Mass problem in the Standard Model

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Introduction

- Models with extra $U(1)$ symmetry are studied as extensions of the Standard Model (SM). There are many motivations to consider this kind of models.
- In supersymmetric extensions, an additional $U(1)$ factor may provides a mechanism to generate the μ_{eff} term through the addition of a scalar singlet. $\mu S \Phi_U \Phi_D$
- Non-supersymmetric extensions give rise to a variety of models like left-right symmetric models, B-L, flip grand unified models, $SU(5)$, $SO(10)$, E_6, etc, which involve theoretical and phenomenological aspects:

1. Flavor physics, Flavor Changing,
2. Neutrino physics,
3. Dark matter,
4. Aditional Higgs,
5. Neutral currents, Z'_μ

- They provide hints to explain the SM mass hierarchy problem, where top quark acquires mass at the EW scale and the other fermions exhibit different low mass values.
- Introduced right handed sterile neutrinos to explain the masses and mixing of the active neutrinos.
- These extensions have Two Higgs Doublet Models in the low energy limit, where two scalar doublets ϕ_1 and ϕ_2 are introduced in order to generate the appropriate Yukawa couplings that provide masses to all fermions.

- A singlet scalar field χ is introduced to break $U(1)$ symmetry and to give masses to exotic particles beyond the SM.
- Anomaly cancellation provides an explanation for the fermion structure of the Standard Model and they determine the $U(1)_Y$ charges.
- In addition of a new neutral gauge boson Z', extended fermion spectrum is necessary in order to obtain an anomaly-free theory.
- Since the new symmetry introduces an additional gauge boson, there arise new couplings that induce nontrivial triangle anomalies
- There are 6 possible combinations $Tr[T'_{SM} T_{SM} T_X] = 0$

Quiral anomalies

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$$

$$\begin{aligned}
[SU(3)_c]^2\, U(1)_X &\rightarrow A_1 = \sum_Q X_{Q_L} - \sum_Q X_{Q_R} \\
[SU(2)_L]^2\, U(1)_X &\rightarrow A_2 = \sum_\ell X_{\ell_L} + 3\sum_Q X_{Q_L}, \\
[U(1)_Y]^2\, U(1)_X &\rightarrow A_3 = \sum_{\ell,Q} \left[Y^2_{\ell_L} X_{\ell_L} + 3\, Y^2_{Q_L} X_{Q_L} \right] \\
&- \sum_{\ell,Q} \left[Y^2_{\ell_R} X_{\ell_R} + 3\, Y^2_{Q_R} X_{Q_R} \right]
\end{aligned}$$

where the sums in Q run over all the quarks (u^i, d^i, T, J^n), and ℓ runs over all leptons $e^i, \nu^i_L, \nu^i_R, N^i_R, E$.

$$
\begin{aligned}
U(1)_Y\,[U(1)_X]^2 &\rightarrow A_4 = \sum_{\ell,Q}\left[Y_{\ell_L}X_{\ell_L}^2 + 3\,Y_{Q_L}X_{Q_L}^2\right] \\
&- \sum_{\ell,Q}\left[Y_{\ell_R}X_{\ell_R}^2 + 3\,Y_{Q_R}X_{Q_R}^2\right] \\
[U(1)_X]^3 &\rightarrow A_5 = \sum_{\ell,Q}\left[X_{\ell_L}^3 + 3X_{Q_L}^3\right] - \sum_{\ell,Q}\left[X_{\ell_R}^3 + 3X_{Q_R}^3\right] \\
[Grav]^2 \otimes U(1)_X &\rightarrow A_6 = \sum_{\ell,Q}\left[X_{\ell_L} + 3X_{Q_L}\right] \\
&- \sum_{\ell,Q}\left[X_{\ell_R} + 3X_{Q_R}\right]
\end{aligned}
$$

Yukawa couplings also constraint the X charges:

$$\Rightarrow -X_{\psi_L} + X_{\psi_R} + X_{\phi} = 0.$$

There are many solutions to anomaly equations. But we consider one which can explain mass hierarchy problem.

Particle contents

Global symmetry: $SU(2)_{q_L} \otimes SU(2)_{l_L} \otimes SU(2)_{U_R} \otimes SU(3)_{D_R} \otimes SU(2)_{l_R}$

Quarks	X	$\mathbf{Z}_2$	Leptons	X	$\mathbf{Z}_2$
$q_L^1 = \begin{pmatrix} U^1 \\ D^1 \end{pmatrix}_L$	$+1/3$	$+$	$\ell_L^e = \begin{pmatrix} \nu^e \\ e^e \end{pmatrix}_L$	0	$+$
$q_L^2 = \begin{pmatrix} U^2 \\ D^2 \end{pmatrix}_L$	0	$-$	$\ell_L^\mu = \begin{pmatrix} \nu^\mu \\ e^\mu \end{pmatrix}_L$	0	$+$
$q_L^3 = \begin{pmatrix} U^3 \\ D^3 \end{pmatrix}_L$	0	$+$	$\ell_L^\tau = \begin{pmatrix} \nu^\tau \\ e^\tau \end{pmatrix}_L$	-1	$+$
$U_R^{1,3}$	$+2/3$	$+$	$e_R^{e,\tau}$	$-4/3$	$-$
U_R^2	$+2/3$	$-$	e_R^μ	$-1/3$	$-$
$D_R^{1,2,3}$	$-1/3$	$-$			
T_L	$+1/3$	$-$	$\nu_R^{e,\mu,\tau}$	$1/3$	$-$
T_R	$+2/3$	$-$	$N_R^{e,\mu,\tau}$	0	$-$
$J_L^{1,2}$	0	$+$	$E_L, \mathcal{E}_R$	-1	$+$
$J_R^{1,2}$	$-1/3$	$+$	$\mathcal{E}_L, E_R$	$-2/3$	$+$

- The $U(1)_X$ symmetry is non-universal in the left-handed SM quark and lepton sectors.
- The right handed down quarks are universal, but the right handed up quarks and leptons are non universal.
- The three extra singlets T and J^n are new up- and down-like quarks, respectively, where $n = 1, 2$.
- The model needs three new neutrinos (ν_R^i)

- We include N_R^i to generate the inverse see-saw neutrino masses in order to obtain a realistic model compatible with oscillation data.
- The model is anomaly free without exotic charge leptons E. But they are needed to generate the electron mass at one loop level.

Particle contents: Bosons

Scalar bosons	X
Higgs Doublets	
$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{h_1+v_1+i\eta_1}{\sqrt{2}} \end{pmatrix}$	2/3
$\phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{h_2+v_2+i\eta_2}{\sqrt{2}} \end{pmatrix}$	1/3
Higgs Singlets	
$\chi = \frac{1}{\sqrt{2}}(\xi_\chi + \nu_\chi + i\zeta_\chi)$	-1/3
σ	-1/3

Table: Non-universal X quantum number for Higgs fields.

- The spectrum includes an additional scalar doublet ϕ_2 identical to ϕ_1 under G_{SM} but with different $U(1)_X$ charges, where the electroweak scale is related to the VEVs by $\nu = \sqrt{\nu_1^2 + \nu_2^2}$.
- An extra scalar singlet χ with VEV ν_χ is required to produce the symmetry breaking of the $U(1)_X$ symmetry. We assume that it happens at a large scale $\nu_\chi \gg \nu$.
- Scalar singlet σ is introduced to give mass at one loop level to light fermions u, d quarks and e.
- Scalar singlet σ can be the DM candidate.
- We define the weak hypercharge Y as usual, where the electric charge is defined by the Gell-Mann-Nishijima relation:

$$Q = T_{3L} + \frac{Y}{2}$$

with T_{3L} the isospin defined for left- and right-handed fermions.

Yukawa Lagrangian

We find the Yukawa Lagrangian compatible with the $G_{SM} \otimes U(1)_X$ symmetry.
For quark sector we find:

$$
\begin{aligned}
-\mathcal{L}_Q \quad = \quad & \sum_j \overline{q_L^u} \left(\widetilde{\phi}_2 h_2^U\right)_{uj} U_R^j + \sum_j \sum_{a=c,t} \overline{q_L^a} (\widetilde{\phi}_1 h_1^U)_{aj} U_R^j \\
+ \quad & \sum_j \overline{q_L^u} \left(\phi_1 h_1^D\right)_{uj} D_R^j + \sum_j \sum_{a=c,t} \overline{q_L^a} \left(\phi_2 h_2^D\right)_{aj} D_R^j \\
+ \quad & \sum_m \overline{q_L^u} (\phi_1 h_1^J)_{um} J_R^m + \sum_m \sum_{a=c,t} \overline{q_L^a} \left(\phi_2 h_2^J\right)_{am} J_R^m \\
+ \quad & \overline{q_L^u} \left(\widetilde{\phi}_2 h_2^T\right)_u T_R + \sum_{a=c,t} \overline{q_L^a} (\widetilde{\phi}_1 h_1^T)_a T_R \\
+ \quad & \sum_{n=1,2} \sum_j \overline{J_L^n} \left(\sigma h_\sigma^D + \chi h_\chi^D\right)_{nj} D_R^j + \sum_j \overline{T_L} \left(\sigma^* h_\sigma^U + \chi^* h_\chi^U\right)_j U_R^j \\
+ \quad & \sum_{n,m} \overline{J_L^n} \left(\chi h_\chi^J\right)_{nm} J_R^m + \overline{T_L} \left(\chi^* h_\chi^T\right) T_R + h.c.,
\end{aligned}
$$

where $\widetilde{\phi}_{1,2} = i\sigma_2 \phi_{1,2}^*$ are conjugate fields,

For lepton sector

$$
\begin{aligned}
-\mathcal{L}_{Y,E} &= \eta\overline{\ell_L^e}\phi_2 e_R^\mu + h\overline{\ell_L^\mu}\phi_2 e_R^\mu + \zeta\overline{\ell_L^\tau}\phi_2 e_R^e + H\overline{\ell_L^\tau}\phi_2 e_R^\tau + \\
&+ q_{11}\overline{\ell_L^e}\phi_1 E_R + q_{21}\overline{\ell_L^\mu}\phi_1 E_R \\
&+ h^E_{\sigma e}\overline{E_L}\sigma e_R^e + h^{\mathcal{E}}_{\sigma\mu}\overline{\mathcal{E}_L}\sigma^* e_R^\mu + h^E_{\sigma\tau}\overline{E_L}\sigma e_R^\tau \\
&+ H_1\overline{E_L}\chi E_R + H_2\overline{\mathcal{E}_L}\chi^*\mathcal{E}_R + \text{h.c.}
\end{aligned}
$$

$$
\begin{aligned}
-\mathcal{L}_{Y,N} &= \sum_{a=e,\mu}\sum_{i=e,\mu,\tau} h^{\nu i}_{2a}\overline{\ell_L^a}\tilde{\phi}_2\nu_R^i + \sum_{i,j=e,\mu,\tau} h^{\nu j}_{\chi i}\overline{\nu_R^{i\,C}}\chi^* N_R^j \\
&+ \sum_{i,j=e,\mu,\tau}\frac{1}{2}\overline{N_R^{i\,C}}M_N^{ij}N_R^j + \text{h.c.},
\end{aligned}
$$

Quark masses

- In particular, the $SU(2)_{q^a}$ symmetry in the left-handed sector remains in the model even after the gauge symmetry breaking. However, the experimental observation shows that this symmetry does not remain if the quark masses are taken into account.
- The extra $U(1)_X$ symmetry is not sufficient to explain the mass spectrum. Thus, we assume the global symmetries, Z_2.

which leads us to the following extended mass matrices:

$$M'_U = \frac{1}{\sqrt{2}} \left(\begin{array}{ccc|c} \upsilon_1 \Sigma_{11} & \upsilon_2 a_{12} & 0 & \upsilon_2 y_1 \\ 0 & \upsilon_1 a_{22} & 0 & \upsilon_1 y_2 \\ \upsilon_1 a_{31} & 0 & \upsilon_1 a_{33} & 0 \\ \hline 0 & \upsilon_\chi c_2 & 0 & \upsilon_\chi h_\chi^T \end{array} \right),$$

$$M'_D = \frac{1}{\sqrt{2}} \left(\begin{array}{ccc|cc} \upsilon_2 \Sigma_{11} & \upsilon_2 \Sigma_{12} & \upsilon_2 \Sigma_{13} & \upsilon_1 j_{11} & \upsilon_1 j_{12} \\ \upsilon_1 \Sigma_{21} & \upsilon_1 \Sigma_{22} & \upsilon_1 \Sigma_{23} & \upsilon_2 j_{21} & \upsilon_2 j_{22} \\ \upsilon_2 B_{31} & \upsilon_2 B_{32} & \upsilon_2 B_{33} & 0 & 0 \\ \hline 0 & 0 & 0 & \upsilon_\chi k_{11} & \upsilon_\chi k_{12} \\ 0 & 0 & 0 & \upsilon_\chi k_{21} & \upsilon_\chi k_{22} \end{array} \right).$$

If $\Sigma_{11} = 0$ then $Det[M'_U] = Det[M'_D] = 0$

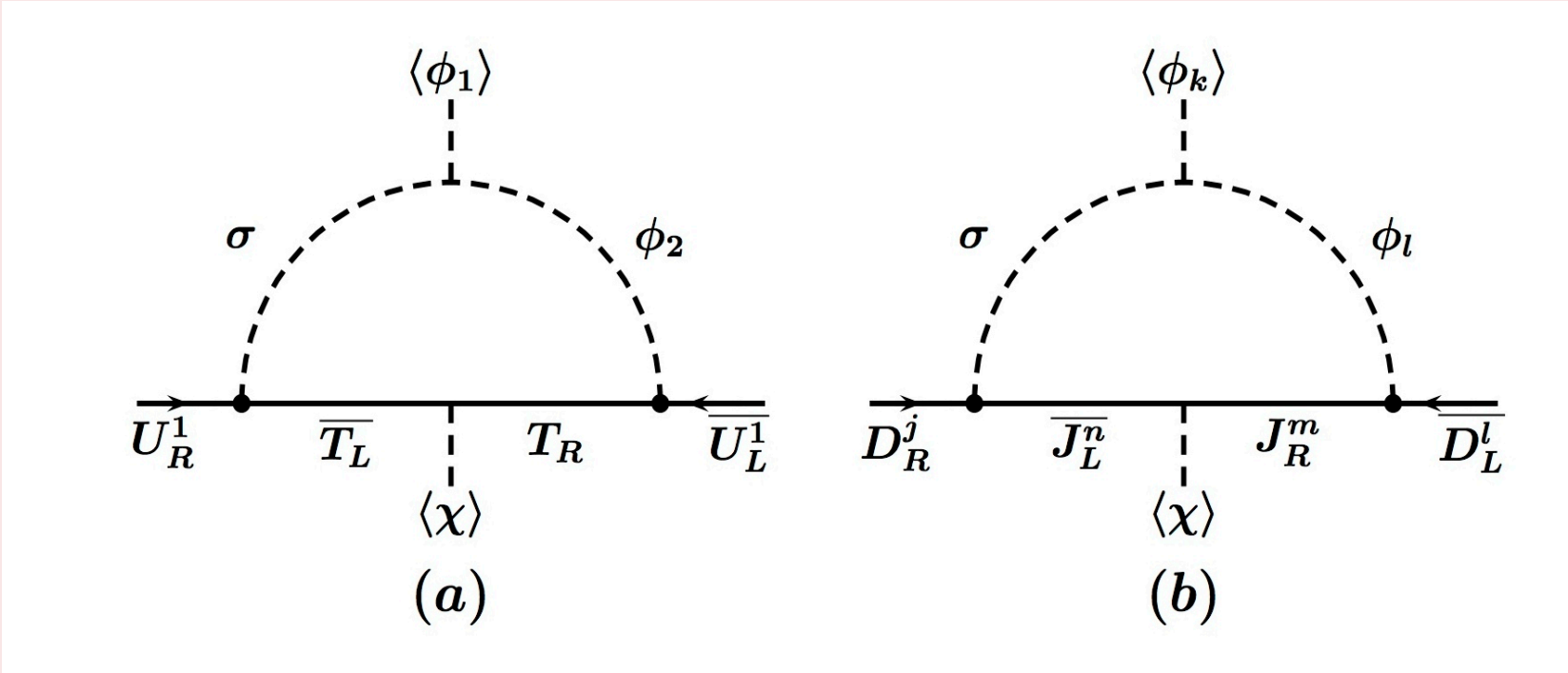

Figure: Mass one-loop correction for (a) up and (b) down - strange quarks, where $l, m, n = 1, 2$ and $j = 1, 2, 3$.

and Σ_{11} the value of the diagram in figure which obey the following analytical expression:

$$\Sigma_{11} = \frac{-1}{16\pi^2} \frac{f' \left(h_\sigma^U\right)_1 \left(h_2^T\right)_1}{\sqrt{2} M_T} C_0 \left(\frac{M_2}{M_T}, \frac{M_\sigma}{M_T}\right), \tag{1}$$

$$\Sigma_{lj} = \frac{-1}{16\pi^2} \frac{f' \left(h_l^J\right)_{lm} \left(h_\sigma^D\right)_{nj}}{\sqrt{2} M_J} C_0 \left(\frac{M_l}{M_J}, \frac{M_\sigma}{M_J}\right). \tag{2}$$

where:

$$\begin{aligned} C_0\left(x_1, x_2\right) &= \frac{1}{\left(1 - x_1^2\right)\left(1 - x_2^2\right)\left(x_1^2 - x_2^2\right)} \\ &\quad \left[x_1^2 x_2^2 \ln\left(\frac{x_1^2}{x_2^2}\right) - x_1^2 \ln x_1^2 + x_2^2 \ln x_2^2\right], \end{aligned} \tag{3}$$

The above matrices are diagonalized through bi-unitary transformation of the form $m_Q = (\mathcal{O}_L^Q)^\dagger M'_Q \mathcal{O}_R^Q$, with m_Q a diagonal matrix with real and positive values.
For the up sector, we find the following approximate eigenvalues:

$$\begin{aligned}
m_u^2 &= \frac{1}{2}\Sigma_{11}^2 \upsilon_1^2 \\
m_c^2 &= \frac{1}{2}\left(r_2^2 \upsilon_1^2 + r_1^2 \upsilon_2^2\right) \approx \frac{1}{2} r_2^2 \upsilon_1^2, \quad \frac{1}{2} r_1^2 \upsilon_2^2 \\
m_t^2 &= \frac{1}{2}\left(a_{31}^2 + a_{33}^2\right) \upsilon_1^2, \\
m_T^2 &\approx \frac{1}{2}\left(c_2^2 + h_\chi^{T2}\right) \upsilon_\chi^2. \qquad (4)
\end{aligned}$$

$$r_1 = \frac{\left(a_{12} h_\chi^T - y_1 c_2\right)}{\sqrt{c_2^2 + h_\chi^{T2}}}, r_2 = \frac{\left(a_{22} h_\chi^T - y_2 c_2\right)}{\sqrt{c_2^2 + h_\chi^{T2}}}. \qquad (5)$$

For the down sector, we find the eigenvalues

$$
\begin{aligned}
m_d^2 &\approx \frac{s_{11}\upsilon_2^2}{2m_b^2}, \\
m_s^2 &\approx \frac{s_{22}\upsilon_1^2}{2m_b^2}, \\
m_b^2 &= \frac{1}{2}\left(B_{31}^2 + B_{32}^2 + B_{33}^2\right)\upsilon_2^2 \\
m_{J1}^2 &= \frac{1}{2}k_{11}^2\upsilon_\chi^2, \\
m_{J2}^2 &= \frac{1}{2}k_{22}^2\upsilon_\chi^2 \\
s_{11} &= \left(\Sigma_{11}B_{32} - \Sigma_{12}B_{31}\right)^2 + \left(\Sigma_{11}B_{33} - \Sigma_{13}B_{31}\right)^2 \\
&+ \left(\Sigma_{12}B_{33} - \Sigma_{13}B_{32}\right)^2, \\
s_{22} &= \left(\Sigma_{21}B_{32} - \Sigma_{22}B_{31}\right)^2 + \left(\Sigma_{21}B_{33} - \Sigma_{23}B_{31}\right)^2 \\
&+ \left(\Sigma_{22}B_{33} - \Sigma_{23}B_{32}\right)^2,
\end{aligned}
$$

Lepton masses and mixing angles

- A leptonic mixing matrix is present in weak charged current which is the product of the rotation matrices of the charged leptons and neutrinos.
- The neutrino oscillation experiments have established that neutrinos are massive and there is lepton flavor violation.
- From the neutrino global analysis the best allow region can be obtained for Δm^2_{21}, Δm^2_{31}, δ_{CP}, $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$ parameters .
- If we consider $\Delta m^2_{21} > 0$ then Δm^2_{31} can be positive or negative. The two cases are called normal ordering and inverted ordering.

Charged lepton masses

For the charged sector in the flavor basis $\mathbf{E} = (e^e, e^\mu, e^\tau, E)$, the mass terms obtained from Lagrangian after the symmetry breaking are

$$-\mathcal{L}_{Y,E} = \overline{\mathbf{E}_L}\mathbb{M}_E\mathbf{E}_R + \frac{H_2 v_\chi}{\sqrt{2}}\overline{\mathcal{E}_L}\mathcal{E}_R + \text{h.c.}, \tag{6}$$

where the lepton mass matrix $\mathbb{M}_E$ has de following form:

$$\mathbb{M}_E = \frac{1}{\sqrt{2}}\begin{pmatrix} \Sigma_{11}\upsilon_2 & \eta\upsilon_2 & \Sigma_{13}\upsilon_2 & | & q_{11}\upsilon_1 \\ \Sigma_{21}\upsilon_2 & h\upsilon_2 & \Sigma_{23}\upsilon_2 & | & q_{21}\upsilon_1 \\ \zeta\upsilon_2 & 0 & H\upsilon_2 & | & 0 \\ - & - & - & - & - \\ 0 & 0 & 0 & | & H_1\upsilon_\chi \end{pmatrix}, \tag{7}$$

which exhibits one massless lepton (the electron). To obtain a massive electron, we include the one-loop correction, which

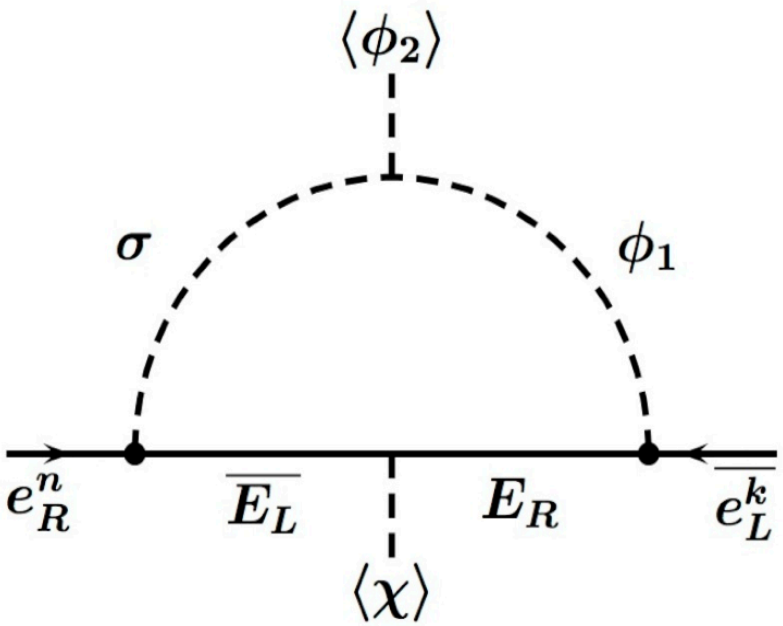

Figure: Mass one-loop correction for charged leptons, where $n = e, \tau$ and $k = e, \mu$.

Since $\mathbb{M}_E$ is not hermitian, there are two rotation matrices $\mathbb{V}_L^E$ and $\mathbb{V}_R^E$ for left- and right-handed leptons. Hence, the left-handed rotation is obtained by diagonalizing $\mathbb{M}_E\mathbb{M}_E^\dagger$ obtaining the corresponding eigenvalues

$$
\begin{aligned}
m_e^2 &= \frac{h^2\Sigma_{11}^2}{\left(\eta^2+h^2\right)}\frac{\upsilon_2^2}{2} \approx \Sigma_{11}^2\frac{\upsilon_2^2}{2},\\
m_\mu^2 &= \left(\eta^2+h^2\right)\frac{\upsilon_2^2}{2} \approx h^2\frac{\upsilon_2^2}{2},\\
m_\tau^2 &= \left(\zeta^2+H^2\right)\frac{\upsilon_2^2}{2} \approx H^2\frac{\upsilon_2^2}{2},\\
m_E^2 &= \frac{H_1^2\upsilon_\chi^2}{2}.
\end{aligned}
\tag{8}
$$

where the corresponding left-handed rotation matrix can be expressed as:

$$\mathbb{V}_L^E = \mathbb{V}_{\mathrm{SS},L}^E \mathbb{V}_{\mathrm{SM},L}^E, \tag{9}$$

which diagonalizes as:

$$\mathbb{M}_E \mathbb{M}_E^\dagger = \frac{1}{2} \begin{pmatrix} \mathcal{M}_{ee}^2 & \mathcal{M}_{eE}^2 \\ \mathcal{M}_{eE}^{2\,\mathrm{T}} & \mathcal{M}_{EE}^2 \end{pmatrix}, \tag{10}$$

The former matrix $\mathbb{V}_{\mathrm{SS},L}^E$ is

$$\mathbb{V}_{\mathrm{SS},L}^E = \begin{pmatrix} I & F^E \\ -F^{E\dagger} & I \end{pmatrix}, \tag{11}$$

with $F^E = \mathcal{M}_{eE}^2 \left(\mathcal{M}_{EE}^2\right)^{-1}$.

The latter rotation is:

$$\mathbb{V}^{E}_{\mathrm{SM},L} = \begin{pmatrix} V^{E}_{\mathrm{SM},L} & 0 \\ 0 & 1 \end{pmatrix}, \tag{12}$$

where the top-left block diagonalizes the SM charged lepton masses

$$V^{E}_{\mathrm{SM},L} \approx \begin{pmatrix} c_{\alpha_{e\mu}} & s_{\alpha_{e\mu}} & \frac{\Sigma_{13}}{H} \\ -s_{\alpha_{e\mu}} & c_{\alpha_{e\mu}} & \frac{\Sigma_{23}}{H} \\ -\frac{\Sigma_{13}}{H} & -\frac{\Sigma_{23}}{H} & 1 \end{pmatrix}. \tag{13}$$

The angle $\alpha_{e\mu}$ is defined by $t_{\alpha_{e\mu}} = \tan\alpha_{e\mu} \approx \eta/h$, which is a free parameter of the model as shown below.

Neutrino masses

The Yukawa langrangian for neutrinos according to the $U(1)_X$ is

$$
\begin{aligned}
-\mathcal{L}_l &= \sum_{j=e,\mu,\tau} h^{ej}_{\nu 2}\bar{\ell}^e_L\tilde{\phi}_2\nu^j_R + h^{\mu j}_{\nu 2}\bar{\ell}^\mu_L\tilde{\phi}_2\nu^j_R \\
&+ \sum_{i,j=e,\mu,\tau} h^{ij}_{\chi N}\nu^{i\mathrm{T}}_R\hat{C}\chi N_R + \frac{1}{2}N^{i\mathrm{T}}_R\hat{C}M^{ij}_N N^j_R + \mathrm{h.c.}
\end{aligned}
$$

which produces a 9×9 mass matrix in the base $\mathbf{N}_L = \left(\nu_L, \nu^c_R, N^c_R\right)^{\mathrm{T}}$

$$
-\mathcal{L}_{Y,\nu} = \frac{1}{2}\overline{\mathbf{N}^c_L}\mathcal{M}_\nu\mathbf{N}_L + \mathrm{h.c.}
$$

$$
\mathcal{M}_\nu = \begin{pmatrix} 0 & m^{\mathrm{T}}_\nu & 0 \\ m_\nu & 0 & m^{\mathrm{T}}_N \\ 0 & m_N & M_N \end{pmatrix}
$$

with the following 3×3 blocks

$$m_\nu = \frac{v_2}{\sqrt{2}} \begin{pmatrix} h^{ee}_{\nu 2} & h^{e\mu}_{\nu 2} & h^{e\tau}_{\nu 2} \\ h^{\mu e}_{\nu 2} & h^{\mu\mu}_{\nu 2} & h^{\mu\tau}_{\nu 2} \\ 0 & 0 & 0 \end{pmatrix} \quad , \quad m_N = h^{ij}_{\chi N} \frac{\nu_\chi}{\sqrt{2}},$$

$$M_N = \mu_N I^{ij}$$

The matrix $\mathcal{M}_\nu$ can be diagonalize by using the inverse see saw mechanism, defining the following blocks

$$\mathcal{M}_{\nu_{6\times 3}} = \begin{pmatrix} m_{\nu_{3\times 3}} \\ 0_{3\times 3} \end{pmatrix}, \mathcal{M}_{N_{6\times 6}} = \begin{pmatrix} 0_{3\times 3} & m^{\mathrm{T}}_{N_{3\times 3}} \\ m_{N_{3\times 3}} & M_{N_{3\times 3}} \end{pmatrix}$$

Then

$$\mathcal{M}_\nu = \begin{pmatrix} 0_{3\times 3} & \mathcal{M}^{\mathrm{T}}_{\nu_{3\times 6}} \\ \mathcal{M}_{\nu_{6\times 3}} & \mathcal{M}_{N_{6\times 6}} \end{pmatrix}$$

In order to diagonalize the neutrino mass matrix $\mathcal{M}_\nu$ by blocks we introduce the W matrix

$$W \approx \begin{pmatrix} \left(1 - \frac{1}{2}FF^\dagger\right)_{3\times 3} & F_{3\times 6} \\ -F^\dagger_{6\times 3} & \left(1 - \frac{1}{2}F^\dagger F\right)_{6\times 6} \end{pmatrix}$$

with

$$F \approx \left(\mathcal{M}_\nu^{\mathrm{T}} \mathcal{M}_N^{-1}\right)^*$$

and two blocks 3×3 and 6×6, respectively

$$\begin{aligned} m_{\mathrm{active}3\times 3} &\approx m_\nu^{\mathrm{T}} \left(m_N\right)^{-1} M_N \left(m_N^{\mathrm{T}}\right)^{-1} m_\nu \\ m_{\mathrm{heavy}6\times 6} &\approx \mathcal{M}_N \end{aligned}$$

where the eigenvalues of m_{heavy} are much larger than the ones of m_{act}.

To diagonalize $m_{heavy} = \mathcal{M}_N$ by blocks we consider the Ω matrix

$$\Omega^{\mathrm{T}} \mathcal{M}_N \Omega = \Omega^{\mathrm{T}} \begin{pmatrix} 0 & m_N \\ m_N^{\mathrm{T}} & M_N \end{pmatrix} \Omega$$

$$= \begin{pmatrix} U^* m_N^{\mathrm{diag}} U^\dagger & 0 \\ 0 & V^* M_N^{\mathrm{diag}} V^\dagger \end{pmatrix}$$

$$\Omega = \begin{pmatrix} 1 - \frac{SS^\dagger}{2} & S \\ -S^\dagger & 1 - \frac{S^\dagger S}{2} \end{pmatrix}$$

$$S = m_N^{-1} M_N$$

where the masses for right handed sterile neutrinos are

$$\begin{aligned} U^* m_N^{\mathrm{diag}} U^\dagger &= -m_N + \frac{M_N}{2} - \frac{1}{8} M_N m_N^{-1} M_N \approx -m_N \\ V^* M_N^{\mathrm{diag}} V^\dagger &= m_N + \frac{M_N}{2} + \frac{1}{8} M_N m_N^{-1} M_N \approx m_N \end{aligned}$$

However, to simplify the model we propose diagonal matrices for m_N and M_N

$$m_N = \begin{pmatrix} h_{\chi N1} & 0 & 0 \\ 0 & h_{\chi N2} & 0 \\ 0 & 0 & h_{\chi N3} \end{pmatrix} \frac{\nu_\chi}{\sqrt{2}}$$

$$M_N = \mu_N \mathbb{I}_{3\times 3}$$

and the the 3×3 light neutrino mass matrix is

$$\begin{aligned} m_{\text{act}} &= \begin{pmatrix} (h^{ee}_{\nu 2})^2 + (h^{\mu e}_{\nu 2})^2 \rho^2 & \times & \times \\ h^{ee}_{\nu 2} h^{e\mu}_{\nu 2} + h^{\mu e}_{\nu 2} h^{\mu\mu}_{\nu 2} \rho^2 & (h^{e\mu}_{\nu 2})^2 + (h^{\mu\mu}_{\nu 2})^2 \rho^2 & \times \\ h^{ee}_{\nu 2} h^{e\tau}_{\nu 2} + h^{\mu e}_{\nu 2} h^{\mu\tau}_{\nu 2} \rho^2 & h^{e\mu}_{\nu 2} h^{e\tau}_{\nu 2} + h^{\mu\mu}_{\nu 2} h^{\mu\tau}_{\nu 2} \rho^2 & (h^{e\tau}_{\nu 2})^2 + (h^{\mu\tau}_{\nu 2})^2 \rho^2 \end{pmatrix} \\ &\times \frac{v_2^2}{\nu_\chi^2} \frac{\mu_N}{h^2_{\chi N1}} \end{aligned}$$

where $\rho = h_{\chi N1}/h_{\chi N2}$.
The light neutrino mass, m_{active}, can be diagonalized by $U_\nu^T m_{\text{active}} U_\nu$

Solutions

- The U_ν and Δm^2_{ij} can be written as function of $h^{ij}_{\nu 2}$ and VEV's.
- Then Pontecorvo Maki Nakagawa Sakata matrix is U_{PMNS}=$U_l \times U_\nu$

$$U_\nu = \begin{pmatrix} c\theta_{12}c\theta_{13} & s\theta_{12}c\theta_{13} & s\theta_{13}e^{-i\delta} \\ -s\theta_{12}c\theta_{23} - c\theta_{12}s\theta_{13}s\theta_{23}e^{i\delta} & c\theta_{12}c\theta_{23} - s\theta_{12}s\theta_{13}s\theta_{23}e^{i\delta} & c\theta_{13}s\theta_{23} \\ s\theta_{12}s\theta_{23} - c\theta_{12}s\theta_{13}c\theta_{23}e^{i\delta} & -c\theta_{12}s\theta_{23} - s\theta_{12}s\theta_{13}c\theta_{23}e^{i\delta} & c\theta_{13}c\theta_{23} \end{pmatrix}$$

- Defining the mixing angles by

$$\sin^2\theta_{12} = \frac{|U_{e2}|^2}{1-|U_{e3}|^2} \quad , \quad \sin^2\theta_{23} = \frac{|U_{\mu 3}|^2}{1-|U_{e3}|^2}$$
$$\sin^2\theta_{13} = |U_{e3}|^2$$

- The analysis of solar, atmospheric, reactor and accelerator neutrino oscillation experiments yields, nu-fit,

Parameter ranges

- NuFIT 3.0 (2016)
- In the case of normal mass ordering $m1 < m2 < m3$,

$$\begin{aligned}
\sin^2\theta_{12} &= 0.306^{+0.012}_{-0.012}\,,\ \sin^2\theta_{13} = 0.02166^{+0.00075}_{-0.00075}, \\
\sin^2\theta_{23} &= 0.441^{+0.027}_{-0.021}; \\
\delta &\approx 261^{+51}_{-59} \\
\Delta m^2_{21} &= 7.50^{+0.19}_{-0.17}\times 10^{-5} eV^2, \Delta m^2_{31} = 2.524^{+0.039}_{-0.040}\times 10^{-3} eV^2
\end{aligned}$$

- In the case of inverted mass ordering $m3 < m1 < m2$

$$\begin{aligned}
\sin^2\theta_{12} &= 0.306^{+0.012}_{-0.012}\,,\ \sin^2\theta_{13} = 0.02179^{+0.00076}_{-0.00076}, \\
\sin^2\theta_{23} &= 0.587^{+0.020}_{-0.024}; \\
\delta &\approx 277^{+40}_{-46} \\
\Delta m^2_{21} &= 7.50^{+0.19}_{-0.17}\times 10^{-5} eV^2, \Delta m^2_{31} = -2.514^{+0.038}_{-0.041}\times 10^{-3} eV^2
\end{aligned}$$

- From charged lepton masses the Yukawa parameters h, H can be adjusted.
- In order to has a model consistent with neutrino oscillation data, the values of the Yukawa parameters $h_{2e}^{\nu e}$, $h_{2e}^{\nu\mu}$, $h_{2e}^{\nu\tau}$, $h_{2\mu}^{\nu e}$, $h_{2\mu}^{\nu\mu}$, $h_{2\mu}^{\nu\tau}$ and $\tan\alpha_{e\mu} = \eta/H$ must be properly adjusted.
- It is worth mentioning that the other two rotation parameters described by Σ_{13}/H and Σ_{23}/H were approximated to m_e/m_τ.
- To achieve this, we implement a MonteCarlo method to generate random numbers in the parameter space, where only the numbers which match up the mass matrix to experimental data are accepted, while the others are rejected.

The outer factor of the mass matrix can be fix at 50 MeV:

$$\frac{\mu_N v_2^2}{{h_{N\chi 1}}^2 {v_\chi}^2} = 50\,\mathrm{meV},$$

if the Yukawa couplings and vev are set by

$$\begin{aligned} h_{N\chi 1} &= 0.15, \\ v_2 &= 7\,\mathrm{GeV}, v_\chi = 7\,\mathrm{TeV}, \mu_N = 1\,\mathrm{keV}. \end{aligned}$$

- The tables show regions where the neutrino Yukawa couplings and the angle $\alpha_{e\mu}$ make consistent this model with neutrino oscillation data reported by nufit at 3σ.
- Yukawa coupling domain which fulfil at 1σ neutrino oscillation data for NO and IO reported by nufit. $h_{2\mu}^{\nu e} = 0$ and $h_{N_\chi 1} = 0.21$ for simplifying the MonteCarlo search

NO	$\alpha_{e\mu} = 0^o$	$\alpha_{e\mu} = 15^o$	$\alpha_{e\mu} = 30^o$
$h_{2e}^{\nu e}$	$0.264 \to 0.278$	$0.285 \to 0.299$	$0.237 \to 0.270$
$h_{2e}^{\nu \mu}$	$-0.707 \to -0.244$	$-0.726 \to -0.335$	$-0.796 \to -0.547$
$h_{2\mu}^{\nu \mu}$	$-0.491 \to -0.190$	$-0,464 \to -0.173$	$-0.342 \to -0.039$
$h_{2e}^{\nu \tau}$	$0.267 \to 0.748$	$0.313 \to 0.677$	$0.140 \to 0.355$
$h_{2\mu}^{\nu \tau}$	$0.130 \to 0.462$	$0.196 \to 0.460$	$0.440 \to 0.510$

IO	$\alpha_{e\mu} = 0^o$	$\alpha_{e\mu} = 1^o$	$\alpha_{e\mu} = 2^o$
$h_{2e}^{\nu e}$	$1.094 \to 1.107$	$1.091 \to 1.105$	$1.090 \to 1.103$
$h_{2e}^{\nu \mu}$	$-0.122 \to -0.106$	$-0.127 \to -0.113$	$-0.128 \to -0.118$
$h_{2\mu}^{\nu \mu}$	$0.970 \to 1.060$	$0.980 \to 1.070$	$1.010 \to 1.080$
$h_{2e}^{\nu \tau}$	$0.110 \to 0.127$	$0.122 \to 0.138$	$0.135 \to 0.149$
$h_{2\mu}^{\nu \tau}$	$0.930 \to 1.030$	$0.920 \to 1.010$	$0.910 \to 0.980$

Bosons	$X^{\pm}$	Quarks	$X^{\pm}$	Leptons	$X^{\pm}$
Scalar Doublets			SM Fermionic Doublets		
$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{h_1+v_1+i\eta_1}{\sqrt{2}} \end{pmatrix}$	$2/3^+$	$q_L^1 = \begin{pmatrix} u^1 \\ d^1 \end{pmatrix}_L$	$1/3^+$	$\ell_L^e = \begin{pmatrix} \nu^e \\ e^e \end{pmatrix}_L$	0^+
$\Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{h_2+v_2+i\eta_2}{\sqrt{2}} \end{pmatrix}$	$1/3^-$	$q_L^2 = \begin{pmatrix} u^2 \\ d^2 \end{pmatrix}_L$	0^-	$\ell_L^\mu = \begin{pmatrix} \nu^\mu \\ e^\mu \end{pmatrix}_L$	0^+
$\Phi_3 = \begin{pmatrix} \phi_3^+ \\ \frac{h_3+v_3+i\eta_3}{\sqrt{2}} \end{pmatrix}$	$1/3^+$	$q_L^3 = \begin{pmatrix} u^3 \\ d^3 \end{pmatrix}_L$	0^+	$\ell_L^\tau = \begin{pmatrix} \nu^\tau \\ e^\tau \end{pmatrix}_L$	-1^-
Scalar Singlets			SM Fermionic Singlets		
$\chi = \frac{\xi_\chi+v_\chi+i\zeta_\chi}{\sqrt{2}}$	$-1/3^+$	$u_R^{1,3}$	$2/3^+$	$e_R^{e,\tau}$	$-4/3^+$
σ	$-1/3^-$	u_R^2	$2/3^-$	e_R^μ	$-1/3^+$
		$d_R^{1,2,3}$	$-1/3^-$		
Gauge bosons		Non-SM Quarks		Non-SM Leptons	
$W_\mu^\pm$	0^+	$\mathcal{T}_L$	$1/3^-$	$\nu_R^{e,\mu,\tau}$	$1/3^+$
W_μ^3	0^+	$\mathcal{T}_R$	$2/3^-$	$\mathcal{N}_R^{e,\mu,\tau}$	0^+
B_μ	0^+	$\mathcal{J}_L^{1,2}$	0^+	$\mathcal{E}_L^1, \mathcal{E}_R^2$	-1^+
Z'_μ	0^+	$\mathcal{J}_R^{1,2}$	$-1/3^+$	$\mathcal{E}_R^1, \mathcal{E}_L^2$	$-2/3^+$

Non-universal X quantum number and $\mathbb{Z}_2$ parity for SM and non-SM fermions.

	Leptons		Quarks	
Family		Mass		Mass
1	ν_L^1	$\frac{\mu_{\mathcal{N}} v_3^2}{\left(h_{\mathcal{N}}^1\right)^2 v_\chi^2} h_{\nu 1}^2$	u	$\frac{h_u^2 - {h'_u}^2}{h_t} \frac{v_3}{\sqrt{2}}$
2	ν_L^2	$\frac{\mu_{\mathcal{N}} v_3^2}{\left(h_{\mathcal{N}}^1\right)^2 v_\chi^2} h_{\nu 2}^2$	c	$\frac{h_c^2 - {h'_c}^2}{h_T} \frac{v_1}{\sqrt{2}}$
3	ν_L^3	$\frac{\mu_{\mathcal{N}} v_3^2}{\left(h_{\mathcal{N}}^1\right)^2 v_\chi^2} h_{\nu 3}^2$	t	$\frac{h_t v_1}{\sqrt{2}}$
Exot	N_L^i	$\frac{h_{\mathcal{N}}^i v_\chi}{\sqrt{2}} \mp \mu_{\mathcal{N}}$	T	$\frac{h_T v_\chi}{\sqrt{2}}$
1	e	$\frac{h_\ell^2 - {h'_\ell}^2}{h_{v1}} \frac{v_3}{\sqrt{2}}$	d	$\frac{\Sigma_d h_d^2}{h_s^2 + {h'_s}^2}$
2	μ	$\frac{h_\ell^2 + {h'_\ell}^2}{h_{v1}} \frac{v_3}{\sqrt{2}}$	s	$\frac{h_s^2 + {h'_s}^2}{h_b} \frac{v_3}{\sqrt{2}}$
3	τ	$\frac{h_\tau v_2}{\sqrt{2}}$	b	$\frac{h_b v_2}{\sqrt{2}}$
Exot	E^1	$\frac{h_{E1} v_\chi}{\sqrt{2}}$	J^1	$\frac{h_{J1} v_\chi}{\sqrt{2}}$
Exot	E^2	$\frac{h_{E2} v_\chi}{\sqrt{2}}$	J^2	$\frac{h_{J2} v_\chi}{\sqrt{2}}$