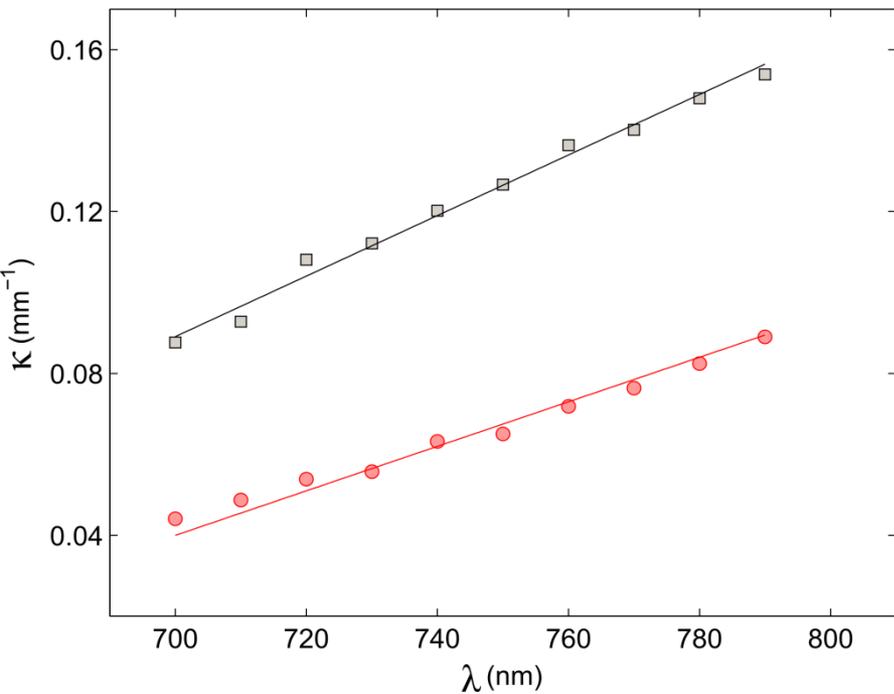
<http://master.basnet.by/lqo>



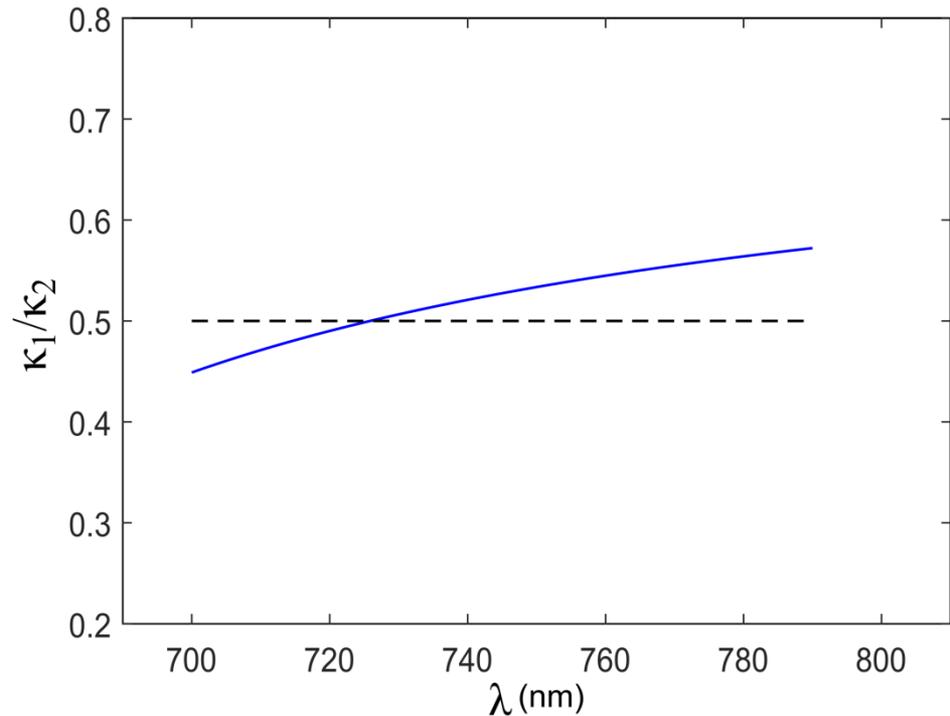




$$\gamma t \leftrightarrow \kappa_1 z \leftrightarrow \kappa_1(\lambda) z_0, \quad \kappa_1/\kappa_2 \approx 0.5$$



$\kappa_1 \rightarrow$  red and  $\kappa_2 \rightarrow$  black



$\kappa_1/\kappa_2$  as a function of  $\lambda$

dashed line:  $\frac{\kappa_1}{\kappa_2} = 0.5$  (desired)

blue line: measured;

maximum deviation from 0.5 is  $\approx \pm 0.05$