Non-perturbative Gauge-Higgs Unification

Nikos Irges, NTUA

ICNFP 2017, Kolymbari, Crete, 28 Aug. 2017

Based on

- **N.I. and F. Knechtli (NP B719, 121, 2005), F. Knechtli, B. Bunk and N.I. (LAT2005), N.I. and F. Knechtli (hep-lat/0604006), N.I. and F. Knechtli (NP B775, 283, 2007), N.I., F. Knechtli and M. Luz (JHEP 0708, 028), N.I., F. Knechtli and K. Yoneyama (NP B865, 541, 2012), N.I., F. Knechtli and K. Yoneyama (PL B722, 378, 2013)**

- **N. I. and F. Knechtli, JHEP 1406 (2014) 070**

- **M. Alberti, N.I., F. Knechtli and G. Moir, JHEP 1509 (2015) 159 and work in progress**

- **N.I. and F. Koutroulis, arXiv:1703.10369 [hep-ph] and work in progress**

- **Work in progress with A. Chatziagapiou**

- **N.I., arXiv:1611.00157 [hep-th]**

- The quantum Higgs mechanism in 4d
- Historical interlude on Extra Dimensions
- Putting things together: NPGHU
- Towards an effective action

The Standard Model

$SU(3) \times SU(2) \times U(1)$
Quantum gauge fields coupled to fermionic matter and Higgs field(s)
in a spontaneously broken EW symmetry phase in 4 space-time dimensions.

The Higgs sector has a naturalness issue that can be resolved by extra symmetry (susy, compositeness, 5d gauge symmetry, etc) or dynamically (e.g. simply by a low cut-off and/or by weird cancellations) or by a combination-correlation of both.
Behind its innocent perturbative nature, subtle non-perturbative effects may be hiding in the details (such as the origin of the relative sign in the Higgs potential):

$$V = -m_0^2 H^2 + \frac{\lambda_0}{6} H^4$$

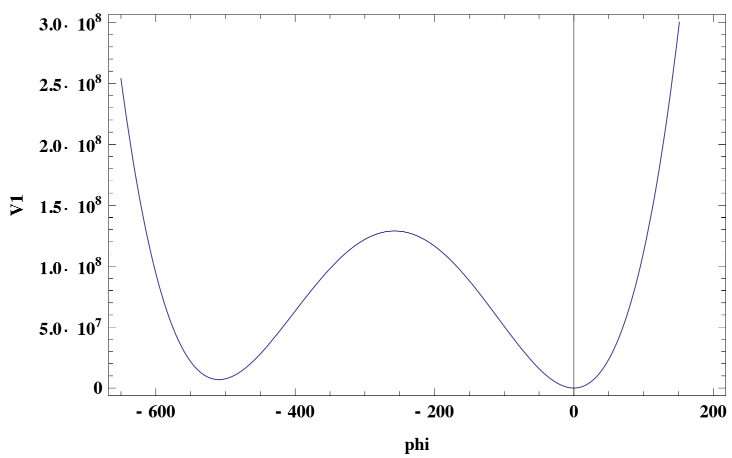
Let us look a bit closer at the Higgs mechanism in the simplest possible 4d context…

A toy model for the EW sector: the Abelian-Higgs model in 4d

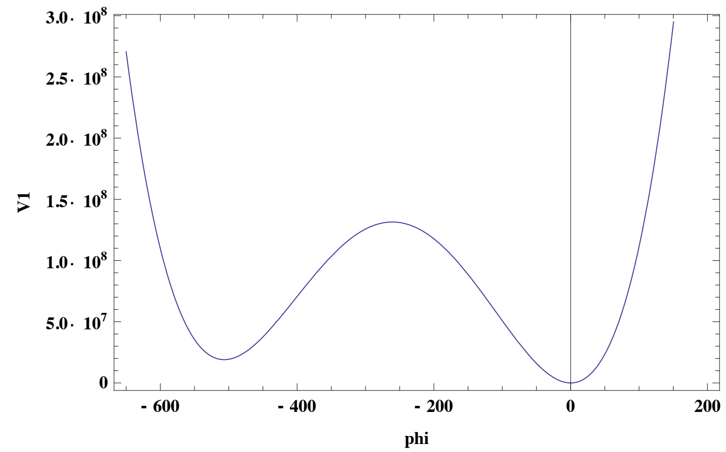
$$\mathcal{L}_{\rm AH} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2 + |D_\mu H|^2 + m_0^2|H|^2 - \frac{\lambda_0}{6}|H|^4 + \text{const.}$$

Gauge invariant 1-loop AHiggs potential

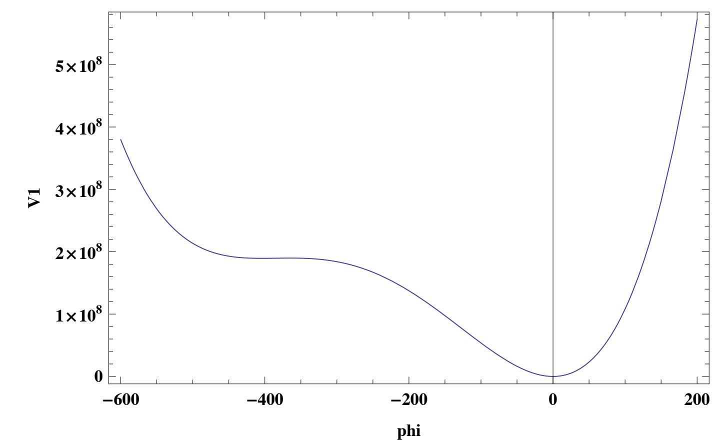
N.I. and F. Koutroulis, arXiv:1703.10369



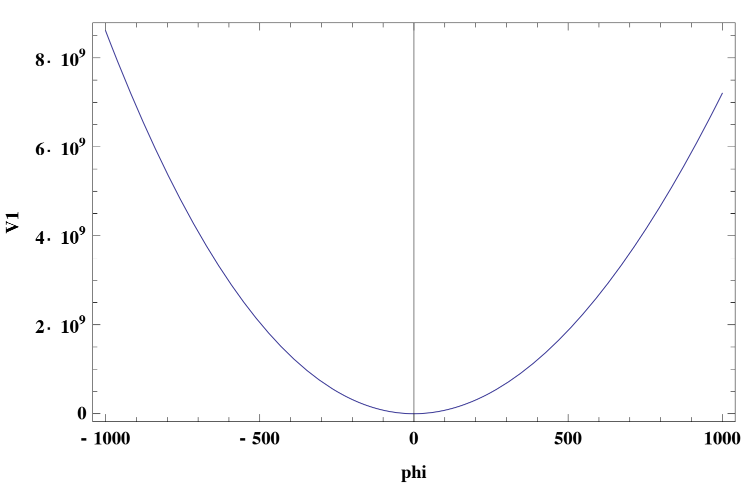
$125\ GeV$



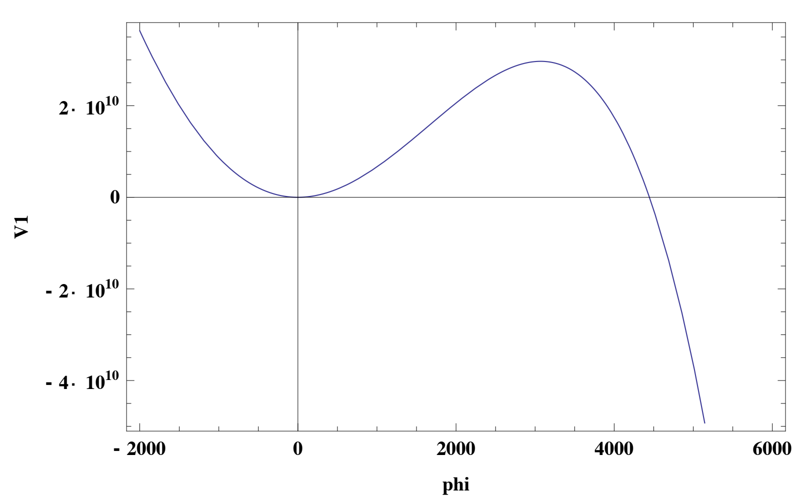
$10^{19}\ GeV$



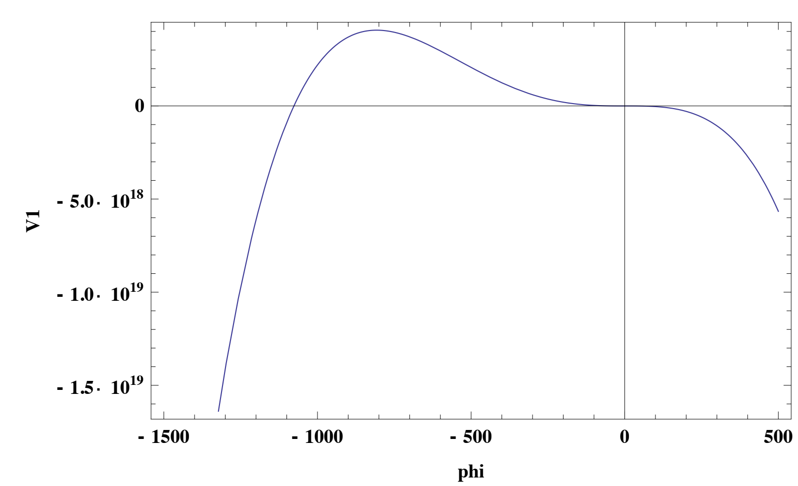
$10^{40}\ GeV$



$3\cdot 10^{46}\ GeV$



$3.1\cdot 10^{46}\ GeV$



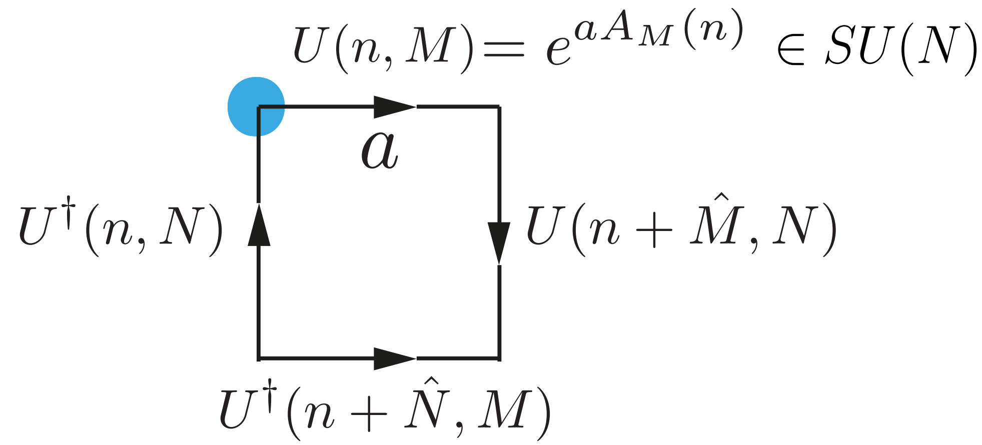
$4.3\cdot 10^{49}\ GeV$

Instability scale Non-perturbative domain Landau Pole Phase transition?

The lattice regularization

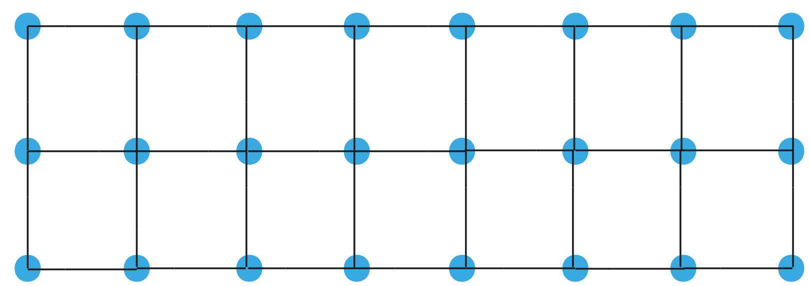
$x \to n \to$ ●

plaquette $U_{\mu\nu}(n)$:

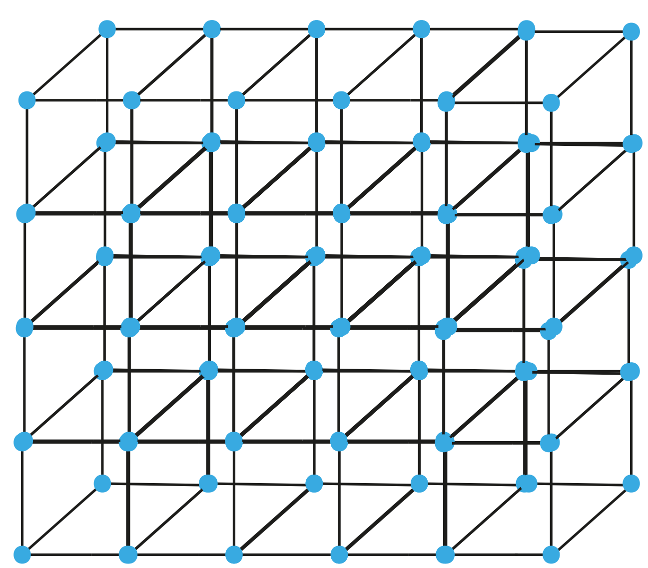


$\to -\frac{1}{4} a^2 F^2_{MN} + O(a^4)$

$2d$



$3d$

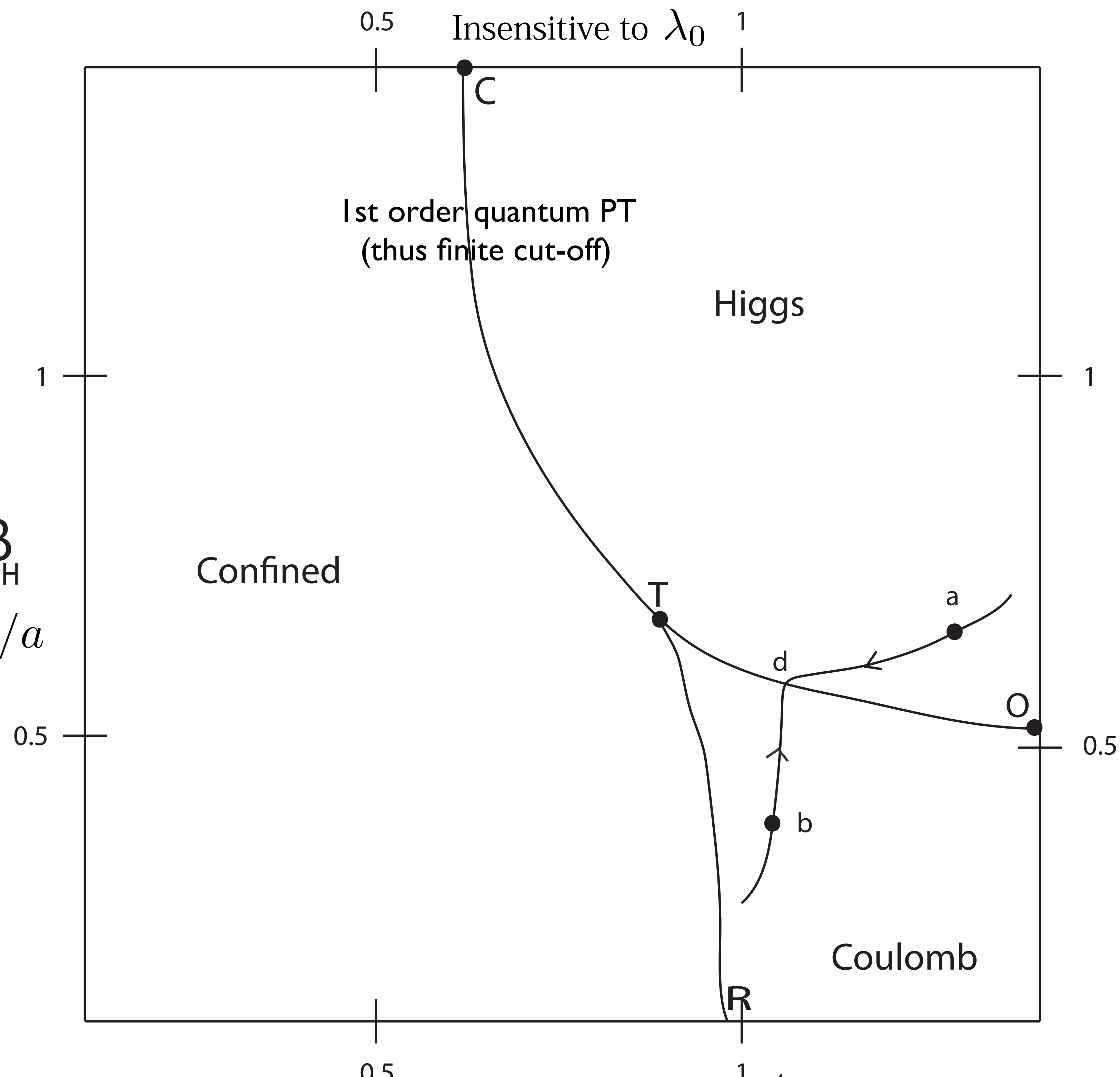


$\cdots 4d, 5d, \cdots$

Now we can approach safely the phase transition

Phase diagram of the Abelian-Higgs model in 4d

Fradkin & Shenker (1979), Callaway & Carson (1982), Evertz et al (1987), Khodayari (2016)



$$\beta_H \equiv \frac{v_0^2}{\Lambda^2}$$

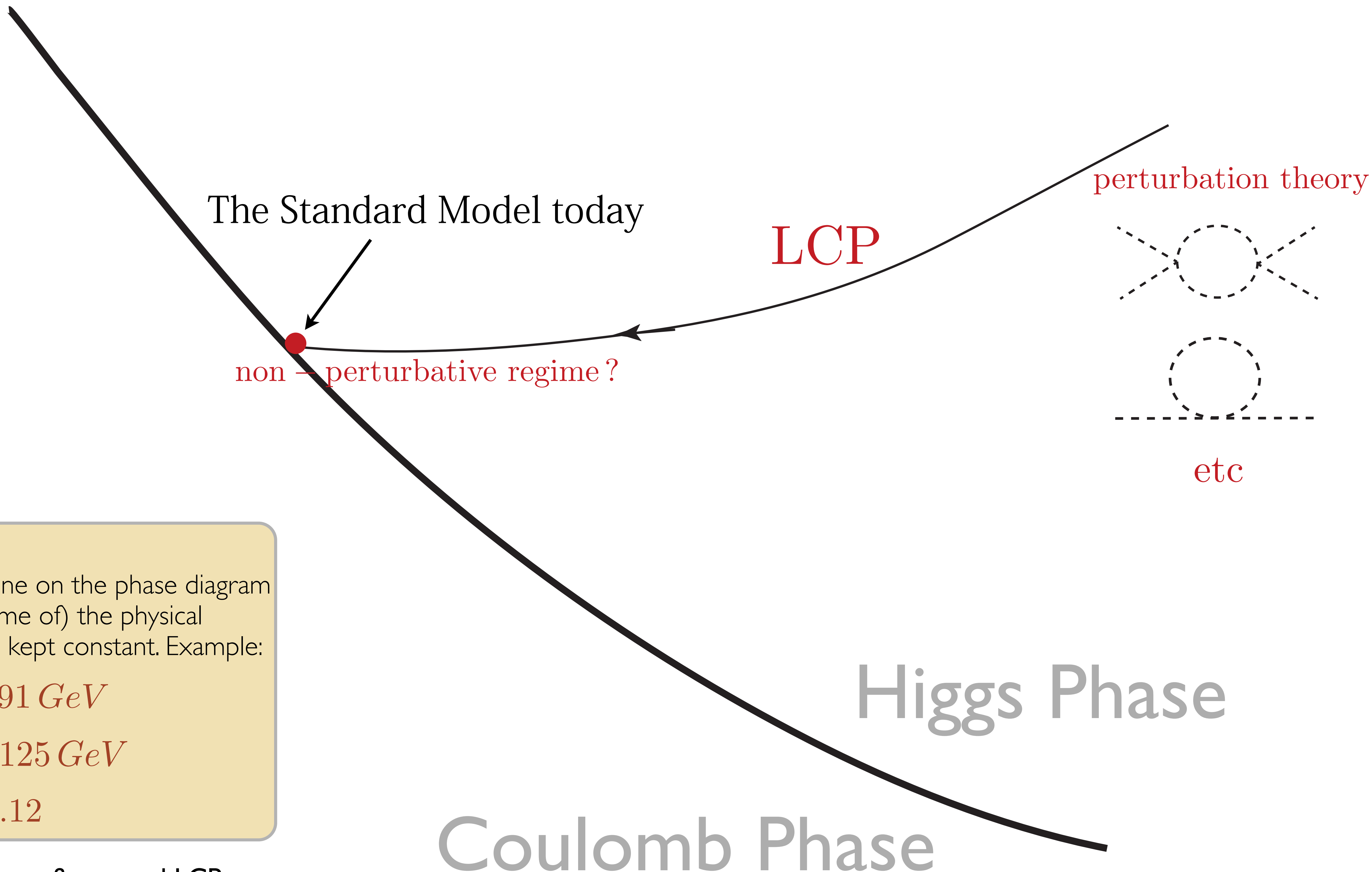
β_H

$$\Lambda = \text{cutoff} \sim 1/a$$

β

$$\beta \equiv \frac{4}{g_0^2}$$

Let us **assume** that the AH model describes the observed Higgs mechanism and that we are sitting near a 1st order quantum phase transition.
Along the LCP the cut-off increases as the phase transition is approached and the LCP terminates on the phase transition at a value of the cut-off that may be lower than its maximal allowed value by perturbation theory (N.I. and F. Koutroulis, arXiv:1703.10369). It is like trying to launch a rocket from inside a room.



Definition:
An LCP is a line on the phase diagram on which (some of) the physical quantities are kept constant. Example:

- $m_Z = 91\, GeV$
- $m_H = 125\, GeV$
- $\lambda_R = 0.12$

Existence of non-fine-tuned LCP equivalent to stabilising the Higgs

Tentative lesson from 4d:
If you want to generate a SM-like Higgs potential without putting it by hand, hope for a solution to the naturalness problem with the SM sitting on the brink of a "bulk" or "quantum" phase transition then find some UV completion without elementary scalars that possesses such phase transition and it has a (dimensionally reduced) Higgs phase where the hierarchy is somehow protected.
Here we have in mind a framework where the mechanism is purely bosonic, that is, not of the susy or technicolor type.

Higher than 4d gauge theories with appropriate boundary conditions turn out to have such properties.

Historical interlude on Extra Dimensions

The basic ideas

- Extra dimensions may hide the origin for the different types of interactions we observe in our 4d world.
  (T. Kaluza (1921), O. Klein (1926))

- Extra dimensions can actually unify forces that look different in 4d.
  (A. Einstein and P. Bergmann (1938))

- String theory and Supergravity compactifications realize naturally this idea.
  (from the 80's until today a huge amount of work; for the basics see for example the textbooks of Green, Schwarz and Witten and that of Polchinski). Some technical and sociological issues have somewhat slowed down this activity during the last few years.

- A fifth dimension is useful to define 4d chiral fermions non-perturbatively via a domain wall construction. Not the first, but the most successful of a series of "localization" ideas.
  (D. B. Kaplan (1992))

- And an independent idea but perhaps relevant to EDs: Some perturbatively non-renormalizable theories may be non-perturbatively renormalizable (S. Weinberg (1979)). The statement has not been proven for either 4d gravity or for higher than four dimensional gauge theories.

The QFT approach I
(perturbative and KK/finite temperature)

- Small and smooth extra dimensions can generate Higgs fields out of gauge fields.
  (N. Manton (1979), Y. Hosotani (1983, 1989))

- The Higgs potential in periodic 5d is a pure quantum/finite temperature effect. The Higgs mass is cut-off independent at one loop (I. Antoniadis, K. Benakli, M. Quiros (2001)). The Higgs mechanism is realized though only in the presence of fermions.

- Orbifold boundary conditions inspired by string theory can yield a Higgs field in the representation that it appears in the SM. Fermions are still necessary for the Higgs mechanism.
  (H. Hatanaka, T. Inami, C. S. Lim (1998))

- Higher than four dimensional QFT's are perturbatively non-renormalizable. Ignoring this fact, one can still perform perturbative QFT computations and obtain for instance a power-like running for gauge couplings (K. R. Dienes, E. Dudas, T. Gherghetta (1998)), a finite Higgs mass via a Coleman-Weinberg mechanism with periodic boundary conditions (I. Antoniadis, K. Benakli, M. Quiros (2001)) or with orbifold boundary conditions (M. Kubo, C. S. Lim, H. Yamashita (2001)) and also via standard Feynman diagram techniques (H. Georgi, A. K. Grant, G. Hailu (2001), G. von Gersdorff, N. Irges, M. Quiros (2002), H-C. Cheng, K. T. Matchev, M. Schmalz (2002), …).

- A huge amount of related work follows, mostly model building. The predictability issue of these theories is too hard to address perturbatively.

- The phenomenology of the perturbative approach is also tough though: the Higgs is generically much lighter than the Z-boson. Fine tuning reappears. The perturbative non-renormalizability casts a shadow over the whole approach…the genre fades…

The QFT approach II
(non-perturbative, periodic and zero temperature)

- A 5d pure SU(2) lattice gauge theory with fully periodic bc's has a 1st order quantum phase transition that separates the Coulomb and Confined phases (M. Creutz (1979)). This phase transition persists also in 6d, 7d and 8d and perhaps in any higher d (N. Irges, G. Koutsoumbas, K. Ntrekis (2015)).

- It is possible to introduce an anisotropy in the 5d theory so that the dimensionality of the phase diagram increases by one. It is argued that for small values of the anisotropy parameter a novel phase emerges where the lattice decomposes into an array of non-interacting 4d slices, the so-called layered phase (Y. Fu, H. B. Nielsen (1984)). The existence of such a phase would be a proof of dynamical (i.e. quantum) gauge field localization on a lower dimensional surface.

- In string theory, D-branes localize gauge interactions (Polchinski 1995). Even though not a QFT idea and not fully non-perturbative, it had a deep influence on many subsequent QFT constructions.

- In QFT, special boundary conditions may localize certain fields on 4d boundaries, while other fields my extend in the higher dimensional bulk. Dimensional reduction is realized by localization, as opposed to compactification (G. Dvali and M. Shifman (1996), I. Antoniadis, N. A-Hamed, S. Dimopoulos, G. R. Dvali (1998), L. Randall, R. Sundrum (1999)). The proofs if any, are classical.

- For large values of the anisotropy parameter the system makes contact with the finite temperature regime (S. Ejiri, J. Kubo, M. Murata (2000)).

- The existence of the layered phase is first investigated on the 5d Abelian-Higgs model and on the 5d adjoint Higgs SU(2) model in a series of works (P. Dimopoulos, K. Farakos, P. de Forcrand, C. P. Korthals-Altes, G Koutsoumbas, S. Nicolis and collaborators (approx. 2000-2002)).

- The non-perturbative phase diagram of the periodic pure gauge, non-Abelian system is determined on the lattice via Monte Carlo methods in a few but heroic efforts (P. de Forcrand, A. Kurkela, M. Panero (2010), K. Farakos, S. Vrentzos (2012), F. Knechtli, M. Luz and A. Rago (2012), L. Del Debbio, A. Hart, E. Rinaldi (2012)). The phase transition separating the Coulomb and Layered phases is a quantum, 1st order transition.

- And the layered phase seems to exist: low temperature dimensional reduction via localization near the Coulomb-layered phase transition is confirmed (F. Knechtli, M. Luz and A. Rago (2012)).

- With fermions, the non-perturbative version of the Hosotani mechanism is investigated (G. Cossu, H. Hatanaka, Y. Hosotani, J-I Noaki (2014)). The periodic theory does not seem to possess a Higgs phase in the absence of fermions.

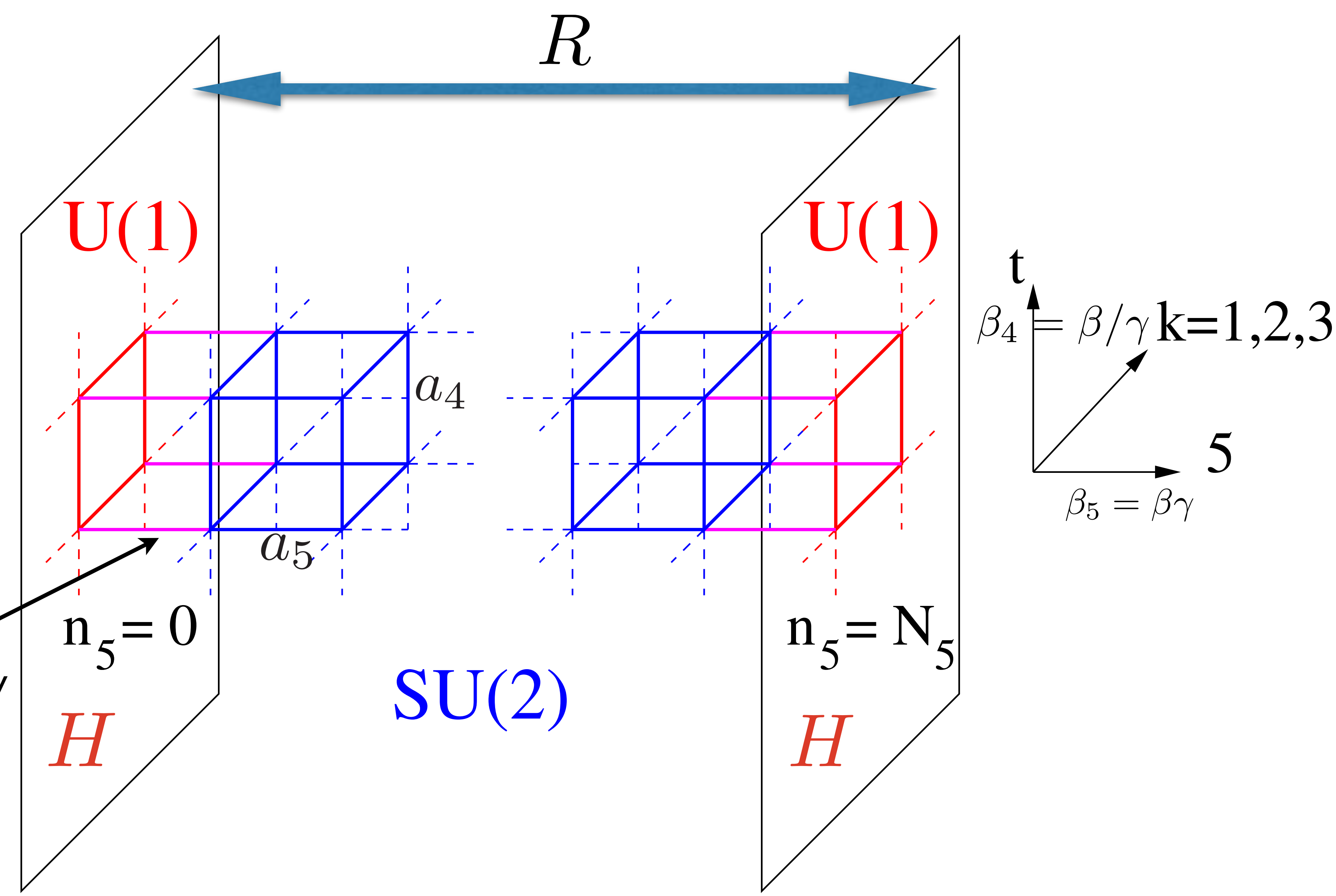
- Monte Carlo simulations are frustratingly hard…most lattice people gradually abandon the approach…

Now let us put everything together

The model

Let us consider now an anisotropic SU(2) 'lattice orbifold'. On the boundaries we have the spectrum of a 4d (Abelian-Higgs) model U(1) + H + excited states. Local and global symmetries are just 'right' so that all phases can be described in a gauge invariant way. We call this new version of GHU, "Non-Perturbative Gauge-Higgs Unification", or NPGHU.

N. I., F. Knechtli (2005, 2007, 2014)



The Higgs mechanism is ensured by the breaking of a global "stick" symmetry, acting on these "hybrid" links.

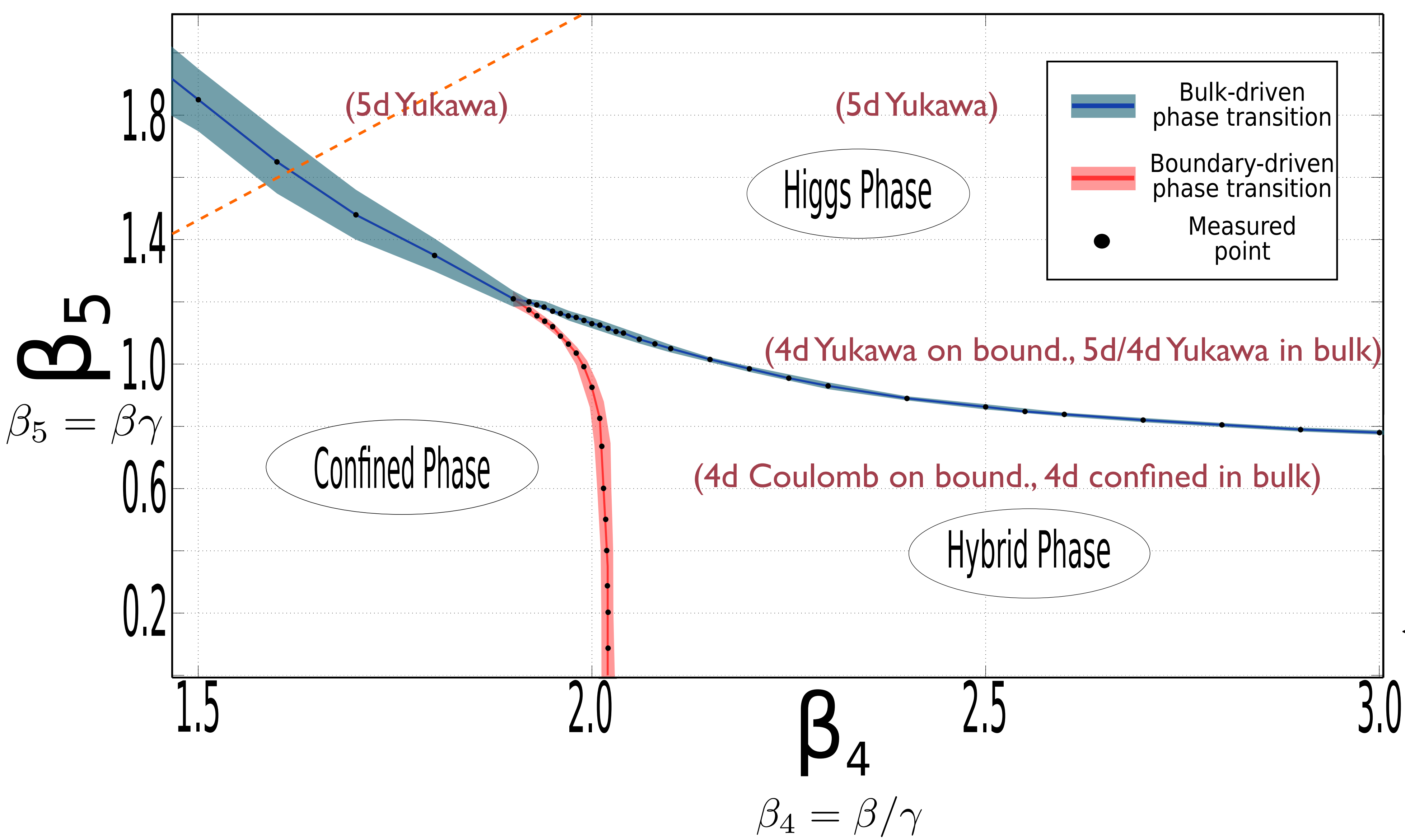
dimension-ful parameters: a_4, a_5, R, g_5

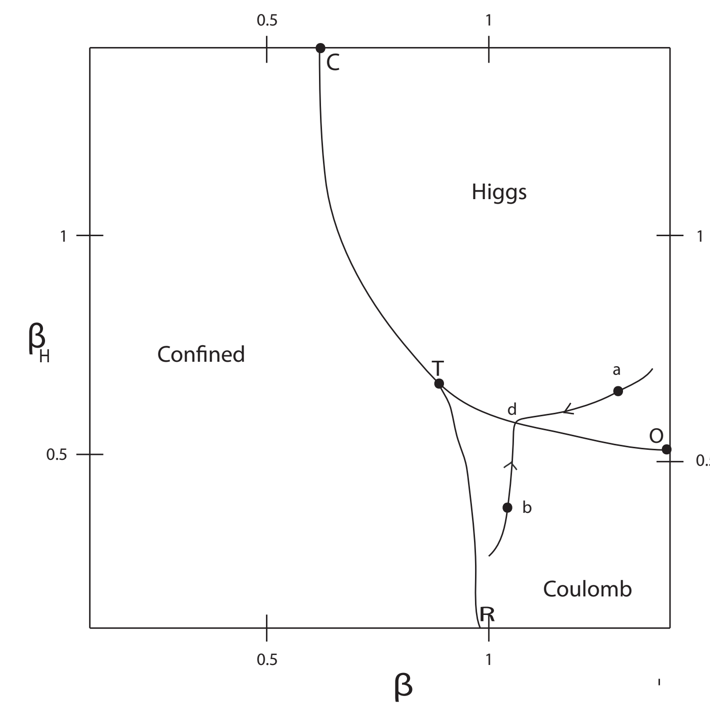
dimensionless parameters: $\beta = 4a_4^2/g_5^2 \ , \gamma = a_4/a_5 \ , N_5 = \pi R/a_5$

The phase diagram

G. Moir, M. Alberti, N. I., F. Knechtli (2015)

dynamics: phases + dimensional reduction (N5 dependence is weak)



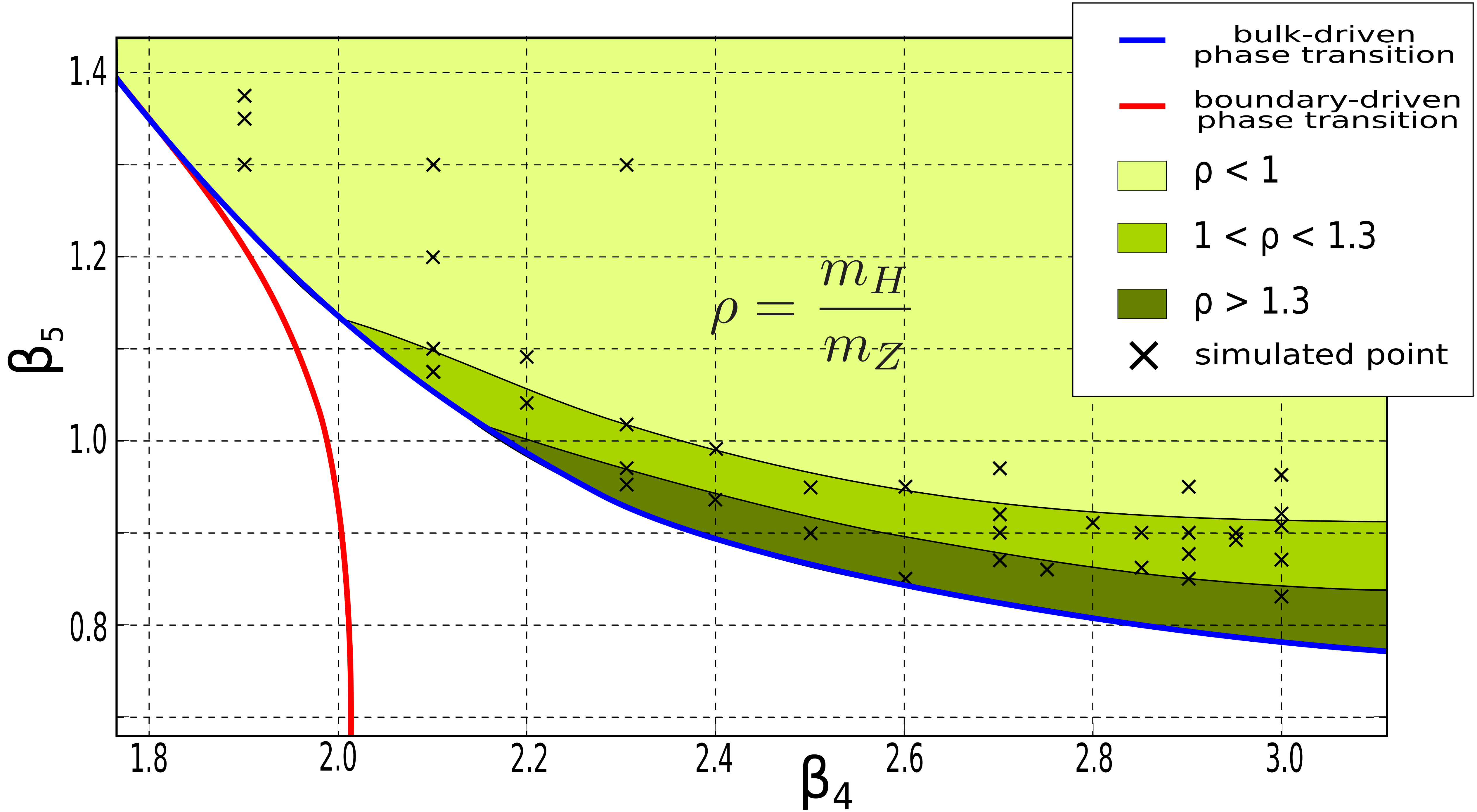


Note: this part of the phase diagram is reminiscent of the phase diagram of the (q=2) 4d Abelian-Higgs model, which is not unexpected since the two models have identical low lying spectra.

All phase transitions 1st order, effective theories must have a finite cut-off.

More dynamics:
Standard Model-like spectrum near the phase transition,
precisely in the 4d Yukawa regime, where the 5d space breaks into an array
of weakly interacting 4d planes. The lattice spacing decreases as the PT is approached from either sides.

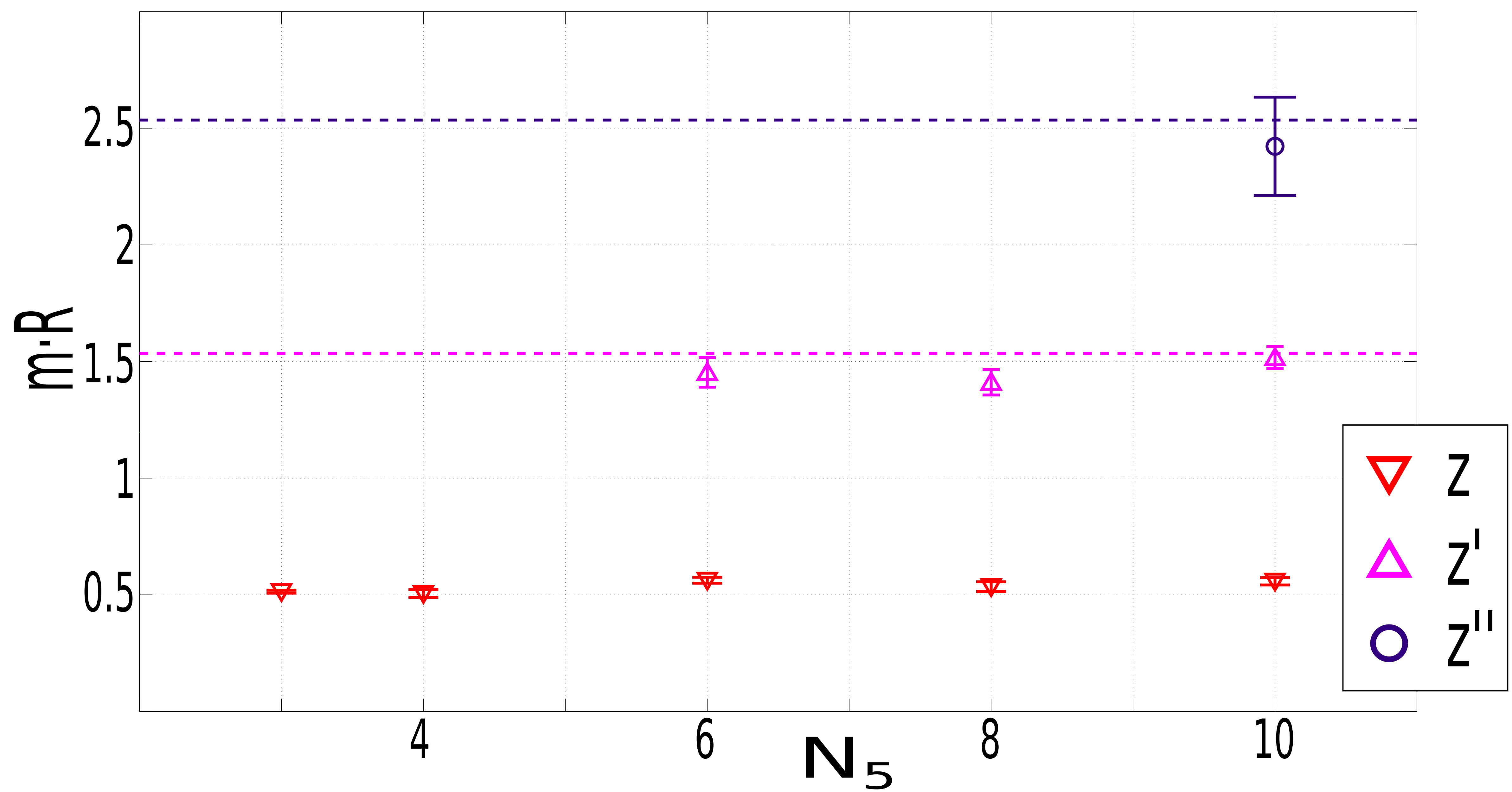
G. Moir, M. Alberti, N. I., F. Knechtli (2015)



quantum + bosonic Higgs mechanism…no other such mechanism in 4 (or higher) dimensions to our knowledge…

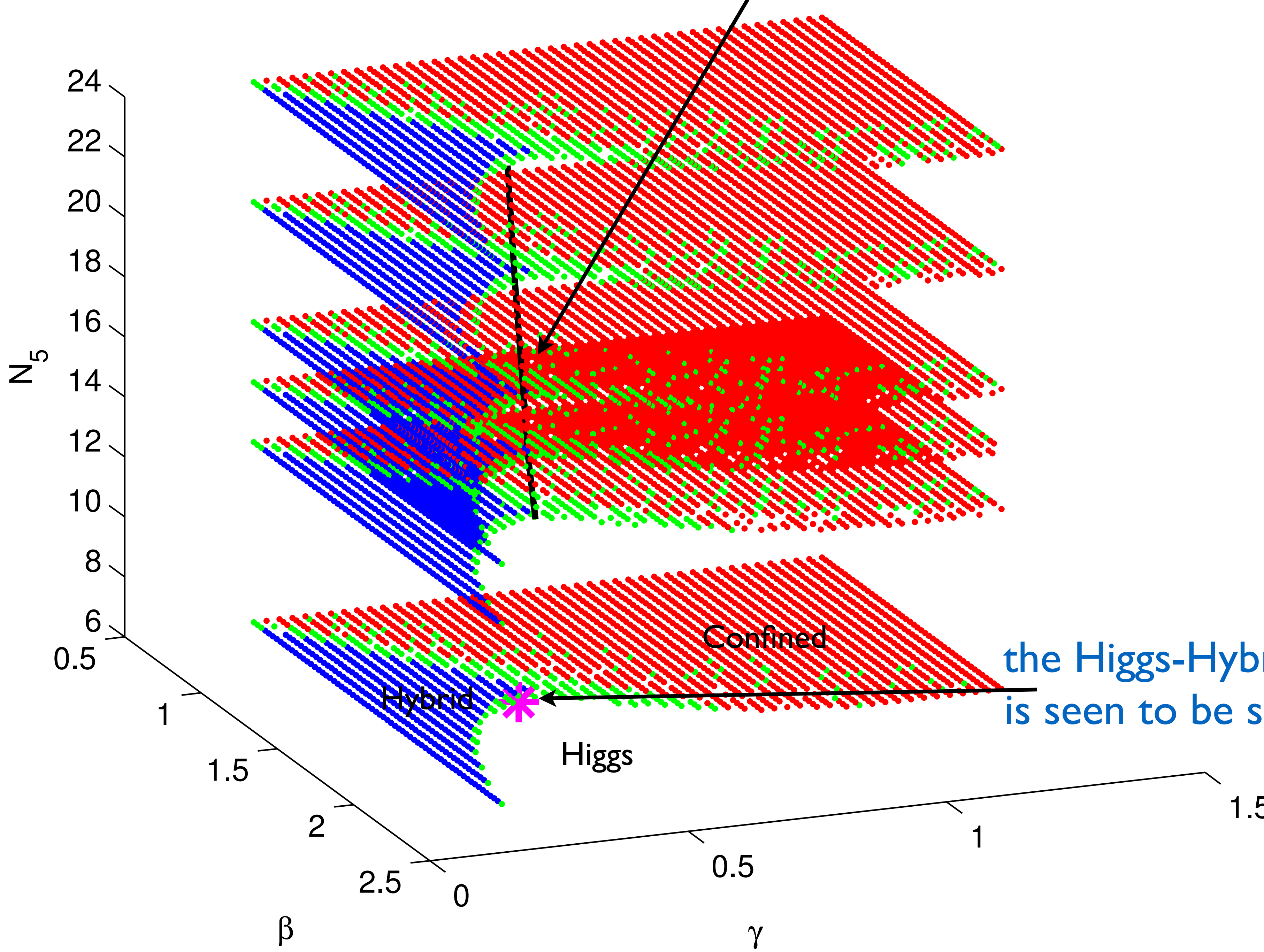
The spectrum in the gauge sector

G. Moir, M. Alberti, N. I., F. Knechtli (2015)



There is a vacuum shift of 1/2 wrt (1-loop) perturbation theory
to explain here…

A semi-analytical approach: The Mean-Field LCP (3d view)

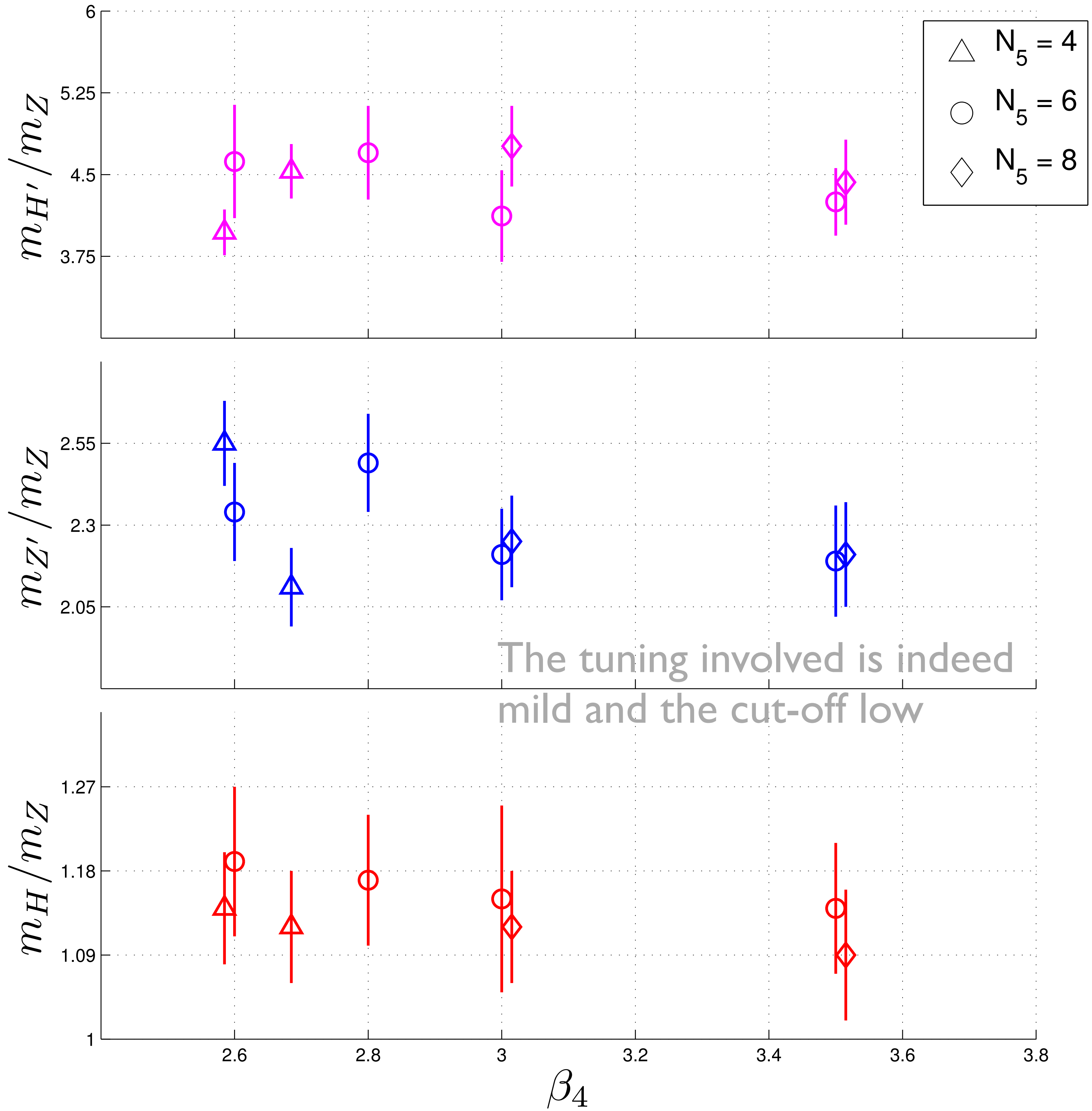


the Higgs-Hybrid phase transition is seen to be second order by MF!

N. I., F. Knechtli and K. Yoneyama (PL B722, 378, 2013)

A Monte Carlo LCP

from M. Alberti's talk at the Lattice 2016 conference

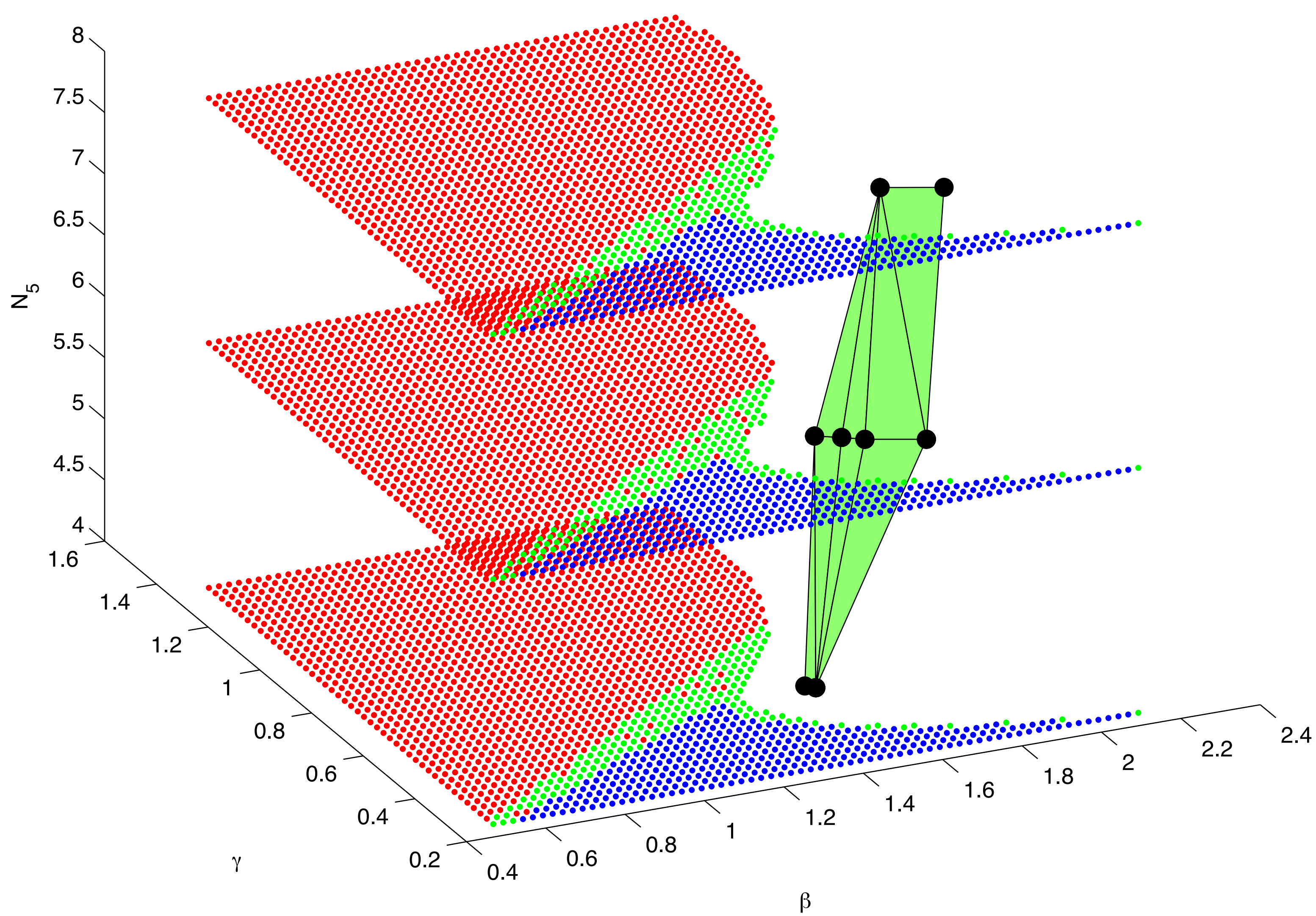


The tuning involved is indeed mild and the cut-off low

The Monte Carlo LCP (3d view)

courtesy of M. Alberti

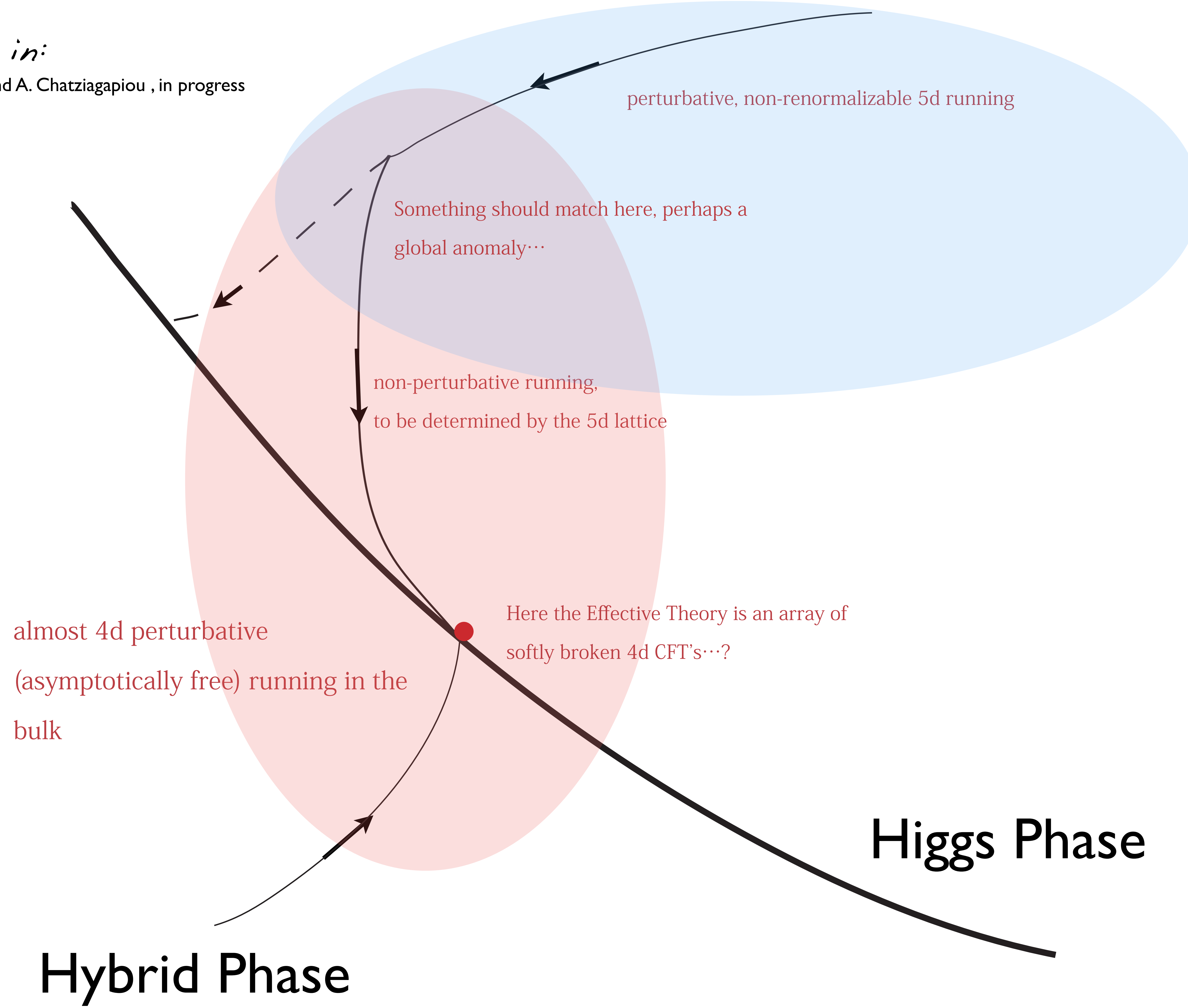
$$m_H/m_Z \simeq 1.15,\ m_{Z'}/m_Z \simeq 2.20$$



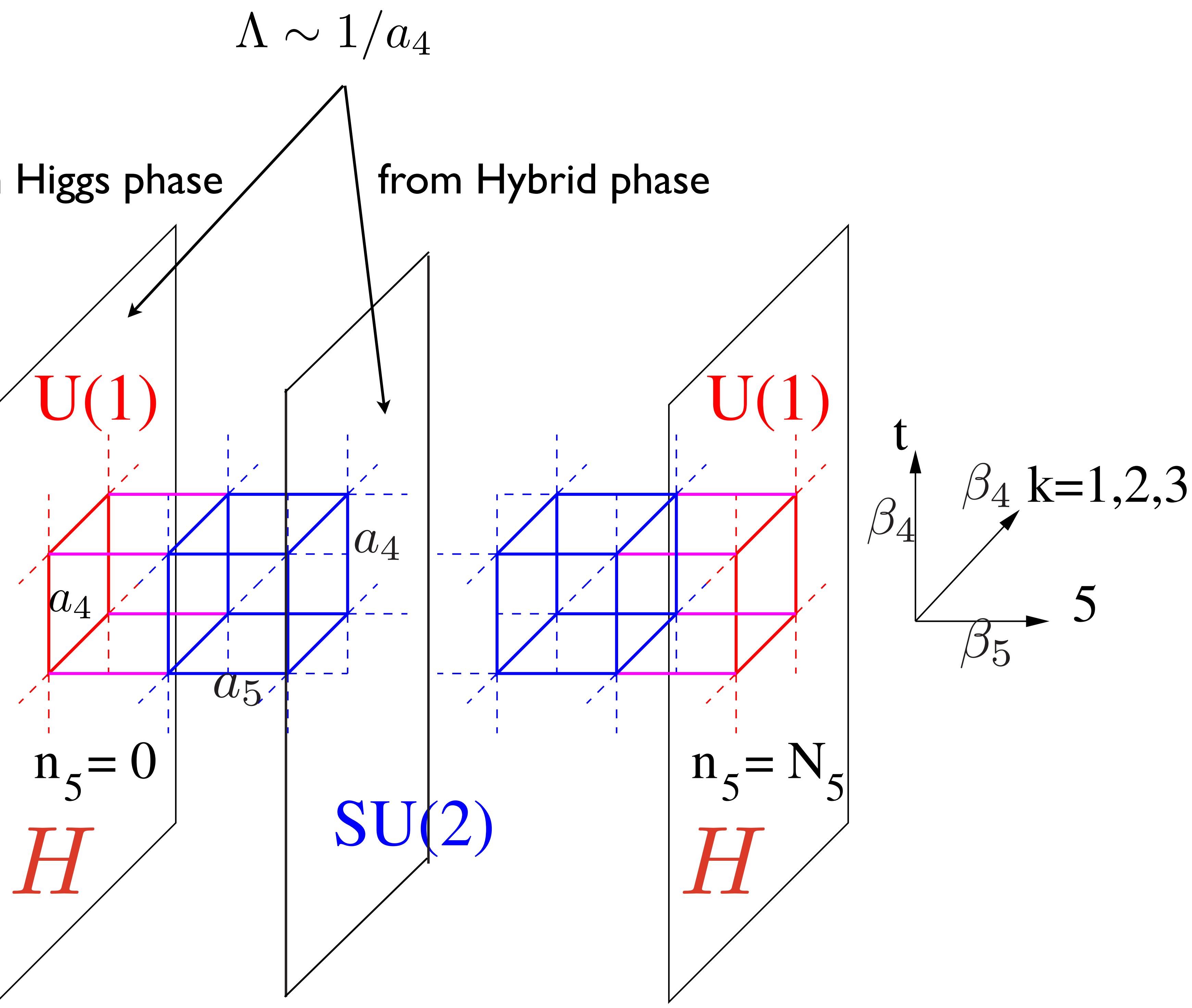
Towards an effective action

Zoom in:

with F. Koutroulis and A. Chatziagapiou , in progress



On the PT these two 4d slices are, to a good approximation, QFT's with the same cut-off,



- This is a gauge invariant way to construct simultaneously two field theories, a confined non-Abelian theory and an Abelian gauge-scalar system with the same cut-off. As the boundary U(1) gauge-scalar theory in the Higgs phase becomes strongly coupled, the confining bulk SU(2) theory in the hybrid phase becomes asymptotically free. There exist LCP's from both sides, notably from the Higgs phase side, meaning that the spectrum, including the Higgs mass is stable as the cut-off varies. The matching of the LCPs on the PT determines the absolute scale, i.e. the cut-off. The "experimental" value of the Sommer scale does not allow the common cut-off to be large.

- There is a yet undetermined RG flow from the UV where the system is 5-dimensional to the IR where we expect to see a system of weakly interacting, softly broken 4d CFT's.

Conclusions and Outlook

- We presented a pure gauge 5d orbifold model where a Higgs mechanism is realised on the weakly coupled 4d boundary in a regime of its phase diagram close to the first order bulk phase transition where the fifth dimension is strongly coupled. Precisely and only in this regime, we simultaneously observe a Higgs slightly heavier than the Z boson and the system being dimensionally reduced to 4-dimensions via localization. The existence of an LCP points to a Higgs mass whose stability is somehow non-perturbatively guaranteed at least within a certain energy regime, by the higher dimensional gauge symmetry. We gave arguments for these statements analytically in the context of a Mean-Field expansion and numerically via extensive Monte Carlo simulations, both on the lattice.

- We described a plausible way to construct a continuum effective field theory (a system of spontaneously broken CFTs perhaps) that will allow us on one hand to answer the question whether this novel Higgs mechanism is in the same class where the Standard Model Higgs mechanism belongs and understand the details of the gauge hierarchy protection dynamics on the other.

Thank you!