Equations of anisotropic hydrodynamics for quark and gluon fluids

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Introduction

- Standard approach to relativistic hydrodynamics might be questioned because it is based on the gradient expansion around equilibrium. At the early stages of heavy ion collisions gradients are large and this leads to substantial modifications of pressure.
- For describing the system with large pressure anisotropies it is better to use anisotropic hydrodynamics.
- So far, anisotropic hydrodynamics has been used mostly to describe simple (one-component) fluids, but we usually deal with mixtures of quarks and gluons.
- This talk is a generalisation of the previous case, where components of mixture were massless.

  W.Florkowski, E.Maksymiuk, R.Ryblewski, L.Tinti, Phys. Rev. C **92**, 054912

  Now we include mass of quarks and quantum statistics of fermions and bosons.

Ideas of the presented model

- One-dimensional and boost-invariant system.
- Mass of quarks is $m_q = 300$ MeV, while mass of gluons is $m_g = 0$ MeV.
- We use zeroth, first and second moment of the kinetic equation in the relaxation time approximation.
- Finding new equations allows us to have a different values of the transverse momentum scale Λ for quarks and gluons.
- In this case baryon chemical potential λ_q is not zero.
- Unknown functions:
  $T(\tau)$, $\xi_q(\tau)$, $\xi_g(\tau)$, $\Lambda_q(\tau)$, $\Lambda_g(\tau)$, $\lambda(\tau)$ and $\mu(\tau)$.

Boltzman equation

General setup

- Boltzmann equation in the relaxation time approximation (RTA)

$$p^\mu \partial_\mu f_s(x,p) = C[f_s(x,p)], \qquad C[f_s] = p\cdot u\, \frac{f_{s,\rm eq} - f_s}{\tau_{\rm eq}}.$$

- Distribution functions $f_s(x,p)$ (Romatschke-Strickland):

$$f_{Q^\pm}(x,p) = \left[\exp\left(\frac{\pm\lambda - \sqrt{(p\cdot U)^2 + \xi_q (p\cdot Z)^2}}{\Lambda_q}\right) + 1\right]^{-1}$$

$$f_G(x,p) = \left[\exp\left(-\frac{\sqrt{(p\cdot U)^2 + \xi_g (p\cdot Z)^2}}{\Lambda_g}\right) - 1\right]^{-1}$$

- Equilibrium distribution functions $f_{s,\rm eq}(x,p)$:

$$f_{Q^\pm,\rm eq}(x,p) = \left[\exp\left(\frac{\pm\mu - p\cdot U}{T}\right) + 1\right]^{-1}$$

$$f_{G,\rm eq}(x,p) = \left[\exp\left(-\frac{p\cdot U}{T}\right) - 1\right]^{-1}$$

$0^{\rm th}$ moment of the Boltzmann eq.

- Zeroth moment of the kinetic equation describes production of the particles

$$\partial_\mu (n_s U^\mu) \quad = \quad \frac{n_{s,\rm eq} - n_s}{\tau_{\rm eq}}$$

- Quarks and antiquarks

$$n_{Q,\rm a} = \frac{2 g_{\rm q} \Lambda_{\rm q}^3 \cosh(\lambda/\Lambda_{\rm q})}{\pi^2 \sqrt{1+\xi_{\rm q}}} \mathcal{N}\left(\frac{m}{\Lambda_{\rm q}}, \frac{\lambda}{\Lambda_{\rm q}}\right), \quad n_{G,\rm a} = \frac{2 g_{\rm g} \Lambda_{\rm g}^3 \zeta(3)}{\pi^2 \sqrt{1+\xi_{\rm g}}}.$$

- Gluons

$$n_{Q,\rm eq} = \frac{2 g_q T^3 \cosh(\mu/T)}{\pi^2} \mathcal{N}\left(\frac{m}{T}, \frac{\mu}{T}\right), \quad n_{G,\rm eq} = \frac{2 g_g T^3 \zeta(3)}{\pi^2}.$$

$$\mathcal{N}(z,\nu) = \frac{1}{2\cosh(\nu)} \int\limits_0^\infty dr\, r^2 \left[\frac{1}{e^{\sqrt{r^2+z^2}-\nu}+1} + \frac{1}{e^{\sqrt{r^2+z^2}+\nu}+1} \right]$$

Baryon number conservation

- Substraction of the equations describing quarks and antiquarks production and deviding by a factor 3

$$\frac{d}{d\tau}\left(\frac{n_q^+ - n_q^-}{3}\right) + \frac{n_q^+ - n_q^-}{3\tau} = \frac{n_{q,\mathrm{eq}}^+ - n_{q,\mathrm{eq}}^- - (n_q^+ - n_q^-)}{\tau_{\mathrm{eq}}}$$

$$\frac{db}{d\tau} + \frac{b}{\tau} = \frac{b_{\mathrm{eq}} - b}{\tau_{\mathrm{eq}}} \qquad b(\tau) = \frac{b_0\tau_0}{\tau} \qquad b_{\mathrm{a}} = b_{\mathrm{eq}}$$

$$\sinh\left(\frac{\mu}{T}\right)\mathcal{B}\left(\frac{\mu}{T}, \frac{m}{T}\right) = \frac{3\pi^2 b_0\tau_0}{4g_{\mathrm{q}}T^3\tau} \tag{1}$$

$$\sinh\left(\frac{\lambda}{\Lambda_{\mathrm{q}}}\right)\mathcal{B}\left(\frac{\lambda}{\Lambda_{\mathrm{q}}}, \frac{m}{\Lambda_{\mathrm{q}}}\right) = \frac{3\pi^2 b_0\tau_0\sqrt{1+\xi_{\mathrm{q}}}}{4g_{\mathrm{q}}\Lambda_{\mathrm{q}}^3\,\tau} \tag{2}$$

$$\mathcal{B}(z,\nu) = \frac{1}{4\sinh(\nu)}\int_0^\infty dr\, r^2\left[\frac{1}{e^{\sqrt{r^2+z^2}-\nu}+1} - \frac{1}{e^{\sqrt{r^2+z^2}+\nu}+1}\right]$$

0^{th} moment of Boltzmann eq.

- Linear combination of the particles production finally has a following form

$$\alpha\left(\frac{dn_q}{d\tau}+\frac{n_q}{\tau}\right)+(1-\alpha)\left(\frac{dn_g}{d\tau}+\frac{n_g}{\tau}\right)=\alpha\,\frac{n_{q,\text{eq}}-n_q}{\tau_{\text{eq}}}+(1-\alpha)\,\frac{n_{g,\text{eq}}-n_g}{\tau_{\text{eq}}}$$

$$\begin{aligned}
&\frac{d}{d\tau}\left[\alpha\,\frac{g_{\text{q}}\Lambda_{\text{q}}^3\cosh\left(\frac{\lambda}{\Lambda_{\text{q}}}\right)}{\sqrt{1+\xi_{\text{q}}}}\mathcal{N}\left(\frac{m}{\Lambda_{\text{q}}},\frac{\lambda}{\Lambda_{\text{q}}}\right)+(1-\alpha)\frac{g_{\text{g}}\Lambda_{\text{g}}^3\zeta(3)}{\sqrt{1+\xi_{\text{g}}}}\right]\\
&+\left(\frac{1}{\tau}+\frac{1}{\tau_{\text{eq}}}\right)\left[\alpha\,\frac{g_{\text{q}}\Lambda_{\text{q}}^3\cosh\left(\frac{\lambda}{\Lambda_{\text{q}}}\right)}{\sqrt{1+\xi_{\text{q}}}}\mathcal{N}\left(\frac{m}{\Lambda_{\text{q}}},\frac{\lambda}{\Lambda_{\text{q}}}\right)+(1-\alpha)\frac{g_{\text{g}}\Lambda_{\text{g}}^3\zeta(3)}{\sqrt{1+\xi_{\text{g}}}}\right]\\
&=\frac{T^3}{\tau_{\text{eq}}}\left[\alpha\,g_{\text{q}}\cosh\left(\frac{\mu}{T}\right)\mathcal{N}\left(\frac{m}{T},\frac{\mu}{T}\right)+(1-\alpha)\,g_{\text{g}}\zeta(3)\right]
\end{aligned}\tag{3}$$

- $\alpha=1$ – quarks; $\alpha=0$ – gluons; $\alpha=1/2$ – quarks and gluons.

$1^{\rm st}$ moment of Boltzmann eq.

- First moment of the kinetic equation describes energy-momentum conservation law

$$\partial_\mu \underbrace{\int dP\, p^\nu p^\mu G}_{T^{\mu\nu}} = \int dP\, p^\nu C = 0 \qquad \frac{d\mathcal{E}}{d\tau} = -\frac{\mathcal{E}+\mathcal{P}_\parallel}{\tau}$$

$$T^{\mu\nu} = (\mathcal{E}+\mathcal{P}_\perp)u^\mu u^\nu - \mathcal{P}_\perp g^{\mu\nu} + (\mathcal{P}_\parallel - \mathcal{P}_\perp)V^\mu V^\nu$$

$$u^\mu = \left(\frac{t}{\tau}, 0, 0, \frac{z}{\tau}\right) \qquad V^\mu = \left(\frac{z}{\tau}, 0, 0, \frac{t}{\tau}\right)$$

- Landau matching condition alows us to find the effective temperature

$$T^4\left[2g_q \cosh\left(\frac{\mu}{T}\right)\widetilde{\mathcal{H}}_3\left(1, \frac{m}{T}, \frac{\mu}{T}\right) + g_g\,\widetilde{\mathcal{H}}\left(1\right)\right] =$$
$$2g_{\rm q}\Lambda_{\rm q}^4 \cosh\left(\frac{\lambda}{\Lambda_{\rm q}}\right)\widetilde{\mathcal{H}}_3\left(\frac{1}{\sqrt{1+\xi_{\rm q}}}, \frac{m}{\Lambda_{\rm q}}, \frac{\lambda}{\Lambda_{\rm q}}\right) + g_{\rm g}\,\Lambda_{\rm g}^4\widetilde{\mathcal{H}}\left(\frac{1}{\sqrt{1+\xi_{\rm g}}}\right) \tag{4}$$

1^{st} moment of Boltzmann eq.

- Energy-momentum conservation law has a form

$$\frac{d\varepsilon}{d\tau} = -\frac{\varepsilon + P_L}{\tau}$$

- It leads to the following equation

$$\frac{d}{d\tau}\left[2g_{\mathrm{q}}\Lambda_{\mathrm{q}}^4 \cosh\left(\frac{\lambda}{\Lambda_{\mathrm{q}}}\right)\widetilde{\mathcal{H}}_3\left(\frac{1}{\sqrt{1+\xi_{\mathrm{q}}}}, \frac{m}{\Lambda_{\mathrm{q}}}, \frac{\lambda}{\Lambda_{\mathrm{q}}}\right) + g_{\mathrm{g}}\,\Lambda_{\mathrm{g}}^4\widetilde{\mathcal{H}}\left(\frac{1}{\sqrt{1+\xi_{\mathrm{g}}}}\right)\right]$$
$$= -\frac{1}{\tau}\left[2g_{\mathrm{q}}\Lambda_{\mathrm{q}}^4 \cosh\left(\frac{\lambda}{\Lambda_{\mathrm{q}}}\right)\widetilde{\mathcal{H}}_3\left(\frac{1}{\sqrt{1+\xi_{\mathrm{q}}}}, \frac{m}{\Lambda_{\mathrm{q}}}, \frac{\lambda}{\Lambda_{\mathrm{q}}}\right) + g_{\mathrm{g}}\,\Lambda_{\mathrm{g}}^4\widetilde{\mathcal{H}}\left(\frac{1}{\sqrt{1+\xi_{\mathrm{g}}}}\right)\right.$$
$$\left. + 2g_{\mathrm{q}}\Lambda_{\mathrm{q}}^4 \cosh\left(\frac{\lambda}{\Lambda_{\mathrm{q}}}\right)\widetilde{\mathcal{H}}_{3\mathrm{L}}\left(\frac{1}{\sqrt{1+\xi_{\mathrm{q}}}}, \frac{m}{\Lambda_{\mathrm{q}}}, \frac{\lambda}{\Lambda_{\mathrm{q}}}\right) + g_{\mathrm{g}}\,\Lambda_{\mathrm{g}}^4\widetilde{\mathcal{H}}_{\mathrm{L}}\left(\frac{1}{\sqrt{1+\xi_{\mathrm{g}}}}\right)\right] \tag{5}$$

2^{nd} moment of the Boltzmann eq.

- Second moment of the kinetic equation in the relaxation time approximation

$$\frac{d}{d\tau}\ln\Theta_I + \theta - 2\theta_I - \frac{1}{3}\sum_J\left[\frac{d}{d\tau}\ln\Theta_J + \theta - 2\theta_J\right]$$
$$= \frac{1}{\tau_{\text{eq}}}\left[\frac{\Theta_{\text{eq}}}{\Theta_I} - 1\right] - \frac{1}{3}\sum_J\left\{\frac{1}{\tau_{\text{eq}}}\left[\frac{\Theta_{\text{eq}}}{\Theta_J} - 1\right]\right\}$$

W.Florkowski, L.Tinti, Phys. Rev. C **89**, 034907 (2014)

- Variables θ_I are equal

$$\theta_X = \theta_Y = 0, \qquad \theta_Z = -1/\tau, \qquad \theta = 1/\tau.$$

- One-dimensional case

$$\frac{d}{d\tau}\ln\Theta_X - \frac{d}{d\tau}\ln\Theta_Z - \frac{2}{\tau} = \frac{\Theta_{\text{eq}}}{\tau_{\text{eq}}}\left[\frac{1}{\Theta_X} - \frac{1}{\Theta_Z}\right]$$

2^{nd} moment of the Boltzmann eq.

Quarks and antiquarks

- Non-equilibrium functions Θ^q_I:

$$\Theta^q_{\mathrm{X,a}} = \frac{8 g_{\mathrm{q}} \Lambda_{\mathrm{q}}^5 \cosh(\frac{\lambda}{\Lambda_{\mathrm{q}}})}{3(2\pi)^2(1+\xi_{\mathrm{q}})^{1/2}} \mathcal{M}\left(\frac{m}{\Lambda_{\mathrm{q}}}, \frac{\lambda}{\Lambda_{\mathrm{q}}}\right)$$

$$\Theta^q_{\mathrm{Z,a}} = \frac{8 g_{\mathrm{q}} \Lambda_{\mathrm{q}}^5 \cosh(\lambda/\Lambda_{\mathrm{q}})}{3(2\pi)^2(1+\xi_{\mathrm{q}})^{3/2}} \mathcal{M}\left(\frac{m}{\Lambda_{\mathrm{q}}}, \frac{\lambda}{\Lambda_{\mathrm{q}}}\right)$$

- Equilibrium functions Θ^q_{eq}

$$\Theta^q_{\mathrm{eq}} = \frac{2 g_{\mathrm{q}} T^5 \cosh\left(\frac{\mu}{T}\right)}{3\pi^2} \mathcal{M}\left(\frac{m}{T}, \frac{\mu}{T}\right)$$

- Finally:

$$\frac{1}{1+\xi_{\mathrm{q}}} \frac{d\xi_{\mathrm{q}}}{d\tau} - \frac{2}{\tau} = -\frac{\xi_{\mathrm{q}}(1+\xi_{\mathrm{q}})^{1/2}}{\tau_{\mathrm{eq}}} \frac{T^5 \cosh\left(\frac{\mu}{T}\right) \mathcal{M}\left(\frac{m}{T}, \frac{\mu}{T}\right)}{\Lambda_{\mathrm{q}}^5 \cosh\left(\frac{\lambda}{\Lambda_{\mathrm{q}}}\right) \mathcal{M}\left(\frac{m}{\Lambda_{\mathrm{q}}}, \frac{\lambda}{\Lambda_{\mathrm{q}}}\right)} \tag{6}$$

$$\mathcal{M}(z,\nu) = \frac{1}{2\cosh(\nu)} \int_0^\infty dr\, r^4 \left[\frac{1}{e^{\sqrt{r^2+z^2}-\nu}+1} + \frac{1}{e^{\sqrt{r^2+z^2}+\nu}+1}\right]$$

2^{nd} moment of the Boltzmann eq.

Gluons

- Non-equilibrium functions Θ^g_I:

$$\Theta^g_{\mathrm{X,a}} = \frac{g_{\mathrm{g}}\Lambda_{\mathrm{g}}^5}{3\pi^2(1+\xi_{\mathrm{g}})^{1/2}} \int\limits_0^\infty \frac{r^4\,dr}{e^r-1} = \frac{8g_{\mathrm{g}}\zeta(5)\Lambda_{\mathrm{g}}^5}{\pi^2(1+\xi_{\mathrm{g}})^{1/2}}$$

$$\Theta^g_{\mathrm{Z,a}} = \frac{8g_{\mathrm{g}}\zeta(5)\Lambda_{\mathrm{g}}^5}{\pi^2(1+\xi_{\mathrm{g}})^{3/2}}$$

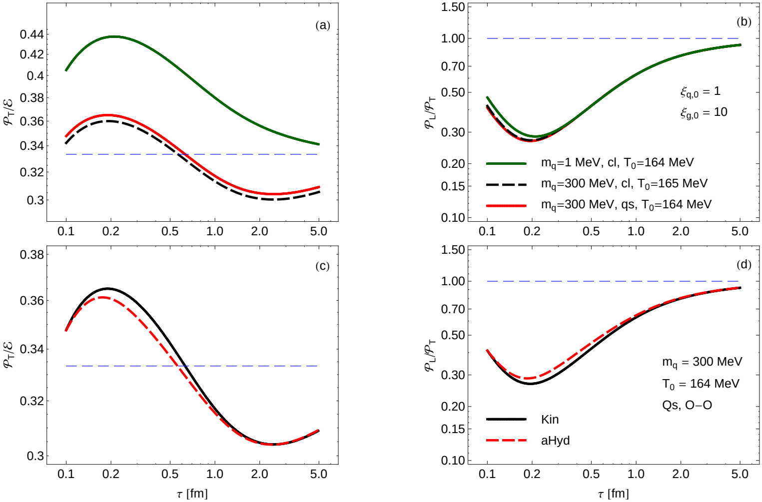
- Equilibrium functions Θ^g_{eq}

$$\Theta^g_{\mathrm{eq}} = \frac{8g_{\mathrm{g}}\zeta(5)T^5}{\pi^2}$$

- It leads to the last equation:

$$\frac{1}{1+\xi_{\mathrm{g}}}\frac{d\xi_{\mathrm{g}}}{d\tau} - \frac{2}{\tau} = -\frac{\xi_{\mathrm{g}}(1+\xi_{\mathrm{g}})^{1/2}}{\tau_{\mathrm{eq}}}\frac{T^5}{\Lambda_{\mathrm{g}}^5} \tag{7}$$

Results for Oblate-Oblate system



Summary

- We have build a model, based on the zeroth, first and the second moments of the kinetic equation for a mixture of quark and gluon fluids. Model allows to find $T(\tau)$, $\xi_q(\tau)$, $\xi_g(\tau)$, $\Lambda_q(\tau)$, $\Lambda_g(\tau)$, $\lambda(\tau)$, $\mu(\tau)$ functions.
- Anisotropic hydrodynamics works very well in the case of mixture of quark and gluon fluids.
- In comparison with previous papers, new formulation of anisotropic hydrodynamics allows to have massive components of the mixture. Quantum statistics have been included.
- We have found a very good agreement between anisotropic hydrodynamics and kinetic theory.

Thank you