ITEP Lattice

Magnetic structure of vector mesons in lattice QCD

E.V. Luschevskaya, O.E. Soloveva,

NRC "Kurchatov Institute"-Institute of Theoretical and Experimental Physics

O.V.Teryaev

Joint Institute of Nuclear Research

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Introduction

In an external abelian constant magnetic field we calculate

- the energies of the ground states of vector $\rho^{\pm}$ and $K^{*\pm}$ mesons;
- g-factor of the vector mesons;
- the magnetic dipole polarizabilities and magnetic hyperpolarizabilities of $\rho^{\pm}$ mesons.

Motivation

g-factor (magnetic moment)

- characterizes the gyromagnetic ratio of an hadron;
- its value reveals the contribution of the strong interactions.

Motivation

Magnetic polarizability and hyperpolarizability

- characterize the response of the quark currents inside a meson to the external magnetic field,
- are the fundamental quantities describing the spin interactions of quarks and the ability to form instantaneous dipoles.

Technique for calculation of energies

Calculate the propagators:

$$D^{-1}(x,y) = \sum_{k<M} \frac{\psi_k(x)\psi_k^\dagger(y)}{i\lambda_k + m_q}.$$

Calculate the correlation functions on the lattice:

$$\langle O_{\rho^+}(x)\bar{O}_{\rho^+}(y)\rangle = -\mathrm{Tr}[\Gamma_1 D_u^{-1}(x,y)\Gamma_2 D_d^{-1}(y,x)],$$

$x = (\mathbf{n}a, n_t a)$, $y = (\mathbf{n}'a, n_t' a), \mathbf{n}, \mathbf{n}' \in \Lambda_3 = \{(n_1, n_2, n_3) | n_i = 0, 1, ..., N-1\}, \Gamma_1, \Gamma_2 = \gamma_\mu$.

$$O_{\rho^+} = \psi^\dagger(x)_d \Gamma_{1,2} \psi(x)_u, \quad O_{\rho^-} = \psi^\dagger(x)_u \Gamma_{1,2} \psi(x)_d$$

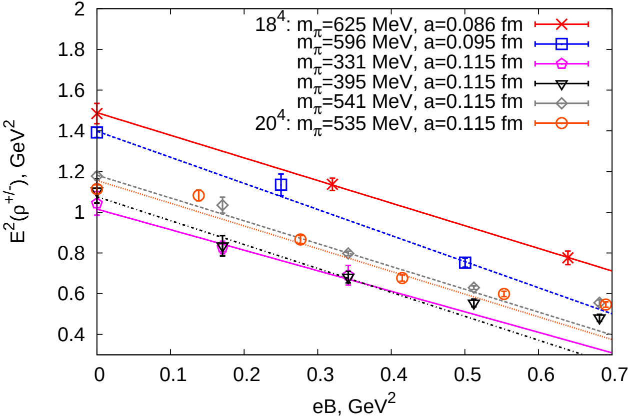
$$C(n_t) = \langle \psi^\dagger(\mathbf{0}, n_t)\Gamma_1 \psi(\mathbf{0}, n_t)\psi^\dagger(\mathbf{0}, 0)\Gamma_2\psi(\mathbf{0}, 0)\rangle_A =$$
$$\sum_k \langle 0|\hat{O}|k\rangle\langle k|\hat{O}^\dagger|0\rangle e^{-n_t a E_k}.$$

$$C(s_z = \pm 1) = C_{11} + C_{22} \pm i(C_{12} - C_{21}), \quad C(s_z = 0) = C_{33}, \quad \vec{B} \parallel {}''3''$$

$$C_{fit}(n_t) = 2A_0 e^{-N_T a E_0/2} \cosh((n_t - \frac{N_T}{2})aE_0)$$

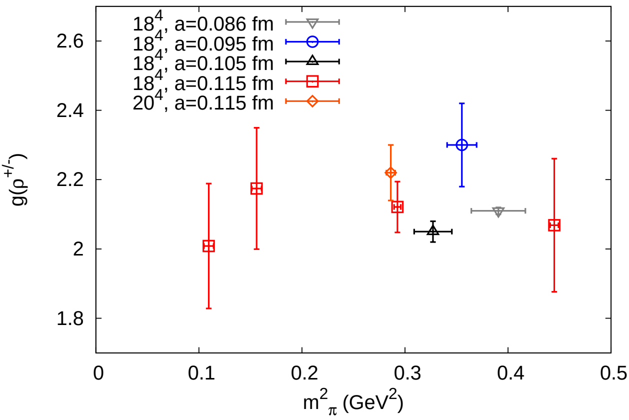
Energy of the $\rho^{+(-)}$ meson for $s_z = +1(-1)$

$$E^2 = |qB| - g s_z qB + m^2$$



g-factor of the $\rho^{\pm}$ meson

V	m_π (MeV)	a(fm)	g-factor	χ^2/d.o.f.	fit, eB (GeV^2)
18^4	331 ± 7	0.115	2.01 ± 0.18	0.826	$[0, 0.35]$
18^4	395 ± 6	0.115	2.17 ± 0.18	0.969	$[0, 0.35]$
18^4	541 ± 3	0.115	2.12 ± 0.07	1.159	$[0, 0.35]$
18^4	667 ± 3	0.115	2.07 ± 0.19	1.695	$[0, 0.35]$
18^4	625 ± 21	0.086	2.11 ± 0.01	0.153	$[0, 0.70]$
18^4	596 ± 12	0.095	2.30 ± 0.12	1.094	$[0, 0.55]$
18^4	572 ± 16	0.105	2.05 ± 0.03	0.644	$[0, 0.45]$
20^4	535 ± 4	0.115	2.22 ± 0.08	1.398	$[0, 0.45]$



$\bar{g}_\rho = 2.10 \pm 0.1_{syst}$, relative systematic error $\sim 5\%$.

Comparison with the other results

BaBar cross section data for the reaction $e^+e^- \to \pi^+\pi^-2\pi^0$:
$g_\rho^{exp} = 2.1 \pm 0.5$,
D.G. Gudino, G.T.Sanchez, Int.J.Mod.Phys.A 30, 1550114 (2015).

2+1 lattice QCD: $g_\rho = 2.21 \pm 0.08$,
B. Owen et.al., Phys. Rev. D 91, 074503 (2015).

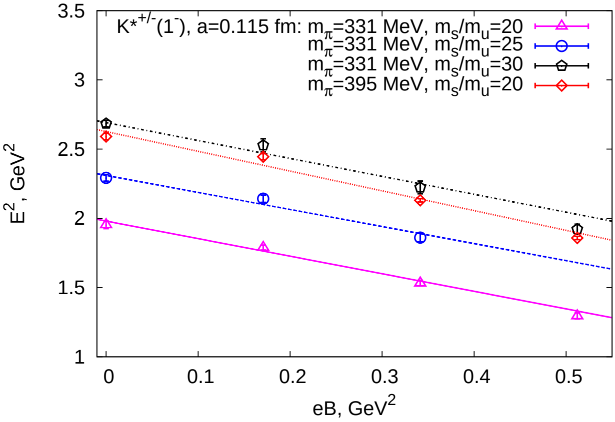
QCD sum rules: $g_\rho = 2.4 \pm 0.4$,
T.M. Aliev et.al., Phys. Lett. B 678, 470 (2009).

covariant quark model: $g_\rho = 2.14$,
J.P.B.C. de Melo and T. Frederico, Phys. Rev. C 55, 2043 (1997).

gravitation theory: $g \approx 2$.
O. V. Teryaev, Front. Phys. (Beijing) 11, 111207 (2016).

Energy of $K^{*+(-)}$ meson for $s_z = +1(-1)$

$$E^2 = |qB| - g s_z qB + m^2$$



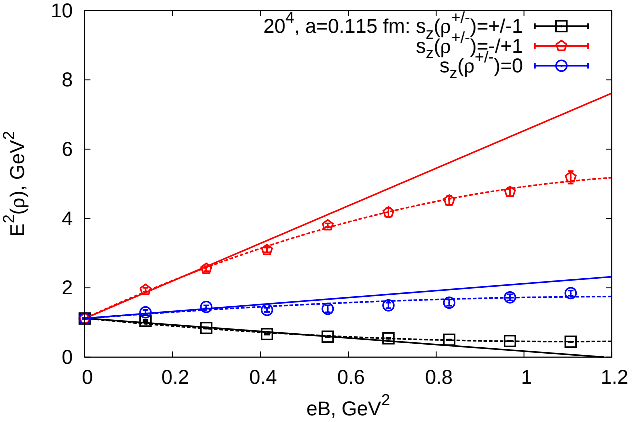
g-factor of the $K^{*\pm}$ meson

m_π(MeV)	m_s/m_u	g-factor	χ^2/d.o.f.	fit, eB (GeV^2)
331 ± 7	20	2.27 ± 0.18	1.845	$[0, 0.35]$
331 ± 7	25	2.23 ± 0.23	1.986	$[0, 0.35]$
331 ± 7	30	2.29 ± 0.19	1.366	$[0, 0.35]$

QCD sum rules: $g_{K^*} = 2.0 \pm 0.4$,
T.M. Aliev et.al., Phys. Lett. B 678, 470 (2009).

gravitation theory: $g_{K^*} \approx 2$,
O.V.Teryaev, Front. Phys. 11, 111207 (2016).

Energy of $\rho^{\pm}$ meson for $s_z = -1, 0, +1$



$$E^2 = |qB| - g s_z qB + m^2 - 4\pi m \beta_m (qB)^2, \quad eB \in [0, 1.2\ \mathrm{GeV}^2]$$

Energy of the $\rho^{\pm}$ meson from theoretical considerations

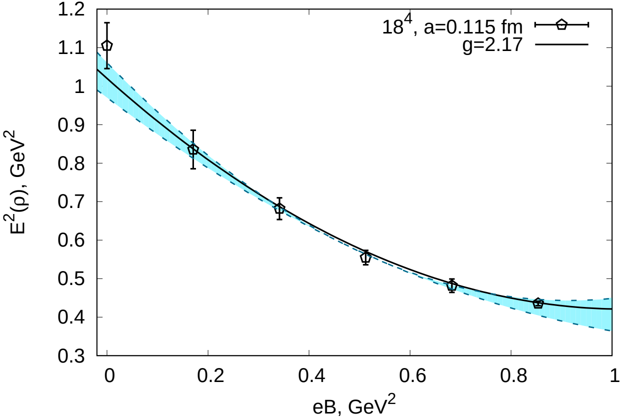
$s_z = 0:$

$$E^2 = |qB| + m^2 - 4\pi m\beta_m (qB)^2 - 4\pi m\beta_m^{h2} (qB)^4 - \dots$$

$s_z = \pm 1:$

$$E^2 = |qB| - g s_z qB + m^2 - 4\pi m\beta_m (qB)^2 - 4\pi m\beta_m^{h1} (qB)^3 - 4\pi m\beta_m^{h2} (qB)^4 - \dots$$

Fixed g-factor, 2-param. fit



$$E^2 = |qB| - g s_z qB + m^2 - 4\pi m \beta_m (qB)^2, \quad g = 2.17 \pm 0.18$$

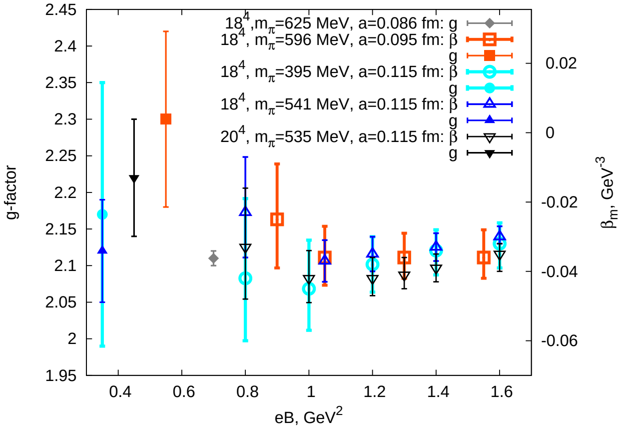
β_m for $|s_z| = 1$ from the 2-param. fit

Fixed g,

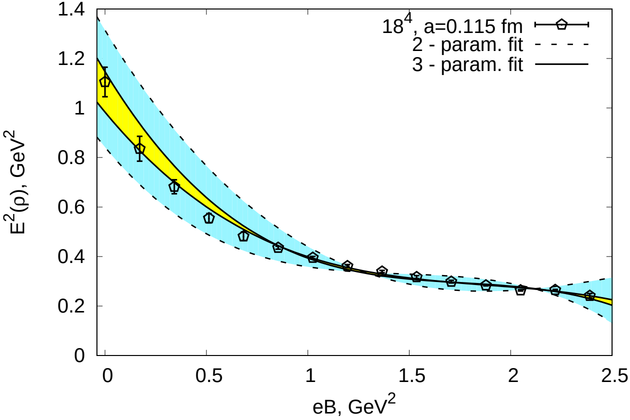
$$E^2 = |qB| - g s_z qB + m^2 - 4\pi m \beta_m (qB)^2$$

V	m_π (MeV)	a (fm)	β_m (GeV^{-3})	$\chi^2/d.o.f$	fit, eB (GeV^2)
18^4	596 ± 12	0.095	$-0.025^{+0.016}_{-0.014}$	1.656	$[0, 0.9]$
18^4	596 ± 12	0.095	$-0.036^{+0.007}_{-0.006}$	1.864	$[0, 1.3]$
18^4	541 ± 3	0.115	$-0.037^{+0.006}_{-0.005}$	2.774	$[0, 1.05]$
20^4	535 ± 4	0.115	$-0.042^{+0.008}_{-0.008}$	2.274	$[0, 1]$
18^4	395 ± 6	0.115	$-0.045^{+0.011}_{-0.012}$	0.823	$[0, 1]$

g-factor and $\beta_m(|s_z| = 1)$ versus the interval of B



Fixed g-factor, 2− and 3−parametric fit



$$E^2 = |qB| - g s_z qB + m^2 - 4\pi m \beta_m (qB)^2 - 4\pi m \beta_m^{h1} (qB)^3, \;\; g = 2.17 \pm 0.18$$

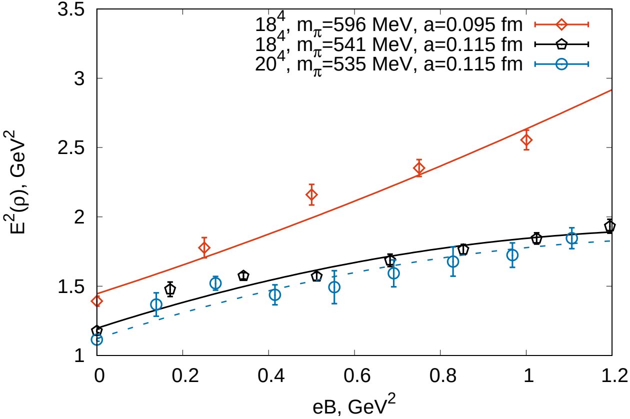
β_m and β_m^{h1} for $\rho^{\pm}(s_z = \pm 1)$ from the 3-param. fit

Fixed g, $eB \in [0, 2.5\ \mathrm{GeV}^2]$

$$E^2 = |qB| - g s_z qB + m^2 - 4\pi m \beta_m (qB)^2 - 4\pi m \beta_m^{h1} (qB)^3$$

V	m_π (MeV)	a (fm)	β_m (GeV^{-3})	β_m^{1h} (GeV^{-5})	$\chi^2/d.o.f.$
18^4	596 ± 12	0.095	$-0.050^{+0.009}_{-0.008}$	$0.009^{+0.002}_{-0.003}$	1.965
18^4	541 ± 3	0.115	$-0.045^{+0.005}_{-0.005}$	$0.009^{+0.001}_{-0.001}$	2.787
20^4	535 ± 4	0.115	$-0.058^{+0.008}_{-0.008}$	$0.013^{+0.002}_{-0.003}$	2.697
18^4	395 ± 6	0.115	$-0.047^{+0.009}_{-0.009}$	$0.009^{+0.002}_{-0.002}$	2.255

Energy of $\rho^{\pm}(s = 0)$



$$E^2 = |qB| + m^2 - 4\pi m\beta_m (qB)^2$$

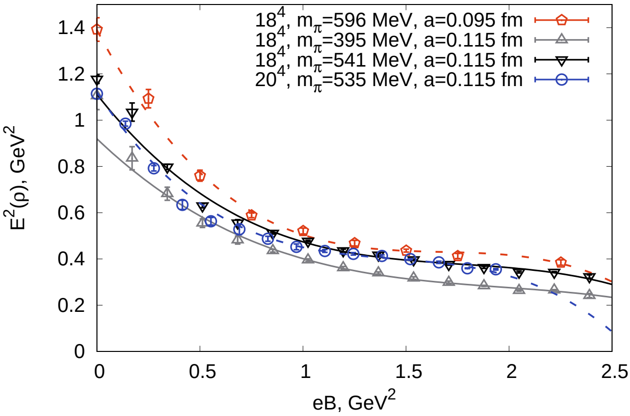
β_m for $s_z = 0$ from the 2-parametric fit

Fit at $eB \in [0, 1.2\ \text{GeV}^2]$

$$E^2 = |qB| + m^2 - 4\pi m\beta_m (qB)^2$$

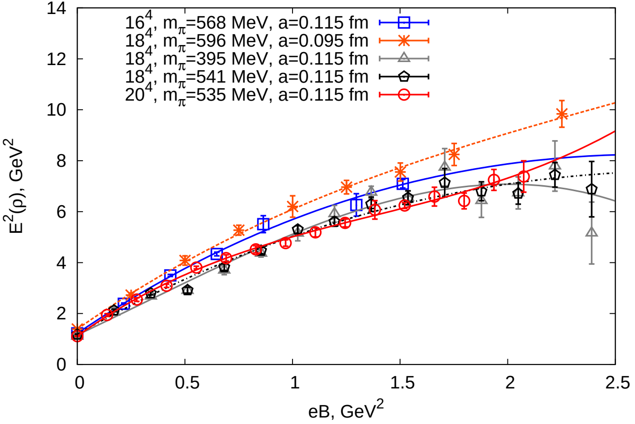
V	m_π(MeV)	a(fm)	β_m(GeV^{-3})	$\chi^2/d.o.f.$
18^4	541 ± 3	0.115	0.026 ± 0.004	1.959
20^4	535 ± 4	0.115	0.026 ± 0.005	1.365

Energy of $\rho^{+(-)}$ for $s_z = +1(-1)$, 4-param. fit



$$E^2 = |qB| - g s_z qB + m^2 - 4\pi m \beta_m (qB)^2 - 4\pi m \beta_m^{h1} (qB)^3$$

Energy of $\rho^{+(-)}$ for $s_z = -1(+1)$



$$E^2 = |qB| - g s_z qB + m^2 - 4\pi m \beta_m (qB)^2 - 4\pi m \beta_m^{h1} (qB)^3$$

Conclusions

1. We calculate the g-factor of the $\rho^{\pm}$ and $K^{*\pm}$ mesons,
2. obtain the magnetic dipole polarizability of the $\rho^{\pm}$ meson for the $|s_z| = 1$,
3. estimate the magnetic dipole polarizability of the $\rho^{\pm}$ meson for the $s_z = 0$,
4. calculate the hyperpolarizability of the $\rho^{\pm}$ for the $s_z = \pm 1$.