$f_1(1285) \to e^+e^-$ decay and direct f_1 production in e^+e^- collisions

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Introduction

$f_1(1285)$ meson: $I^G(J^{PC}) = 0^+(1^{++})$

$$m_f = 1282.0 \pm 0.5 \text{ MeV}, \ \Gamma_f = 24.1 \pm 1.0 \text{ MeV} \ \text{[PDG'16]}$$

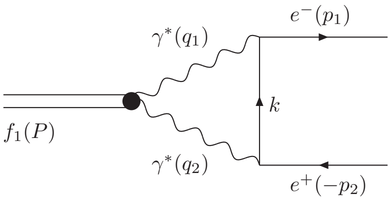
C-even meson $\to e^+e^-$ decay proceeds via two virtual photons and therefore Γ is suppressed by a factor of α^4

Experimental limits:
$\Gamma(\eta'(958) \to e^+e^-) < 0.002$ eV (90% CL)
$\Gamma(f_2(1270) \to e^+e^-) < 0.11$ eV (90% CL)
$\Gamma(a_2(1320) \to e^+e^-) < 0.56$ eV (90% CL)

Limit on $\Gamma(f_1(1285) \to e^+e^-)$ is still not obtained

The process $e^+e^- \to f_1 \to mesons$ is planned to be studied at VEPP-2000 e^+e^- collider in Novosibirsk

Simple estimate of $f_1 \to e^+e^-$ decay width



There is only one P- and C-even invariant amplitude (if $m_e = 0$)

$$M(f_1 \to e^+e^-) = F_A \alpha^2 \widetilde{e}_\mu \bar{u}\gamma^\mu\gamma^5 v \quad (1)$$

where F_A is the dimensionless coupling constant.

The decay width: $\Gamma(f_1 \to e^+e^-) = \dfrac{\alpha^4 |F_A|^2}{12\pi} m_f \quad (2)$

It is natural to assume that $|F_A| \sim 1 \Rightarrow \Gamma(f_1 \to e^+e^-) \sim 0.1$ eV

Model-independent description of $f_1 \to e^+e^-$ amplitude

To calculate $\Gamma(f_1 \to e^+e^-)$ we should know $f_1 \to \gamma^*\gamma^*$ amplitude.

$f_1 \to \gamma^*\gamma^*$ amplitude is parameterized in general by two dimensionless form factors, $F_1(q_1^2, q_2^2)$ and $F_2(q_1^2, q_2^2)$, e.g.

$$M(f_1 \to \gamma^*\gamma^*) = \frac{\alpha}{m_f^2} F_1(q_1^2, q_2^2) i\epsilon_{\mu\nu\rho\sigma} q_1^\mu e_1^{*\nu} q_2^\rho e_2^{*\sigma} \widetilde{e}^\tau (q_1 - q_2)_\tau +$$
$$+ \frac{\alpha}{m_f^2} \Big\{ F_2(q_1^2, q_2^2) i\epsilon_{\mu\nu\rho\sigma} q_1^\mu e_1^{*\nu} \widetilde{e}^\rho \left[q_2^\sigma e_2^{*\lambda} q_{2\lambda} - e_2^{*\sigma} q_2^2 \right] +$$
$$+ F_2(q_2^2, q_1^2) i\epsilon_{\mu\nu\rho\sigma} q_2^\mu e_2^{*\nu} \widetilde{e}^\rho \left[q_1^\sigma e_1^{*\lambda} q_{1\lambda} - e_1^{*\sigma} q_1^2 \right] \Big\} \quad (3)$$

e_1, e_2 and $\widetilde{e}$ are the polarization vectors of photons and f_1 meson.

F_1 corresponds to transversal photons (TT),
F_2 describes a combination of TT and LT polarization states.
The polarization state LL (when both virtual photons are longitudinal) does not exist.

Model-independent description of $f_1 \to e^+e^-$ amplitude

$$M(f_1 \to \gamma^*\gamma^*) = \frac{\alpha}{m_f^2} F_1(q_1^2, q_2^2) i\epsilon_{\mu\nu\rho\sigma} q_1^\mu e_1^{*\nu} q_2^\rho e_2^{*\sigma} \tilde{e}^\tau (q_1 - q_2)_\tau +$$

$$+ \frac{\alpha}{m_f^2} \left\{ F_2(q_1^2, q_2^2) i\epsilon_{\mu\nu\rho\sigma} q_1^\mu e_1^{*\nu} \tilde{e}^\rho \left[q_2^\sigma e_2^{*\lambda} q_{2\lambda} - e_2^{*\sigma} q_2^2 \right] + \right.$$

$$\left. + F_2(q_2^2, q_1^2) i\epsilon_{\mu\nu\rho\sigma} q_2^\mu e_2^{*\nu} \tilde{e}^\rho \left[q_1^\sigma e_1^{*\lambda} q_{1\lambda} - e_1^{*\sigma} q_1^2 \right] \right\}$$

Due to Bose symmetry form factor $F_1(q_1^2, q_2^2)$ must be antisymmetric, $F_1(q_1^2, q_2^2) = -F_1(q_2^2, q_1^2)$.

$f_1 \to \gamma\gamma$ decay is forbidden by Landau-Yang theorem $\Rightarrow$ the amplitude vanishes when both photons are on-shell.
The first term vanishes because $F_1(0,0) = 0$, while all other terms vanish because $q^2 = 0$ and $e^\lambda q_\lambda = 0$ for real photons.

After substitution of this $f_1 \to \gamma^*\gamma^*$ amplitude into the one-loop diagram:

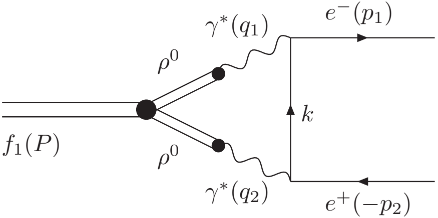
$$M(f_1 \to e^+e^-) = -\frac{16\pi i\alpha^2}{m_f^2}\tilde{e}^{\mu} P^{\nu} \bar{u}\gamma^{\lambda}\gamma^5 v \int \frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu k_\lambda}{k^2 q_1^2 q_2^2} F_1(q_1^2, q_2^2) -$$
$$-\frac{8\pi i\alpha^2}{m_f^2}\tilde{e}^{\mu}\bar{u}\gamma^{\nu}\gamma^5 v \int \frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu}{k^2 q_1^2 q_2^2}\left\{F_2(q_1^2, q_2^2) q_2^2 + F_2(q_2^2, q_1^2) q_1^2\right\} +$$
$$+\frac{4\pi i\alpha^2}{m_f^2}\tilde{e}_{\mu}\bar{u}\gamma^{\mu}\gamma^5 v \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 q_1^2 q_2^2}\times$$
$$\times \left\{F_2(q_1^2, q_2^2)\left[k^2(p_1p_2 + p_1k - p_2k) - 2q_2^2(p_1k) + 2q_2^2k^2\right] +\right.$$
$$\left. + F_2(q_2^2, q_1^2)\left[k^2(p_1p_2 + p_1k - p_2k) + 2q_1^2(p_2k) + 2q_1^2k^2\right]\right\} \quad (4)$$

where $q_1 = p_1 - k$ and $q_2 = p_2 + k$.

Constants of $f_1 \to \rho^0\gamma$ decay from experimental data

One can not calculate $\Gamma(f_1 \to e^+e^-)$ in model-independent way, because explicit form of $F_1(q_1^2, q_2^2)$ and $F_2(q_1^2, q_2^2)$ is unknown. So, we have to choose some reasonable model.

We assume that the main contribution to the amplitude $M(f_1 \to e^+e^-)$ comes from the diagram, where both virtual photons are coupled with f_1 meson via intermediate ρ^0 mesons.



Constants of $f_1 \to \rho^0\gamma$ decay from experimental data

Arguments for this model — dimensional analysis shows that form factors F_1 and F_2 should decrease rapidly with increasing momentum k in order to avoid divergences in (4).
This is the hint that both virtual photons couple with f_1 meson via some massive vector mesons.
In such case form factors F_1 and F_2 behave as $1/k^4$ and the amplitude (4) does not diverge.

Experimental data show that one of the main f_1 decay channels, $f_1 \to 4\pi$ [$\mathcal{B}(f_1 \to 4\pi) \approx 33\%$], proceeds mainly via the intermediate $\rho\rho$ state.
Other evidence of this mechanism is a large (5.5%) branching ratio of radiative $f_1 \to \rho^0\gamma$ decay.

Some parameters of the model can be constrained from experimental data on $f_1 \to \rho^0\gamma$ decay

$$M(f_1 \to \rho^0\gamma) = \frac{\alpha}{m_f^2} g_1 i\epsilon_{\mu\nu\rho\sigma} p^\mu \epsilon^{*\nu} q^\rho e^{*\sigma} \widetilde{e}^\tau (p-q)_\tau - \\ - \frac{\alpha m_\rho^2}{m_f^2} g_2 i\epsilon_{\mu\nu\rho\sigma} \widetilde{e}^\mu \epsilon^{*\nu} q^\rho e^{*\sigma} \quad (5)$$

This amplitude contains two complex coupling constants, g_1 and g_2. g_1 corresponds to T polarization state of ρ^0 in the f_1 rest frame, and g_2 corresponds to a combination of L and T polarization states.

Constants of $f_1 \to \rho^0\gamma$ decay from experimental data

The width of $f_1 \to \rho^0\gamma$ decay

$$\Gamma(f_1 \to \rho^0\gamma) = \frac{\alpha^2}{96\pi} m_f (1-\xi)^3 \times \\ \times \left[(1-\xi)^2 |g_1|^2 + \xi(1+\xi)|g_2|^2 + 2\xi(1-\xi)|g_1||g_2|\cos\delta\right] \quad (6)$$

where $\xi = m_\rho^2/m_f^2 \approx 0.37$

Since the parameters g_1 and g_2 do not correspond to different polarization states, the interference term does not vanish after summation over polarizations, and expression (6) contains $\delta = \phi_1 - \phi_2$, which is the relative phase of the complex constants g_1 and g_2.

Constants of $f_1 \to \rho^0\gamma$ decay from experimental data

In addition to $\Gamma(f_1 \to \rho^0\gamma)$ one more relation was derived from the polarization experiment. The ratio of the contributions of two ρ^0 helicity states, $r = \rho_{LL}/\rho_{TT} = 3.9 \pm 0.9 \pm 1.0$, was determined in the VES experiment from the analysis of angular distributions in the reaction $f_1 \to \rho^0\gamma \to \pi^+\pi^-\gamma$:

$$|M(f_1 \to \rho^0\gamma \to \pi^+\pi^-\gamma)|^2 \sim \rho_{LL}\cos^2\theta + \rho_{TT}\sin^2\theta \qquad (7)$$

where ρ_{LL} and ρ_{TT} are density matrix elements corresponding to longitudinal and transverse ρ^0 mesons, respectively; θ is the angle between π^+ and γ momenta in the ρ^0 rest frame.
Calculation of $|M(f_1 \to \rho^0\gamma \to \pi^+\pi^-\gamma)|^2$ with our amplitude (5) leads to the following ratio of the coefficients at $\cos^2\theta$ and $\sin^2\theta$:

$$r = \frac{2\xi|g_2|^2}{(1-\xi)^2|g_1|^2 + \xi^2|g_2|^2 + 2\xi(1-\xi)|g_1||g_2|\cos\delta} \qquad (8)$$

Constants of $f_1 \to \rho^0\gamma$ decay from experimental data

It is possible to find from (6) and (8) the magnitude of coupling constant g_2,

$$\alpha|g_2| = 1.5 \pm 0.2 \tag{9}$$

However, it is impossible to extract the magnitude of the constant g_1 and/or the phase δ from the experimental data.

Taking into account that $-1 \leq \cos\delta \leq 1$, we obtain

$$0.16 \lesssim \alpha|g_1| \lesssim 1.9 \tag{10}$$

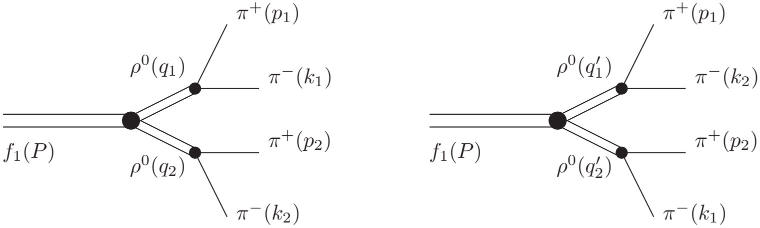
It is seen that there is a large uncertainty in the value of $|g_1|$.

In what follows we treat δ as a free parameter!

Calculation of $f_1 \to e^+e^-$ decay width

Now let us consider $f_1 \to \pi^+\pi^-\pi^+\pi^-$ decay. Experimental data indicate that the main contribution to it is given by the intermediate state with two virtual ρ mesons.

Certainly, the form factors of our model should meet the requirement that the result of calculation of $f_1 \to \pi^+\pi^-\pi^+\pi^-$ decay width should be in a good agreement with the experiment.



Calculation of $f_1 \to e^+e^-$ decay width

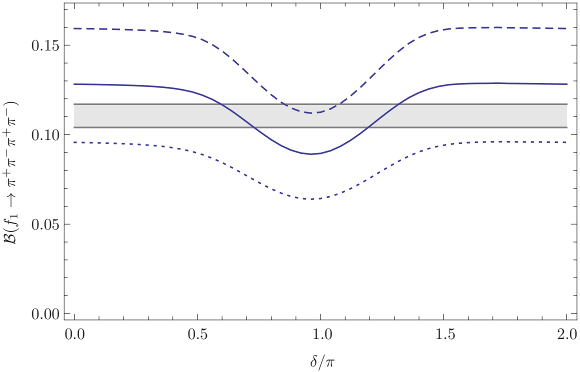
Taking into account all the requirements we write the form factors F_1 and F_2 as

$$F_1(q_1^2, q_2^2) = \frac{g_1 \cdot g_{\rho\gamma}(m_\rho^2 - im_\rho\Gamma_\rho)(q_2^2 - q_1^2)}{(q_1^2 - m_\rho^2 + im_\rho\Gamma_\rho)(q_2^2 - m_\rho^2 + im_\rho\Gamma_\rho)} \quad (11)$$

$$F_2(q_1^2, q_2^2) = \frac{g_2 \cdot g_{\rho\gamma}(m_\rho^2 - im_\rho\Gamma_\rho)(-m_\rho^2)}{(q_1^2 - m_\rho^2 + im_\rho\Gamma_\rho)(q_2^2 - m_\rho^2 + im_\rho\Gamma_\rho)} \quad (12)$$

$m_\rho = 775.26$ MeV, $\Gamma_\rho = 147.8$ MeV are ρ^0-meson mass and width, $g_{\rho\gamma}$ is the coupling constant of the transition $\rho^0 \to \gamma^*$, $M(\rho^0 \to \gamma^*) = g_{\rho\gamma}(q^2 g_{\mu\nu} - q_\mu q_\nu)\epsilon^\mu e^{*\nu} \Rightarrow g_{\rho\gamma} = \sqrt{\frac{3\Gamma(\rho^0 \to e^+e^-)}{\alpha m_\rho}} \approx 0.06$.

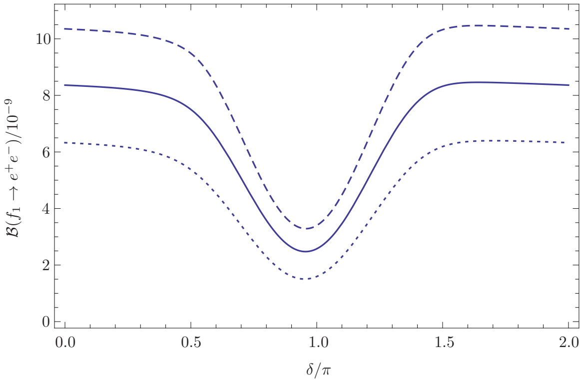
Calculation of $f_1 \to e^+e^-$ decay width



$\mathcal{B}(f_1 \to \pi^+\pi^-\pi^+\pi^-)$ in our model

The solid line — $\mathcal{B}(f_1 \to \pi^+\pi^-\pi^+\pi^-)$ calculated using the central values: $r = 3.9$ and $\mathcal{B}(f_1 \to \rho^0\gamma) = 5.5\%$. Dashed and dotted lines indicate 1σ deviations. The shaded horizontal band denotes value allowed experimentally, $\mathcal{B}(f_1 \to \pi^+\pi^-\pi^+\pi^-) = \left(11.0^{+0.7}_{-0.6}\right)\%$.

Calculation of $f_1 \to e^+e^-$ decay width



$\mathcal{B}(f_1 \to e^+e^-)$ in our model

The solid line — $\mathcal{B}(f_1 \to e^+e^-)$ calculated for the central values: $r = 3.9$ and $\mathcal{B}(f_1 \to \rho^0\gamma) = 5.5\%$. The dashed and dotted lines indicate 1σ deviations.

Calculation of $f_1 \to e^+e^-$ decay width

It is seen from the Figures above that in our model the branching ratio $\mathcal{B}(f_1 \to e^+e^-)$ should be taken in the range from $3 \cdot 10^{-9}$ for $\delta \simeq \pi$ to $8 \cdot 10^{-9}$ for $\delta = 0$,

$$\mathcal{B}(f_1 \to e^+e^-) \simeq (3 \div 8) \cdot 10^{-9} \tag{13}$$

and the corresponding decay width is

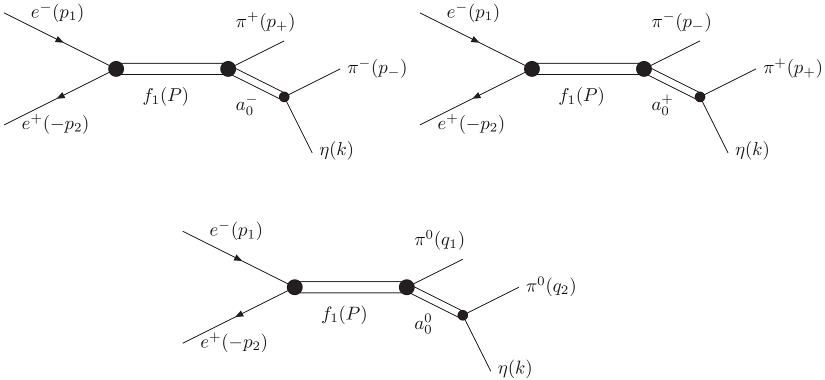
$$\Gamma(f_1 \to e^+e^-) \simeq 0.07 \div 0.19 \text{ eV} \tag{14}$$

The naive estimate $\Gamma \sim 0.1$ eV is in good agreement with (14).

Estimate of $e^+e^- \to f_1 \to \eta\pi\pi$ cross section

The process $e^+e^- \to f_1 \to \eta\pi\pi$ can be used for study of direct f_1 production in e^+e^- collisions.

$f_1 \to \eta\pi\pi$ decay proceeds approximately with 70% probability through the intermediate $a_0(980)$ meson.



Estimate of $e^+e^- \to f_1 \to \eta\pi\pi$ cross section

Using the experimental value for the branching ratio $\mathcal{B}(f_1 \to a_0\pi) = 0.36 \pm 0.07$ and the result of our calculations for $\mathcal{B}(f_1 \to e^+e^-)$, we obtain

$$\sigma(e^+e^- \to f_1 \to a_0\pi) \simeq 7.8 \div 30 \text{ pb} \quad (15)$$

Isospin symmetry:

$$\sigma(e^+e^- \to f_1 \to a_0^\pm\pi^\mp \to \eta\pi^+\pi^-) \simeq 5.2 \div 20 \text{ pb} \quad (16)$$

$$\sigma(e^+e^- \to f_1 \to a_0^0\pi^0 \to \eta\pi^0\pi^0) \simeq 2.6 \div 10 \text{ pb} \quad (17)$$

$e^+e^- \to f_1 \to \eta\pi^0\pi^0$ process

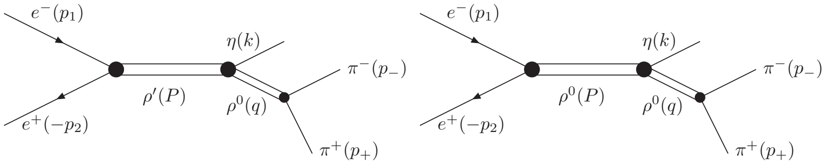
$e^+e^- \to \eta\pi^0\pi^0$ reaction proceeds only through two-photon annihilation, since C parity of $\eta\pi^0\pi^0$ final state is positive.

Therefore, there is no background from one-photon annihilation, and $e^+e^- \to f_1 \to \eta\pi^0\pi^0$ cross section can be measured directly.

According to our estimate (17), the lower bound on this cross section is quite small ($\approx 3\text{pb}$), nevertheless it can be measured at the VEPP-2000 collider in Novosibirsk.

Charge asymmetry in $e^+e^- \to \eta\pi^+\pi^-$ process

On the contrary, $e^+e^- \to \eta\pi^+\pi^-$ reaction proceeds mainly through one-photon annihilation, which is described quite well by the VMD model with intermediate $\rho'(1450)$ and $\rho^0(770)$ mesons.



The measured $e^+e^- \to \eta\pi^+\pi^-$ Born cross section is about 500 pb at $\sqrt{s} = m_f$. According to our estimate (16), $e^+e^- \to f_1 \to a_0^{\pm}\pi^{\mp} \to \eta\pi^+\pi^-$ cross section constitutes $\simeq 5.2 \div 20$ pb, so its measurement is rather complicated task.

One possibility to overcome this difficulty is to investigate the two-photon annihilation channel $e^+e^- \to f_1 \to \eta\pi^+\pi^-$ through C-odd effects, which arise from interference of C-odd one-photon and C-even two-photon amplitudes.

This interference is P- and C-odd, therefore it does not contribute to the total cross section, but it can lead to the charge asymmetry in the differential cross section.

Let us define the charge asymmetry in $e^+e^- \to \eta\pi^+\pi^-$ process as

$$A = \left. \frac{\sigma_{tot}(\cos\theta_\pi > 0) - \sigma_{tot}(\cos\theta_\pi < 0)}{\sigma_{tot}(\cos\theta_\pi > 0) + \sigma_{tot}(\cos\theta_\pi < 0)} \right|_{\cos\theta_\eta > 0} \tag{18}$$

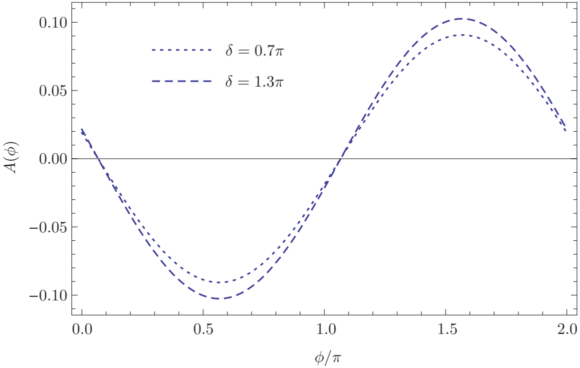
Here θ_η is the angle between η meson 3-momentum and e^+ beam axis in the e^+e^- center-of-mass frame, θ_π is the angle between π^+ meson and η meson 3-momenta in the $\pi^+\pi^-$ center-of-mass system.

Charge asymmetry — different numbers of π^+ and π^- mesons propagating in some direction.

Charge asymmetry in $e^+e^- \to \eta\pi^+\pi^-$ process

The interference term contains one additional free parameter ϕ – relative phase arising from the complex coupling constants,

$$F_A g_{f\pi a} g_{a\pi\eta} f^*_{\rho\pi\pi} = |F_A g_{f\pi a} g_{a\pi\eta} f_{\rho\pi\pi}| e^{i\phi} \qquad (19)$$



Charge asymmetry in $e^+e^- \to \eta\pi^+\pi^-$ process may be up to $\pm 10\%$

Conclusions

- The width of $f_1(1285) \to e^+e^-$ decay is calculated in the vector meson dominance model. The result depends on the relative phase δ between two coupling constants describing $f_1 \to \rho^0\gamma$ decay,
  $\Gamma(f_1 \to e^+e^-) \simeq 0.07 \div 0.19$ eV
  $[\mathcal{B}(f_1 \to e^+e^-) \simeq (3 \div 8) \cdot 10^{-9}]$
- In our model
  $\sigma(e^+e^- \to f_1 \to a_0^{\pm}\pi^{\mp} \to \eta\pi^+\pi^-) \simeq 5.2 \div 20$ pb, and
  $\sigma(e^+e^- \to f_1 \to a_0^0\pi^0 \to \eta\pi^0\pi^0) \simeq 2.6 \div 10$ pb
- $e^+e^- \to f_1 \to \eta\pi^0\pi^0$ cross section can be measured directly.
- Measurement of $e^+e^- \to f_1 \to \eta\pi^+\pi^-$ cross section is rather complicated task, because of background from the one-photon annihilation $\Rightarrow$ to investigate the charge asymmetry.
  It may be quite large, up to $\pm 10\%$

$$M(f_1 \to \rho^{0*}\rho^{0*}) = \frac{1}{m_f^2} h_1(q_1^2, q_2^2) i\epsilon_{\mu\nu\rho\sigma} q_1^\mu e_1^{*\nu} q_2^\rho e_2^{*\sigma} \tilde{e}^\tau (q_1 - q_2)_\tau +$$

$$+ \frac{1}{m_f^2} \left\{ h_2(q_1^2, q_2^2) i\epsilon_{\mu\nu\rho\sigma} q_1^\mu e_1^{*\nu} \tilde{e}^\rho \left[q_2^\sigma e_2^{*\lambda} q_{2\lambda} - e_2^{*\sigma} q_2^2 \right] + \right.$$

$$\left. + h_2(q_2^2, q_1^2) i\epsilon_{\mu\nu\rho\sigma} q_2^\mu e_2^{*\nu} \tilde{e}^\rho \left[q_1^\sigma e_1^{*\lambda} q_{1\lambda} - e_1^{*\sigma} q_1^2 \right] \right\} \quad (20)$$

$$\alpha F_{1,2}(q_1^2, q_2^2) = \frac{g_{\rho\gamma}^2 q_1^2 q_2^2}{(q_1^2 - m_\rho^2 + i m_\rho \Gamma_\rho)(q_2^2 - m_\rho^2 + i m_\rho \Gamma_\rho)} h_{1,2}(q_1^2, q_2^2) \quad (21)$$

$$\lim_{q_2^2 \to 0} q_2^2 h_1(m_\rho^2, q_2^2) = -\frac{\alpha g_1}{g_{\rho\gamma}} \left(m_\rho^2 - i m_\rho \Gamma_\rho \right) \quad (22)$$

$$\lim_{q_2^2 \to 0} q_2^2 h_2(q_2^2, m_\rho^2) = -\frac{\alpha g_2}{g_{\rho\gamma}} \left(m_\rho^2 - i m_\rho \Gamma_\rho \right) \quad (23)$$

Backup

$$F_A \simeq -\alpha g_1 \, (0.22 + 0.25i) - \alpha g_2 \, (0.75 + 0.57i) \qquad (24)$$

$$|F_A|^2 \simeq \left| e^{i\delta} \cdot \alpha |g_1| \cdot (0.22 + 0.25i) + \alpha |g_2| \cdot (0.75 + 0.57i) \right|^2 \qquad (25)$$