Supergravity solutions from the double copy

Silvia Nagy

University of Nottingham

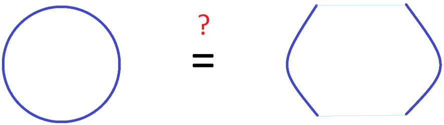
based on work done in collaboration with:
A. Anastasiou, L. Borsten, G. Cardoso,
M. Duff, G. Inverso, S. Nampuri and M. Zoccali

August 28, 2017

Table of Contents

The double copy

- Basic ingredints of string theory: closed strings, open strings.
- Can we construct a closed string from two open strings ?



The Double Copy

- KLT relations for vertex operators in string theory. [Kawai,Lewellen,Tye '85]

$$
\begin{array}{ccc}
V_{closed} & = V^{open}_{left} & \times V^{open}_{right} \\
\downarrow & \downarrow & \downarrow \\
\text{gravity} & = \text{gauge theory} & \times \text{gauge theory}
\end{array}
$$

low energy limit

The Double Copy

- KLT relations for vertex operators in string theory.[Kawai,Lewellen,Tye '85]

$$\begin{array}{cccc} V_{closed} & = V_{left}^{open} & \times V_{right}^{open} & \\ \downarrow & \downarrow & \downarrow & \text{low energy limit} \\ Supergravity & = SYM & \times SYM & \end{array}$$

when we have supersymmetry (this gives interesting BH solutions as well!).

Table of Contents

tensoring helicity states

field	momentum states	helicity
$g_{\mu\nu}(x)$	$g^{++}(p), g_{--}(p)$	$+2, -2$
$\psi_\mu(x)$	$\psi^+(p), \psi^-(p)$	$+3/2, -3/2$
$A_\mu(x)$	$A^+(p), A^-(p)$	$+1, -1$
$\lambda(x)$	$\lambda^+(p), \lambda^-(p)$	$+1/2, -1/2$
$\phi(x)$	$\phi(p)$	0

Examples:

$$A^+ \otimes \tilde{A}^+ = g^{++} \qquad A^- \otimes \tilde{A}^- = g^{--}$$

$$A^+ \otimes \tilde{A}^- = \phi \qquad \lambda^+ \otimes \tilde{\lambda}^- = \varphi$$

$$A^+ \otimes \tilde{\lambda}^+ = \psi^+ \qquad \lambda^+ \otimes \tilde{\lambda}^+ = A^+$$

tensoring multiplets

$$(\mathcal{N}=0)_{YM} \otimes (\mathcal{N}=0)_{YM} = \text{gravity } + 2 \text{ scalars}$$
$$(\mathcal{N}=2)_{SYM} \otimes (\mathcal{N}=0)_{YM} = (\mathcal{N}=2)_{sugra} + (\mathcal{N}=2)_{vector}$$
$$(\mathcal{N}=4)_{SYM} \otimes (\mathcal{N}=4)_{SYM} = (\mathcal{N}=8)_{sugra}$$

Where does it show up?

- Essential in computing scattering amplitudes in (super)gravity theories, particularly N=8 in 4 dimensions, up to 5 loops now[Bern,Carrasco,Chen, Johansson, Roiban], using color-kinematiks replacement rule. A naked singularity solution (JNW) has been constructed perturbatively based on this replacement rule [Luna, Monteiro, Nicholson, Ochirov, O'Connell, Westerberg, White 16]
- Ultimate goal: Is (all of) supergravity just a double copy?
    - Can we achieve gauge covariance? (BCJ requires a particular gauge, can we make the double copy gauge-independent)[Anastasiou,Borsten,Duff,Hughes,SN,Zoccali 14] Can we reproduce global symmetries of supergravity? [Anastasiou,Borsten,Duff,Hughes,Marrani,SN 13-17]
    - Can we construct (all) supergravity solutions from the double copy?[Cardoso,SN,Nampuri 16] Then generate new solutions in (modified) gravity, e.g. AdS?

Table of Contents

Table of Contents

Local symmetries

- We tensor left and right SYM multiplets with arbitrary non-Abelian gauge groups G_L and G_R
- Naively:

$$h_{\mu\nu} = A^{i}_{(\mu} \otimes \tilde{A}^{j'}_{\nu)}$$

- This can't reproduce the symmetries correctly!

Local symmetries

At linear level, want to reproduce

$$\text{graviton:} \quad \delta h_{\mu\nu} = \partial_{(\mu}\xi_{\nu)}$$
$$\text{gravitino:} \quad \delta\psi_\mu = \partial_\mu\eta$$
$$\text{gauge field:} \quad \delta V_\mu = \partial_\mu\Lambda$$

from the (linearised) YM gauge field

$$\delta A^i_\mu = \partial_\mu\alpha^i + f^i{}_{jk}A^j_\mu\theta^k$$

Local symmetries

- We define[Anastasiou, Borsten, Duff, Hughes, SN 2014]:

$$g_{\mu\nu}(x) = [A_\mu{}^i(L) \star \Phi_{ii'} \star \tilde{A}_\nu{}^{i'}(R)](x)$$

where $\Phi_{ii'}$ is the "spectator" bi-adjoint scalar field introduced by [Hodges 2013] and [Cachazo 2014]

- The convolution is defined as

$$[f \star g](x) = \int d^4y f(y) g(x-y).$$

and is a consequence of the momentum-space origin of squaring: product in momentum space is convolution in position space!

- Importantly, is **doesn't** obey the Leibnitz rule:

$$\partial_\mu(f \star g) = (\partial_\mu f) \star g = f \star (\partial_\mu g)$$

Component Formalism

- Field dictionary:

$$\begin{aligned} h_{\mu\nu} &= A_{(\mu}{}^{i} \star \Phi_{ii'} \star \tilde{A}_{\nu)}{}^{i'} \\ \psi_{\nu} &= \chi^{i} \star \Phi_{ii'} \star \tilde{A}_{\nu}{}^{i'} \\ V_{\nu} &= D^{i} \star \Phi_{ii'} \star \tilde{A}_{\nu}{}^{i'}, \end{aligned}$$

- Parameter dictionary:

$$\xi_{\mu} = A_{\mu}{}^{i} \star \Phi_{ii'} \star \tilde{\alpha}^{i'} + \alpha^{i} \star \Phi_{ii'} \star \tilde{A}^{i'}_{\mu}$$

$$\eta = \chi^{i} \star \Phi_{ii'} \star \tilde{\alpha}^{i'}$$

$$\Lambda = D^{i} \star \Phi_{ii'} \star \tilde{\alpha}^{i'}$$

Degrees of freedom and spectator scalar

Off-shell, degrees of freedom are number of independent components - gauge freedom. Look at

$$Z_{\mu\nu} = h_{\mu\nu} + B_{\mu\nu} = A_{\mu}{}^{i} \star \Phi_{ii'} \star \tilde{A}_{\nu}{}^{i'}$$

We want $Z_{\mu\nu}$ to have 16 independent components (before we start substracting gauge freedom). If $G_L = G_R = U(1)$:

$$Z_{\mu\nu} = \begin{pmatrix} A_0 \star \tilde{A}_0 & A_0 \star \tilde{A}_1 & A_0 \star \tilde{A}_2 & A_0 \star \tilde{A}_3 \\ A_1 \star \tilde{A}_0 & A_1 \star \tilde{A}_1 & A_1 \star \tilde{A}_2 & A_1 \star \tilde{A}_3 \\ A_2 \star \tilde{A}_0 & A_2 \star \tilde{A}_1 & A_2 \star \tilde{A}_2 & A_2 \star \tilde{A}_3 \\ A_3 \star \tilde{A}_0 & A_3 \star \tilde{A}_1 & A_3 \star \tilde{A}_2 & A_3 \star \tilde{A}_3 \end{pmatrix}$$

Degrees of freedom and spectator scalar

Off-shell, degrees of freedom are number of independent components - gauge freedom. Look at

$$Z_{\mu\nu} = h_{\mu\nu} + B_{\mu\nu} = A_\mu{}^i \star \Phi_{ii'} \star \tilde{A}_\nu{}^{i'}$$

We want $Z_{\mu\nu}$ to have 16 independent components (before we start substracting gauge freedom). If $G_L = G_R = U(1)$:

$$Z_{\mu\nu}(p) = \begin{pmatrix} A_0\tilde{A}_0 & A_0\tilde{A}_1 & A_0\tilde{A}_2 & A_0\tilde{A}_3 \\ A_1\tilde{A}_0 & A_1\tilde{A}_1 & A_1\tilde{A}_2 & A_1\tilde{A}_3 \\ A_2\tilde{A}_0 & A_2\tilde{A}_1 & A_2\tilde{A}_2 & A_2\tilde{A}_3 \\ A_3\tilde{A}_0 & A_3\tilde{A}_1 & A_3\tilde{A}_2 & A_3\tilde{A}_3 \end{pmatrix}$$

Degrees of freedom and spectator scalar

Off-shell, degrees of freedom are number of independent components - gauge freedom. Look at

$$Z_{\mu\nu} = h_{\mu\nu} + B_{\mu\nu} = {A_\mu}^i \star \Phi_{ii'} \star {\tilde{A}_\nu}^{i'}$$

We want $Z_{\mu\nu}$ to have 16 independent components (before we start substracting gauge freedom). If G_L and G_R are non-abelian:

$$Z_{\mu\nu} = \begin{pmatrix} A_0^i \star \phi_{ii'} \star \tilde{A}_0^{i'} & A_0^i \star \phi_{ii'} \star \tilde{A}_1^{i'} & A_0^i \star \phi_{ii'} \star \tilde{A}_2^{i'} & A_0^i \star \phi_{ii'} \star \tilde{A}_3^{i'} \\ A_1^i \star \phi_{ii'} \star \tilde{A}_0^{i'} & A_1^i \star \phi_{ii'} \star \tilde{A}_1^{i'} & A_1^i \star \phi_{ii'} \star \tilde{A}_2^{i'} & A_1^i \star \phi_{ii'} \star \tilde{A}_3^{i'} \\ A_2^i \star \phi_{ii'} \star \tilde{A}_0^{i'} & A_2^i \star \phi_{ii'} \star \tilde{A}_1^{i'} & A_2^i \star \phi_{ii'} \star \tilde{A}_2^{i'} & A_2^i \star \phi_{ii'} \star \tilde{A}_3^{i'} \\ A_3^i \star \phi_{ii'} \star \tilde{A}_0^{i'} & A_3^i \star \phi_{ii'} \star \tilde{A}_1^{i'} & A_3^i \star \phi_{ii'} \star \tilde{A}_2^{i'} & A_3^i \star \phi_{ii'} \star \tilde{A}_3^{i'} \end{pmatrix}$$

Table of Contents

Unsourced Equations of Motion

Fields decouple at linear level - v simple!

- SYM

$$\partial_\mu F^{\mu\nu a} = \partial_\mu ({}^* F)^{\mu\nu a} = 0 \; ,$$
$$\not{\partial}\lambda_i^a = 0 \; ,$$
$$\Box\sigma^a = 0 \; ,$$

- Supergravity

$$R_{\mu\nu} = 0$$
$$\gamma^{\mu\nu\rho}\partial_\nu\psi_\rho^i = 0 \Rightarrow \gamma^\mu\psi_{\mu\nu} = 0$$
$$\ldots$$

- In the absence of sources, we can give a field strength dictionary for (super)gravity coupled to matter, where the LHS gauge theory is unconstrained, and the RHS theory is in Lorenz gauge [Cardoso,SN,Nampuri 16]

Sourced e.o.m.

Working at linear level, the quations of motion for the relevant fields are:

- SYM

$$\partial_\mu F^{\mu\nu a} = \partial_\mu ({}^*F)^{\mu\nu a} = j^{\nu a}_{(A)} ,$$
$$\not{\partial}\lambda^a_i = j^a_{(\lambda)_i} ,$$
$$\Box\sigma^a = j^a_{(\sigma)} ,$$

- Supergravity

$$R^{(lin)}_{\mu\nu} = T_{\mu\nu}$$
$$\gamma^{\mu\nu\rho}\partial_\nu\psi^i_\rho = j^\mu_{(\psi)}$$
$$\ldots$$

$\mathcal{N} = 2$ supergravity solutions

- Admit BPS black hole solutions.
- Scalars are solutions to attractor equations.
- We linearize:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$
$$X^I = \langle X^I \rangle + \delta X^I$$

Start with a simple example: tensor on-shell $\mathcal{N}=2$ SYM with a gauge field. At the level of helicity states, we get the content of the $\mathcal{N}=2$ supergravity multiplet coupled to a vector multiplet:

	$\tilde{A}^-$	$\tilde{A}^+$
A^-	g^{--}	φ_0
λ_i^-	ψ_i^-	χ_i^+
σ^0, σ^1	$A_{0,1}^-$	$A_{0,1}^+$
λ_i^+	χ_i^-	ψ_i^+
A^+	φ_1	g^{++}

The ansatz

- Make an ansatz for the graviton:

$$h_{\mu\nu} = A_\mu \star \tilde{A}_\nu + A_\nu \star \tilde{A}_\mu - \eta_{\mu\nu} \frac{1}{\Box} j_{(A)\rho} \star \tilde{j}^\rho_{(A)}$$

- This gives the correct (linearised) gauge transformation $\delta h_{\mu\nu} = \partial_{(\mu}\xi_{\nu)}$ with parameter dictionary:

$$\xi_\mu = A_\mu \star \tilde{\alpha} + \alpha \star \tilde{A}_\mu$$

- AND it solves the Einstein equation, with source dictionary

$$T_{\mu\nu} = -j_{(\mu} \star \tilde{j}_{\nu)}$$

- Rest can be obtained by SUSY transformations!

Full Dictionary

Reproduces both (linearised) local transformations and e.o.m. :

$$h_{\mu\nu} = A_\mu \star \tilde{A}_\nu + A_\nu \star \tilde{A}_\mu - \eta_{\mu\nu} \frac{1}{\Box} j_{(A)\rho} \star \tilde{j}^{\rho}_{(A)}$$

$\updownarrow$ *susy*

$$\psi^i_\mu = \varepsilon^{ij} \lambda_j * \tilde{A}_\mu - \frac{1}{2} \frac{1}{\Box} \varepsilon^{ij} \gamma_\mu \gamma_\rho \gamma^\sigma \partial_\sigma j_{(1/2)j} * \tilde{j}^\rho$$

$\updownarrow$

$$\mathcal{T}^-_{\mu\nu} = -4\sigma \star \tilde{F}^-_{\mu\nu}$$

$\updownarrow$

...

Source dictionary

We also get a dictionary relating the gravitational sources to the YM ones:

$$T_{\mu\nu} = -j_{(\mu} \star \tilde{j}_{\nu)}$$
$$J^{\mu}_{(3/2)} = -2j_{(1/2)} \star \tilde{j}^{\mu}$$
$$\dots$$

Schematics

Field dictionary:

$$\Psi_{grav} = \Psi_{YM} \star \tilde{\Psi}_{YM} + \frac{1}{\Box} j_{YM} \star \tilde{j}_{YM}$$

Source dictionary:

$$j_{grav} = j_{YM} \star \tilde{j}_{YM}$$

Reproduces equations of motion and local transformations.

Without SUSY

Just get graviton and two scalars (one of which can be dualised into a two-form):

$$h_{\mu\nu} = A_\mu \star \tilde{A}_\nu + A_\nu \star \tilde{A}_\mu - \eta_{\mu\nu} \tfrac{1}{\Box} j_{(A)\rho} \star \tilde{j}^{\rho}_{(A)}$$

$$\partial_\mu \bar{X} = b \tfrac{1}{2} \left[F^{-}_{\mu\rho} \star \tilde{A}^{\rho} - \frac{1}{\Box} j_\mu \star \partial_\rho \tilde{A}^{\rho} \right]$$

interesting solutions[Cardoso,SN,NAmpuri 16]

$\mathcal{N}=2$ gauge theory	$\mathcal{N}=0$ gauge theory	$\mathcal{N}=2$ BH solution
BPS configuration with electric fields	point source	BPS black hole with electric and magnetic charges
BPS configuration with electric and magnetic fields	point source	multi-centered BPS BH with electric and magnetic charges

Table of Contents

current/future work

- Go to higher orders pertubartively (recursive relations: each oreder in perturbation theory sources the next).
- Goal: solution generating technique (generalize to different backgrounds, gauged supergravity).
- BRST quantization: ghost/gauge fixing dictionary -¿ can it shed light on aplitudes?
- Asymptotic symmetries - boundary terms in convolution...

Thank You !