Using the Tsallis distribution for hadron spectra in *pp* collisions: Pions and quarkonia at $\sqrt{s}$ = 5 – 13000 GeV

(Based on: S.Grigoryan, Phys.Rev.D95 (2017) 056021, arXiv:1702.04110 [hep-ph])

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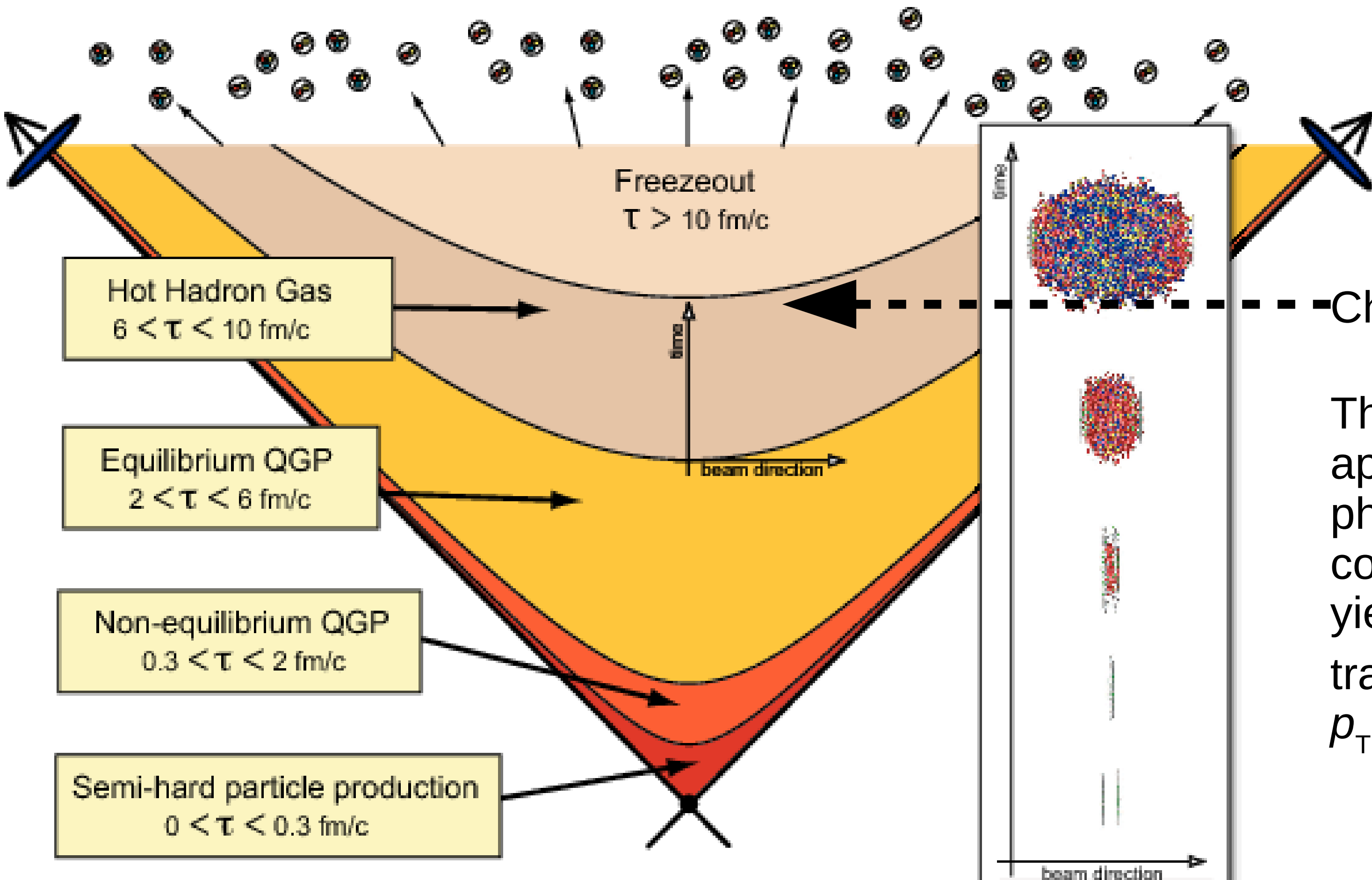
Outline

- Thermal-statistical approach to hadronic interactions
- Boltzmann-Gibbs distribution (BGD) vs Tsallis distribution (TD)
- Thermal distribution + blast-wave model (BWM)
- Model for hadronic yields $dN/dp_T dy$ in *pp* collisions
- Data fits and parametrization of the model parameters
- Results for neutral and charged pions: π^0, $\pi^\pm$ (most abundant data)
- Results for quarkonia: ϕ, J/ψ, $\psi(2S)$, $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$
- Conclusion

Thermal-statistical approach to hadronic interactions

- Thermodynamical/Hydrodynamical models have long history: E.Fermi 1950, L.Landau 1953, R.Hagedorn 1965
- The modern approach assumes that initial collision (*pp* or *AA*) criates a thermalized quark-gluon fireball, which expands, cools, hadronizes and goes through the chemical freeze-out and finally the kinetic freeze-out, when it decays into the free-streaming hadrons. So $T_{\text{ch}} > T_{\text{k}}$. Drawing shows some typical values for the proper time τ.



Chemical freeze-out

This approach has many applications in hadronic physics. Here I will consider only differential yields $dN/dp_T dy$ versus transverse momentum p_T and rapidity y in *pp* collisions.

BGD vs TD

- BGD for a static fireball of thermalized hadronic gas: ok for integrated yields and p_T spectra at low p_T but underestimates data at $p_T \gtrsim 2$ GeV/c ; y spectra too narrow. Different models: 80 MeV < T_{BGD} < 160 MeV.
Expanding (flowing) fireball, like in BWM (E.Schnedermann et al., Phys.Rev.C, 1993) can describe y spectra and improve p_T spectra up to p_T = 4–5 GeV/c.

$$E\frac{d^3N}{d^3p} = \frac{gVE}{(2\pi)^3}\, e^{-\frac{E-\mu}{T}},$$
$$\frac{d^2N}{m_T dm_T dy} = \frac{gVm_T\cosh y}{(2\pi)^2}\, e^{-\frac{m_T\cosh y-\mu}{T}}$$

V gas volume, T temperature (k_B=1), E particle energy, m_T transverse mass $m_T = (m^2 + p_T^2)^{1/2}$, μ chemical potential (related to conserved numbers: B, Q, S,… in chemical equilibrium), g = 2J+1

- TD is based on Tsallis non-extensive statistics (C.Tsallis, J.Stat.Phys. **52**, p.479, 1988) has new parameter $1 < q \lesssim 1.2$ (11/9). TD → BGD at $q \to 1$ (TD results from averaging of T-fluctuations in BGD, q - 1 is strength of fluctuations : G.Wilk et al., PRL **84**, p.2770, 2000)

$$E\frac{dN}{d^3p} = gVE\frac{1}{(2\pi)^3}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}$$

TD provides nearly exponential p_T spectra at low p_T and power-law form, like in QCD, at high p_T

TD with static fireball: ok for p_T spectra at any p_T. Hundreds of papers on p_T-data fits, $T < T_{BGD}$. But y spectra too narrow ⇒ need flowing fireball.

Thermal distribution + BWM

General formula for single-particle invariant yield from flowing fireball with thermal distribution $f(X)$ is Cooper-Frye integral over the kinetic freeze-out space-time hypersurface Σ_f (F.Cooper, G.Frye, Phys.Rev. D**10**, p.186, 1974)

$$E\frac{d^3N}{d^3p} = \frac{g}{(2\pi)^3}\, p_\nu \int_{\Sigma_f} d^3\Sigma^\nu f(X)$$

$$N = \frac{g}{(2\pi)^3}\, V_f \int d^3p\, f(\frac{E-\mu}{T})$$

$$V_f = \int_{\Sigma_f} d^3\Sigma^\nu\, u_\nu(x)\,.$$

$X = (p_\nu u^\nu - \mu)/T$, u_ν collective flow 4-velocity ($u_\nu u^\nu = 1$) depends on 4-coordinate $x_\nu = (t, x, y, z)$. T and μ also can depend on x_ν, but for simplicity I let them be const. Then due to Lorentz invariance, the invariant volume V_f can be factored out in the expression of integrated yield N.

Fireball flow & geometry in BWM are azimuthally symmetric and dominated by boost along z-axis as expected at high energy *pp* (or central *AA*) collisions. So instead of Cartesian coordinates one can introduce radial vector $(r\cdot\cos\phi,\ r\cdot\sin\phi)$ and Bjorken longitudinal proper time $\tau = (t^2 - z^2)^{1/2}$ and space-time rapidity $\eta = 0.5\cdot\ln((t+z)/(t-z))$. In BWM: $u_\nu = \gamma_r\cdot(\cosh\eta,\ v_r\cos\phi,\ v_r\sin\phi,\ \sinh\eta)$,
$\gamma_r = (1 - v_r^2)^{-1/2}$, v_r radial flow velocity. We obtain $p_\nu u^\nu = \gamma_r\cdot(m_T\cosh(y-\eta) - p_T v_r \cos(\phi_p - \phi))$.
→ BWM defines Σ_f by condition that freeze-out happens at $\tau = \tau_f = \text{const}$. Then one gets:
$d^3\Sigma^\nu = \tau_f\cdot(\cosh\eta, 0, 0, \sinh\eta)d\eta d^2r$. Geometry of Σ_f is fixed as: $-\eta_{max} < \eta < \eta_{max}$, $0 < r < R(\eta)$.
→ For $f(X)$ I use TD with quantum statistics $\xi = \pm 1$ (for Bosons/Fermions)

$$f(X) = \left[[1+(q-1)X]^{\frac{1}{q-1}} - \xi\right]^{-q}$$

J.M.Conroy et al., Phys.Lett. A**374**, 2010
J. Cleymans et al., J.Phys. G**39**, 2012

Correction is important only for pions due to small *m*. Then I compute 1st integral using binomial series in powers of ξ (assuming μ < m).

Model for hadronic yields $dN/dp_T dy$ in *pp* collisions

Calculations of integrals with FD give the main formula of the model :

$$\frac{d^2N}{p_T dp_T dy} = g\frac{3V_0}{8\pi^2}\sum_{k=0}^{\infty}\xi^k\binom{q-1+k}{k}\int_{-\eta_{max}}^{\eta_{max}}\frac{d\eta}{\eta_{max}}$$
$$\times\int_0^{R(\eta)}\frac{rdr}{R_0^2}\frac{m_T\cosh(y-\eta)\,a^{\frac{q+k}{q-1}}}{[1+\frac{\gamma_r m_T\cosh(y-\eta)-\mu}{T/(q-1)}]^{\frac{q+k}{q-1}}}P_{\frac{k+1}{q-1}}(a)\,,$$

where $a = 1/\sqrt{1-b^2}$,

$$b = \frac{\gamma_r v_r p_T}{T/(q-1)+\gamma_r m_T\cosh(y-\eta)-\mu}$$

here $V_0 = 4/3\,\pi R_0^2\tau_f\eta_{max}$

P_ν (a) Legendre function of 1st kind.

One can verify that at $q \to 1$ this formula reproduces usual BWM formulae when BGD is used.

I have chosen :

$R(\eta) = R_0(1 - \eta^2/\eta^2_{max})^{1/2}$ important for *y* spectra

$v_r(r) = v_s\cdot(r/R_0)^2$ important for p_T spectra

→ First 3 terms of the series are almost sufficient for accurate computation of pion spectra.
→ For other hadrons the first term is enough and one can use $\xi = 0$.
→ It can be seen that effect of radial flow diminishes with increasing *y*. Owing to this, the model describes the experimental fact that particle p_T spectra become softer ($\langle p_T\rangle$ become smaller) with increase of its rapidity.
→ Resonance decay contributions into pions are not calculated, assuming same spectra for direct and secondary pions (like in other TD models).

Then I fit the pion and quarkonia data.

If data given in terms of σ, we convert to *N* using $\sigma = N\sigma_{in}$, σ_{in} is *pp* inelastic cross section.

Fit parameters are: T, q, μ, V_0, η_{max}, v_s

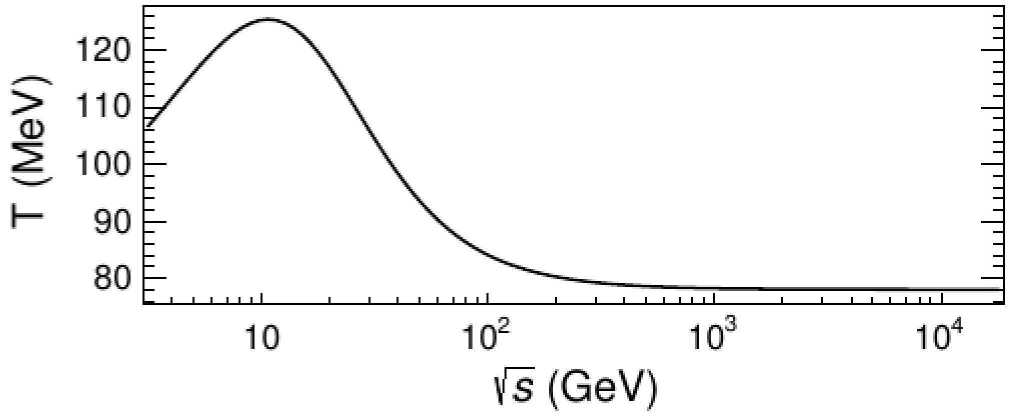
Generally, they can depend on √s and hadron type.

I assume same *T* for all hadrons (while T_{ch} expected to rise with hadron mass)

Data fits and parametrization of the model parameters (1)

I follow two aims: 1) build a model with possibly small set of parameters, which fits well the existing data on dN/dp_Tdy for different particles and √s in *pp* collisions;
2) systematize fit results and provide meaningful parametrizations for √s dependence of model parameters, allowing predictions for future experiments.

I started with fits of most abundant pion data and obtained this form of T(s) :



$$T = T_\infty \left(1 + \frac{1.33\sqrt{x} - 0.21}{1 + x^2}\right)$$

where x=√s/(16 GeV),
T_∞ = 78 MeV value of T at $\sqrt{s} \to \infty$.
This T(s) is used then in the fits of other hadrons.

Resulting values for η_{max} , v_s and V_0 are parametrized as:

$$\eta_{max} = 0.89 y_m - 0.32 - 1.18 \frac{y_m}{y_b} + \frac{1.86}{y_b} - \frac{0.17}{y_m} + \frac{0.24\, Q}{1 + \sqrt{s}/e_0} y_b \,,$$

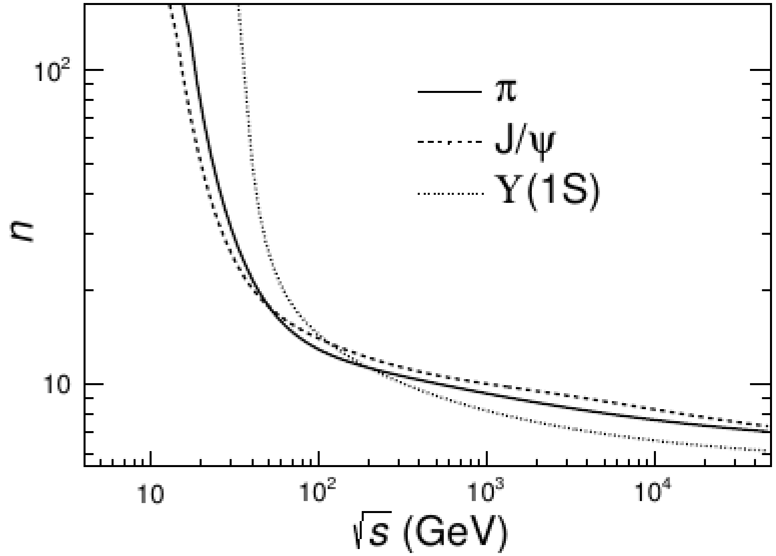
$$v_s = 0.78 \left(1 - \frac{1.31}{y_m} - \frac{0.09}{y_m}\frac{m}{m_p} + \frac{0.20\, Q}{1 + \sqrt{s}/e_0}\right),$$

$$V_0 = \widetilde{V}\, \eta_{max}\, y_b\, (T_\infty / T)^{1.92}$$

Where e_0 = 3.7 GeV, m_p proton mass,
y_m = ln(√s/m) maximum rapidity of hadron,
y_b = ln(√s/m_p) beam rapidity at high energy.
In the model, η_{max} defines y-spectrum width.
Terms ~ Q (charge) describe the difference between spectra of different charge pions vs y (η_{max}) and vs p_T (v_s) at low √s.

Data fits and parametrization of the model parameters (2)

Fitted values of q grow with √s and depend on hadron type. But they vary in the very small interval 1 – 11/9. More convenient is parameter $n = q/(q-1)$, controling large-p_T behavior of $dN/dp_T dy$, $n > n_\infty = 11/2$. Values of n for different hadrons are parametrized as :



$$n = \frac{n_\infty}{1+p_1 x} + \frac{p_3/\ln x}{\ln x - p_2} + p_4 x^{0.37} - p_5 x$$

where $x = e_1/\sqrt{s} < 1$, parameters e_1, $p_1 - p_5$ listed in the table of a next slide.

Figure shows n(s) for 3 particles. For ϕ, ψ(2S) and Y(2S), Y(3S) it is similar to one for π, J/ψ and Y(1S), respectively.

Values of μ are proportional to m ($\mu < m$) and vanish with increasing √s. Parametrized as:

$$\mu = p_6 \left(\ln \frac{e_2}{\sqrt{s}} - p_7 \frac{1-\sqrt{s}/e_2}{1+\sqrt{s}/e_3}\right) m$$

For quarkonia (see e_2, e_3, p_6, p_7 in the table)

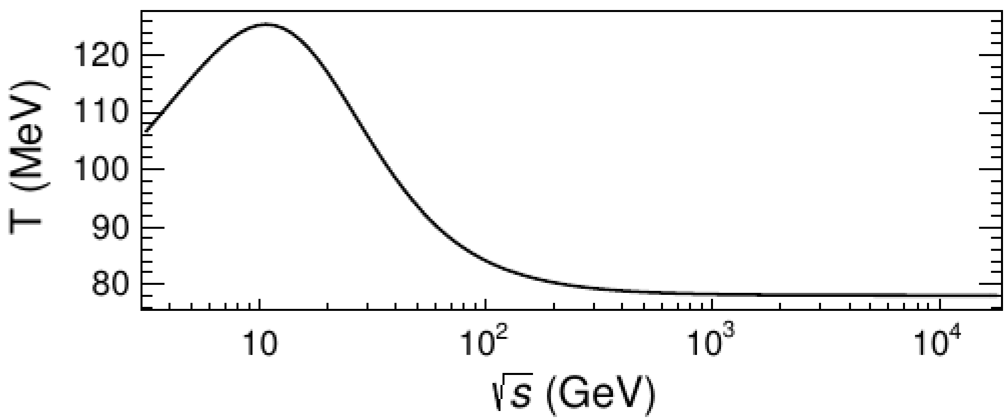
$$\mu_\pi = \left(0.80\, e^{-\sqrt{s}/(5\,\mathrm{GeV})} + \frac{0.65\, Q}{(1+\sqrt{s}/e_0)^{1.5}}\right) m_\pi$$

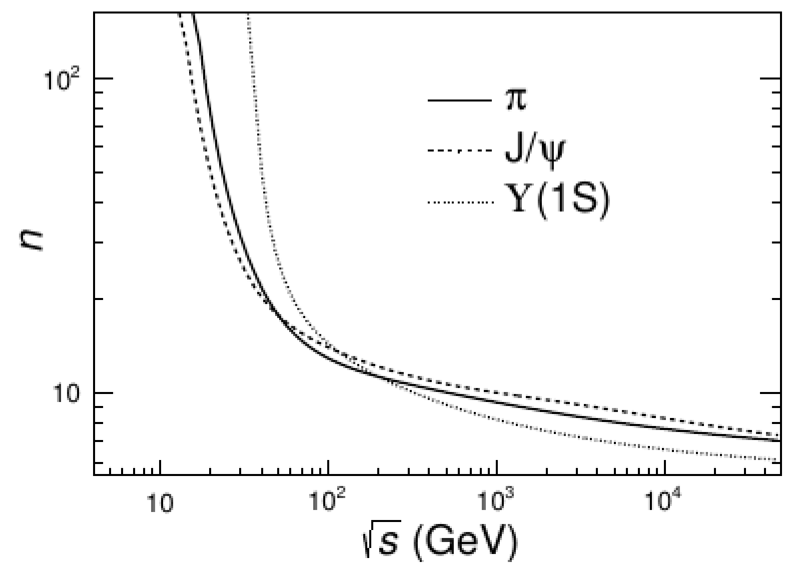
For pions, m_π is averaged mass for pion 3 charge states.

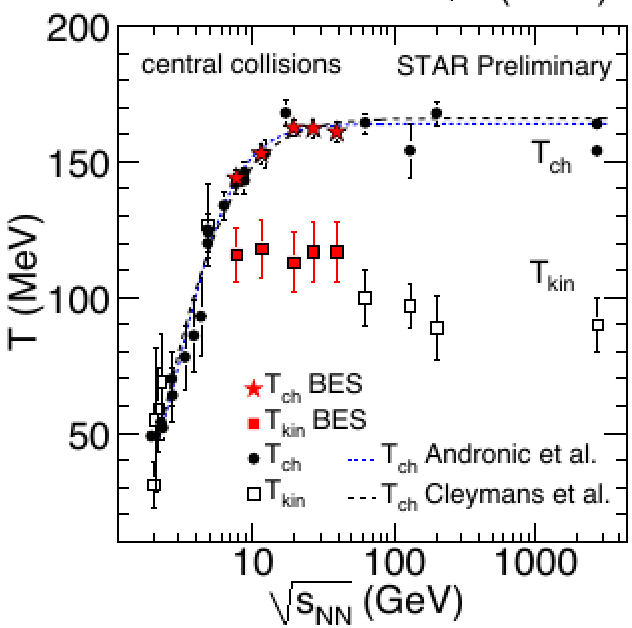
μ_π vanishes rapidly with increasing √s. The term ~ Q describes difference of $\pi^\pm$, π^0 yields in pp collisions at low √s.
These yields practically coincide at √s = 62.4 GeV according to RHIC data.

Data fits and parametrization of the model parameters (3)

Note that parameter n and kinetic freeze-out temperature T, show a remarkable behavior at energies $\sqrt{s} \sim 10$ GeV : $n \rightarrow \infty$, i.e. TD reduces to BGD, and T reaches its max. This is the energy region of the new accelerators NICA /FAIR, where the quark-hadron phase transition is expected.







Similar behavior was observed for T_{kin} , obtained from analysis of central A-A collisions data, using thermal models with BGD and BWM.
Plot is from : L.Kumar (for the STAR Collaboration) Nucl.Phys. A931 (2014) 1114.

Data fits and parametrization of the model parameters (4)

Results of combined fits for pion data and each quarkonium data (in pp and high energy $p\bar{p}$ collisions) using parametrizations of the model parameters given above :

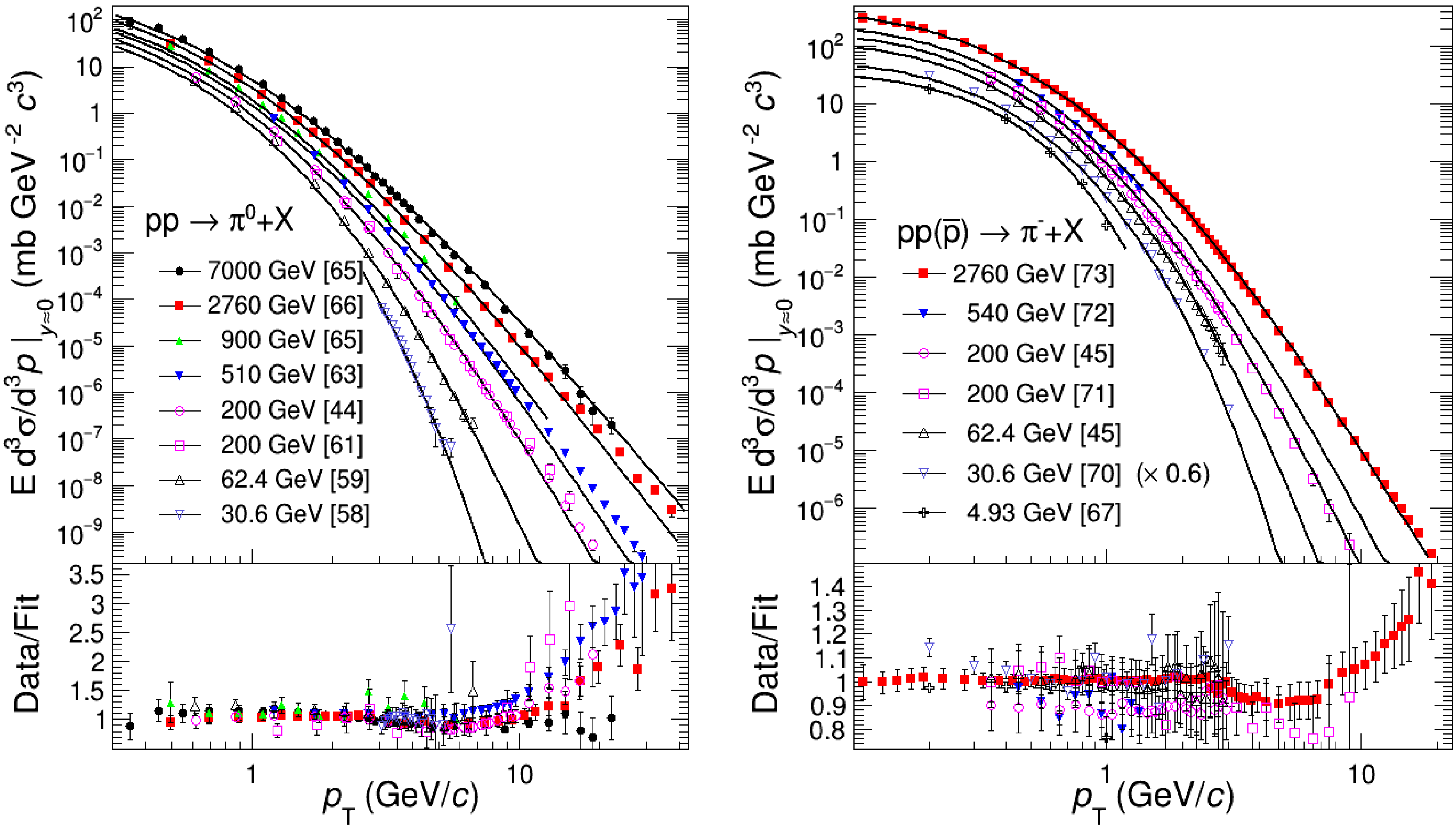
	π	ϕ	J/ψ	ψ(2S)	Υ(1S)	Υ(2S)	Υ(3S)
$\tilde{V}$GeV^{-3}	5028.6 $\pm$6.0	561.1 $\pm$5.2	107.2 $\pm$0.4	19.2 $\pm$0.1	0.244 $\pm$0.001	0.094 $\pm$0.001	0.034 $\pm$0.001
e_1 GeV	12.5	12.5	7.8	7.8	30.0	30.0	30.0
p_1	3.8	3.5	76.1	128.1	0	0	0
p_2	2.0	3.1	0	0	0	0	0
p_3	133.4	166.4	56.3	56.3	3.0	0.6	0.4
p_4	0	0	29.2	29.2	8.6	7.5	5.8
p_5	50.2	46.7	87.9	94.0	-4.3	-8.0	-8.0
e_2 GeV	-	8786	13900	35500	$1{\cdot}10^9$	$1{\cdot}10^9$	$1{\cdot}10^9$
e_3 GeV	-	225	63.1	63.1	16000	16000	16000
p_6	-	0.047	0.072	0.060	0.0580	0.0548	0.0531
p_7	-	2.50	2.05	0.51	3.52	3.52	3.52
χ^2	10905	238	5975	1566	5759	3439	3000
NDF	2514	223	1707	974	622	491	478

→ New parameters for nonprompt J/ψ and ψ(2S) (from B-hadron decays) are given further.
→ Rather large ratios χ^2 / NDF are due to large amount of data included in fits which use √s-dependent parametrizations for model parameters. Since quality and normalization of different data sets for given hadron do not always agree well, the combined fit gives larger χ^2 / NDF than individual fits for each data set. Moreover, in the case of Υ mesons, two different measurements of the LHCb at √s = 7 TeV do not agree well (see below).

→ Then I will illustrate the results of combined fits for pions and quarkonia.

Results for neutral and charged pions (1)

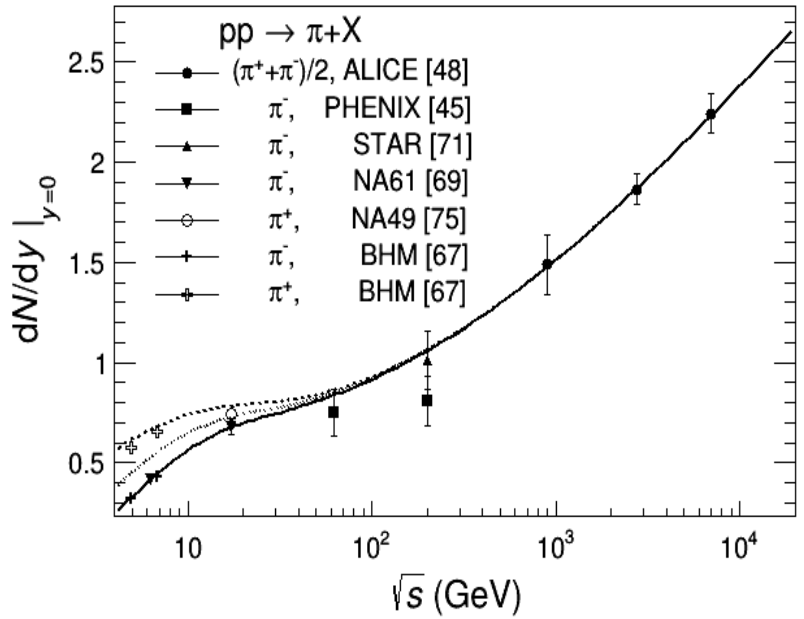
Fitted $\pi^{\pm}$ (π^0) data for *pp* collisions at $\sqrt{s}$ = 4.93 (30.6) GeV – 7 TeV and π^- data for $p\bar{p}$ collisions at $\sqrt{s}$ = 540 GeV. Here and in next plots, [x] are data Refs. from paper S.G. PRD95 2017. Some data and respective lines multiplied (or shifted) by a number, given in parentheses, for visibility.

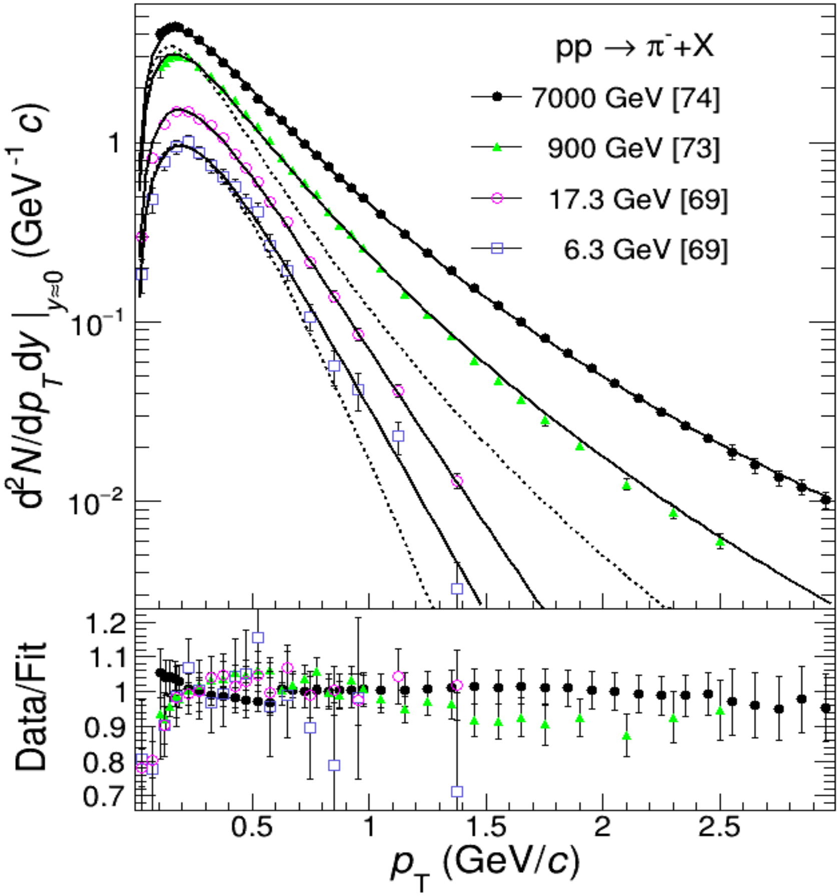


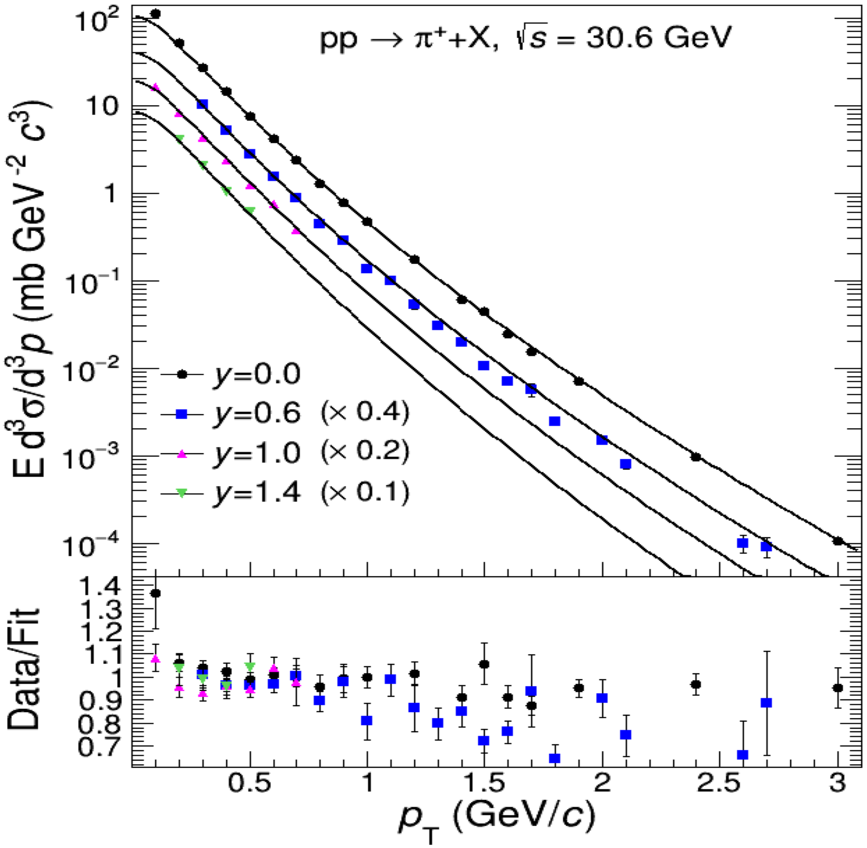
Results for neutral and charged pions (2)

Data and lines at y = 0.6, 1.0, 1.4 are multiplied by 0.4, 0.2, 0.1 for a better visibility.

Dashed lines in the plot below are for $v_s = 0$ at √s = 6.3, 900 GeV to show the effect of radial flow.

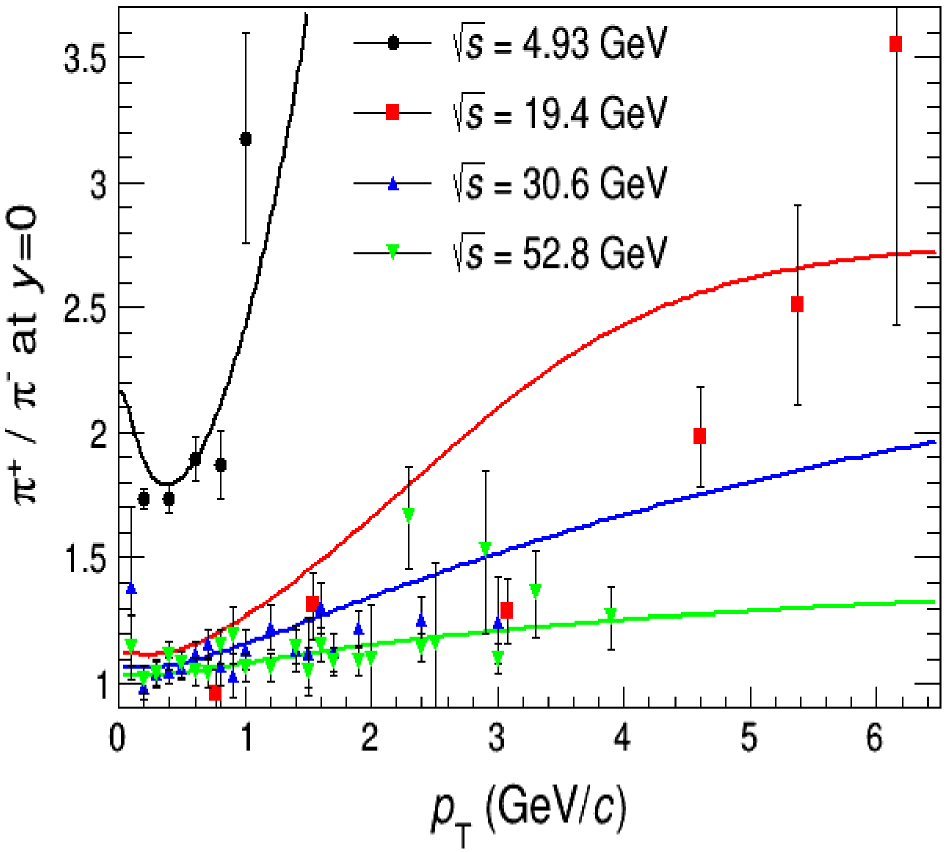


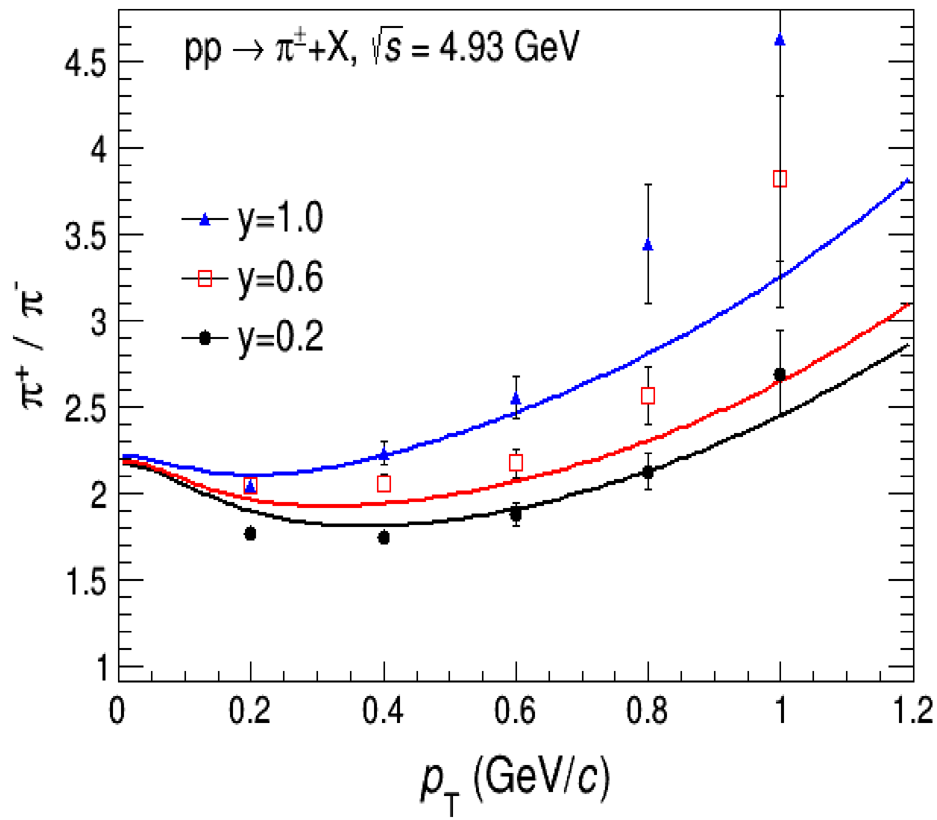




Results for neutral and charged pions (3)

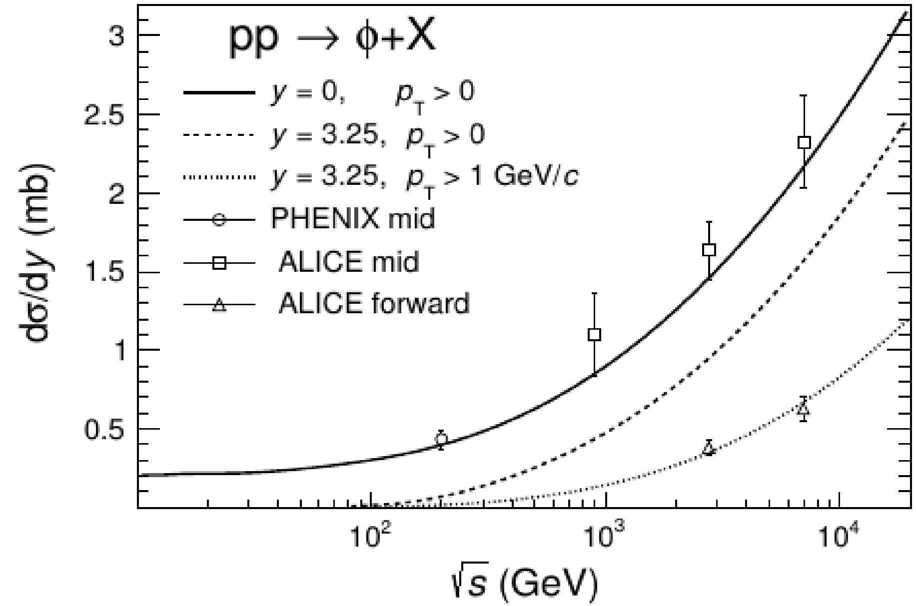
Some plots for π^+ / π^- ratio vs p_T at different values of √s (left) and y (right)

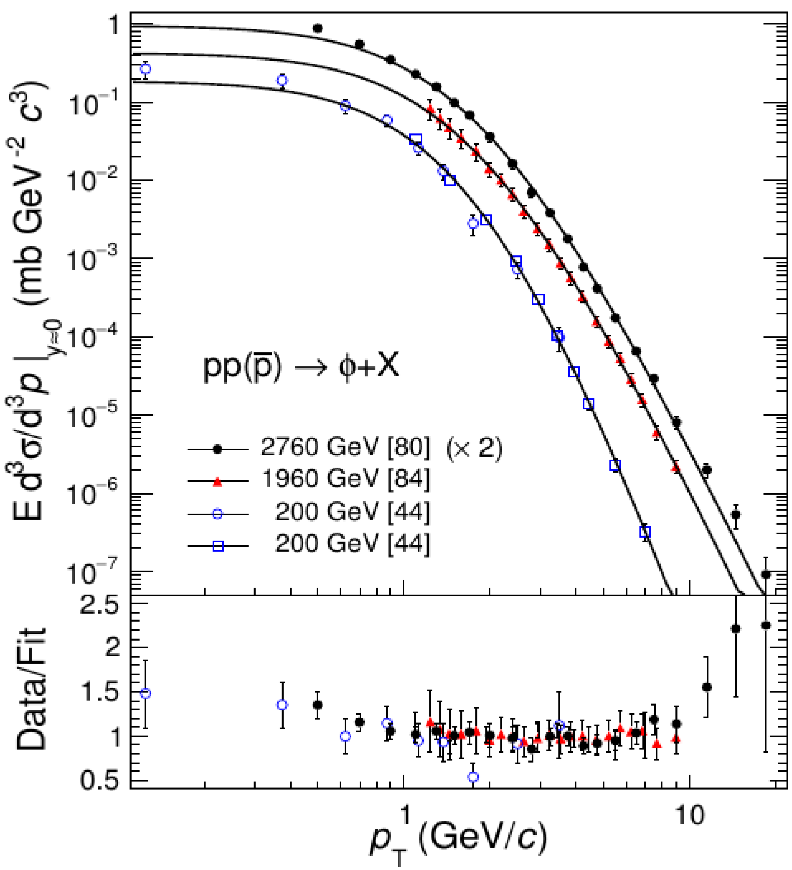


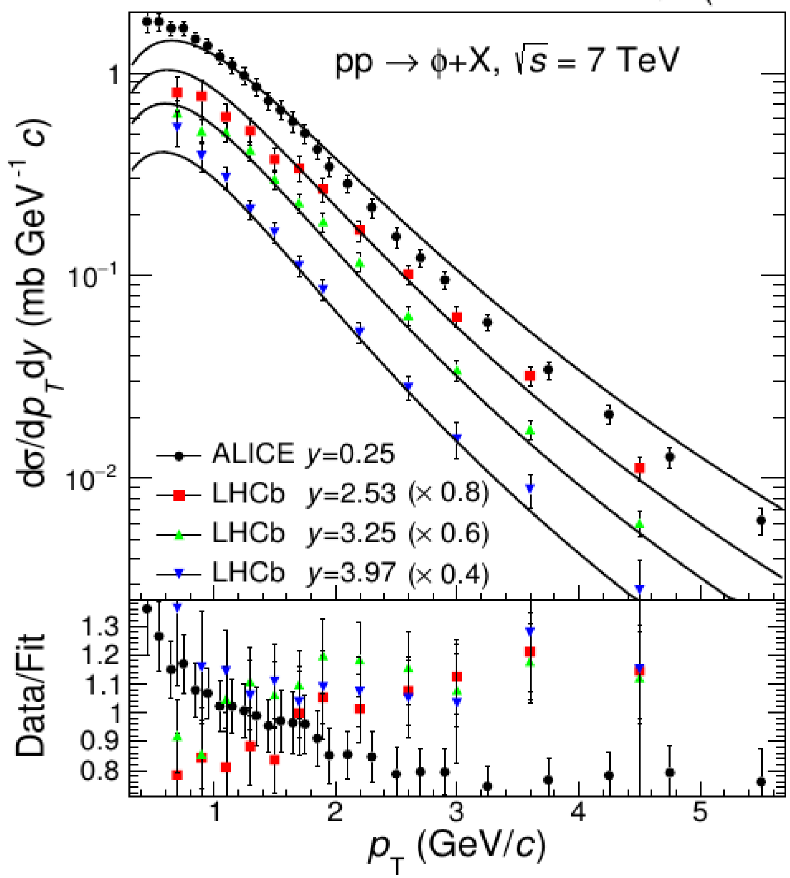


Results for φ(1020) meson

Fitted data for *pp* at $\sqrt{s}$ = 17.3 GeV – 7 TeV
and for $p\bar{p}$ at $\sqrt{s}$ = 1.96 TeV.







Results for J/ψ meson (1)

Inclusive J/ψ consistes of prompt (direct production and radiative decays of heavier charmonia) and nonprompt (weak decays of B-hadrons) components.
Nonprompt fraction (f_B) is negligible at $\sqrt{s} < 100$ GeV but rises with $\sqrt{s}$ and p_T.
At LHC energies $f_B \gtrsim 0.1$.
Nonprompt J/ψ has significantly harder p_T-spectrum and narrower y-spectrum. I utilize for it same T, n, μ, v_s, as for prompt, and new parameters to describe the difference:
Replace $m \to c_m \cdot m$ (in m_T of formula for $dN/dp_T dy$), $\eta_{max} \to c_\eta \cdot \eta_{max}$ and use

$$\widetilde{V} = \widetilde{V}_{NP}\left(1 + \frac{c_1}{1 + (c_2/p_T)^4}\right)$$

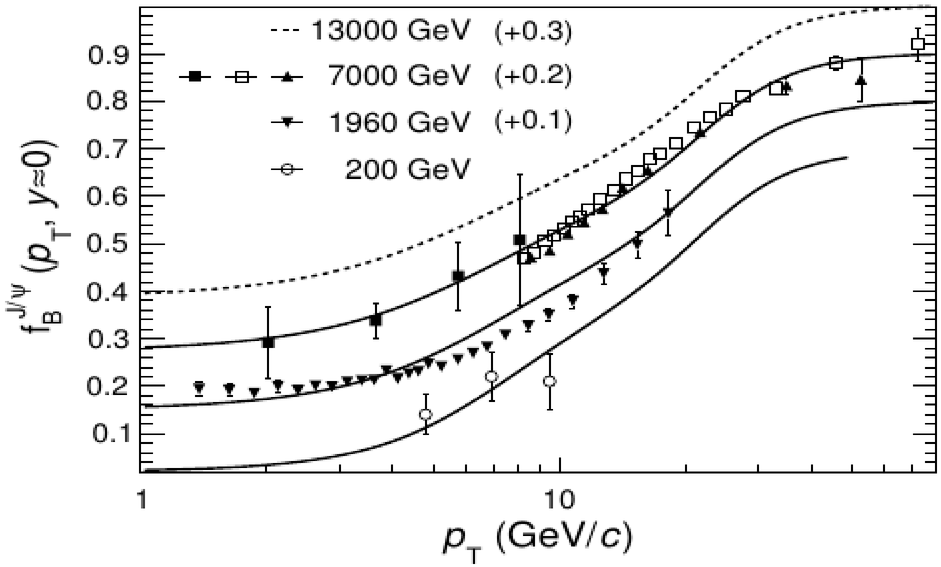
Fitting of existing prompt and nonprompt or inclusive J/ψ data gives, in addition to the values for parameters, χ^2 and NDF listed in the table above, the following values for nonprompt component new parameters:

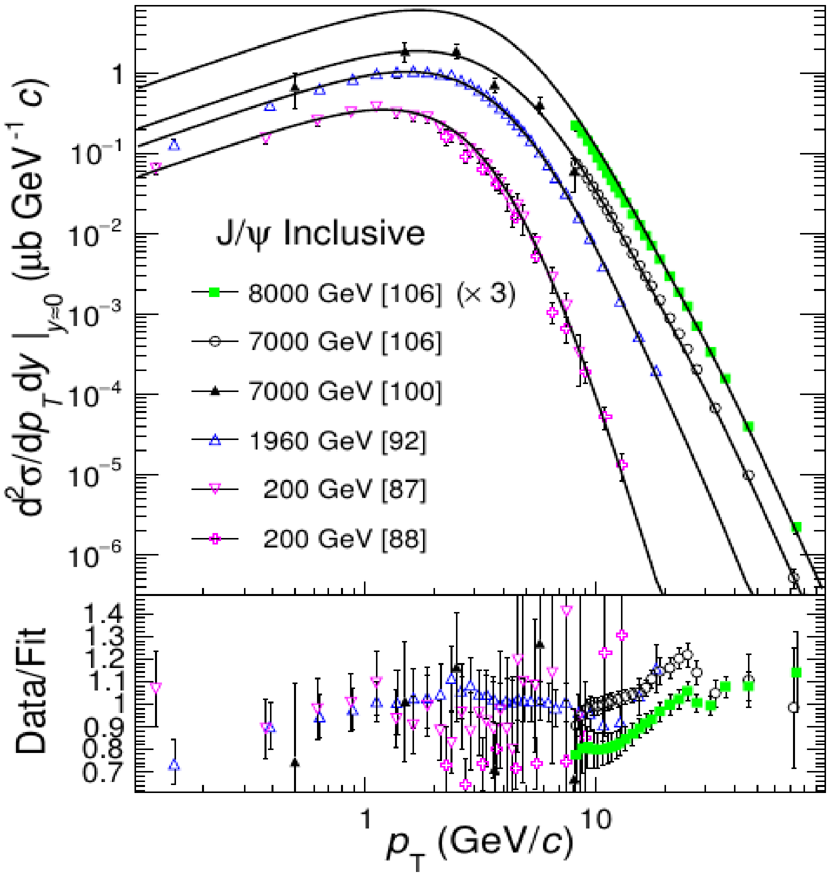
$$\overline{V}_{NP} = 82.0 \pm 0.7 \text{ GeV}^{-3}, \quad c_2 = 26.3 \text{ GeV}$$

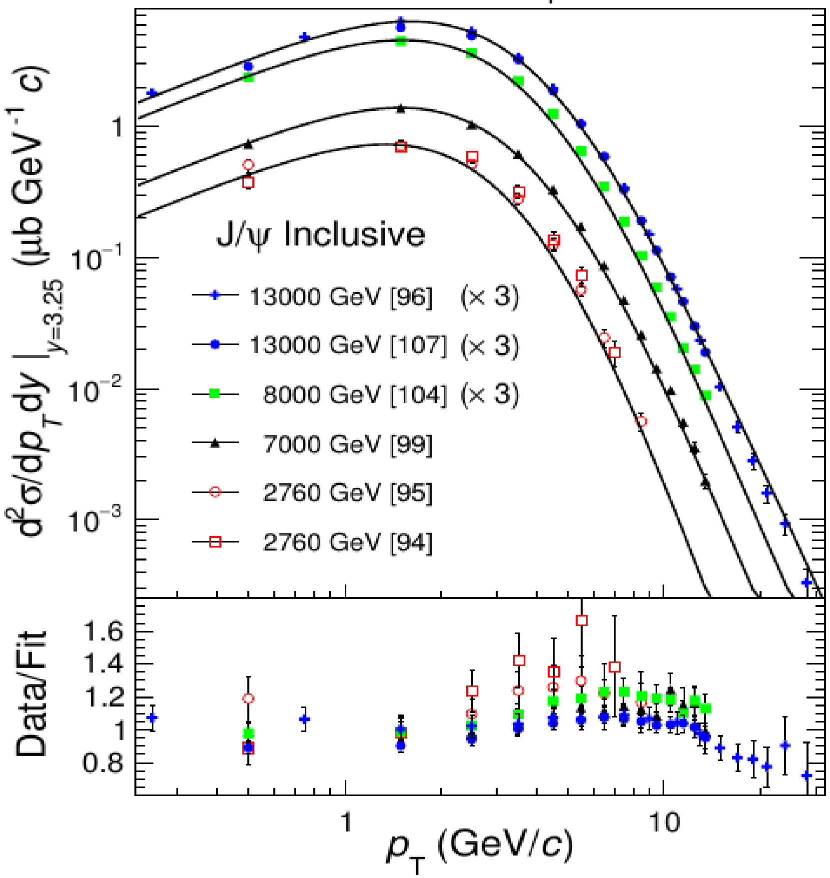
$$c_1 = 2.1, \quad c_m = 1.4, \quad c_\eta = 0.82$$

Results for J/ψ meson (2)

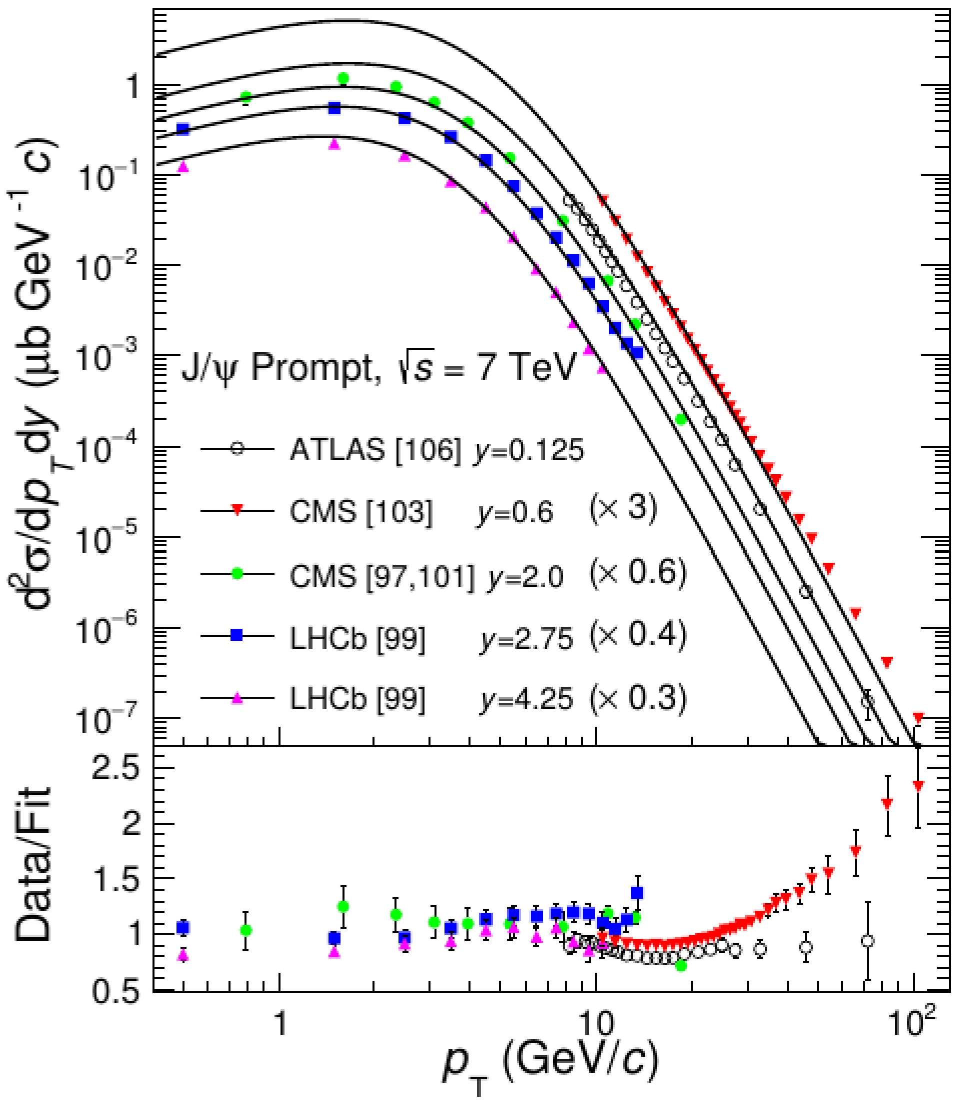
Fitted data for pp at $\sqrt{s}$ = 19.4 GeV – 13 TeV and for $\bar{p}p$ at $\sqrt{s}$ = 1.8, 1.96 TeV.

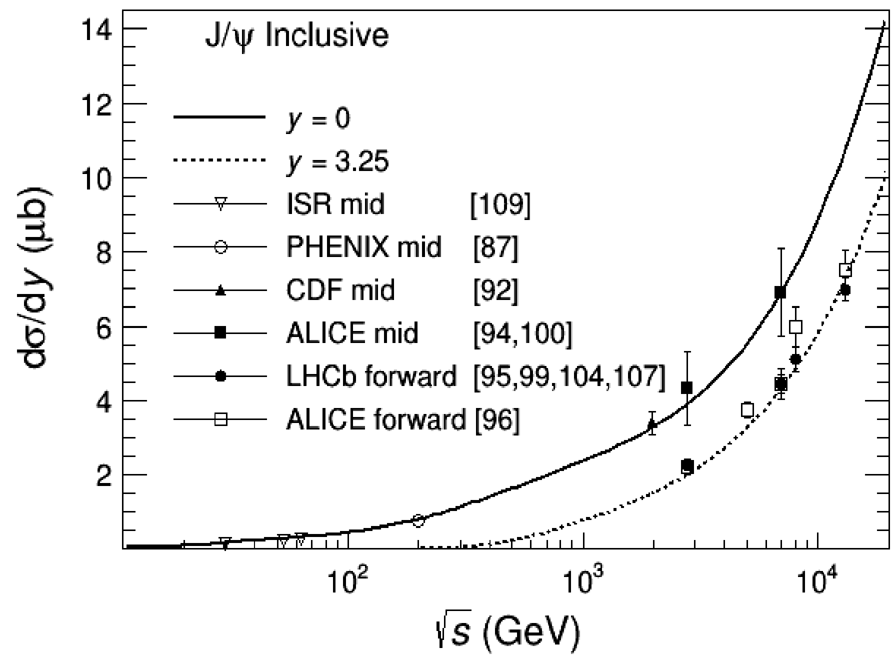


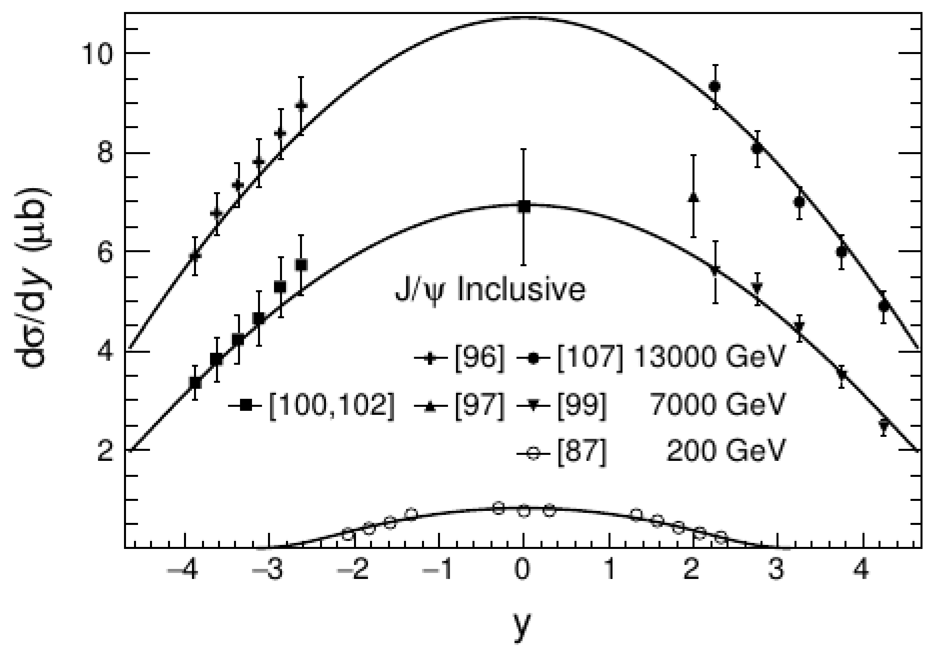




Results for J/ψ meson (3)





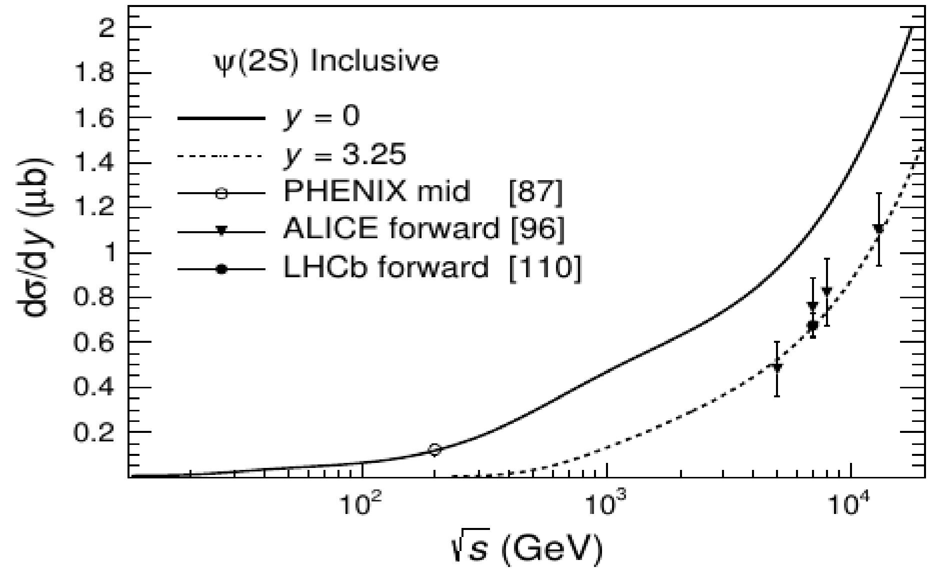


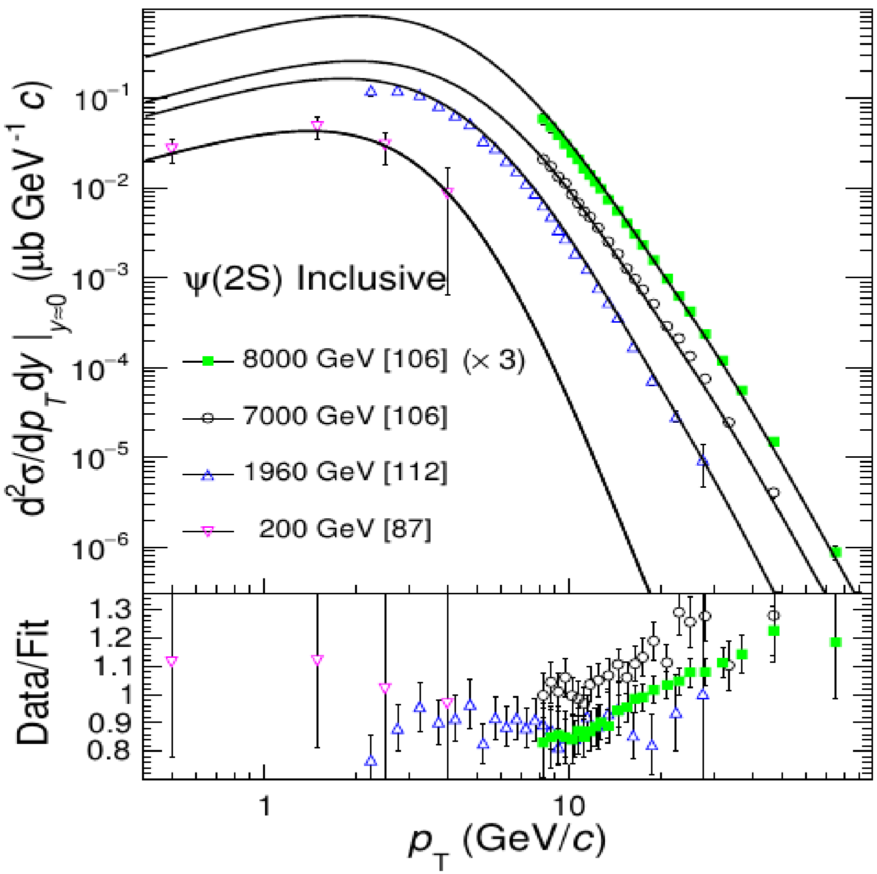
Results for ψ(2S) meson

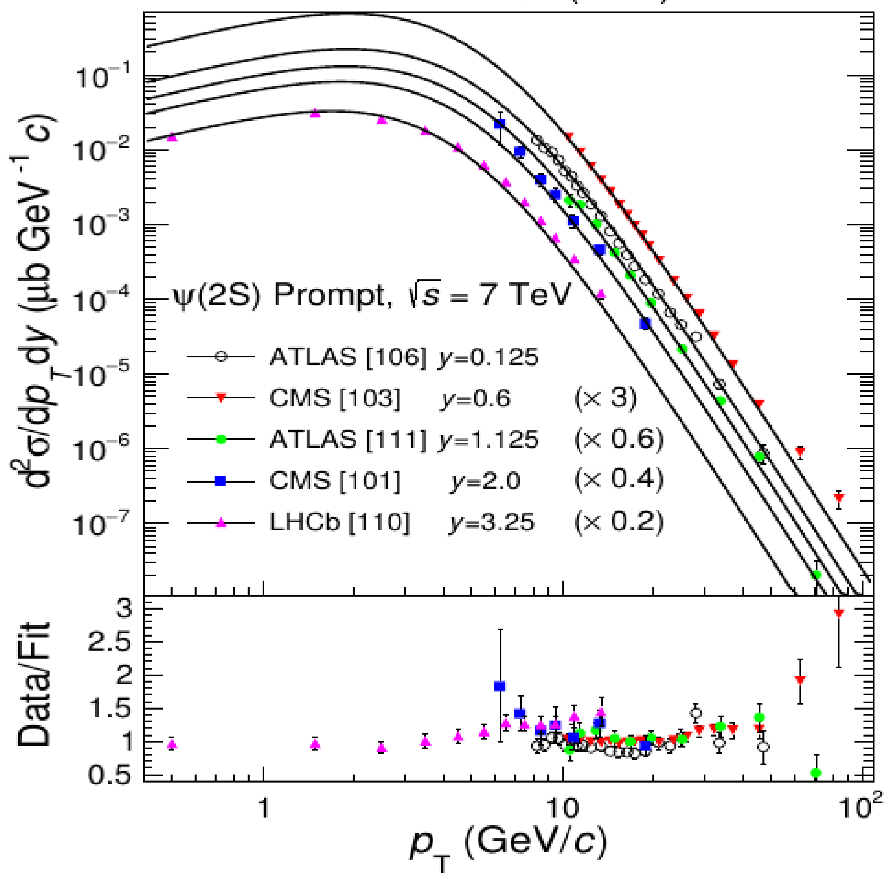
Similarly to J/ψ case we get:

$$\bar{V}_{NP} = 15.9 \pm 0.1\ \mathrm{GeV}^{-3}\ ,\ c_2 = 26.3\ \mathrm{GeV}$$
$$c_1 = 2.1,\ c_m = 1.3,\ c_\eta = 0.82$$

Fitted data for pp at $\sqrt{s}$ = 200 GeV – 13 TeV
and for $p\bar{p}$ at $\sqrt{s}$ = 1.8, 1.96 TeV.



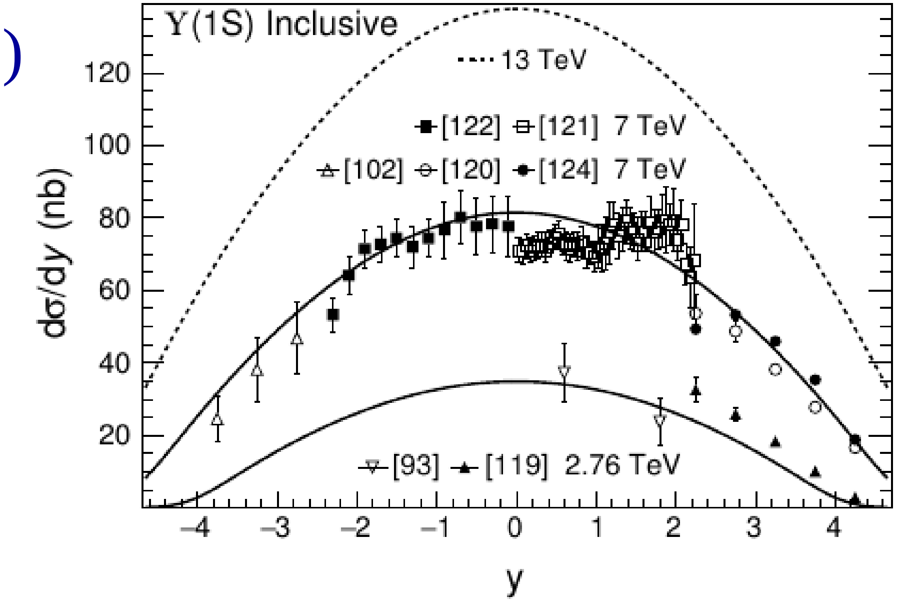


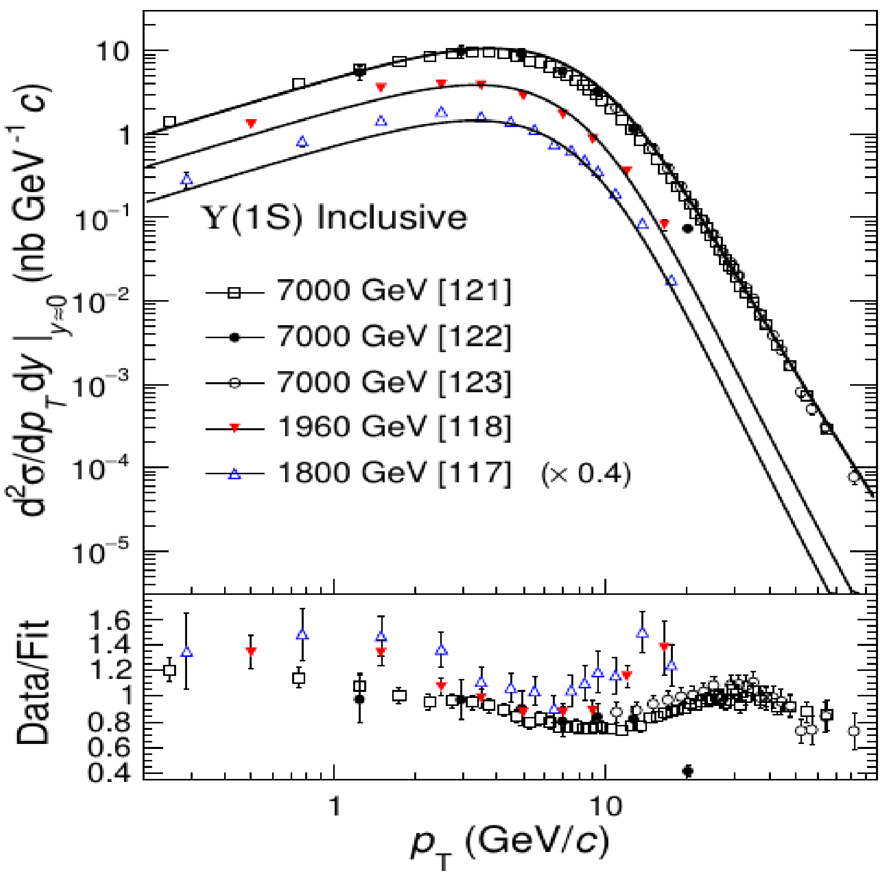


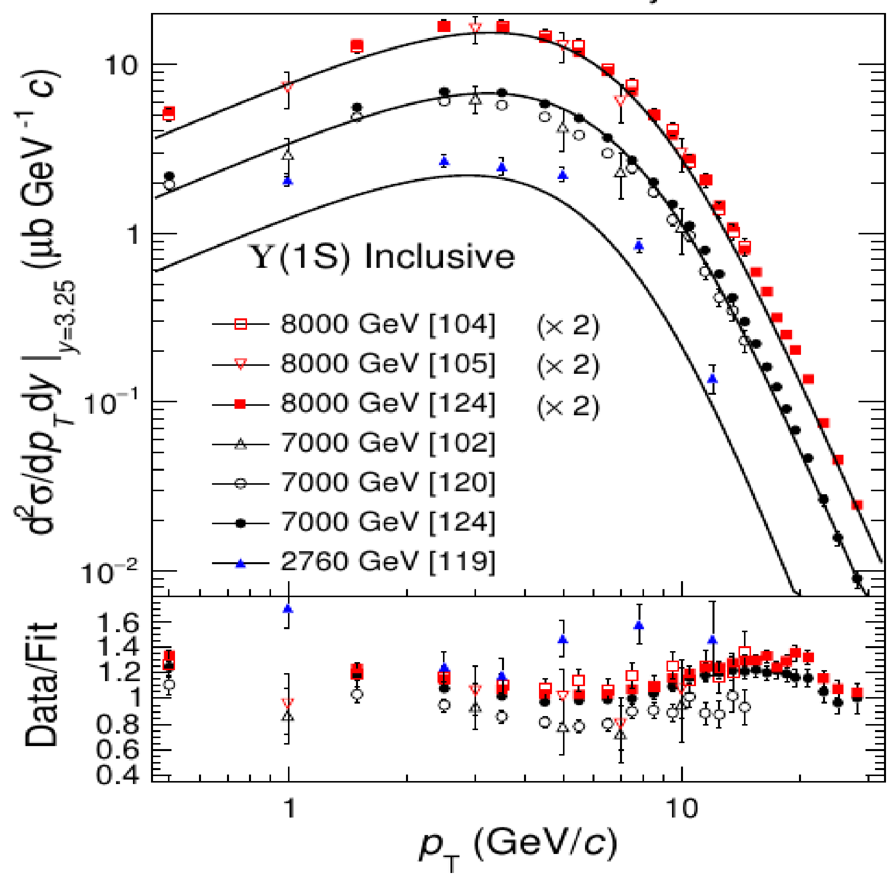
Results for Υ(1S), Υ(2S), Υ(3S) mesons (1)

Fitted data for *pp* at √s = 38.8 GeV – 8 TeV and for $p\bar{p}$ at √s = 1.8, 1.96 TeV. Mostly Υ(1S) data

Note a poor match between two data sets of LHCb for √s = 7 TeV [120,124], giving large χ^2 / *NDF*, as mentioned above.



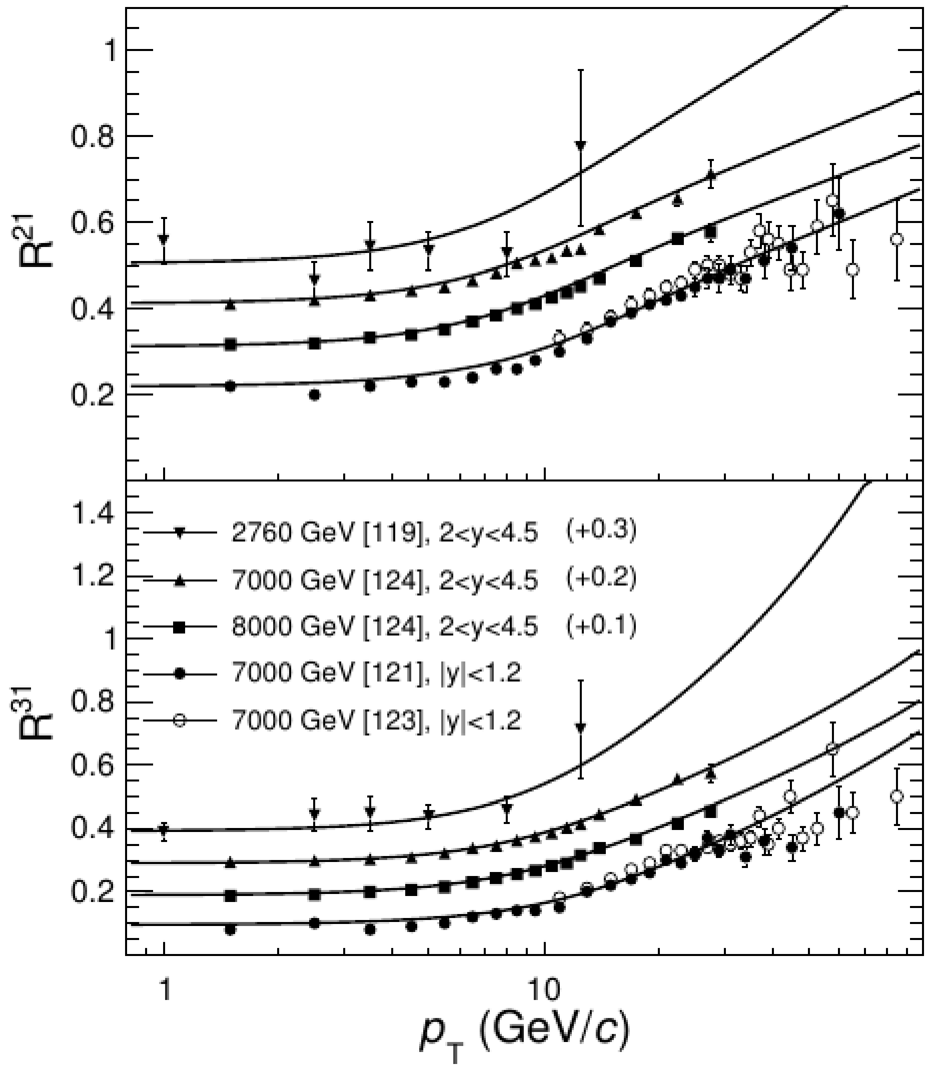


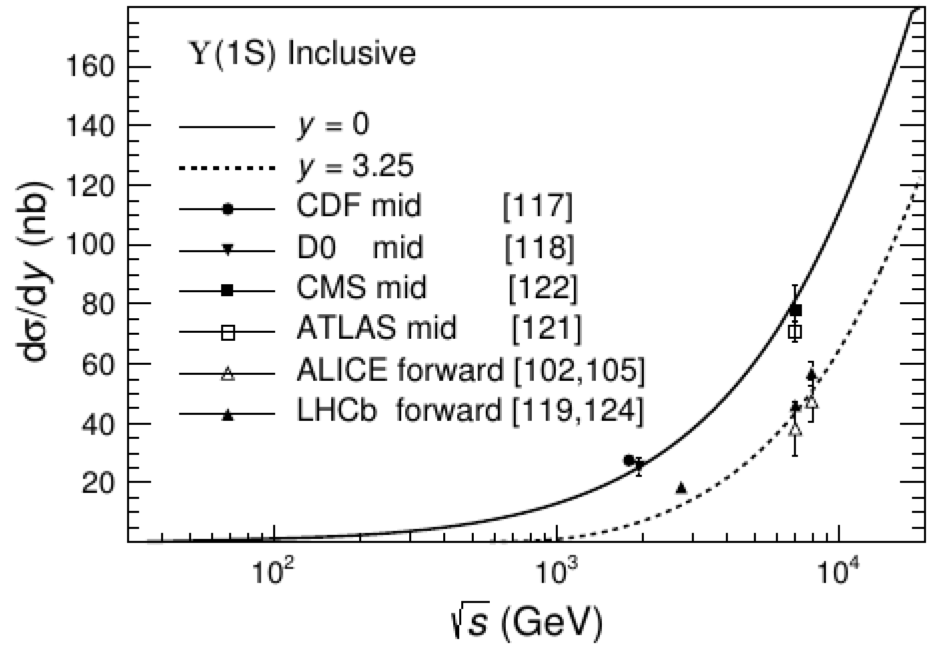


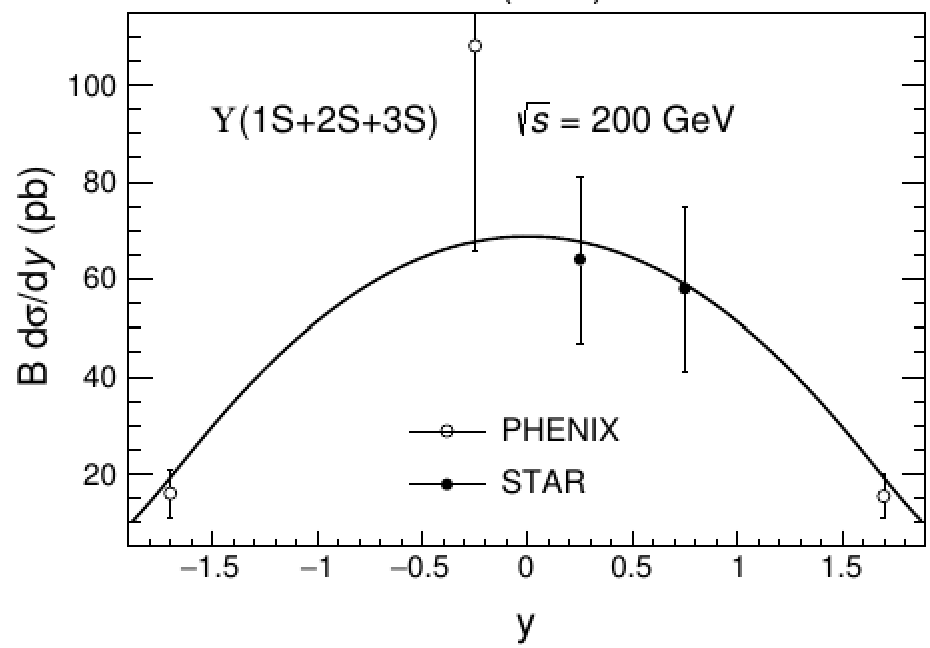
Results for Υ(1S), Υ(2S), Υ(3S) mesons (2)

Denote $A^i = B(\Upsilon(iS) \to l^+l^-)\, dN^i/dp_T dy$

$R^{21} = A^2/A^1$, $R^{31} = A^3/A^1$







Conclusion

- Thermal model is proposed, describing well almost all data on pion and quarkonia double-differential yields in *pp* at $\sqrt{s} \geq 5$ GeV. Main features:
  - same kinetic freeze-out temperature T(s) for all particle types
  - particle chemical potential vanishes with increasing $\sqrt{s}$ and can be interpreted as a measure of its chemical non-equilibrium
  - describes softening of particle p_T spectra with increase of its rapidity
- Simple parametrizations are given for $\sqrt{s}$ dependence of the model parameters, allowing predictions for new energies of existing and future accelerators (e.g. NICA)
- An example script https://www.dropbox.com/s/5zocgyqafh7avpe/yields.C is provided, showing how to compute particle yields in the ROOT
- Since model includes all ingredients of the thermal fireball, it can be used for the pion Bose-Einstein correlation studies (à la A.Bialas et al., 2014)
- Other particles besides pions and quarkonia can be also considered
- Model can be generalized for *pA* and *AA* collisions

Thank you !