Worldline colour fields and quantum field theory

James P. Edwards

ICNFP Crete
Aug 2017

Based on **[arXiv:1603.07929 [hep-th]]** and **[arXiv:1607.04230 [hep-th]]**

In collaboration with **Olindo Corradini**, *Università degli Studi di Modena e Reggio Emilia*

Outline

Recent applications of worldline techniques

Christian Schubert gave an outline[1] of the development of the worldline formalism in his earlier talk. Yet first quantisation is proving useful for exploring currently trending topics in high energy physics, such as

- *Gravitational and axial anomalies* [Alvarez-Gaumé, Witten, Nuclear Physics B234]
- *Higher spin fields and differential forms* [Bastianelli, Corradini, Latini arXiv:0701055 [hep-th]], [Bastianelli, Bonezzi, Iazeolla arXiv:1204.5954 [hep-th]]
- **Non-Abelian quantum field theory** [Bastianelli et. al arXiv:1504.03617 [hep-th]], [Ahmadiniaz et. al arXiv:1508.05144 [hep-th]]
- *QFT in non-commutative space-time* [Ahmadiniaz, Corradini, JPE, Pisani]

These applications have something in common: internal degrees of freedom are represented by additional, **auxiliary fields** in the worldline theory. In the non-Abelian case, these supplementary **"colour fields"** generate the Hilbert space associated to the gauge group degrees of freedom.

[1] Schubert, Phys.Rept. 355

Worldline description of QED

The **phase space action** for spin $\frac{1}{2}$ matter (we consider massless particles for simplicity) coupled to a $U(1)$ gauge potential, $A(x)$, is given in the worldline formalism by[2]

$$S\left[\omega, p, \psi, e, \chi\right] = \int_0^1 d\tau \left[p \cdot \dot{\omega} + \frac{i}{2}\psi \cdot \dot{\psi} - eH - i\chi Q\right],$$

where

$$H \equiv \frac{1}{2}\pi^2 + \frac{i}{2}\psi^\mu F_{\mu\nu}\psi^\nu; \qquad Q \equiv \psi \cdot \pi; \qquad \pi^\mu = p^\mu - A^\mu.$$

Here ω^μ is the embedding of a particle trajectory in Minkowski space and p^μ its momentum, whilst ψ^μ are Grassmann functions that represent the spin degrees of freedom of the particle.

[2] Strassler, Nucl. Phys. **B385**

SUSY

There is a local worldline supersymmetry associated to the einbein, $e(\tau)$, and the gravitino, $\chi(\tau)$, whose algebra closes as

$$\{Q, Q\}_{PB} = -2iH; \qquad \{H, Q\}_{PB} = 0; \qquad \{H, H\}_{PB} = 0.$$

These Poisson brackets follow from the canonical simplectic relations

$$\{\omega^\mu, p_\nu\}_{PB} = \delta^\mu_\nu; \qquad \{\psi^\mu, \psi^\nu\}_{PB} = -i\eta^{\mu\nu}.$$

Field transformations follow from Poisson brackets $\delta\bullet = \{\bullet, G\}_{PB}$, with the generator

$$G(\tau) = \xi(\tau)H + i\eta(\tau)Q,$$

providing

$$\begin{aligned} \delta\omega^\mu &= \xi p^\mu + i\eta\psi^\mu \\ \delta\psi^\mu &= -\eta p^\mu \\ \delta e &= \dot{\xi} + 2i\chi\eta \\ \delta\chi &= \dot{\eta} \end{aligned}$$

Canonical quantisation

After gauge fixing the translation invariance and super-symmetry, the equations of motion for the worldline fields $e(\tau) = T$ and $\chi(\tau) = 0$ still imply **constraints** that must be imposed on the physical states of the Hilbert space,

Canonical quantisation

After gauge fixing the translation invariance and super-symmetry, the equations of motion for the worldline fields $e(\tau) = T$ and $\chi(\tau) = 0$ still imply **constraints** that must be imposed on the physical states of the Hilbert space,

- $H\,|\text{phys}\rangle = 0$ implies the mass shell condition $\left[(p - A)^2 + \frac{i}{4}[\gamma^\mu, \gamma^\nu]F_{\mu\nu}\right]|\text{phys}\rangle = 0$.
- $Q\,|\text{phys}\rangle = 0$ provides the covariant Dirac equation $\gamma \cdot (p - A)\,|\text{phys}\rangle = 0$.
- Note that in canonical quantisation the anti-commutation relations for the Grassmann fields are solved by setting $\hat{\psi}^\mu \longrightarrow \frac{1}{\sqrt{2}}\gamma^\mu$.

We can think of these constraints as projecting unwanted states out of the Hilbert space, leaving us with the correct subspace of physically meaningful states.

Thus far, however, we have described only an Abelian theory, so how can we modify the worldline theory for a particle that transforms in a given representation of $SU(N)$?

Non-Abelian symmetry group - Wilson loops

In the case of an $SU(N)$ symmetry group the vector potential is Lie algebra valued

- Gauge covariance demands that the worldline interaction take on a path ordering prescription, since $A_\mu = A^a_\mu T^a$.
- Physical information of the field theory can be expressed in terms of **Wilson lines**

$$W(T) := \mathscr{P}\left\{\exp\left(i\int_0^T \mathcal{A}^a(\tau)T^a d\tau\right)\right\}.$$

- Here, $\mathcal{A} = A\cdot\omega - \frac{1}{2}\psi^\mu F_{\mu\nu}\psi^\nu$ with $F = d\wedge A - iA\wedge A$.

Non-Abelian symmetry group - Wilson loops

In the case of an $SU(N)$ symmetry group the vector potential is Lie algebra valued

- Gauge covariance demands that the worldline interaction take on a path ordering prescription, since $A_\mu = A_\mu^a T^a$.
- Physical information of the field theory can be expressed in terms of **Wilson lines**

$$W(T) := \mathscr{P}\left\{\exp\left(i\int_0^T \mathcal{A}^a(\tau)T^a d\tau\right)\right\}.$$

- Here, $\mathcal{A} = A\cdot\omega - \frac{1}{2}\psi^\mu F_{\mu\nu}\psi^\nu$ with $F = d\wedge A - iA\wedge A$.

Problem!

- The naïve replacement $\pi_\mu \to p_\mu - A_\mu^a T^a$ provides Poisson brackets

$$\frac{i}{2}\{Q,Q\}_{PB} = \frac{1}{2}\pi^2 + \frac{i}{2}\psi^\mu \partial_{[\mu}A_{\nu]}\psi^\nu \overset{?!}{=} H.$$

This generates only the "Abelian" part of the field strength tensor, so it seems we must abandon the supersymmetric formulation....?

Auxiliary variables

Building upon recent work on higher spin fields we introduce additional worldline fields to represent the degrees of freedom associated to the colour space of the matter field.

- Take N pairs of **Grassmann** fields $\bar{c}^r$, c_r with Poisson brackets $\{\bar{c}^r, c_s\}_{PB} = -i\delta^r_s$. They transform in the (conjugate-)fundamental of $SU(N)$.
- Consider the Poisson brackets for the new objects $R^a \equiv \bar{c}^r (T^a)_r{}^s c_s$,

$$\left\{R^a, R^b\right\}_{PB} = f^{abc} R^c \ .$$

- These **colour fields** provide us with a (classical) representation of the gauge group algebra.

We may use them to absorb the gauge group indices attached to the potential. They also produce the **path ordering** automatically, greatly simplifying the organisation of perturbative calculations.

Colour space

The Hilbert space of the colour fields is described by wavefunction components which transform in fully **anti-symmetric representations** of the gauge group. In canonical quantisation we can use a coherent states basis,

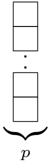
$$\langle \bar{u}| = \langle 0| e^{\bar{u}^r \hat{c}_r}; \qquad \langle \bar{u}| \hat{c}^{\dagger r} = \bar{u}^r \langle \bar{u}|; \qquad \langle \bar{u}| \hat{c}_r = \partial_{\bar{u}^r} \langle \bar{u}|,$$

to write wavefunctions as

$$\Psi(x, \bar{u}) = \psi(x) + \psi_{r_1}(x)\bar{u}^{r_1} + \psi_{r_1 r_2}(x)\bar{u}^{r_1}\bar{u}^{r_2} + \ldots + \psi_{r_1 r_2 \ldots r_N}(x)\bar{u}^{r_1}\bar{u}^{r_2}\cdots\bar{u}^{r_N} \;,$$

where

$$\psi_{r_1 r_2 \ldots r_p} \sim$$



So we will need some way of picking out contributions from only one of these irreducible representations.

Arbitrary matter multiplets

Can we also describe matter that does not transform in a fully anti-symmetric representation?

- We need to enrich the colour Hilbert space to include wavefunction components that transform in less-trivial representations
- Achieve by using multiple copies of the colour fields – using F families of fields leads to the wavefunction being described by components transforming in the F-fold **tensor product**

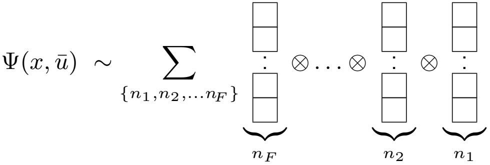
$$\Psi(x,\bar{u}) \sim \sum_{\{n_1,n_2,\ldots n_F\}} \underbrace{\begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}\atop{\vdots}\atop\begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}}_{n_F} \otimes \ldots \otimes \underbrace{\begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}\atop{\vdots}\atop\begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}}_{n_2} \otimes \underbrace{\begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}\atop{\vdots}\atop\begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}}_{n_1}$$

How do we project onto just *one* irreducible representation from this space?

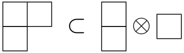
Arbitrary matter multiplets

Can we also describe matter that does not transform in a fully anti-symmetric representation?

- We need to enrich the colour Hilbert space to include wavefunction components that transform in less-trivial representations
- Achieve by using multiple copies of the colour fields – using F families of fields leads to the wavefunction being described by components transforming in the F-fold **tensor product**



How do we project onto just *one* irreducible representation from this space?



Generalised worldline action

Incorporating F families of the colour fields into the worldline dynamics yields the phase space action

$$S\left[\omega, p, \psi, e, \chi, \bar{c}, c\right] = \int_0^1 d\tau \left[p \cdot \dot{\omega} + \frac{i}{2}\psi \cdot \dot{\psi} + \boldsymbol{i\bar{c}^{r}_{f}\dot{c}_{fr}} - e\widetilde{H} - i\chi\widetilde{Q}\right],$$

where

$$\widetilde{H} = \widetilde{\pi}^2 + \frac{i}{2}\psi^\mu F^a_{\mu\nu}\psi^\nu \boldsymbol{\bar{c}^{r}_{f}(T^a)_r{}^s c_{fs}}; \qquad \widetilde{Q} = \psi \cdot \widetilde{\pi}; \qquad \widetilde{\pi}^\mu = p^\mu - A^{a\mu}\boldsymbol{\bar{c}^{r}_{f}(T^a)_r{}^s c_{fs}}.$$

The anti-commutating nature of the colour fields has restored the supersymmetry

$$\left\{\widetilde{Q}, \widetilde{Q}\right\}_{PB} = -2i\widetilde{H}$$

where now in $\widetilde{H}$, we have **completed** $F_{\mu\nu}$ to the full, **non-Abelian** field strength tensor.

In fact, the supersymmetry of the matter and spinor fields can be extended to incorporate the colour fields to form a $1D$ super-gravity with novel interactions.

$U(F)$ worldline symmetry.

There is a global $U(F)$ symmetry that rotates between the families of colour fields:

$$c_{fr} \to \Lambda_{fg} c_{gr}; \qquad \bar{c}^r_f \to \bar{c}^r_g \Lambda^\dagger_{gf}.$$

Gauging this symmetry will allow a projection onto chosen representations. The generators of the symmetry are $L_{fg} := \bar{c}_f{}^r c_{gr}$, satisfying the algebra

$$\{L_{fg}, L_{f'g'}\}_{PB} = i\delta_{fg'} L_{f'g} - i\delta_{f'g} L_{fg'}.$$

We choose to **partially gauge** the $U(F)$ symmetry, introducing gauge fields $a_{fg}(\tau)$ for the generators L_{fg} *only for* $f \geqslant g$. This leads us to the worldline action ($s_f = n_f - \frac{N}{2}$)

$$S'[\omega, p, \psi, e, \chi, \bar{c}, c, a] = \int_0^1 d\tau \Big[p \cdot \dot{\omega} + \frac{i}{2}\psi \cdot \dot{\psi} + i\bar{c}^r_f \dot{c}_{fr} - e\widetilde{H} - i\chi\widetilde{Q} - \sum_{f=1}^{F} a_{ff}\left(L_{ff} - s_f\right) - \sum_{g<f} a_{fg} L_{fg} \Big].$$

$U(F)$ worldline symmetry.

There is a global $U(F)$ symmetry that rotates between the families of colour fields:

$$c_{fr} \to \Lambda_{fg} c_{gr}; \qquad \bar{c}^r_f \to \bar{c}^r_g \Lambda^\dagger_{gf}.$$

Gauging this symmetry will allow a projection onto chosen representations. The generators of the symmetry are $L_{fg} := \bar{c}_f{}^r c_{gr}$, satisfying the algebra

$$\{L_{fg}, L_{f'g'}\}_{PB} = i\delta_{fg'} L_{f'g} - i\delta_{f'g} L_{fg'}.$$

We choose to **partially gauge** the $U(F)$ symmetry, introducing gauge fields $a_{fg}(\tau)$ for the generators L_{fg} *only for* $f \geqslant g$. This leads us to the worldline action ($s_f = n_f - \frac{N}{2}$)

$$S'[\omega, p, \psi, e, \chi, \bar{c}, c, a] = \int_0^1 d\tau \Big[p \cdot \dot{\omega} + \frac{i}{2}\psi \cdot \dot{\psi} + i\bar{c}^r_f \dot{c}_{fr} - e\widetilde{H} - i\chi\widetilde{Q} - \sum_{f=1}^{F} a_{ff}\left(L_{ff} - s_f\right) - \sum_{g<f} a_{fg} L_{fg}\Big].$$

- **Partial gauging** has allowed the *Chern-Simons* terms $\sum_{f=1}^{F} a_{ff} s_f$
- These will provide us with the projection that we need!

Constraints on physical states

In a coherent state basis the $U(F)$ generators become $\hat{L}_{fg} = \bar{u}^r_f \partial_{\bar{u}^r_g}$. The equations of motion for the diagonal elements a_{ff} impose constraints on the state space:

- $\left(\hat{L}_{ff} + \frac{N}{2}\right)|\Psi\rangle = n_f\,|\Psi\rangle \longrightarrow \left(\bar{u}^r_f \frac{\partial}{\partial \bar{u}^r_f} - n_f\right)\Psi(x, \bar{u}) = 0$

Similarly for the off-diagonal elements:

- $\hat{L}_{fg}\,|\Psi\rangle = 0 \;\longrightarrow\; \bar{u}^r_f \frac{\partial}{\partial \bar{u}^r_g}\Psi(x, \bar{u}) = 0$

Here's an example with $F = 2$ families and $\Psi \sim \sum_{n_1, n_2} \underbrace{\begin{array}{c}\square\\ \vdots \\ \square\end{array}}_{n_2} \otimes \underbrace{\begin{array}{c}\square\\ \vdots \\ \square\end{array}}_{n_1}$.

Constraints on physical states

In a coherent state basis the $U(F)$ generators become $\hat{L}_{fg} = \bar{u}^r_f \partial_{\bar{u}^r_g}$. The equations of motion for the diagonal elements a_{ff} impose constraints on the state space:

- $\left(\hat{L}_{ff} + \frac{N}{2}\right)|\Psi\rangle = n_f\,|\Psi\rangle \longrightarrow \left(\bar{u}^r_f \frac{\partial}{\partial \bar{u}^r_f} - n_f\right)\Psi(x,\bar{u}) = 0$

Similarly for the off-diagonal elements:

- $\hat{L}_{fg}\,|\Psi\rangle = 0 \;\longrightarrow\; \bar{u}^r_f \frac{\partial}{\partial \bar{u}^r_g}\Psi(x,\bar{u}) = 0$

Here's an example with $F = 2$ families and $\Psi \sim \sum_{n_1,n_2} \underbrace{\begin{array}{c}\square\\ \vdots\\ \square\end{array}}_{n_2} \otimes \underbrace{\begin{array}{c}\square\\ \vdots\\ \square\end{array}}_{n_1}$.

$$\left(\hat{L}_{22} + \frac{N}{2}\right)|\Psi\rangle = 2\,|\Psi\rangle \;\text{ and }\; \left(\hat{L}_{11} + \frac{N}{2}\right)|\Psi\rangle = |\Psi\rangle \Longrightarrow \Psi \sim \begin{array}{c}\square\\ \square\end{array} \otimes \square.$$

$$\hat{L}_{21}\,|\Psi\rangle = 0 \Longrightarrow \Psi \sim \begin{array}{cc}\square & \square\\ \square & \end{array}$$

Functional quantisation on S^1.

Let's check our work by calculating the Wilson-loop interaction for an arbitrary matter multiplet. The part of the action involving the colour fields is

$$S = \int_0^1 d\tau \Big[\bar{c}^r_f \dot{c}_{fr} - i\bar{c}^r_f \mathcal{A}^a (T^a)_r{}^s c_{fs} + \sum_{f=1}^{F} i a_{ff} \left(\bar{c}^r_f c_{fr} - s_f \right) + \sum_{g<f} i a_{fg} \bar{c}^r_f c_{gr} \Big].$$

We also need to **gauge fix** the local $U(F)$ symmetry associated to the colour fields:

- Choose $\hat{a}_{fg} = \mathrm{diag}\,(\theta_1, \theta_2, \ldots, \theta_F)$.
- Introduce the Faddeev-Popov **determinant** that maintains gauge invariance.

Functional quantisation on S^1.

Let's check our work by calculating the Wilson-loop interaction for an arbitrary matter multiplet. The part of the action involving the colour fields is

$$S = \int_0^1 d\tau \left[\bar{c}_f^r \dot{c}_{fr} - i\bar{c}_f^r \mathcal{A}^a (T^a)_r{}^s c_{fs} + \sum_{f=1}^{F} i a_{ff} \left(\bar{c}_f^r c_{fr} - s_f \right) + \sum_{g<f} i a_{fg} \bar{c}_f^r c_{gr} \right].$$

We also need to **gauge fix** the local $U(F)$ symmetry associated to the colour fields:

- Choose $\hat{a}_{fg} = \mathrm{diag}\left(\theta_1, \theta_2, \ldots, \theta_F\right)$.
- Introduce the Faddeev-Popov **determinant** that maintains gauge invariance.

$$\mu\left(\{\theta_k\}\right) = \prod_{h<g} 2i \sin\left(\frac{\theta_g - \theta_h}{2}\right).$$

- Interpret as a measure on the $U(F)$ moduli that remain to be integrated over:

$$\int \mathscr{D}a_{fg} \longrightarrow \prod_{f=1}^{F} \int_0^{2\pi} \frac{d\theta_f}{2\pi} \mu\left(\{\theta_k\}\right)$$

Integrating over colour fields

The integration over the colour degrees of freedom factorises and provides a product of **functional determinants**

$$\prod_{f=1}^{F} \underset{\text{ABC}}{\text{Det}}\left(i\left(\frac{d}{d\tau}+i\theta_f+i\mathcal{A}\right)\right)$$

Firstly, we evaluate the product of the eigenvalues and find the regular determinant[3]

$$\prod_{k=1}^{F} \det\left(\sqrt{e^{i\theta_k}W(2\pi)}+1/\sqrt{e^{i\theta_k}W(2\pi)}\right).$$

It is then necessary to express this determinant in terms of **group invariants**:

$$\prod_{k=1}^{F}\left(\mathrm{tr}W(\,\cdot\,)+\mathrm{tr}W(\square)e^{i\theta_k}+\mathrm{tr}W\left(\begin{smallmatrix}\square\\\square\end{smallmatrix}\right)e^{2i\theta_k}+\ldots+\mathrm{tr}W\left(\begin{smallmatrix}\square\\\square\\\vdots\\\square\\\square\end{smallmatrix}\right)e^{(N-1)i\theta_k}+\mathrm{tr}W(\,\cdot\,)e^{iN\theta_k}\right).$$

[3] JPE arXiv:1411.6540

Integrating over colour fields

The integration over the colour degrees of freedom factorises and provides a product of **functional determinants**

$$\prod_{f=1}^{F} \underset{\text{ABC}}{\mathrm{Det}}\left(i\left(\frac{d}{d\tau}+i\theta_f+i\mathcal{A}\right)\right)$$

Firstly, we evaluate the product of the eigenvalues and find the regular determinant[3]

$$\prod_{k=1}^{F} \det\left(\sqrt{e^{i\theta_k}W\left(2\pi\right)}+1/\sqrt{e^{i\theta_k}W\left(2\pi\right)}\right).$$

It is then necessary to express this determinant in terms of **group invariants**:

$$\prod_{k=1}^{F}\left(\mathrm{tr}W(\,\cdot\,)+\mathrm{tr}W(\square)e^{i\theta_k}+\mathrm{tr}W\left(\begin{array}{c}\square\\\square\end{array}\right)e^{2i\theta_k}+\ldots+\mathrm{tr}W\left(\begin{array}{c}\square\\\square\\\vdots\\\square\\\square\end{array}\right)e^{(N-1)i\theta_k}+\mathrm{tr}W(\,\cdot\,)e^{iN\theta_k}\right).$$

How do we extract a Wilson loops transforming in a single irreducible representation?

[3] JPE arXiv:1411.6540

Picking out an irrep.

The integral over the $U(F)$ **moduli** and their Faddeev-Popov **measure** will provide the irreducibility we seek. Putting everything together, we must determine

$$\prod_{k=1}^{F}\int_0^{2\pi}\frac{d\theta_k}{2\pi}\,e^{-in_k\theta_k}\prod_{j<k}\left(1-e^{-i\theta_k}e^{i\theta_j}\right)\times$$

$$\prod_{k=1}^{F}\left(\mathrm{tr}W(\cdot)+\mathrm{tr}W(\square)e^{i\theta_k}+\mathrm{tr}W\left(\begin{smallmatrix}\square\\\square\end{smallmatrix}\right)e^{2i\theta_k}+\ldots+\mathrm{tr}W\left(\begin{smallmatrix}\square\\\square\\\vdots\\\square\\\square\end{smallmatrix}\right)e^{(N-1)i\theta_k}+\mathrm{tr}W(\cdot)e^{iN\theta_k}\right).$$

If we introduce the worldline Wilson-loop variables $z_k=e^{i\theta_k}$ then we can recast this expression as a contour integral in the complex plane:

$$\prod_{k=1}^{F}\oint\frac{dz_k}{2\pi i}\prod_{j<k}\left(1-\frac{z_j}{z_k}\right)\prod_{k=1}^{F}\sum_{p=0}^{N}\frac{\mathrm{tr}W_p}{z_k^{n_k+1-p}}$$

where $W_p\sim\begin{smallmatrix}\square\\\square\\\vdots\\\square\\\square\end{smallmatrix}$ transforms in the representation with p fully anti-symmetrised indices.

A demonstration

Let's return to our illustrative example to see how this works. We take $F = 2$ and $n_2 = 2$, $n_1 = 1$. We compute, for gauge group $SU(N)$,

$$\oint \frac{dz_1}{2\pi i} \oint \frac{dz_2}{2\pi i} \left(1 - \frac{z_1}{z_2}\right) \sum_{p=0}^{N} \frac{\mathrm{tr} W_p}{z_1^{2-p}} \sum_{p=0}^{N} \frac{\mathrm{tr} W_p}{z_2^{3-p}}.$$

The contour integrals just pick out the simple poles at $z = 0$:

A demonstration

Let's return to our illustrative example to see how this works. We take $F = 2$ and $n_2 = 2$, $n_1 = 1$. We compute, for gauge group $SU(N)$,

$$\oint \frac{dz_1}{2\pi i} \oint \frac{dz_2}{2\pi i} \left(1 - \frac{z_1}{z_2}\right) \sum_{p=0}^{N} \frac{\mathrm{tr} W_p}{z_1^{2-p}} \sum_{p=0}^{N} \frac{\mathrm{tr} W_p}{z_2^{3-p}}.$$

The contour integrals just pick out the simple poles at $z = 0$:

$$\begin{aligned} &\mathrm{tr} W\left(\yng(1,1)\right) \mathrm{tr} W\left(\yng(1)\right) - \mathrm{tr} W\left(\yng(1,1,1)\right) \mathrm{tr} W\left(\cdot\right) \\ &= \mathrm{tr} W\left(\yng(2,1)\right) \end{aligned}$$

as desired!

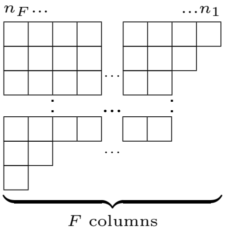
In general

Putting the colour space information back in to the full expression for the worldline formulation of the field theory's partition function we arrive at

$$\Gamma_\Psi\left[A\right] = \int_0^\infty \frac{dT}{T} \oint \mathscr{D}\omega \mathscr{D}\psi \, e^{-\frac{1}{2}\int_0^{2\pi} \frac{\dot{\omega}^2}{T} + \psi\cdot\dot{\psi}} \, \mathrm{tr}_{\mathcal{R}} \mathscr{P} \exp\left(i\int_0^{2\pi} \mathcal{A}[\omega(\tau), \psi(\tau)] d\tau\right)$$

where the Wilson-loop interaction generated by the colour fields transforms in the representation $\mathcal{R}$ specified by our choice of F and the F-tuple $(n_1, n_2, \ldots n_F)$ so that the spinor wavefunction has Young Tableau:

$$\Psi(x, \bar{u}) \sim$$



Outline of (ongoing) applications

So far, applications of this technique to non-Abelian quantum field theory include

- Vacuum polarisation: scalar and spinor contributions to the gluon self energy at one loop order for a matter transforming in fully (anti-)symmetric representations of the gauge group.
- Scalar propagator in a non-Abelian background: again the matter was chosen to transform in a fully (anti-)symmetric representation of the gauge group.
- Wilson-loop interactions for spinor matter transforming in an arbitrary representation of the gauge group.

Ongoing work includes

- Extending the tree-level and one-loop amplitudes to arbitrary representations using the families of colour fields presented here.
- Describing the spinor propagator in a non-Abelian background in the worldline formalism.
- Application of the same techniques to Lorentz group structure of effective action for $U(N)$ Yang-Mills theory in non-commutative space-time.

Conclusion

Auxiliary worldline fields can be used to encode Lie group degrees of freedom for matter fields in the worldline approach to QFT.

1. Grassmann worldline fields span a Hilbert space described by (reducible) **tensor products** of fully anti-symmetric representations of the gauge group.
2. Partially gauging a $U(F)$ symmetry enforces physical states to transform in an **irreducible** representation.
3. Although I haven't shown it, one may achieve completely analogous results using **bosonic** colour fields with only minor modifications.
4. Very **versatile technique** is easy to apply to scattering amplitudes, higher-loop effective actions, confinement, tensor decomposition of vertices....

Conclusion

Auxiliary worldline fields can be used to encode Lie group degrees of freedom for matter fields in the worldline approach to QFT.

1. Grassmann worldline fields span a Hilbert space described by (reducible) **tensor products** of fully anti-symmetric representations of the gauge group.
2. Partially gauging a $U(F)$ symmetry enforces physical states to transform in an **irreducible** representation.
3. Although I haven't shown it, one may achieve completely analogous results using **bosonic** colour fields with only minor modifications.
4. Very **versatile technique** is easy to apply to scattering amplitudes, higher-loop effective actions, confinement, tensor decomposition of vertices....

Thank you for your attention.