Complexity and Unification in Physical Theory

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ΔΗΜΟΚΡΙΤΕΙΟ
ΠΑΝΕΠΙΣΤΗΜΙΟ
ΘΡΑΚΗΣ

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ICNFP2017

Physical Theory from Past to Future

Mechanistic reductionism, microscopical causality
(from bottom to top)

Holistic multiscale distributed causality
(from bottom to top and from top to bottom)

Irreversibility of time, novelty and creativity

It is said that …

- **Φύσις ουδέν μάτην ποιεί. (Αριστοτέλης)**

- **Aristotle**: mathematics are abstraction from material objects
- **Aristotle**: discrimination between potential and energetic reality
- **Plato**: material objects are realization of mathematical forms
- **Einstein**: space time ,finite or infinite dimensional, participates in the physical reality while fields and particles are geometrical properties the space-time manifold
- **Bohr Born, Heinsenberg**: probabilism, as possibility or potentiality ,is prior to the observed reality (ontological probabilism according to Popper)
- **K.Wilson**: physicist try to explain nature by using only one scale but forget that nature is a multi scale dynamical production
- **Prigogine**: material forms as dissipative structures as well physical laws are novelty emergence through the non equilibrium and non reversible physical self-organization process
- **G.Nicolis**: ordering and development of long range correlations in non equilibrium systems cannot be explained by local forces.
- **P.Davies and C. Castro**: in nature acts a global ordering principle

Non - Linear Physics

Dynamical Systems

Non-Equilibrium Thermodynamics

Stochastic Processes

Tsallis Theory Non-Equilibrium Statistics

Renormalization Group Theory (RGT) Scale Invariance

Fractional Extension Of Dynamics

Tsallis Extension of Statistics

Nonextensive Statistical Mechanics

Microscopic Level → Macroscopic Level

Quantum Complexity
Quantum Phase
Transition
(Q.P.T.)

Equilibrium Phase Transition
(E.P.T.)

Non – Equilibrium
Phase Transition
(N.E.P.T.)

Gaussian Statistical Mechanics

Liouville Equation – BBGKY Hierarchy
Langevin Equations
Fokker Planck Equation
(Boltzmann – Vlasov Theory)
Normal Diffusion Theory
Statistical Entropy

→ **Partition Function Z**

↓

Thermodynamical Theory
(Equilibrium Thermodynamics
Fluctuation Theory
Central Limit Theorem)

Langevin Equation

$$dX_t = \mu(X_t, t)dt + \sqrt{2D(X_t, t)}dW_t$$

Fokker Planck Equation

$$\frac{\partial}{\partial t} f(x,t) = -\frac{\partial}{\partial x}\left[\mu(x,t) f(x,t)\right] + \frac{\partial^2}{\partial x^2}\left[D(x,t) f(x,t)\right].$$

Distributed Dynamics

q-entropy extremization (Sq=max)
Basic principle of nature

⇓

- Multifractal phase space structuring
  - **Random – intermittent timeseries - signals**
  - **q-CLT theory**
  - **q-triplet**
- Power laws, multiscale self-organization
- Long range spatial – temporal correlations
- Fractal – multifractal spatial distribution of fields – particles
  - **(Non-equilibrium stationary states, NESS)**
- Fractional Dynamics – Strange dynamics on fractals
  - **Anomalous diffusion**
  - **Fractional Langevin process – fractional random walk**
  - **Fractional Maxwell equations**
  - **FHD – FMHD**
  - **Fractional accelerations – anomalous diffusions in velocity space**
- K-energy distributions (non-equilibrium energy spectra)
- Singular functions – multifractal timeseries

What is Complexity Theory ?

Irreversibility of time, dissipative structures, Entropy production and dynamics of correlations (I.Prigogine)

Indications from statistical physics

$$\frac{\partial \hat{f}(t)}{\partial t} = \hat{L}\,\hat{f}(t)$$

BBGKY Hierarchy

$\hat{f}(t) \rightarrow$ **Probability density function**

$$\hat{f} = \{ f_0, f_1(x_1), \ldots, f_\delta(x_1, x_2, \ldots, x_\delta), \ldots \}$$

$$\hat{f} = V\,\hat{f} + C\,\hat{f} =$$ *Vaccum + Correlation components of pdf*

$$\hat{f} = \hat{U}(t)\,\hat{f}(0) = e^{\hat{L}t}\,\hat{f}(0) \;\Rightarrow\; \frac{dS^{en}}{dt} > 0$$

Entropy production Subdynamics

Mathematical Extension

- Hilbert Space
- Euclidean Space
- Gaussian Statistics
- Smooth Diff. Functions

→

- Rigged Hilbert Space
- Non-Euclidean , Fractal Space
- Non-Gaussian Levy Statistics
- Singular Fractal. Functions

Smooth ODE-PDE → Fractional Dynamics

Complex Laws of Nature – Time Irreversibility

Gregoire Nicolis

Classical science is a marvelous algorithm explaining natural phenomena in terms of building blocks of the universe and their interactions. Physical phenomena reducible to a few fundamental interactions

Complexity

- A new attitude concerning the description of nature
- Self-organization phenomena on a macroscopic scale
- Spatial patterns in on temporal rythms
- Scale orders of magnitude much larger than the range of fundamental interactions
- Order, regularization, information
- Dissipative Structures, Symmetry breaking → sudden transition from simple to complex

Indications from Quantum Theory

- Commutative operators
- Coherence
- Entaglement / Superposition

- Non Locality of quantum states
- Quantum Processes
- Quantum Vaccum

$$|\psi\rangle = \int |x\rangle\langle x|\psi\rangle dx = \int \psi(x)\,|x\rangle dx \quad , \quad |\psi\rangle = \int |p\rangle\langle p|\psi\rangle dp = \int \psi(p)\,|p\rangle dp$$

$$|\psi\rangle = \sum_i |\varepsilon_i\rangle\langle\varepsilon_i|\psi\rangle + \int |\varepsilon\rangle\langle\varepsilon|\psi\rangle d\varepsilon$$

All the physical space $\{|x\rangle\}$, the momentum space $\{|p\rangle\}$ and the energy space $\{|\varepsilon\rangle\}$ are included in the quantum state – physical objects.

$|\psi\rangle \approx$ Physical object-system

Holistic description of nature

$$\psi(\vec{r}_1,\vec{r}_2,\dots,\vec{r}_N) \neq \psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\dots,\psi_N(\vec{r}_N)$$ ⟹

Non local Quantum Entaglement

Quantum Processes as Complex Processes

Quantum continous creativity of Nature at every scale, creation and destruction

$$\hat{a}(k_i)|n_1, n_2, ..., n_i, ...\rangle = n_i^{1/2}|n_1, n_2, ..., n_i - 1, ...\rangle$$

$$\hat{a}^+(k_i)|n_1, n_2, ..., n_i, ...\rangle = (n_i + 1)^{1/2}|n_1, n_2, ..., n_i + 1, ...\rangle$$

$$|k_1, k_2, ..., k_N\rangle = \hat{a}^+(k_N) ... \hat{a}^+(k_2)\hat{a}^+(k_1)|0\rangle, \quad [\hat{a}(k_j), \hat{a}^+(k_i)] = \delta_{ij}$$

$$\psi(t) = \sum \psi_{n_1, n_2, ...}(t)|n_1, n_2, ...\rangle$$

Quantum Field Theory : states operators and observables

$$\hat{\psi}(\mathbf{x}) = (2\pi)^{-3/2} \int \hat{a}(k) e^{i\mathbf{k}\cdot\mathbf{x}} d\mathbf{k} \Longrightarrow \textit{Destruction}$$

$$\hat{\psi}^+(\mathbf{x}) = (2\pi)^{-3/2} \int \hat{a}^+(k) e^{-i\mathbf{k}\cdot\mathbf{x}} d\mathbf{k} \Longrightarrow \textit{Creation}$$

Quantum Vaccum ⟶ {states of patricles-fields-matter}

$$|0\rangle \xrightarrow{\textit{Creation}} \{ |\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N\rangle, |\mathbf{k}_1, \mathbf{k}_2, ..., \mathbf{k}_N\rangle, |n_1, n_2, ... n_i, ...\rangle \}$$ ***Complex System***

Quantum Complexity (M-Theory)

Quantum Vaccum

$|0\rangle$

Operators →

$\hat{a}^{+}, \hat{a}, \hat{\mathbf{x}}$

Quantum Objects, Quantum States

Complex System

- Supersymmetry
- Super-gravity
- Super-strings
- Gauge Fields
- Symmetry Breaking
- Renormalization

- D-dimensional Space
- Point Particles
- Strings
- p - branes
- Universe
- Multiverse

- Quantum Unification

- Time Reversibility

Quantum Time Evolution

- Space-Time Symmetries
- Internal Symmetries
- Super Symmetry

$$\hat{L}_{unif} \Leftrightarrow \hat{H}_{unif}$$

Time Evolution

$$i\frac{\partial}{\partial t}|\psi\rangle = \hat{H}|\psi\rangle \quad , \quad |\psi,t\rangle = \hat{U}(t,t_0)|\psi,t_0\rangle$$

$$\hat{U}(t,t_0) = \hat{1} - i\int_{t_0}^{t} \hat{H}\,\hat{U}(\tau,t_0)\,d\tau$$

Schroendinger-Heiseberg–Dirac Formulation

$$\langle\psi',t'|\psi,t\rangle = N\int D\varphi \exp[\frac{i}{\hbar}\int_{t}^{t'} L(\varphi,\dot{\varphi})\,d\mathbf{x}]$$

Feynman Path Integral Formulation

Reductionistic Laws of Nature

- **Basic Laws (time reversible) -- Classical Theory**

$$\delta S^{(action)} = \delta \int L\, dt = 0$$

(particle and fields gravity)

$$L = L_{particles} + L_{fields} + L_{gravity}$$

- **Probablilistics (Probability Amplitudes Quantum Theory)**

Feynman Path Integral

N-point quantum particle corrleation amplitude (expectation value)

$$\langle q, t | \hat{T}[\hat{q}(t_1), \hat{q}(t_2), \ldots, \hat{q}(t_N)] | q_0, t_0 \rangle = N \int Dq[q(t_1), q(t_2), \ldots, q(t_N)] \exp[\frac{i}{\hbar} \int_t^{t'} L\, d\mathbf{x}]$$

N-point quantum field corrleation amplitude (expectation value)

$$\langle 0 | \hat{T}[\hat{\varphi}(\mathbf{x}_1 t_1), \ldots, \hat{\varphi}(\mathbf{x}_N t_N)] | 0 \rangle = N \int D\varphi[\hat{\varphi}(\mathbf{x}_1 t_1), \ldots, \hat{\varphi}(\mathbf{x}_N t_N)] \exp[\frac{i}{\hbar} \int_t^{t'} L(\varphi, \dot{\varphi})\, d\mathbf{x}]$$

Produced-Phenomenological Laws

Fundamental level (no entropy production)

- **Microscopic Time Reversible Dynamics**

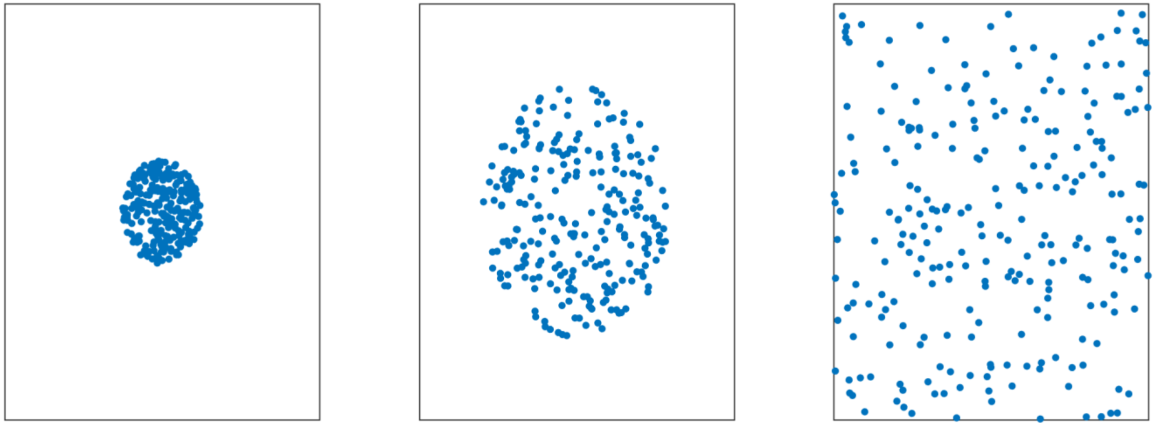
Statistical → Mechanics

Phenomenological Level Entropy Production

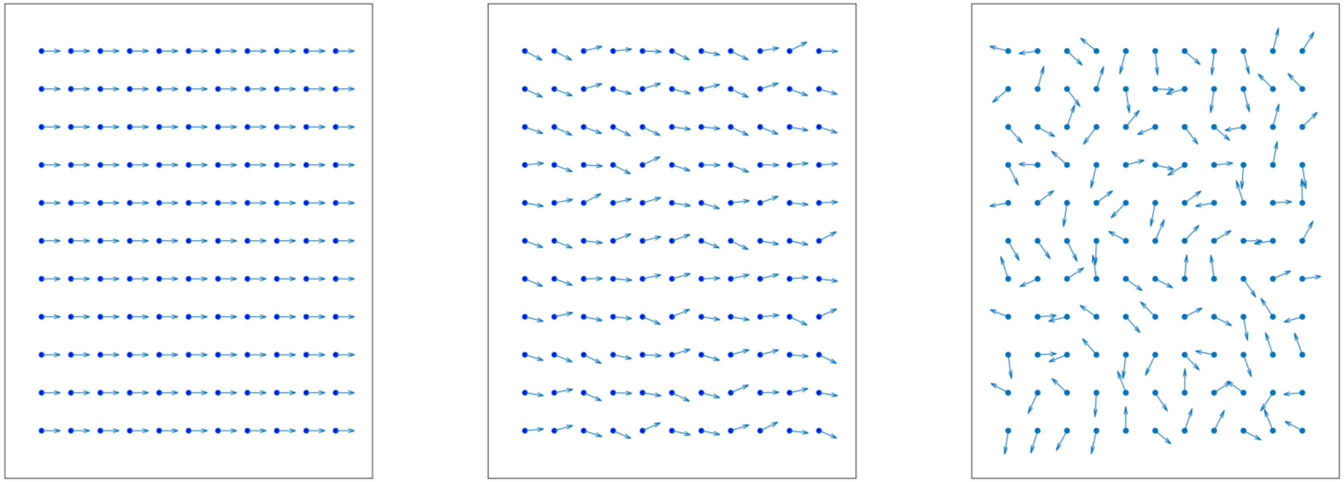
- **Macroscopic Time irreversible Laws-Phases**
- **Probabilistic or Deterministic Linear or non Linear Dynamics**
- **Diffusion-Convection Fluids or Chemical Dynamics**
- **Population Dynamics**
- **Climate Dynamics , etc.**

The Entropy Principle as the Fundamental Law of Nature

Irreversible Diffusion in physical space – Holistic processes

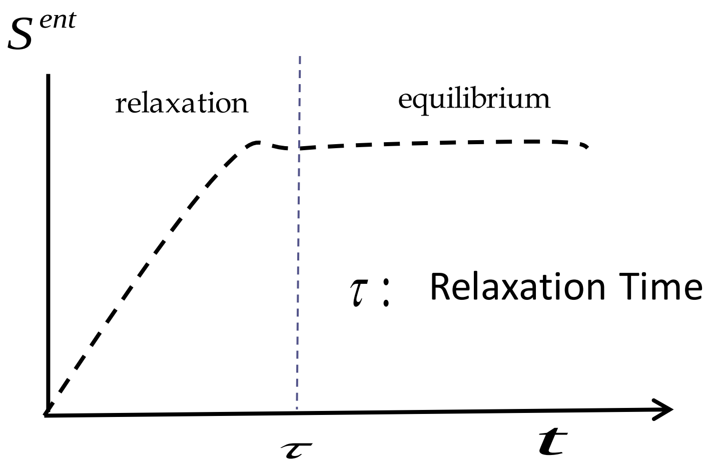


Irreversible Diffusion in velocity space – Holistic processes



The Entropy Principle as the Fundamental Law of Nature

Entropy production



Entropy Principle

$$\frac{dS^{ent}}{dt} \geq 0 \;,\; S^{ent} = -k\int f(\mathbf{x},\mathbf{v},t)\ln f(\mathbf{x},\mathbf{v},t)\,d\mathbf{x}d\mathbf{v}$$

Classical Science

- **Deterministic Fundamental Laws Time Reversible**

$$\frac{dS^{en}}{dt} = 0$$

Entropy Principle $\frac{dS^{en}}{dt} \geq 0$ **Complexity Theory** → **Deterministic *Time-Reversible* Dynamical Laws**

Fundamental Principle

Global Holistic Entropy Dynamics

→

Derived Laws, Local Interactions

Thermodynamics as Fundamental Theory of Nature

$$\frac{d\Omega}{dt} \geq 0$$

Cantor – Kelvin – Clausious – Boltzmann – Gibbs - Prigogine

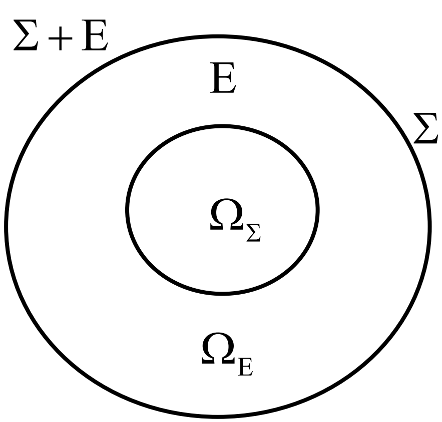
Entropy Principle through Boltzmann Complexous

$$S^{ent} = k \ln \Omega$$ **Fundamental Definitnion (Boltzmann 1872)**

$\Omega =$ **Number of microscopic states of the thermodynamic system**

Fundamental law of Nature

“ Ω increases to the maximum value for isolated systems (microcanonical ensemble) “



$\Sigma =$ Canonical Thermodynamic System

$\mathrm{E} =$ Environment

$\Sigma + \mathrm{E} =$ Isolated microcanonical system

$$\Omega_{all} = \Omega_E \, \Omega_\Sigma \rightarrow S^{ent} = S_E^{ent} + S_\Sigma^{ent}$$

$$dS^{ent} = d^r S_\Sigma^{ent} + d^i S_\Sigma^{ent}$$

$d^r S_\Sigma^{ent} =$ reversible flow of entropy

$d^i S_\Sigma^{ent} =$ irreversible production of entropy

Entropy Principle

$$\frac{dS_{all}^{ent}}{dt} \geq 0 \quad \approx \quad \frac{d\Omega_{all}}{dt} \geq 0$$

Equillibrium State

$$d^r S_\Sigma^{ent} = d^i S_\Sigma^{ent} = dS_E^{ent} = 0$$

Thermodynamic Potentials and Extremum Principle

Fundamental Laws - Entropy Law

I. $dS \geq 0$ **Isolated System (microcanonical)**

$$U = \text{int}.energy = const.$$

$$V = volume = const.$$

II. $dU \leq 0$ **when** $S, V = const.\,,\ S = entropy$

III. $dF \leq 0$ **when** $T, V = const, T = temperature$

$$F = U - TS\,,\ Helmholtz's\ \ free\,Energy$$

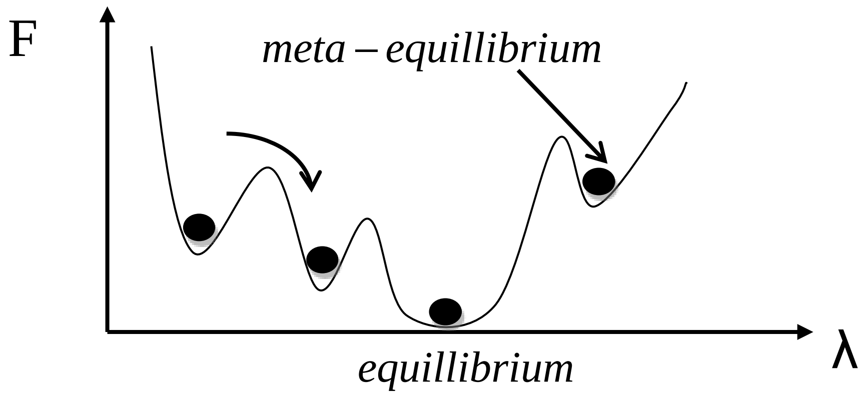
IV. $dH \leq 0$ **when** $S, P = const.\,,\ P = pressure$

$$H = U + PV = enthalpy$$

V. $dG \leq 0$ **when** $T, P = const.\,,\ P = pressure$

$$G = U - TS + PV,\ Gibb's\ \ free\,Energy$$

Equillibrium-Metaequillibrium Thermodynamic States



- **Local minimums of Free Energy**
- **Local Maximums of Entropy**

Equillibrium-Metaequillibrium Phase Transitions

Three stages of Thermodynamic Processes

First Stage : Thermodynamics of Equillibrium $S = S_{max}$

$$p_i = \frac{1}{Z} e^{-E_i/\kappa T}$$ (Bolzmann Probability Distribution Law)

$p_i =$ probablitiy of microstate (i), $E_i =$ energy of the system at microstate (i)

$Z = \sum_i e^{-E_i/\kappa T} =$ partition function of the system (canonical statistics)

Physical Meaning of partition function Z for canonical system

$$Z = \sum_i e^{-E_i/\kappa T} = \sum_\tau \Omega(E_\tau) e^{-\beta E_\tau} \ , \ E_i \le E_\tau \le E_i + \Delta E \ , \ \beta = 1/\kappa T$$

$$U = \langle E \rangle = \sum_i p_i E_i = -\partial \ln Z / \partial \beta$$ (internal energy)

$$P = \langle P_i \rangle = \sum_i p_i P_i = \beta^{-1} \partial \ln Z / \partial \beta$$ (pressure)

$$F = -\kappa T \ln Z \rightarrow Z = e^{-F/\kappa T}$$
(free energy)

$$S = \kappa \ln Z + U T^{-1} = -\kappa \sum_i p_i \ln p_i$$
(Entropy)

Three stages of Thermodynamic Processes

First Stage : Thermodynamics of Equillibrium

$$S = -\sum_i p_i \ln p_i$$ Entropy as functional of probablity

For small Fluctuations ΔE of E from mean U :

$$Z = \Omega(U)\,e^{-\beta U} \Rightarrow \begin{cases} \ln Z = \ln \Omega(U) - \beta U \\ S = \kappa \ln Z + U T^{-1} = \kappa \ln \Omega(U) \end{cases}$$

$\Omega(U)$: number of microstates for canonical statistics

Second Stage : Near Equillibrium Thermodynamics

$$S_\Sigma = S_{0,\Sigma} + \delta S_\Sigma + \delta^2 S_\Sigma$$

$S_{0,\Sigma}$ = equllibrium state , Σ = canonical system

$$\delta S_\Sigma = 0 \ , \ \delta^2 S_\Sigma \geq 0$$ Theorem of minimum Entropy production

Three stages of Thermodynamic Processes

Second Stage : Near Equillibrium Thermodynamics

Normal Fokker - Planck Equation

$$\frac{\partial}{\partial t}[\underbrace{f(x,t)}_{prob.density}] = \underbrace{-\frac{\partial}{\partial x}[A(x,t)\, f(x,t)]}_{Drift} + \underbrace{\frac{\partial^2}{\partial x^2}[D(x,t)\, f(x,t)]}_{Diffusion}$$

Ornstein – Uhlenbeck Process

$$\frac{\partial}{\partial t} f(x,t) = \frac{\partial}{\partial x}[a\; f(x,t)] + \frac{1}{2}\sigma^2 \frac{\partial^2}{\partial x^2}[f(x,t)]$$

Gaussian Evolution

$$f(x,t) = (2\pi D t)^{-1/2} e^{-(x-x_0)^2/2Dt}$$

Gaussian Stationary Solution

$$f_\infty(x) = \left(\frac{a}{\pi\sigma^2}\right)^{1/2} e^{-ax^2/2\sigma^2}$$

Boltzmann - Gibbs entropy extremization

$$S_{BG} = -\kappa \sum_i p_i \ln p_i \xrightarrow[\sum_i p_i E_i = const.]{} p_i = Z^{-1} e^{-E_i/\kappa T}$$

Three stages of Thermodynamic Processes

Third Stage : Nonequillibrium Statistical Mechanics
Boltzmann-Gibbs, Tsallis and other entropic forms

Far from equillibrium Thermodynamics

$$S_\Sigma = S_{0,\Sigma} + \delta S_\Sigma + \delta^2 S_\Sigma \qquad \delta^2 S_\Sigma < 0$$

decrease of entropy

- **Self-organization**
- **Dissipative Structures**
- **Metaequillibrium stationary states , (NESS)**

Tsallis q-Entropy

$$S_q = -\kappa \frac{1-\sum_i p_i^q}{1-q} \quad \xrightarrow{\sum_i p_i E_i = const.} \quad p_i = \frac{1}{Z_q} e_q^{-\beta_q E_i}$$

$$\beta_q = \beta / \sum_i p_i^q$$

q-Gaussian Disribution

q-Thermodynamics

$$Z_q = \sum_i e_q^{-\beta_q E_i} \;,\; S_q = \kappa \ln_q Z_q \;,\; T^{-1} = \partial S_q / \partial U_q \;,\; U_q = \sum_i p_i^q E_i / \sum_i p_i^q$$

$$U_q = -\partial \ln_q Z_q / \partial \beta \;,\; F_q = U_q - T S_q = -\beta^{-1} \ln_q Z_q$$

q=1 → Boltzmann-Gibbs Statistics

Three stages of Thermodynamic Processes

Third Stage : Nonequillibrium Statistical Mechanics – Tsallis and other entropic forms - Fractional Processes

Fractional Kinetics

- Fractional FPE – Anomalous Diffusion
- Fractional Dynamics on Fractals
- Fractional Space-Time Derivatives-Integration

$$\Rightarrow \overbrace{\left(\frac{\partial^{\alpha}}{\partial t^{\alpha}}, \frac{\partial^{\beta}}{\partial x^{\beta}}\right)}^{fractional\ \ order}$$

Fractal Time Contraction Processes

$$\frac{\partial^{\alpha}}{\partial t^{\alpha}}$$

- *Supper (persistent) diffusion 1<α<2*
- *Sub (antipersistent) diffusion 0<α<1*
- *Power law waiting time distribution*
- *Non-Markovian memory property*
- *Long range time correlations*

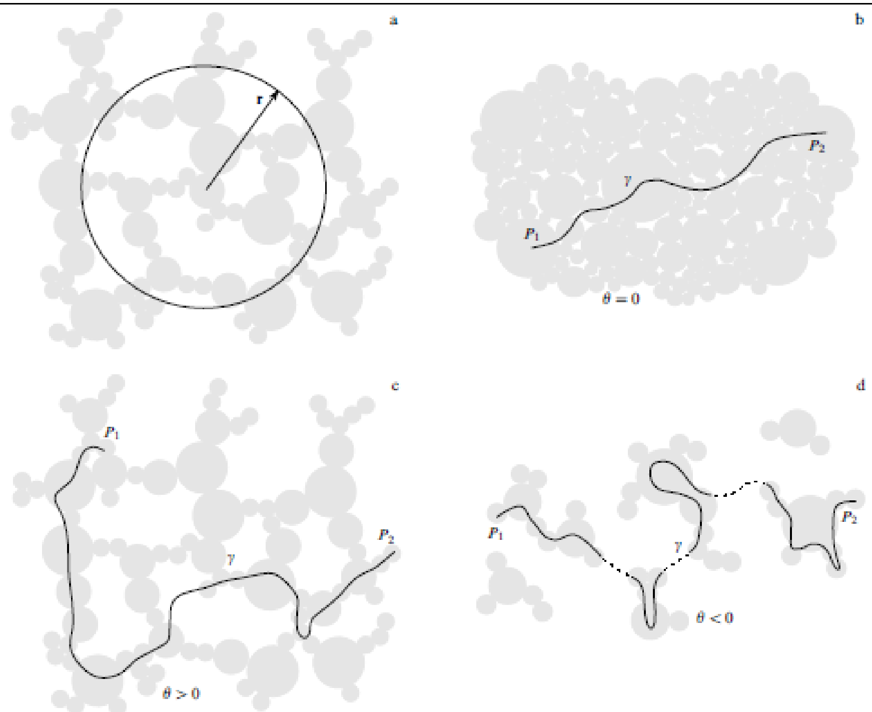
Fractal Space Contraction Processes

$$\frac{\partial^{\beta}}{\partial x^{\beta}}$$

- *Levy Flights/walks , Levy-Tsallis*
- *Levy non local processes*
- *Power law distributions*
- *Fractal Geometry (Physical/Phase)*
- *Fractal Matter-Fields distributions*

Fractal topology and strange kinetics: from percolation theory to problems in cosmic electrodynamics

L M Zelenyĭ, A V Milovanov *Physics – Uspekhi* **47** (8) 749 – 788 (2004)



Mean – square displacement
Normal – Anomalous Diffusion
Non-equilibrium Stationary States,
Percolation Critical States

$$< x^2(t) > \cong Dt^{\mu} = Dt^{2H}, 0 \leq \mathrm{H} \leq 1$$

$$\mu = \frac{\beta}{\alpha} = \frac{ds}{df} = \frac{2}{2+\theta},\ 1 < \mu \leq 2$$

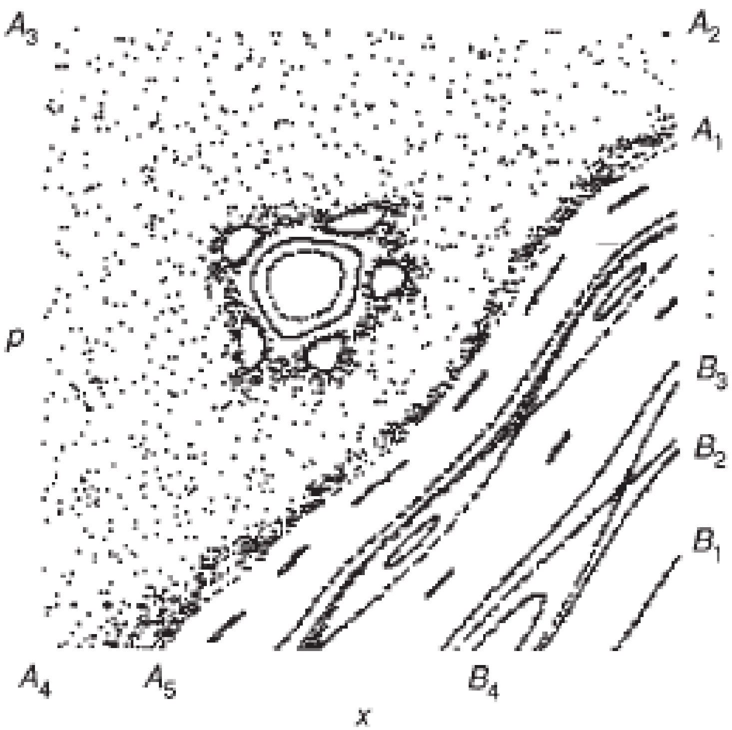
(Non Gaussian dynamics)

$$\mu \neq 1, \begin{Bmatrix} \theta > 0, \mu < 1 \leftrightarrow \text{subdiffusion} \\ \theta < 0, \mu > 1 \leftrightarrow \text{superdiffusion} \end{Bmatrix}$$

$\mu = 2H$, d_s=spectral fractal dimension
θ=connectivity index
d_f= Hausdorff fractal dimension of space
H=Hurst exponent
d_w=1/H=fractal dimension of trajectories
$\mu = 1, \mathrm{H} = 1/2, \mathrm{dw} = 2 \leftrightarrow$ Normal diffusion (Gaussian dynamics)
μ={competition between FTRW – Levy process}

Strange kinetics

Michael F. Shlesinger, George M. Zaslavsky & Joseph Klafter NATURE · VOL 363 · 6 MAY 1993



Poincare Section of Phase Space
Intermittency in Phase Space

- Stochastic Sea
- Fractal Set (Island, Cantori)
- Trapping – Stickiness – Levy flights
- Lyapunov Exponents ≥ 0
- Strange Topology – Strange Dynamics
- Scale Invariance (RGT)

Levy Distribution – Anomalous Diffusion

$$\langle R^2(t)\rangle \sim t^{\gamma} \qquad p_n(k) = \exp\left(-\text{constant} \times n|k|^{\alpha}\right)$$

$$\langle |R| \rangle \sim t^{\mu} \qquad (t \leftarrow \infty) \qquad p_n(x) \sim \text{constant} \times n/x^{1+\alpha}$$

$$\mathcal{B} = \lim_{\Delta t \to 0, \Delta x \to 0} |\Delta x|^{2\alpha}/|\Delta t|^{\beta} = \text{constant}$$

Scale Invariant Distributed Dynamics

BBGKY Hierarchy → **Boltzmann - Vlassov, MHD Theory, HD Theory**

Scale Invariance - Singularities

$$\vec{\tau}' = \lambda \vec{\tau} \qquad t' = \lambda^{1-\alpha/3} t = \lambda^{1-h_t} \qquad \vec{u}' = \lambda^{\alpha/3} \vec{u} = \lambda^{h} u \qquad \vec{b}' = \lambda^{\alpha/3} \vec{b}'$$

Fractional Extension

Fractional Real - Space Derivative

$$\frac{\partial^\beta}{\partial x_i^\beta} f(t,\vec{r}) = \frac{1}{\Gamma(1-\beta)} \frac{\partial}{\partial x_i} \int_{-\infty}^{x} \frac{dx_i'}{(x_i - x_i')^\beta} f(t,\vec{r})$$

$$\nabla_{\vec{r}}^{\beta} \equiv \frac{\partial^\beta}{\partial \vec{r}^{\beta}} \equiv \sum_i \hat{e}^i \frac{\partial^\beta}{\partial x_i^{\beta}}$$

$$\nabla_{\vec{r}}^{2\beta} \equiv \frac{\partial^{2\beta}}{\partial \vec{r}^{2\beta}} \equiv \sum_{i,k} \delta^{i,k} \frac{\partial^{2\beta}}{\partial x_i^\beta \partial x_k^\beta}$$

Riesz – Weyl operator

Levy flights – Levy walks

Power Law distribution

$$\varphi(\ell) = \frac{1}{\ell^{1+2\beta}}$$

Fractional Time Derivative

$$\frac{\partial^\alpha}{\partial t^a} f(t,\vec{r}) = \frac{1}{\Gamma(m-a)} \frac{\partial^m}{\partial t^m} \int_0^t \frac{d\theta}{(t-\theta)^{1+\alpha-m}} f(\theta,\vec{r})$$

Riemann – Liouville operator

Fractal – time random walks (FTRW)

Fractal active time (Cantor set)

$1 \le a \le r$ persistent (super-diffusive) process

$0 \le a \le 1$ anti-persistent (sub-diffusive) process

Waiting time Power Law distribution

$$\phi(r) \sim \frac{1}{r^{1+a}}$$

Strange Dynamics in Phase Space

FRACTIONAL KINETIC EQUATION
(FRACTIONAL FOKKER–PLANCK–KOLMOGOROV EQUATION)

Zaslavsky G.M., Chaos, fractional kinetics, and anomalous transport, *Physics Reports 371, 461-580*, 2002.

$$\frac{\partial^\beta P(x,t)}{\partial t^\beta} = \frac{\partial^\alpha}{\partial(-x)^\alpha}(\mathscr{A}(x)P(x,y)) + \frac{\partial^{\alpha_1}}{\partial(-x)^{\alpha_1}}(\mathscr{B}(x)P(x,y))$$

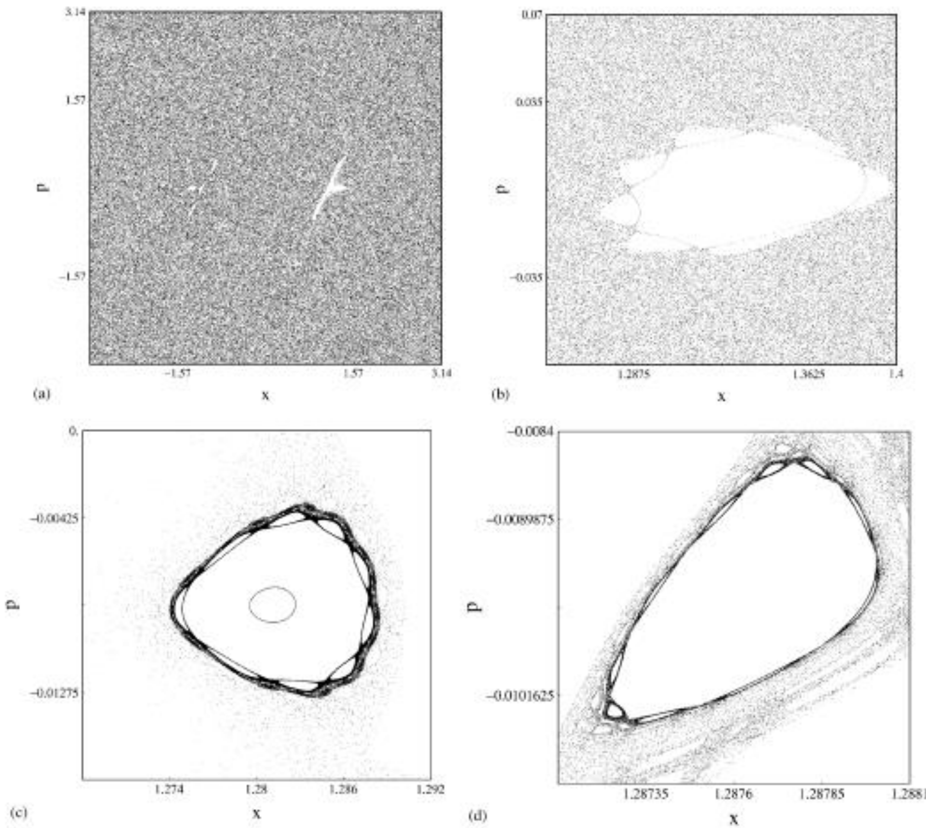
$$\mathscr{B}(x) = \frac{1}{\Gamma(2+\alpha)} \lim_{\Delta t \to 0} \frac{\langle\langle |\Delta x|^{\alpha+1} \rangle\rangle}{(\Delta t)^\beta},$$

$$\mathscr{A}(x) = \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta t \to 0} \frac{\langle\langle |\Delta x|^{\alpha} \rangle\rangle}{(\Delta t)^\beta}$$

$$\langle R^2(t) \rangle \sim t^\gamma$$

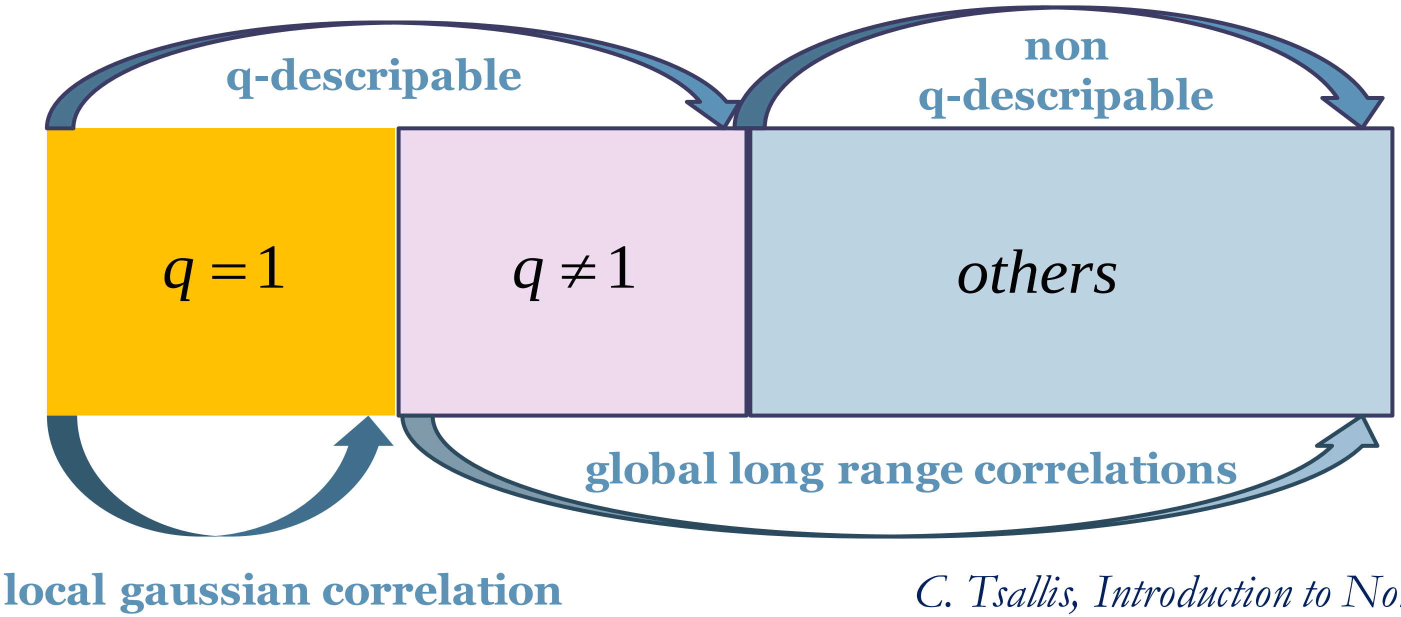
$$p_n(k) = \exp(-\text{constant} \times n|k|^\alpha)$$

$$p_n(x) \sim \text{constant} \times n / x^{1+\alpha}$$



Three stages of Thermodynamic Processes

Entropic Principles



C. Tsallis, Introduction to Nonextensive Statistical Mechanics, Springer, 2009

Other Entropic Forms

- **Renyi , Curado**
- **Arneodo – Plastino**
- **Landsberg-Vedral**
- **Abe entropy**

- **Kaniadakis κ-entropy**
- **Sharma-Mittal**
- **Beck – Cohen superstatistics**
- **Spectral Statistics**

Theoretical Concepts

Near Thermodynamic Equilibrium

Euclidean Geometry-Topology
Smooth Functions – Smooth differential Equations
Normal derivatives-integrals

HD – MHD – Vlason-Blotzman theory
Normal diffusion-Brownian motion
Gaussian statistics-dynamics
Normal Langevin-FP equations
Extensive statistics – BG entropy
Infinite dimensional noise
White-colored noise
Normal Central Limit Theorem (CLT)

<u>Normal Liouville theory</u>
Locality in space and time
Separation of time-spatial scales
Microscopic-macroscopic locality
Equilibrium RG

Far from Thermodynamic Equilibrium

Fractal Geometry -singular functions
Fractional differential-integral equations

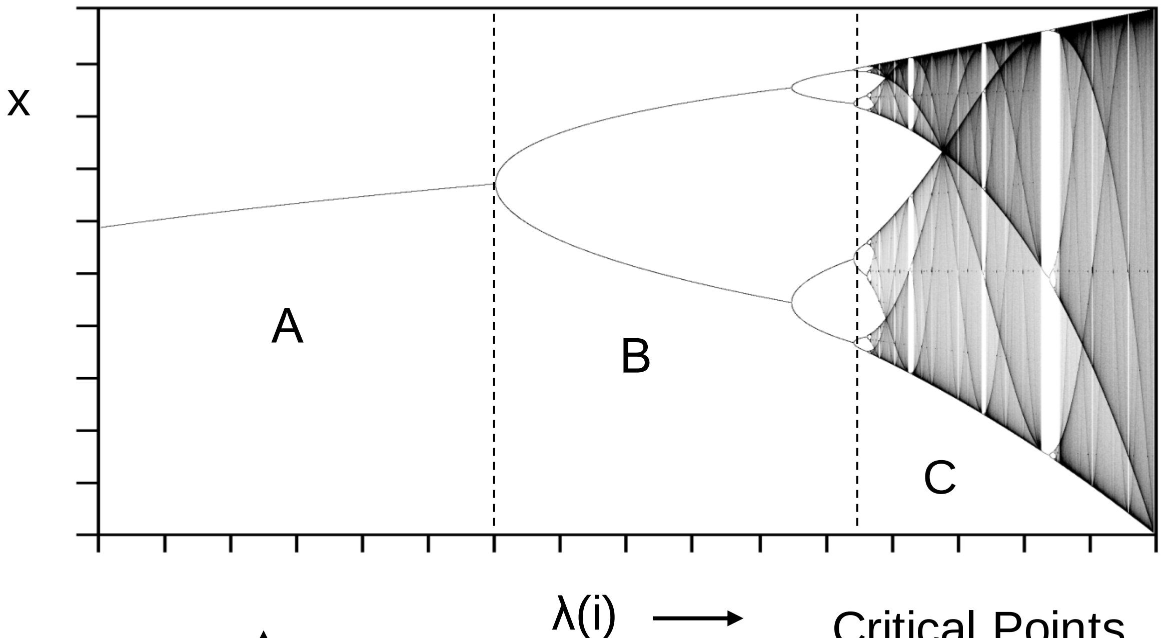
Fractional HD - MHD theory
Anomalous diffusion – motion
Strange Kinetics, Fractional Accelerations, K-distributions
Non-Gaussian statistics-dynamics
Fractional Langevin-FP equations
Non-Extensive statistics – Extremization of Tsallis entropy
q-extended CLT
Intermittent turbulence

<u>Fractional Liouville theory</u>
(multi)Fractal Topology
Power laws – mulitscale processes
Memory – long range correlations
Nonlocality in space and time
Non-Equilibrium RG

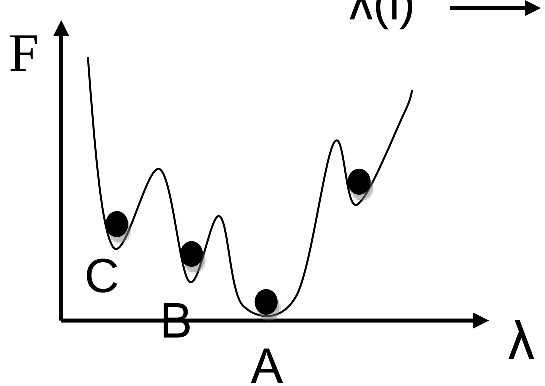
Non-Equilibriun Dynamical Phase Transitions

$$dx/dt = f(x, \lambda)$$

Non – Linear Dynamics



Critical Points
Bifurcation Points



(A)
Gaussian Equilibrium
Critical States
Equilibrium Phase
Transition
Power Laws

(B)
Self-Organization Structure
Dissipative Structures
Long Range Correlations

(C)
Spatiotemporal Chaos
Strange Dynamics (Attractors)
Levy Processes
Scale Invariance
Intermittent Turbulence
Tsallis Entropy
Fractal Topology
Anomalous Diffusion
Fractional Acceleration
K-Distributions

q-Extension of Statistics

Complexity Theory ⟷ **Entropy Principle**

Nonequilibrium self-organization

$$S_q = -k\sum_i p_i^q \ell n_q p_i = k\frac{1-\sum p_i^q}{q-1} \qquad S_q = k\ell n_q W = \max$$

$$F_q = U_q - TS_q = -\frac{1}{\beta}\ell n_q Z_q = \min \qquad Z_q = \sum e_q^{-\beta E}$$

F_q (Free energy) = minimum => Nonequilibrium stationary states (NESS)

Tsallis Non-extensive Statistical Mechanics

q-Gaussian distributions:

Energy spectra, timeseries magnitudes

$$P_{opt} = \frac{[1-(1-q)\beta E]^{1/1-q}}{Z_q} = \frac{e_q^{-\beta E}}{Z_q} \qquad k = 1/(q-1)$$

$$p(x) \propto \left[1-(1-q)\beta_{q_{stat}} x^2\right]^{1/1-q_{stat}}$$

q=1 => Boltzman – Gibbs Theory

Kappa distributions

High Energy Spectra

Especially, for energetic (nonthermal) particle populations the q-exponential probability distributions take the form of kappa distributions of two main types:

$$P^{(1)}(\varepsilon_\kappa) \sim \left[1 + \frac{1}{\kappa_*} \cdot \frac{\varepsilon_\kappa - U(T_*, \kappa_*)}{k_B T_*}\right]^{-\kappa}, \quad P^{(2)}(\varepsilon_\kappa) \sim \left[1 + \frac{1}{\kappa} \cdot \frac{\varepsilon_\kappa - U(T)}{k_B T}\right]^{-\kappa - 1}$$

Where U is the main kinetic energy $U = \langle \varepsilon_\kappa \rangle$ (Livadiotis, 2015a). According to Livadiotis and McComas (2009), the connection between kappa distributions and the entropic index q of Tsallis non-extensive statistical mechanics is given by the transformation $k = 1/(q-1)$.

Tsallis Theory
Non-extensive Statistical Mechanics

q - Calculus

$$\frac{dy}{dx} = y^q$$

$$y = [1 + (1-q)x]^{1/(1-q)} \equiv e_q^x \quad (e_1^x = e^x) \, .$$

$$y = \frac{x^{1-q} - 1}{1-q} \equiv \ln_q x \quad (x > 0; \ln_1 x = \ln x) \, ,$$

q-Product, q-Sum

$$x \otimes_q y \equiv \left[x^{1-q} + y^{1-q} - 1 \right]_+^{\frac{1}{1-q}} \quad (x \geq 0, \;\; y \geq 0), ^7$$

$$x \oplus_q y \equiv x + y + (1-q)xy$$

q-Fourier Transform (FT)

$$F_q[f](\xi) \equiv \int dx \, e_q^{i \xi x} \otimes_q f(x)$$

Tsallis Theory
Non-extensive Statistical Mechanics

q-entropy

$$S_q = k \ln_q W \quad (S_1 = S_{BG}) . \qquad S_q = k \langle \ln_q (1/p_i) \rangle . \qquad S_q = k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1} .$$

$$\frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\frac{S_q(A)}{k}\frac{S_q(B)}{k} .$$

$$p_{ij} = p_i^A p_j^B \ (\forall (i, j))$$

Independents Sub-systems

$$W = W_A W_B$$

$$S_q[A+B] = S_q[A] + S_q[B|A] + (1-q) S_q[A] S_q[B|A] ,$$

Statistically Dependent Sub-systems

where $S_q[A+B] \equiv S_q(\{p_{ij}\}), \ S_q[A] \equiv S_q(\{p_i^A\})$

conditional entropy $$S_q[B|A] \equiv \frac{\sum_{i=1}^{W_A} (p_i^A)^q S_q[B|A_i]}{\sum_{i=1}^{W_A} (p_i^A)^q} \equiv \langle S_q[B|A_i] \rangle_q$$

where $$S_q[B|A_i] \equiv \frac{1 - \sum_{j=1}^{W_B} (p_{ij}/p_i^A)^q}{q-1} \quad (i = 1, 2, \ldots, W_A)$$

Tsallis Theory
Non-extensive Statistical Mechanics

Generalized Fokker-Planck Equations

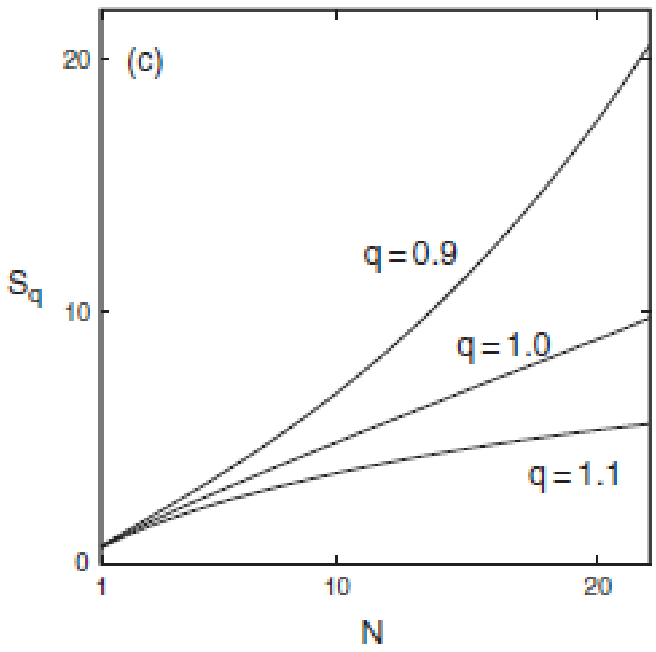
$$\frac{\partial^{\beta} p(x,t)}{\partial |t|^{\beta}} = D_{\beta,\gamma,q} \frac{\partial^{\gamma}[p(x,t)]^{2-q}}{\partial |x|^{\gamma}} \; (0 < \beta \leq 1; 0 < \gamma \leq 2) \,.$$

$$\frac{\partial p(x,t)}{\partial t} = D_{\gamma,q} \frac{\partial^{\gamma}[p(x,t)]^{2-q}}{\partial |x|^{\gamma}} \; (0 < \gamma \leq 2; q < 3).$$

$$L_{\gamma}(x) \propto \frac{1}{|x|^{1+\gamma}} \quad (|x| \to \infty; 0 < \gamma < 2) \,,$$ Levy Distribution

$$p_q(x) \propto \frac{1}{|x|^{2/(q-1}} \quad (|x| \to \infty;\; 1 < q < 3) \,.$$ q-Gaussian Distribution

$$\gamma = \begin{cases} 2 & \text{if } q \leq 5/3, \\ \frac{3-q}{q-1} & \text{if } 5/3 < q < 3, \end{cases}$$



q-extension of Thermodynamics

$$Z_q = \sum_{conf} e_q^{-\beta q(E_i - V_q)}$$

$$F_q \equiv U_q - TS_q = -\frac{1}{\beta}\ln qZ_q$$

$$\beta_q = \beta / \sum_{conf} p_i^q \qquad \beta = 1/KT$$

$$U_q = \frac{\partial}{\partial\beta}\ln qZ_q, \frac{1}{T} = \frac{\partial S_q}{\partial U_q}$$

$$\langle E \rangle_q \equiv \sum_{conf} p_i^q E_i / \sum_{conf} p_i^q = U_q$$

$$C_q \equiv T\frac{\partial \delta_q}{\partial T} = \frac{\partial U_q}{\partial T} = -T\frac{\partial^2 F_q}{\partial T^2}$$

Non-equilibrium q-points correlations

$$G_q^n(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n) \equiv \langle \varphi(\vec{x}_1)\varphi(\vec{x}_2)\ldots\varphi(\vec{x}_n)\rangle_q = \frac{1}{Z}\frac{\delta}{\delta J(\vec{x}_1)}\cdots\frac{\delta}{\delta J(\vec{x}_n)} Z_q(J)$$

q-extension of Central Limit Theorem (q-CLT)

$$F_q[X+Y](\xi) = F_q[X](\xi) \otimes_{\frac{1+q}{3-q}} F_q[Y](\xi)\,, \qquad F_q[f](\xi) \equiv \int dx\, e_q^{i\,\xi\,x} \otimes_q f(x)\,.$$

where $h(x, y)$ is the joint distribution. Therefore, q*-independence* means *independence* for $q = 1$ (i.e., $h(x, y) = f_X(x) f_Y(y)$), and it means *strong correlation* (of a certain class) for $q \neq 1$ (i.e., $h(x, y) \neq f_X(x) f_Y(y)$).

$$q\text{-parameters} \quad \{\ldots, q_{-2}, q_{-1}, q_0, q_1, q_2, \ldots\}$$

$$q_k = \frac{2q + k(1-q)}{2 + k(1-q)} \quad k = 0, \pm 1, \pm 2, \ldots$$

$$(P_{att}, P_{cor}, P_{scl}) \equiv (q_{k-1}, q_k, q_{k+1}) \equiv (q_{stat}, q_{rel}, q_{sen})$$

Theory of q-triplet of Tsallis

The essence of the q-CLT (q-extension of CLT) concerns the possibility of q-generalization of the standard CLT by allowing the random variables that are being summed to be correlated through an attractor as follows. For a sequence of $x_1, x_2, ..., x_N$ of q_k $(k \in Z)$ independent and identically distributed random variables with a finite q-mean and a finite second q-moment then their sum $Z_N = x_1 + x_2 + ... + x_N$ is g-convergent to the attractor of a q_{k-1} Gaussian distribution $Gq_{k-1}(\beta; x)$. The q-FT of the attractor $Gq_{k-1}(\beta; x)$ is the distribution $Gq_k(\beta_*; x)$ (q-Gaussian). Thus, the correlations are introduced through the q_k products of q_k - Fourier Transforms. The q-CLT includes three physically significant q-parameters known as the q-triplet: (P_{att}, P_{cor}, P_{scl}) identified by the relation:

$$(P_{att}, P_{cor}, P_{scl}) \equiv (q_{k-1}, q_k, q_{k+1}) \equiv (q_{stat}, q_{rel}, q_{sen}) \qquad (39)$$

The parameter $P_{att} \equiv P_{k-1} \equiv q_{stat}$, describes the statistical attractor $Gq_{k-1}(\beta; x)$ of the q-independent random variables. The $P_{cor} \equiv P_k \equiv q_{rel}$, describes the q-correlated random variables of the system and the relaxation process of fluctuations toward the attracting stationary state. The $P_{scl} \equiv P_{k+1} \equiv q_{sen}$, describes the scale invariance of the multifractal structure of the system according to the asymptotically scaling form:

$$N^D P_x(x) \sim G\left(\frac{x}{N^D}\right) \qquad (40)$$

where $P_x(x)$ is the probability function of the self-similar attractor (q-Gaussian) and D is the scaling exponent characterizing the anomalous diffusion process (Baldovin and Stella, 2007):

$$< x^2 > \sim x^{2D} = x^{\delta/2} \qquad (41)$$

where $\delta = q_{k+1} = q_{sen}$. For non-Gaussian dynamics ($q_{k+1} \neq 1$) the statistical attractor of the system dynamics creates through the q-entropy extremization multiscale correlations and multifractal structures in the phase space as well as in the physical space. From this point of view the q_{sens} parameter describes the non-ergodic q-entropy production of the multiscale correlated process as the system shifts to the state of the q-Gaussian attractor where the q-entropy is extremized in accordance with the q-generalization of the Pesin's theorem (Tsallis, 2004b). The q_{sen} parameter, characterizes the intermittent turbulence character of the spatially distributed complex and non-extensive system as in the case of the solar wind system.

Multiplicative Processes in Phase Space – Physical Space

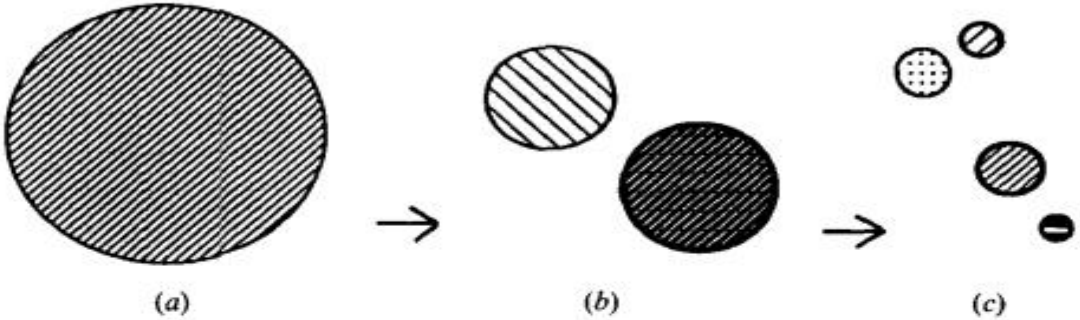


FIGURE 2. Schematic representation of an isotropic multiplicative process. A large piece, (*a*), is divided into two smaller pieces, (*b*). Both pieces or 'blobs' may have a different density of measure, as indicated by the different shading. After the next iteration of the multiplicative process, each piece of (*b*) is divided into even smaller pieces, (*c*), etc.

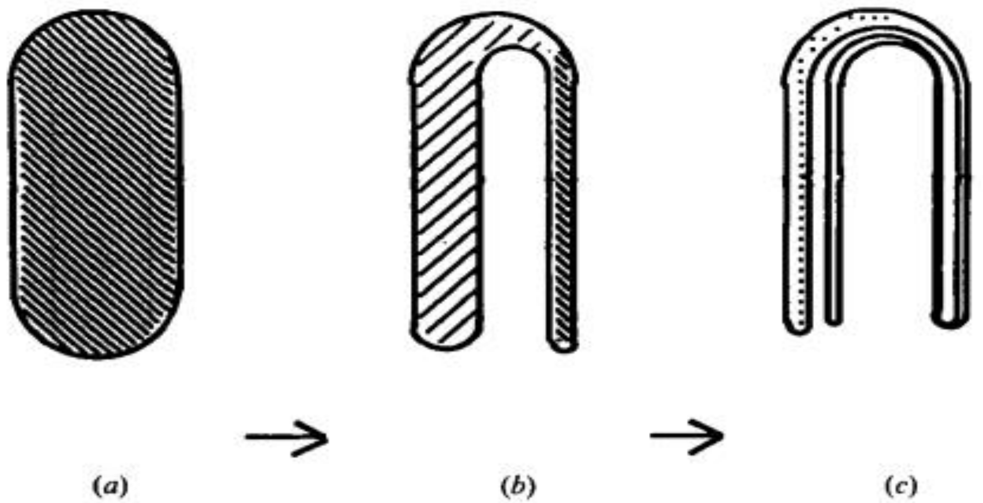


FIGURE 3. Schematic representation of three stages of a stretching and folding process. A piece (*a*), is stretched in the vertical direction, contracted unequally (thus accounting for the unequal thickness and measure) and then folded back to form the piece, (*b*). After another similar step, (*c*) is obtained.

C. Meneveau and K. R. Sreenivasan *J. Fluid Mech.* (1991), *vol.* 224, *pp.* 429–484

Multiplicative Processes (Holistic timeseries production)

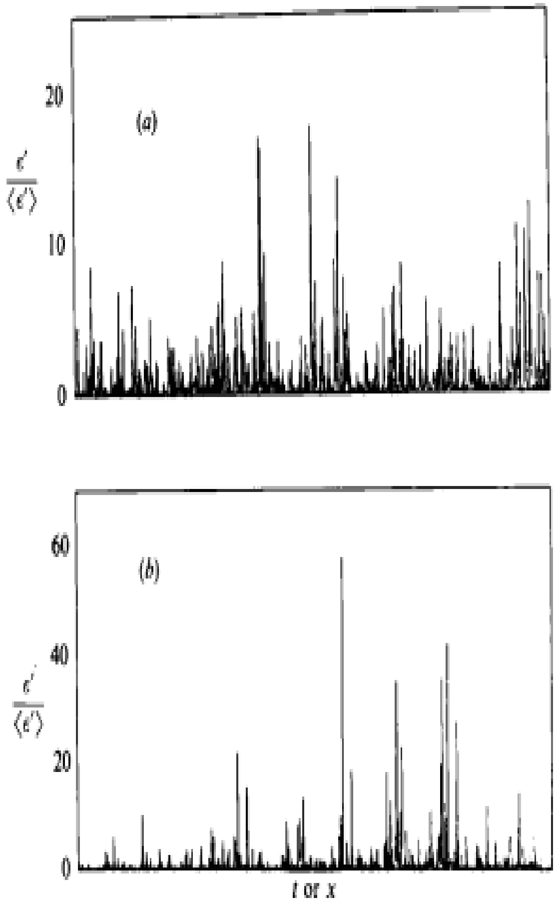


FIGURE 1. Typical signals of a representative component of ϵ, namely $\epsilon' \sim (du_1/dt)^2$ normalized by the mean: (a) was obtained in a laboratory boundary layer at a moderate Reynold number, and (b) in the atmospheric surface layer at a high Reynolds number. For a description of the experimental conditions, see §3.1 and table 1.

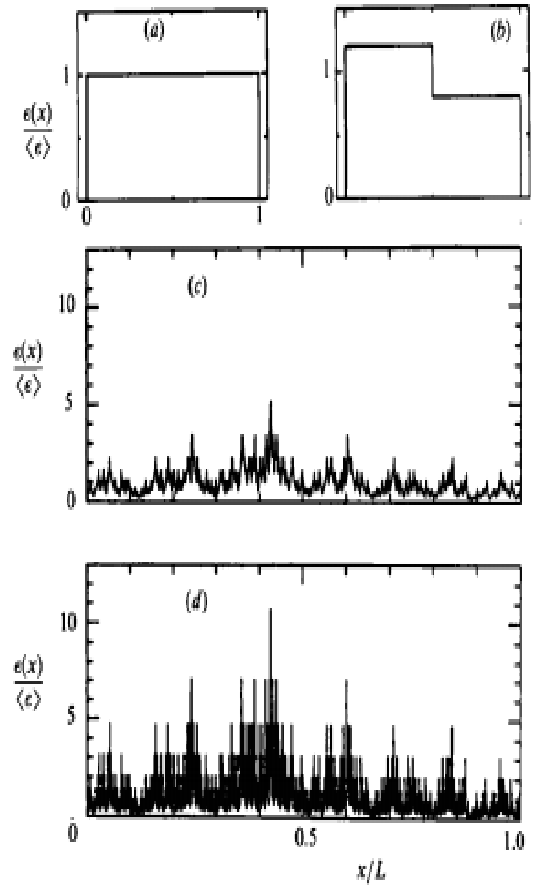


FIGURE 4. Binomial measure $\epsilon(x)/\langle\epsilon\rangle$ on the unit interval, using $M = 0.6$ or 0.4. (a) The original uniform distribution of density, and (b) after one fragmentation. The total dissipation on the two sides are 0.6 and 0.4, and the corresponding densities of $\epsilon(x)$ are 1.2 and 0.8. (c) $\epsilon(x)$ after 9 steps and (d) after 13 steps.

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Multifractal Theory

Theiler J., Vol. 7, No. 6/June 1990/J. Opt. Soc. Am. A, 1055

generalized dimensions

$$D_q = \frac{1}{q-1} \lim_{r \to 0} \frac{\log \sum_i P_i^q}{\log r}.$$

most-dence points

$$D_\infty = \lim_{r \to 0} \frac{\log\left(\max_i P_i\right)}{\log r},$$

least-dence points

$$D_{-\infty} = \lim_{r \to 0} \frac{\log\left(\min_i P_i\right)}{\log r}.$$

Information entropy

$$S(r) = -\sum_i P_i \log_2 P_i,$$

$$D_I = \lim_{r \to 0} \frac{-S(r)}{\log_2 r}$$

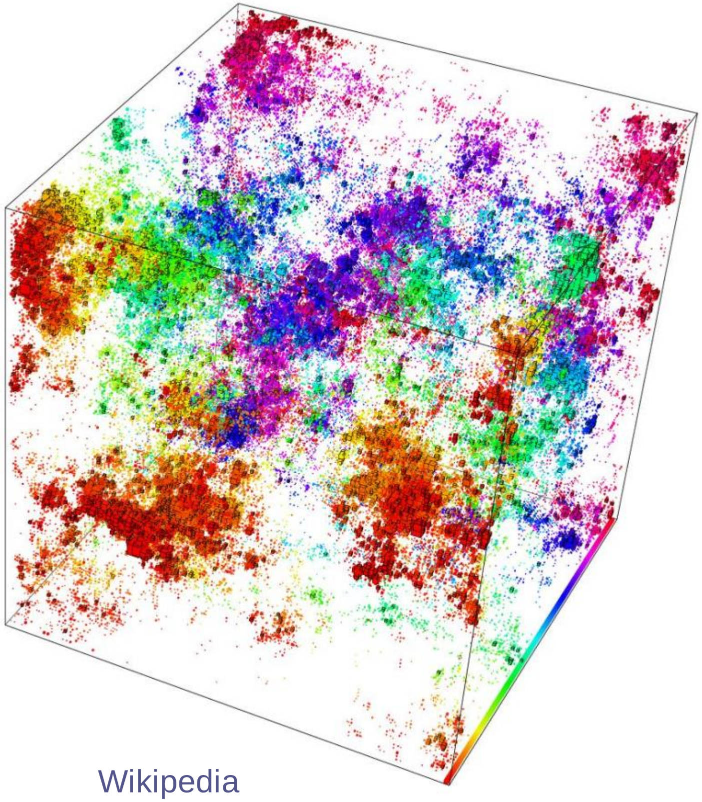
Information dimension

$$= \lim_{r \to 0} \frac{\sum_i P_i \log_2 P_i}{\log_2 r}.$$

Rényi entropy

$$S_q(r) = \frac{1}{q-1} \log \sum_i P_i^q,$$

$$D_q = \lim_{r \to 0} \frac{-S_q(r)}{\log r} = \frac{1}{q-1} \lim_{r \to 0} \frac{\log \sum_i P_i^q}{\log r},$$



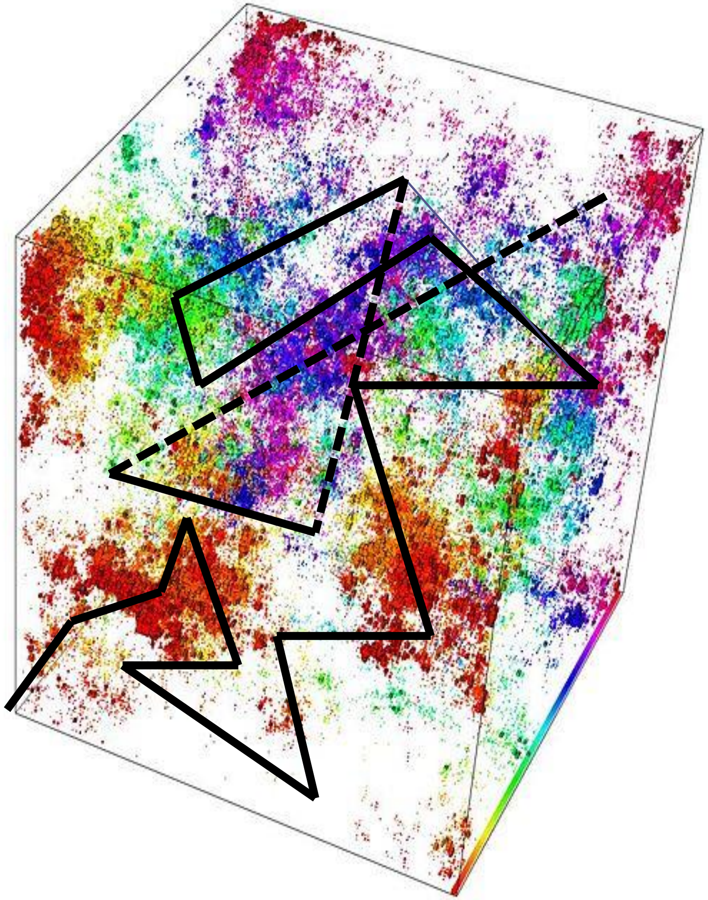
Wikipedia

Sq maximization produces multifractal phase space

$$\frac{\partial^\alpha \psi}{\partial t^\alpha} = \nabla_{\mathbf{r}}^{2\beta}(\mathcal{B}\psi)\,.$$

$$\frac{\partial^\alpha \psi}{\partial t^\alpha} = \nabla_{\mathbf{w}}^{2\beta}\psi$$

- Fractional Maxwell equations, fractional distributions
- Fractional Langevin- FP equations
- Fractional- multi scale acceleration
- K distributions, singular- multi fractal time series, q-triplet



Renormalization Group Theory (RGT)

Non-Equilibrium – Non-linear Complex Dynamics (Chang, 1992)

$$\frac{\partial \varphi_i}{\partial t} = f_i\left(\boldsymbol{\varphi}, \mathbf{x}, t\right) + n_i\left(\mathbf{x}, t\right) \qquad i = 1, 2, \ldots$$

Generalized Langevin Stochastic Equation

$$P\left(\boldsymbol{\varphi}\left(\mathbf{x}, t\right)\right) =$$

$$\int D\left(\mathbf{x}\right) \exp\left\{-i \cdot \int L\left(\dot{\boldsymbol{\varphi}}, \boldsymbol{\varphi}, \mathbf{x}\right) d\mathbf{x}\right\} dt$$

Random Field Distribution Function

Stochastic Lagrangian Dynamics

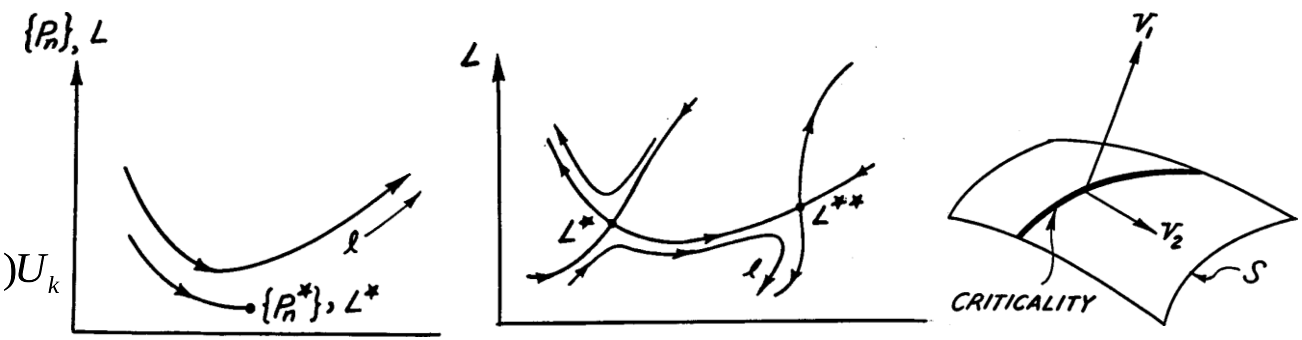
Scale Invariance - RGT - Fixed Points (SOC, Chaos, Intermittency, NESS, etc.)

$$\partial L / \partial l = RL$$

$$\partial L' / dl = R_L L' \qquad \dot{\eta}$$

$$dP'_m / dl = \Sigma (R_L)_{mn} P'_n$$

$$L'(l) = \Sigma V_k(l) U_k = \Sigma V_{k0} \exp(\lambda_k l) U_k$$



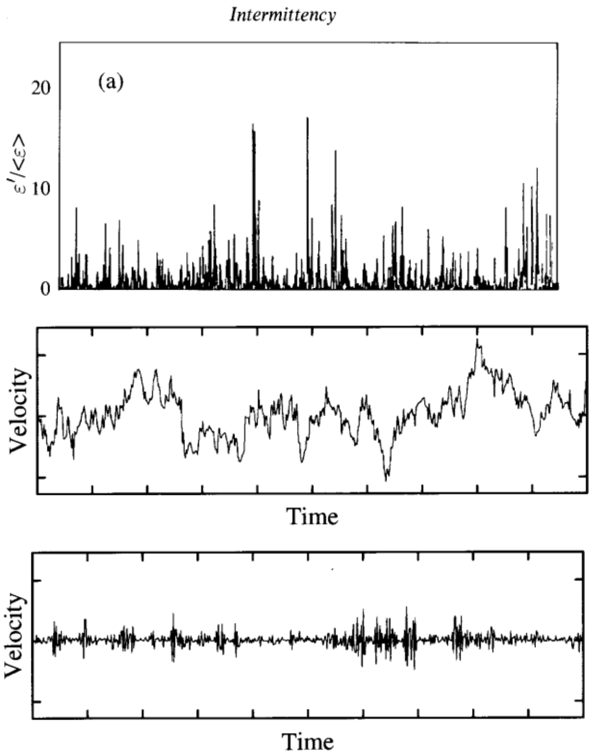
Non-equilibrium – Non-extensive Random Field Theory (Partition Function Theory)

$$G_N^q\left(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_N\right) = \frac{1}{Z} \frac{\delta^N Z_q\left(J\left(\vec{x}\right)\right)}{\delta J\left(\vec{x}_1\right) \cdot \delta J\left(\vec{x}_2\right) \cdot \cdots \cdot \delta J\left(\vec{x}_N\right)}$$

$$Z_q = \lim_{J \to 0} Z\left(J\left(\vec{x}\right)\right)$$

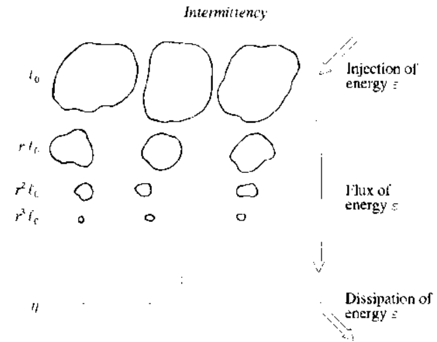
$$Z\left(J\left(\vec{x}\right)\right) = \int D\left[\Psi\right] \cdot e^{-\int F_q(J) d^D x}$$

Turbulence – Intermittent Turbulence



Uriel Frisch – Turbulence (The Legacy of A.N. Kolmogorov)

'mother – eddy' → 'daughters



β-model cascade (eddies without space filling)

$$\tau(q) = (q-1)d(q)$$

(β-model)

$$\beta \ (0 < \beta < 1).$$

$$\ell = r^n \ell_0$$

Tsallis q-entropy principle

probability density P(α)

singularity spectrum f(α)

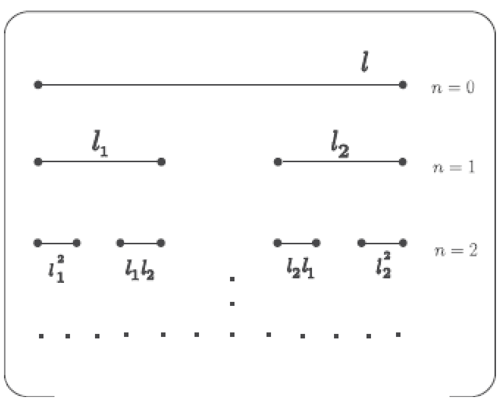
turbulence mass exponent τ(q)

$$P(a) = Z_q^{-1}[1-(1-q)\frac{(a-a_0)^2}{2X/\ln 2}]^{\frac{1}{1-q}}$$

$$f(a) = D_0 + \log_2[1-(1-q)\frac{(a-a_o)^2}{2X/\ln 2}]/(1-q)^{-1}$$

$$\tau(\bar{q}) = \bar{q}a_0 - 1 - \frac{2X\bar{q}^2}{1+\sqrt{C_{\bar{q}}}} - \frac{1}{1-q}[1-\log_2(1+\sqrt{C_{\bar{q}}})]$$

'p-model



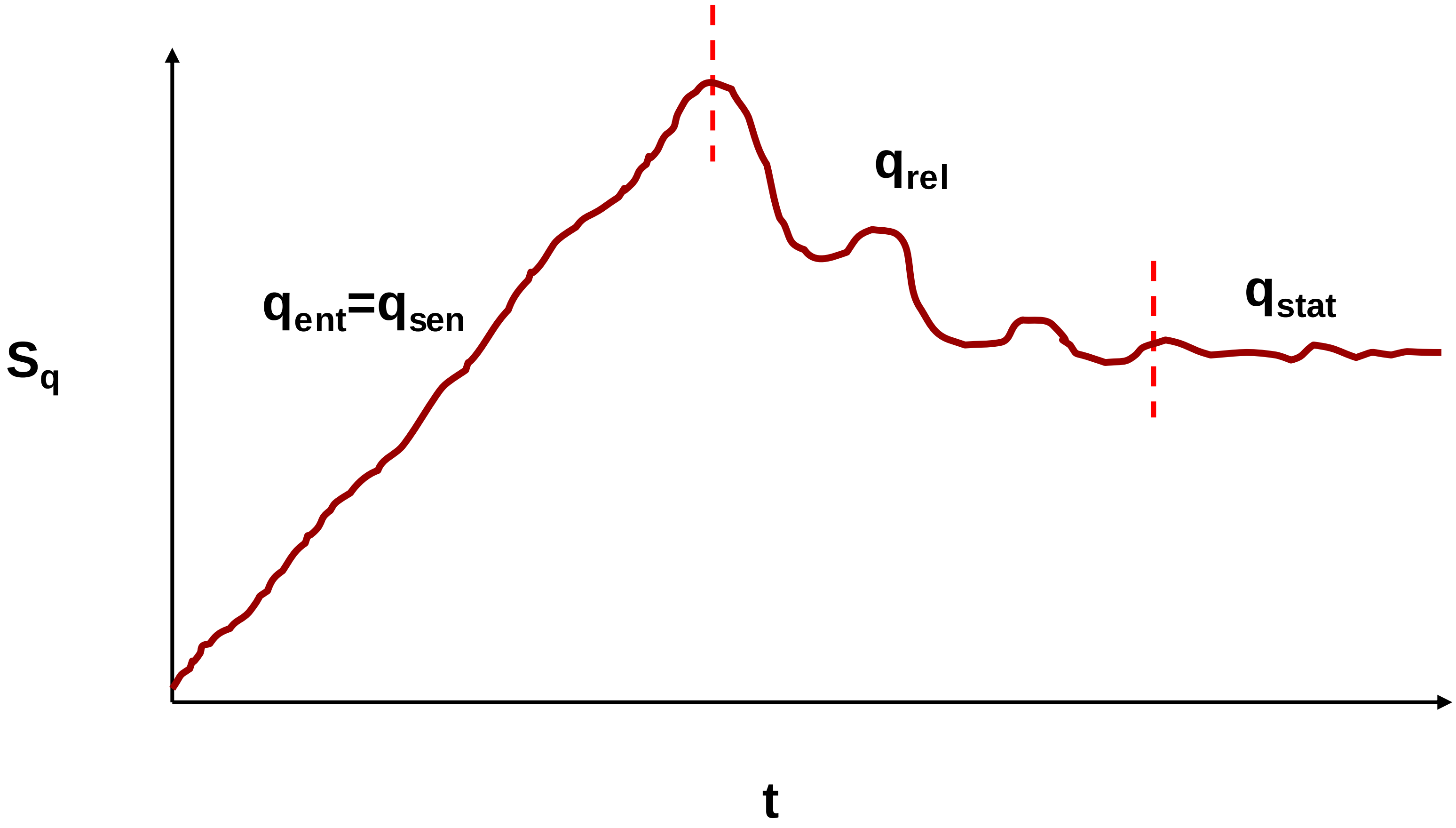
Two-scale Cantor set

$$\frac{p^q}{l_1^{\tau(q)}} + \frac{(1-p)^q}{l_2^{\tau(q)}} = 1$$

$$f(\alpha) = q\alpha(q) - \tau(q)$$

$$\tau(\bar{q}) = (\bar{q}-1)d(\bar{q})$$

q-triplet Tsallis
One-Dimensional Systems (timeseries)



The q-triplet of Tsallis

q - CLT → (q_{k-1}, q_k, q_{k+1})= qstat, qrel, qsen)

Gaussian-BG equilibrium (qstat=qsen=qrel=1) → Nonequilibrium (qstat, qsen, qrel)

$\frac{dy}{dx} = y^q$, $(y(0)=1, q \in \Re)$ → (qstat, qsen, qrel)

Equilibrium PDF → Metaequilibrium PDF

(q-stationary) $\frac{d(p_i Z_{stat})}{dE_i} = -\beta q_{stat}(p_i Z_{stat})^{q_{stat}}$ → $p(x) \propto \left[1-(1-q)\beta_{q_{stat}} x^2\right]^{1/1-q_{stat}}$

Equilibrium BG entropy production → Metaequilibrium q-entropy production

$K_q \equiv \lim_{t\to\infty} \lim_{W\to\infty} \lim_{N\to\infty} \frac{<S_q>(t)}{t}$ → $\frac{1}{1-q_{sen}} = \frac{1}{\alpha_{min}} - \frac{1}{\alpha_{max}}$ $f(a_{\min}) = f(a_{\max}) = 0$

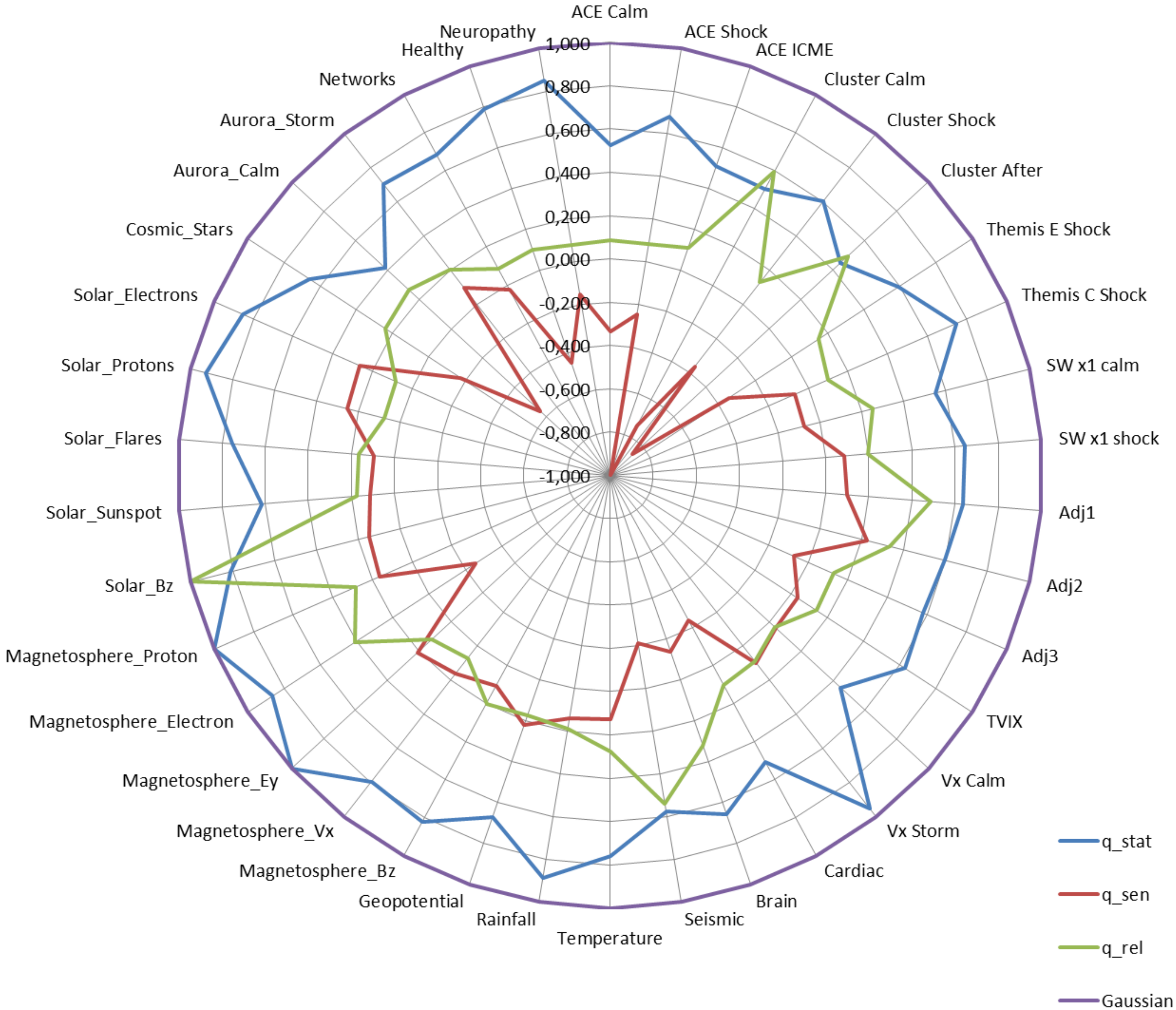
(q-sensitivity) $\frac{d\xi}{dt} = \lambda_{q_{sen}} \xi^{q_{sen}}$, → $\xi = e_{q_{sen}}^{\lambda_{q_{sen}} t}$, $\xi \equiv \lim_{\Delta x(0)\to 0} \frac{\Delta x(t)}{\Delta x(0)}$,

Equilibrium relaxation process → Metaequilibrium nonextensive relaxation process

(q-relaxation) $\Omega(t) \equiv [O(t) - O(\infty)]/[O(0) - O(\infty)]$ $\frac{d\Omega}{dt} = -\frac{1}{T_{q_{rel}}}\Omega^{q_{rel}}$ → $\Omega(t) \simeq e_{q_{rel}}^{-t/\tau_{rel}}$

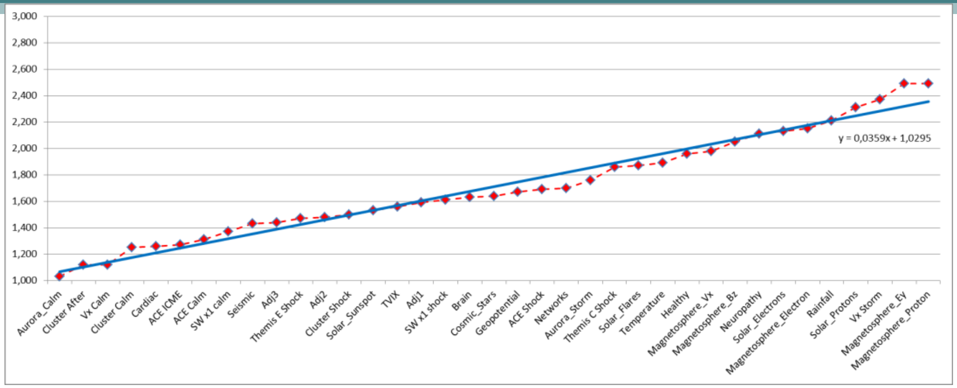
Experimental Verifications

Universality of Tsallis q triplet

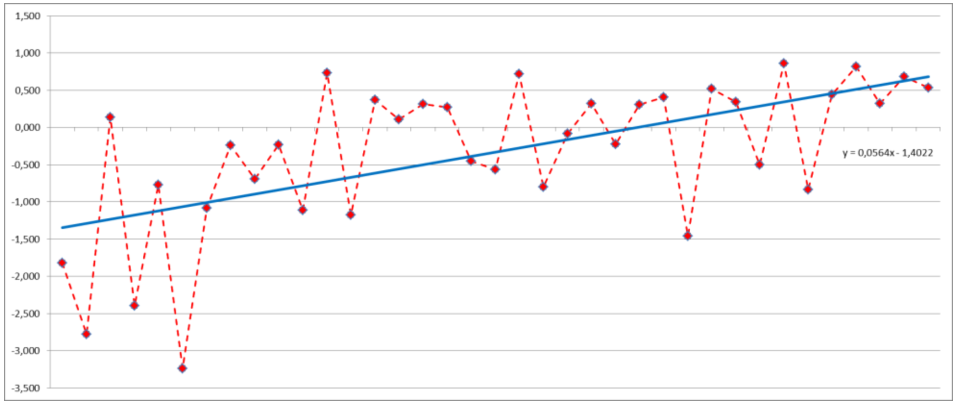


- Cosmic Stars
- Space Plasmas
- Turbulence
- Energetic Particles
- Climate
- Seismicity
- Biology
- Human Body
- Economic Systems
- Information Systems

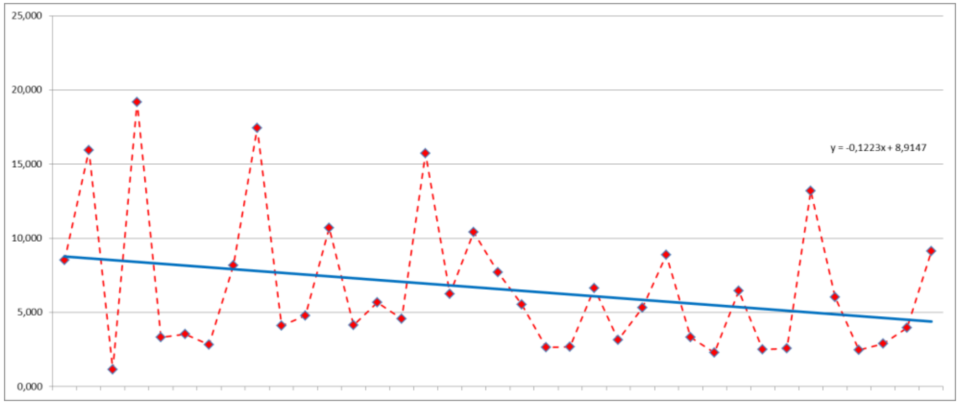
Universality of Tsallis q triplet



q_stat



q_sen

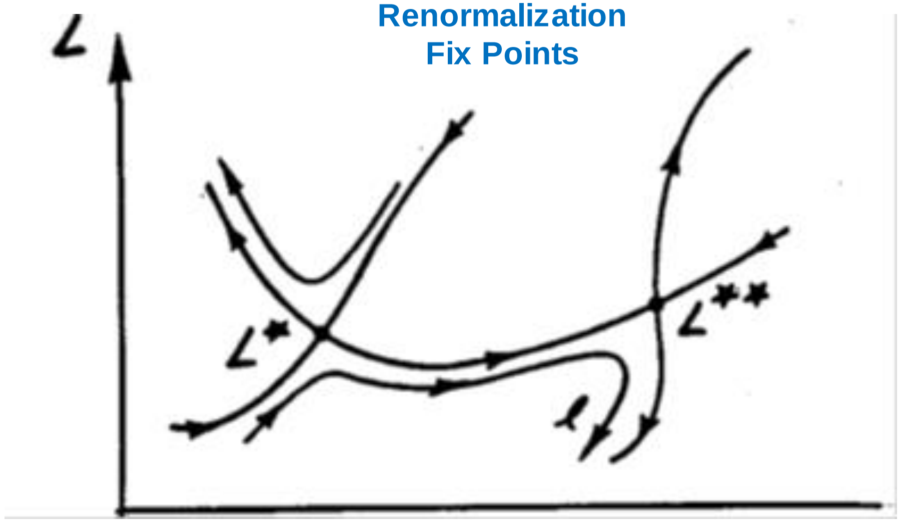


q_rel

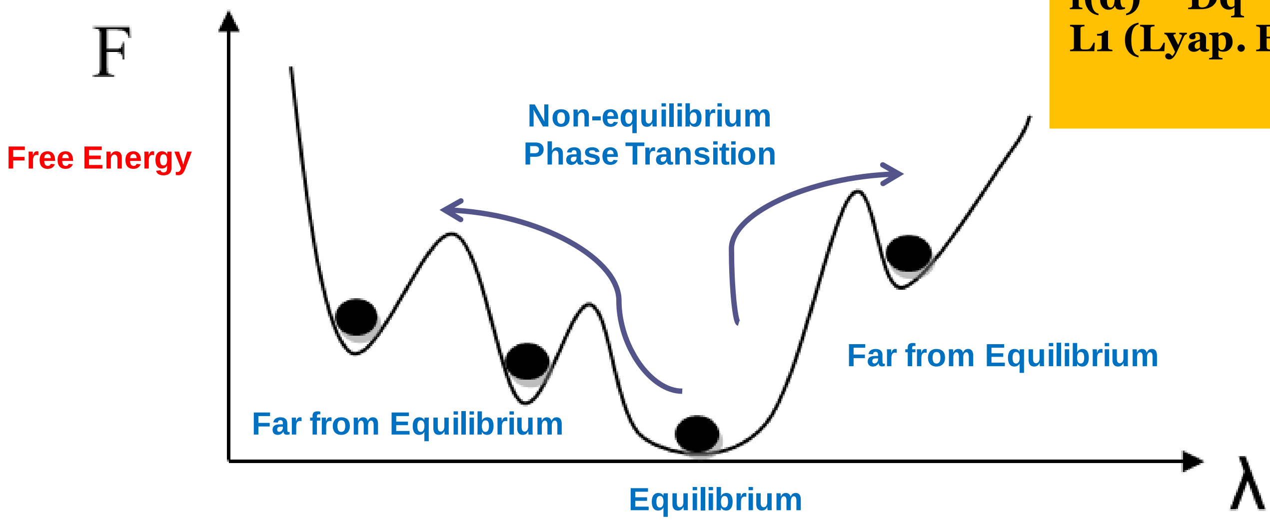
q-triplet	Cross Correlation
q_stat, q_sen	**0,590099636**
q_stat, q_rel	**-0,101510833**

Universality of Non-equilibrium Phase Transition

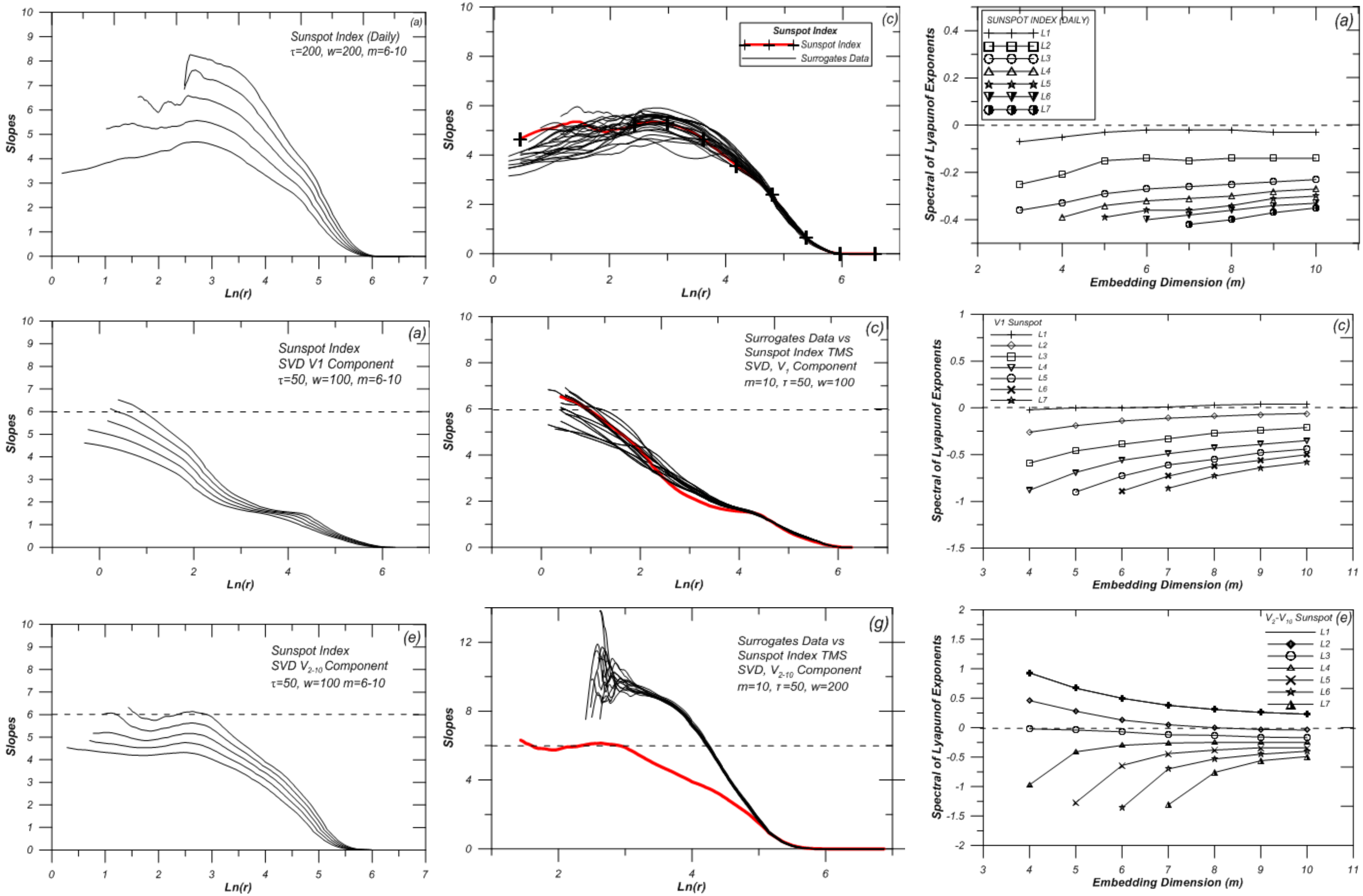
Non-equilibrium Renormalization Fix Points



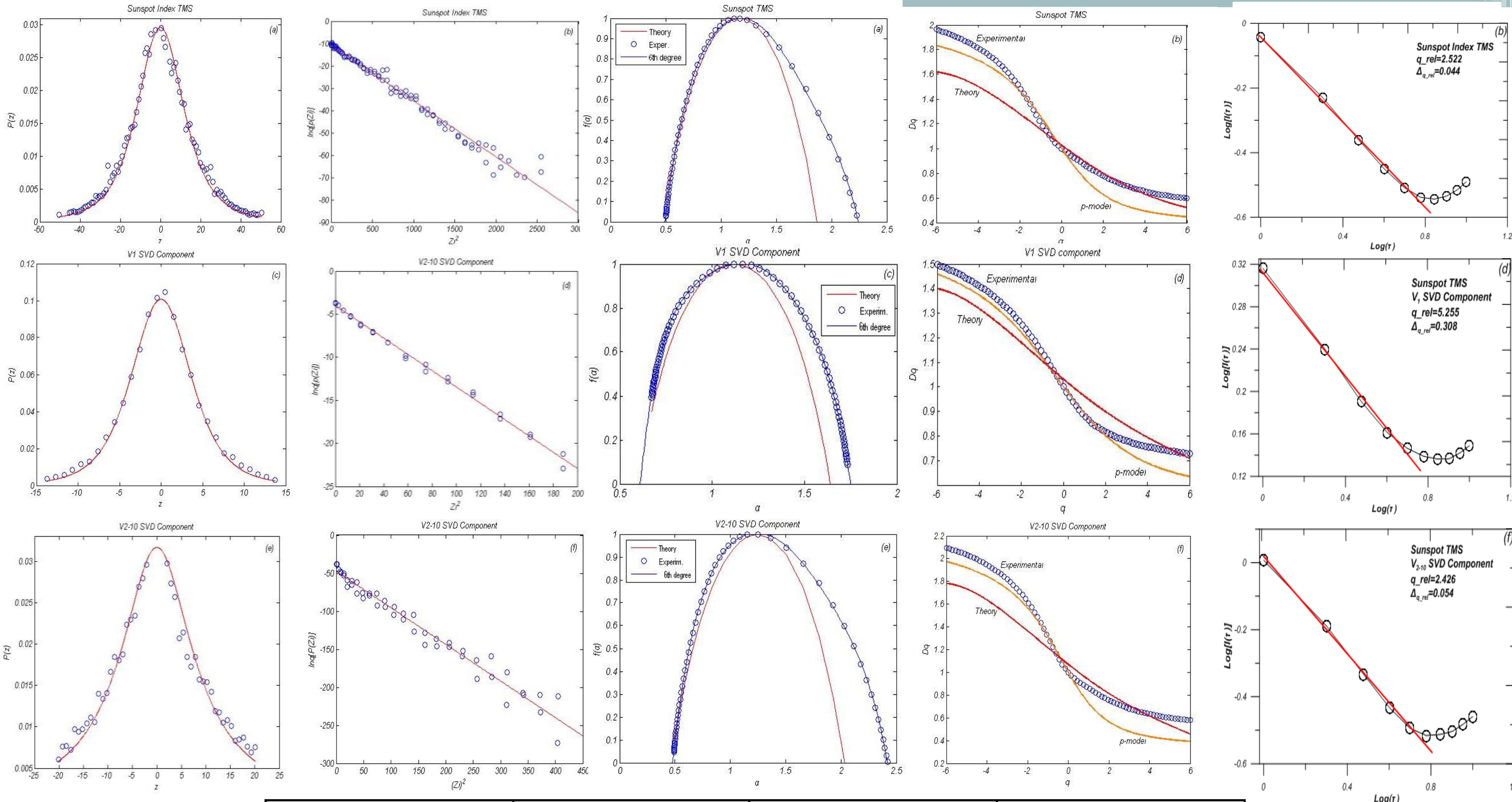
Increase	Decrease
F_{min}	Sq
q_stat	Cor. Dim. (D_2)
q_sen	q_rel
f(α) – Dq	
L_1 (Lyap. Expon)	



CHAOTIC ANALYSIS (SUNSPOT DYNAMICS)

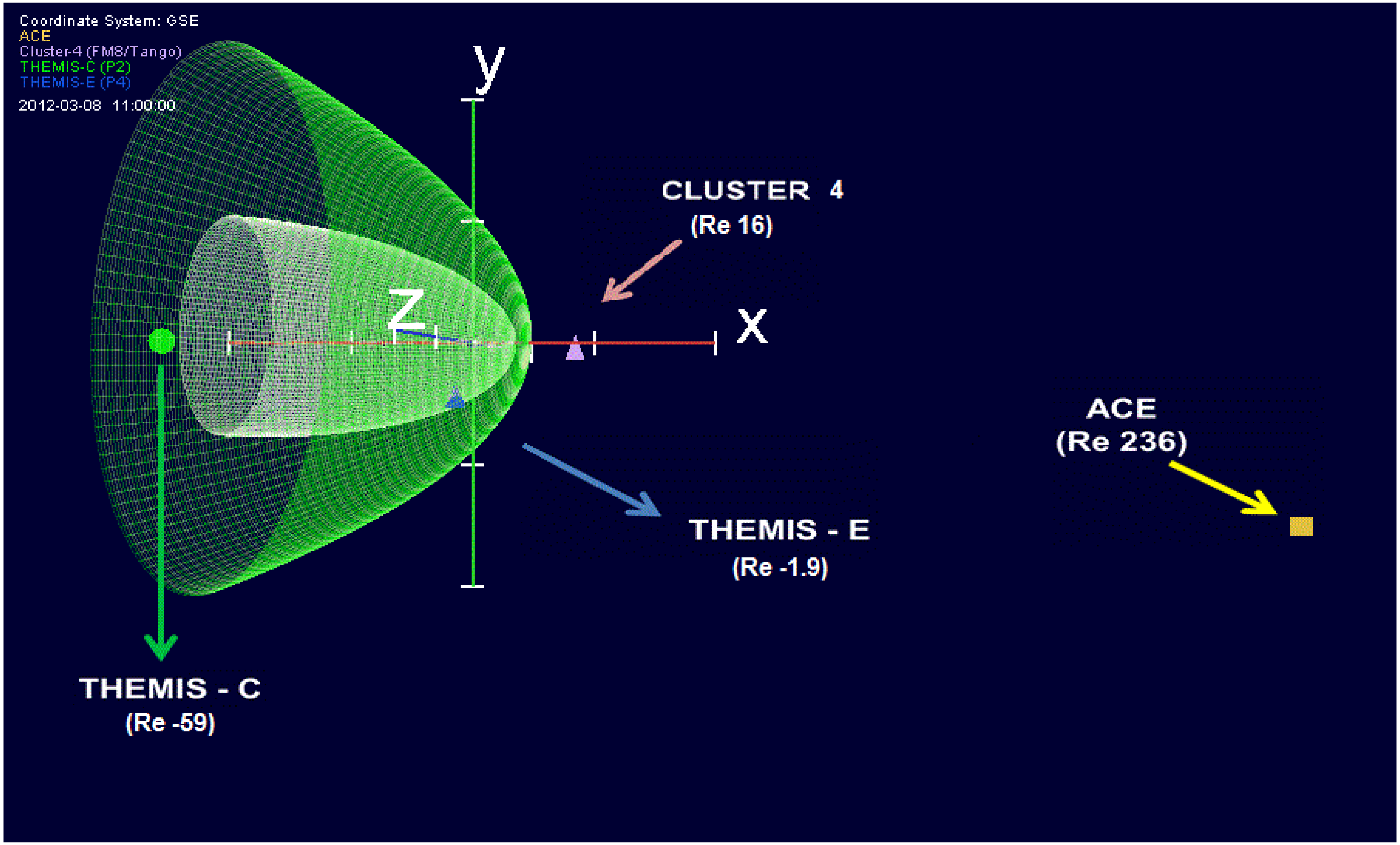


TSALLIS STATISTICS (SUNSPOT DYNAMICS)



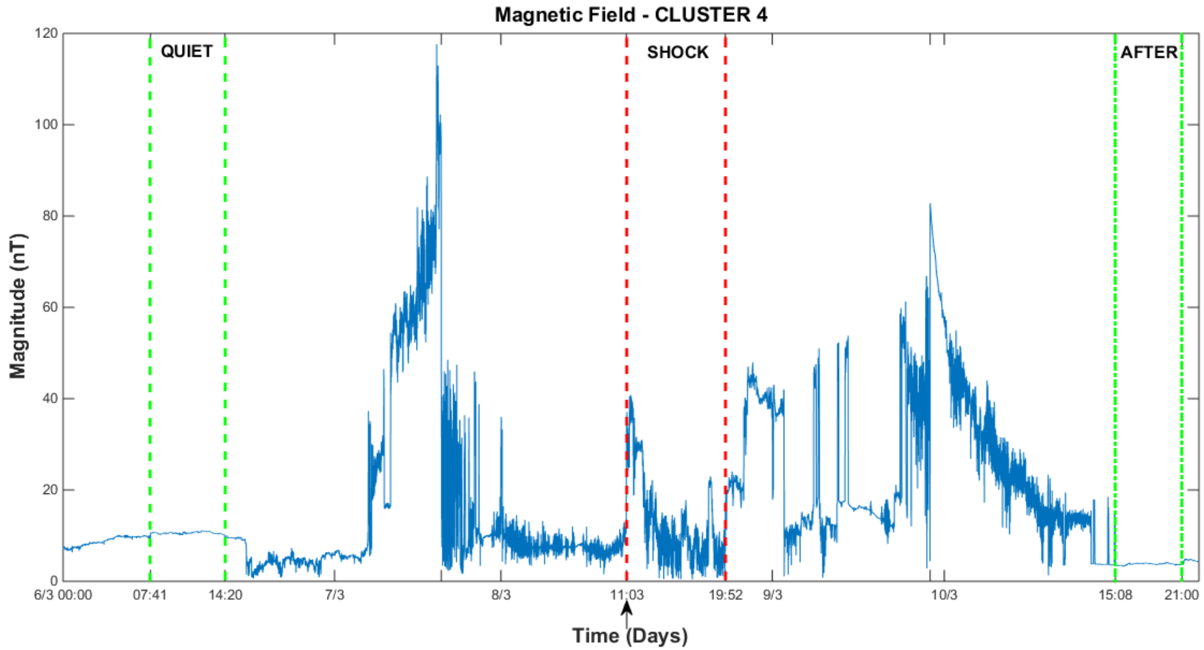
	Sunspot TMS	V1 Component	V2-10 Component
q relaxation	2.522±0.044	5.255±0.308	2.426±0.054
q stationary	1.53±0.04	1.40±0.08	2.12±0.20
q sensibility	0.368± 0.005	0.055± 0.009	0.407±0.029
$\Delta\alpha=\alpha_{max}-\alpha_{min}$	1.752± 0.003	1.133± 0.009	1.940±0.029

Interplanetary Observations (8 March 2012)

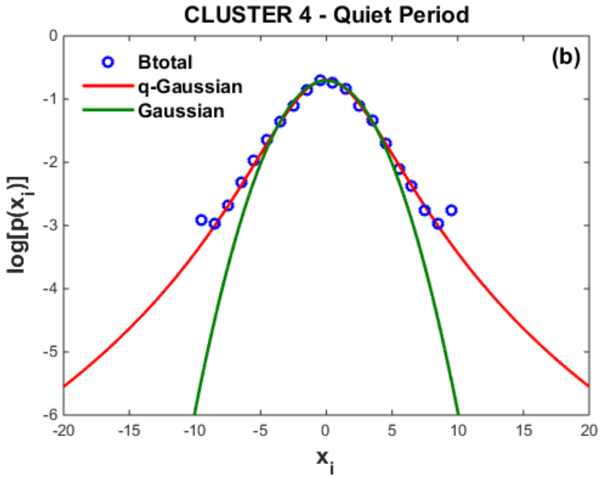


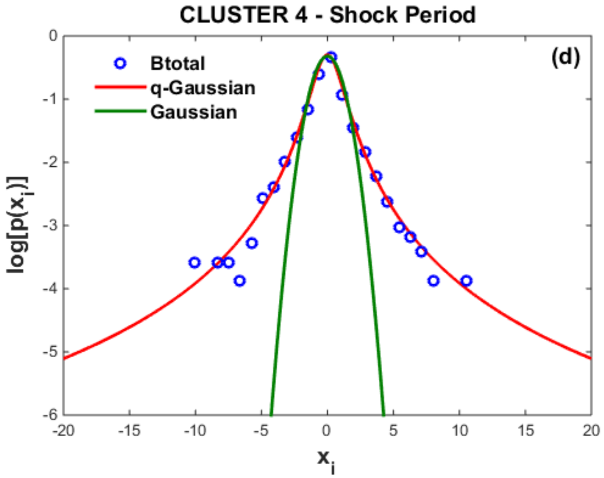
The locations of spacecraft ACE, CLUSTER 4, THEMIS-E and C, on 8 March 2012 11:00 UT are shown. The green bow-shaped structure corresponds to the bow-shock, while the white-shaped structure to magnetopause and its axis are in the Geocentric Solar Ecliptic (GSE) system.

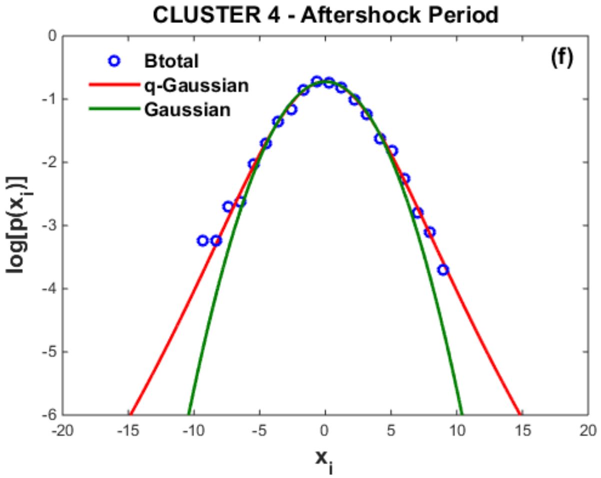
Solar Wind – Magnetic Field



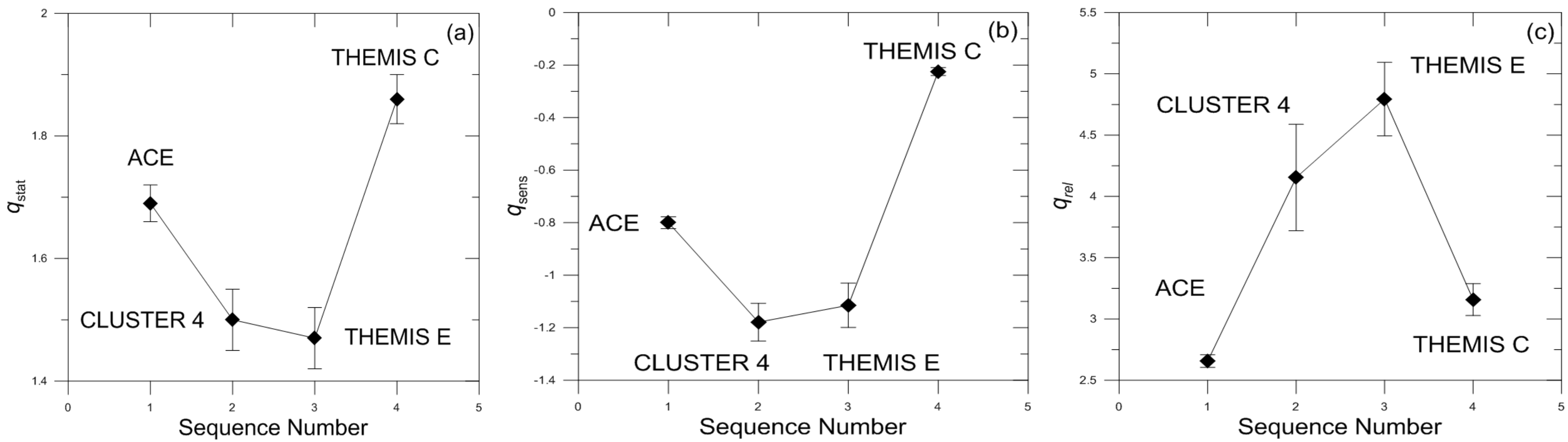
Calm - qstat= 1.25 ± 0.06 | **Shock - qstat=1.50 ± 0.05** | **ICME - qstat = 1.12 ± 0.05**





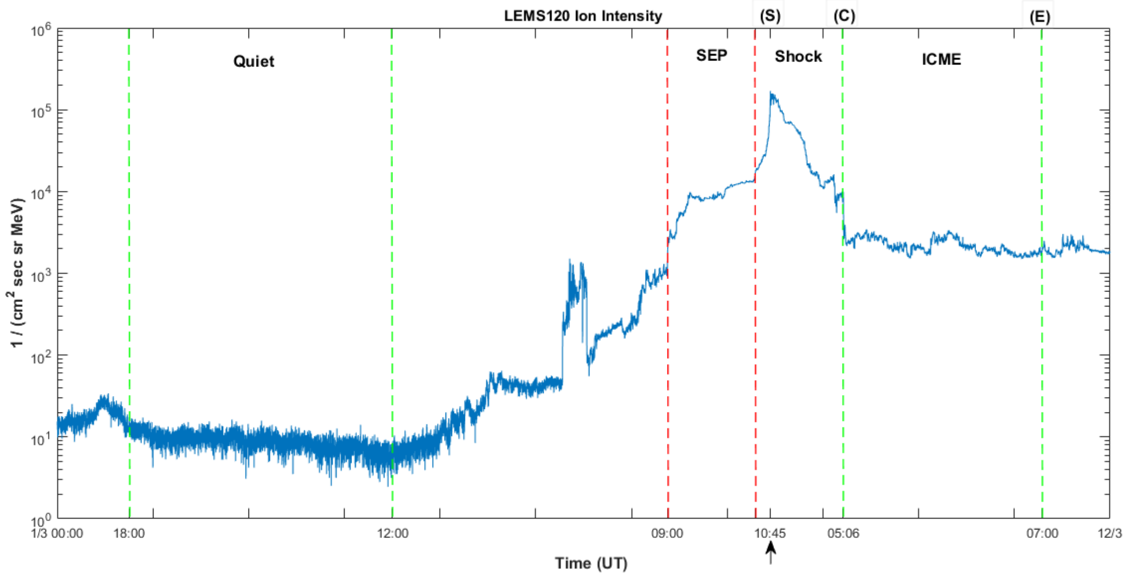


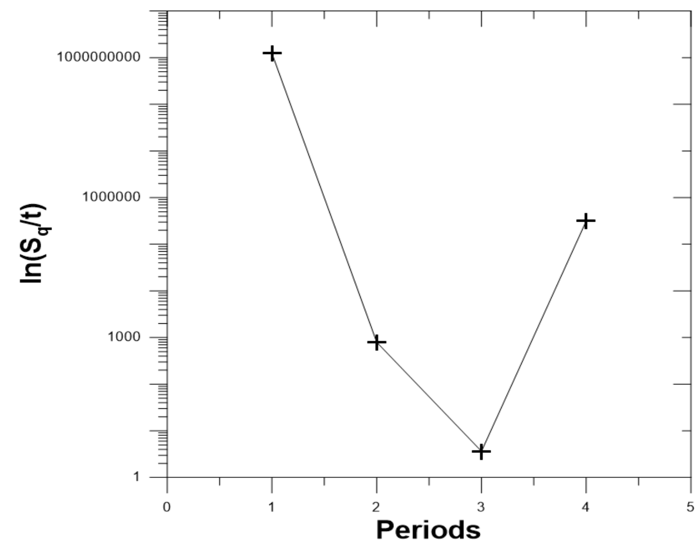
Multi-spacecraft approach (Shock Period)



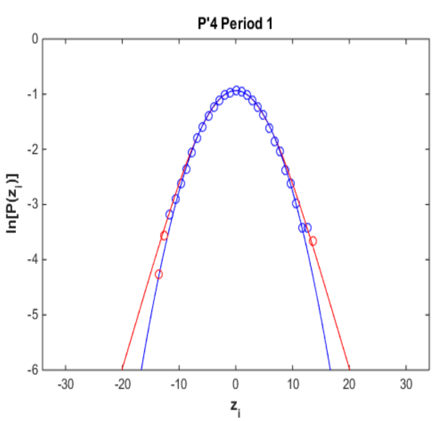
	ACE (Shock Period)	CLUSTER 4 (Shock Period)	THEMIS E (Shock Period)	THEMIS C (Shock Period)
$q_{stationary}$	1.69 ± 0.03	1.50 ± 0.05	1.47 ± 0.05	1.86 ± 0.04
$q_{sensitivity}$	-0.8002 ± 0.0223	-1.1787 ± 0.0717	-1.1145 ± 0.0843	-0.2246 ± 0.0149
$q_{relaxation}$	2.656 ±0.052	4.154 ± 0.434	4.794 ±0.345	3.155 ±0.175

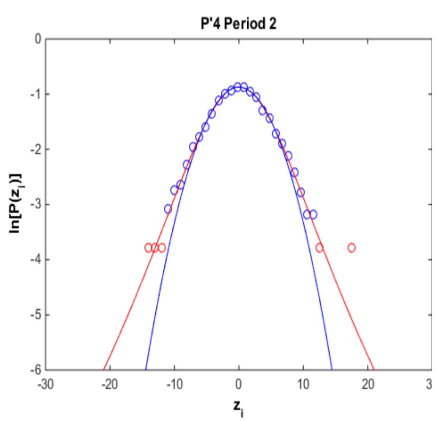
Solar Energetic Particle

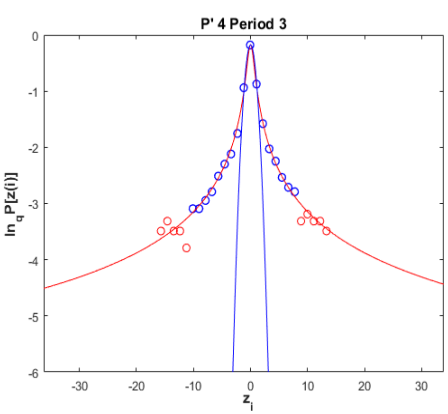


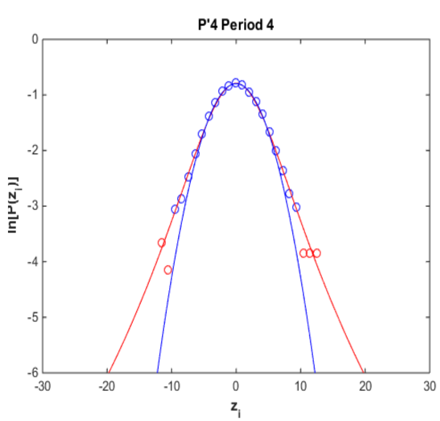


Quiet **- qstat=1.08±0.03** **SEP- qstat=1.18 ±0.05** **Shock - qstat=1.77 ±0.09** **ICME - qstat=1.16 ±0.04**



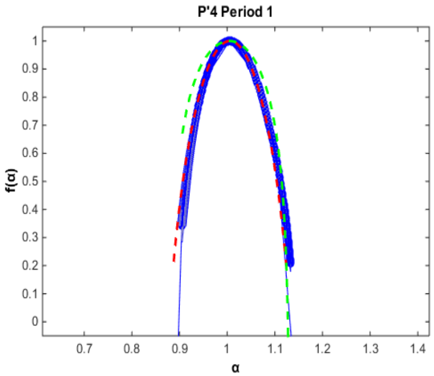


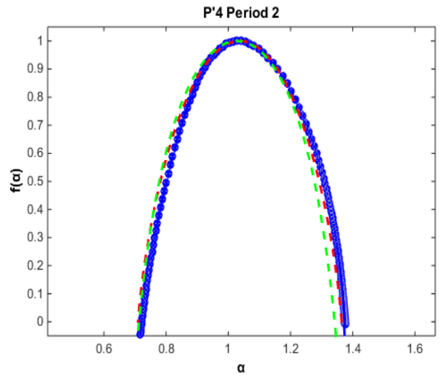


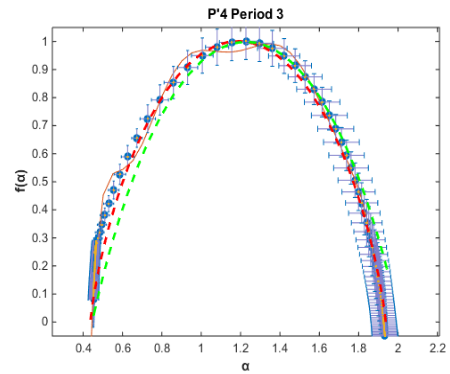


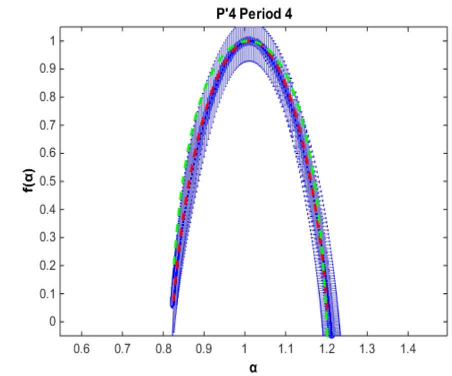
Solar Energetic Particle (Multifractal Spectrum)

Quiet **-** $q_{sen}=-3.049$ $\Delta\alpha=0.251$	**SEP -** $q_{sen}=-0.521$ $\Delta\alpha=0.653$	**Shock -** $q_{sen}=0.427$ $\Delta\alpha=1.483$	**ICME -** $q_{sen}=-1.563$ $\Delta\alpha=0.388$

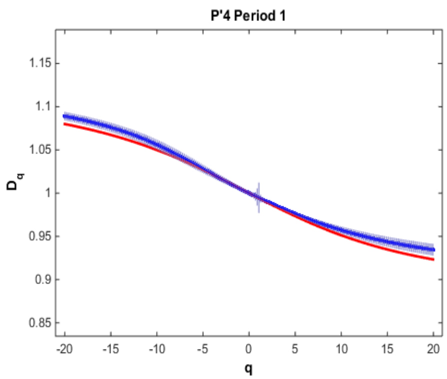


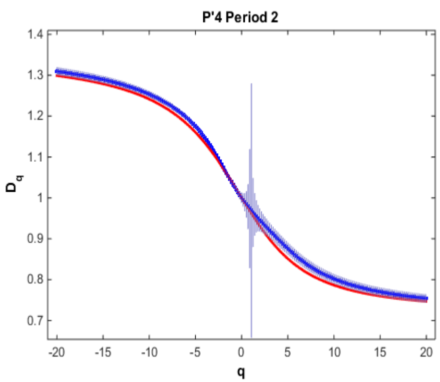


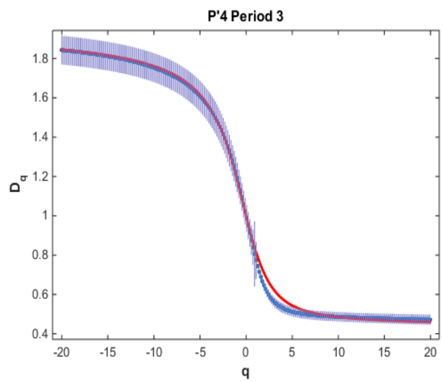


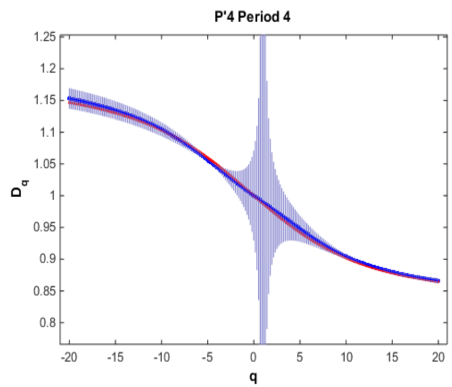


Quiet **-** $\Delta D_q=0.155$ **p-model=0.546**	**SEP -** $\Delta D_q=0.556$ **p-model=0.612**	**Shock -** $\Delta D_q=1.371$ **p-model=0.753**	**ICME -** $\Delta D_q=0.287$ **p-model=0.566**

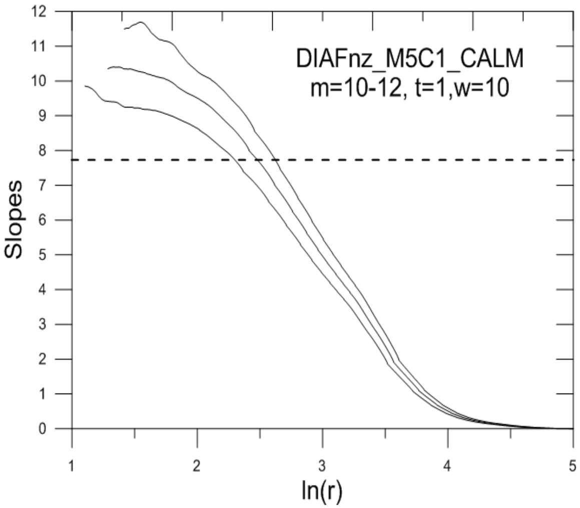


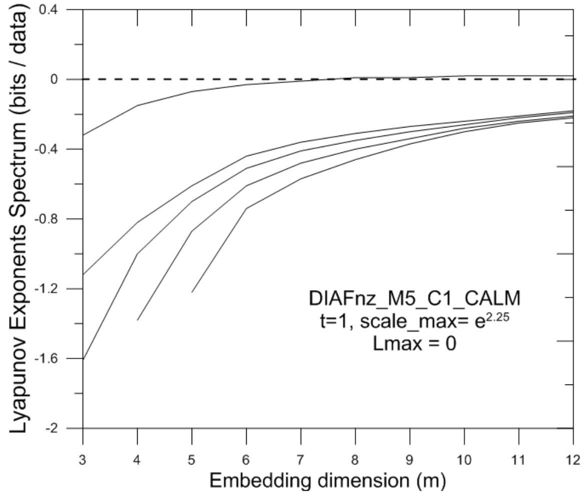


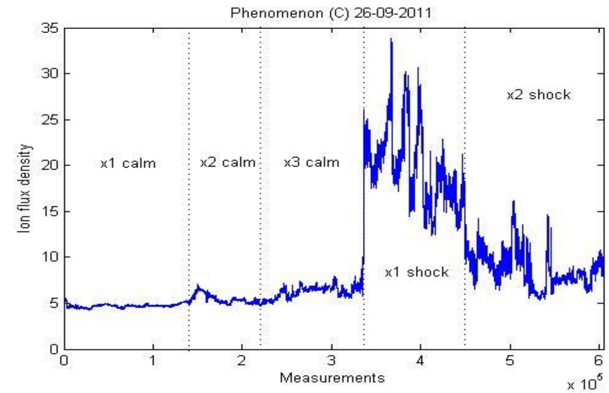


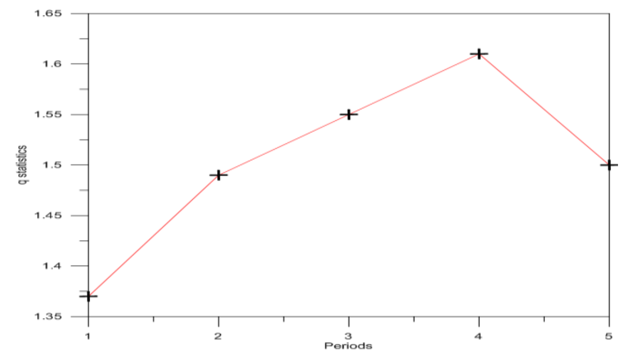


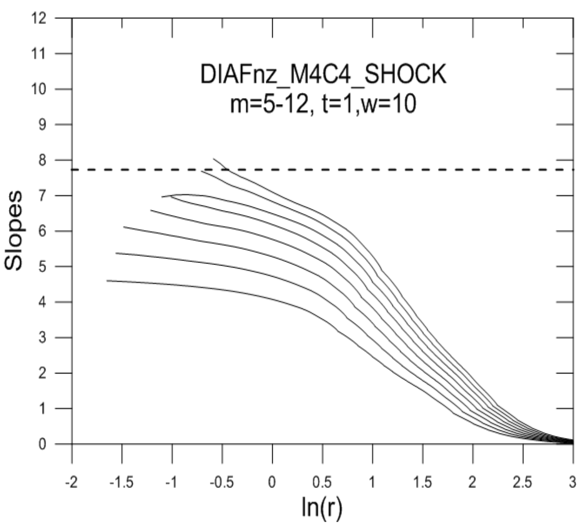
Solar Wind – Ion Flux

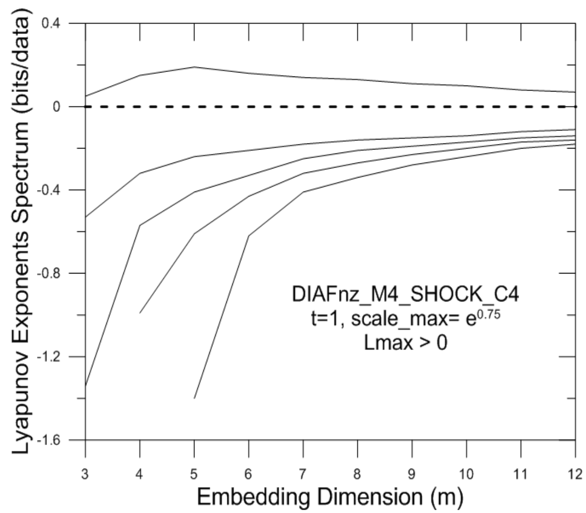


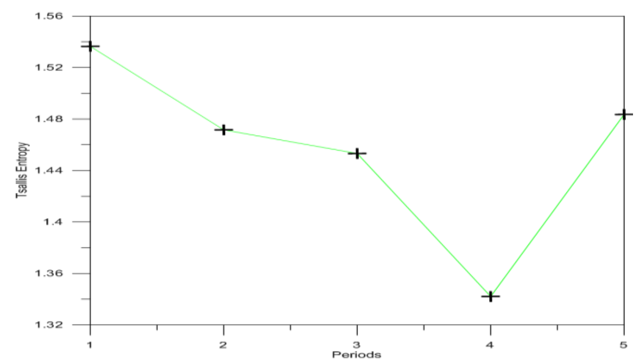












Extension of Thermodynamics

Near Thermodynamic Equilibrium

- *Boltzmann-Gibbs Entropy S_{BG} and Statistics.*
- *Gaussian probability density functions.*
- *Normal Classical and Quantum diffusion. Langevin, Fokker-Planch Equations.*
- *Classical Mechanics-Field Theory and QFT.*
- *Unified Quantum Fields, Electromagnetic Weak, Nuclear.*
- *Smooth Space-Time Manifolds Euclidean, Riemannian.*
- *Critical Self-Organized States.*
- *Seperation of Dynamics and Thermodynamics.*
- *Matter ~ Energy*
- *Equllibrium RGT and Reduction of dimensionality (D-finite).*
- *Normal Central Limit Theorem.*

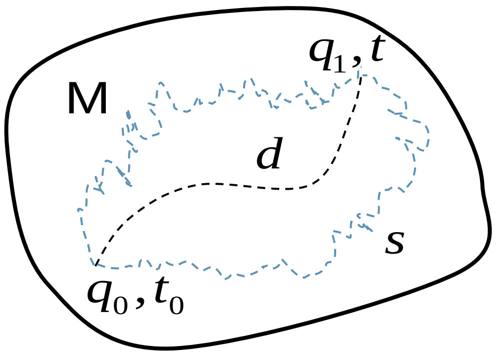
Far from Thermodynamic Equilibrium

- *Tsallis Entropy S_q and other Entropies – Statistics.*
- *Levy – Tsallis non Gaussian distributions.*
- Fractional *Classical and Quantum Diffusion, Langevin, Fokker-Planch Equations.*
- *Fractional Mechanics-Field Theory and QFT.*
- *Unified E-W-N+ gravity .*
- *Fractal Space-Time Sets.*
- *Critical Dissipative Self-Organized Structures.*
- *Unification of Dynamics and Thermodynamics.*
- *Matter ~ Energy ~ Information*
- *Non Equllibrium RGT and Reduction of dimensionality (D-infinite).*
- *q-Gaussian Central Limit Theorem.*

From Thermodynamics to Classical and Quantum Dynamics

d: smooth trajectory

s : stochastic trajectory



- **State Space Manifold** M
- **Classical Mechanics** M = $\Re^N$
- **Quantum Theory** M **= Hilbert space**
- **Nonequillibrium Dynamics** M **= extended fractal space**

ρ: probablility density in phase space

$\hat{\rho}$: density operator in quantum states

Ω: number of microstates

S : entropy of the system

$\hat{L}^c$: Liouville operator in classical mechanics

$\hat{L}^q$: Liouville operator in quantum mechanics

$\hat{L}^c_{FP}$: classical Fokker-Planck operator

$\hat{L}^q_{FP}$: quantum Fokker-Planck operator

No entropy production

Deterministic time reversible Dynamics

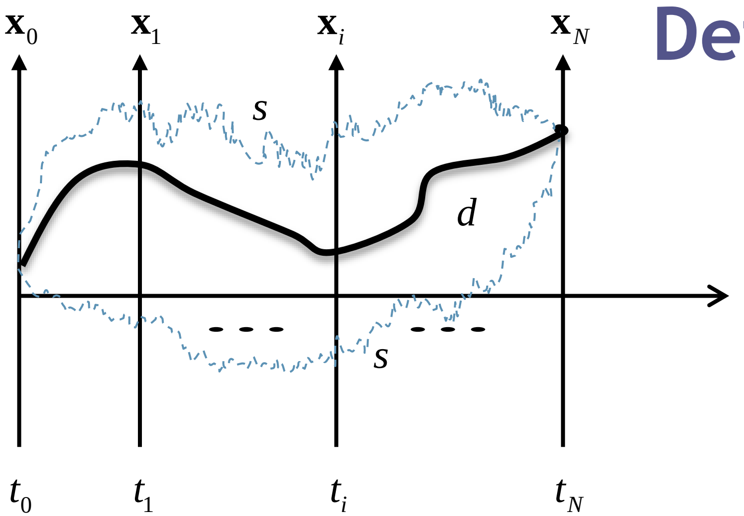
$$\approx \begin{cases} \dfrac{\partial \rho}{\partial t} = \{H, \rho\}_\rho = \hat{L}^c \rho \\ i\hbar \dfrac{\partial \hat{\rho}}{\partial t} = [\hat{H}, \hat{\rho}] = \hat{L}^q \hat{\rho} \end{cases} \Rightarrow \frac{d\Omega}{dt} = 0 \,,\; \frac{dS}{dt} = 0$$

Entropy production

Probabilistic time irreversible Dynamics

$$\approx \begin{cases} \dfrac{\partial \rho}{\partial t} = \hat{L}^c_{FP} \rho \\ \dfrac{\partial \hat{\rho}}{\partial t} = \hat{L}^q_{FP} \hat{\rho} \end{cases} \Rightarrow \frac{d\Omega}{dt} \ge 0 \,,\; \frac{dS}{dt} \ge 0$$

From Probabilism and Stochasticity to Determinism



$\hat{L}^s$: Stochastic Lagrangian for Fokker-Planch equation

Path Integral Formulation

$$\lim_{N\to\infty}\int\ldots\int d\mathbf{x}_0 d\mathbf{x}_1 \ldots d\mathbf{x}_N$$

$$= \lim_{N\to\infty}\int\prod_{i=0}^{N} d\mathbf{x}_i \equiv \int D\mathbf{x}$$

Fokker-Planck Process

$$f(x,t) = \int D\mathbf{x}\ \exp[\int_{t_0}^{t} L^s(\mathbf{x},\frac{d\mathbf{x}}{dt})\,d\tau]\ f(\mathbf{x}_0,t_0)$$

Quantum Mechanics (Feynman Formulation)

$$\langle x,t;x_0,t_0\rangle = prob.amplitude,\quad |\langle x,t;x_0,t_0\rangle|^2 = probability\ ,\quad \langle x,t;x_0,t_0\rangle = \int D\mathbf{x}\ \exp[\frac{i}{\hbar}\int_{t_0}^{t_1} L(\mathbf{x},\dot{\mathbf{x}})\,d\tau]$$

Probability = max ⟶ Classical deterministic trajectory in phase space

Extremiztion of probablility ⟶ Reversible deterministic Dynamics

L^s stochastic Lagrangian ⟶ L deterministic Lagrangian

Correlations and Entropy Production

Stable Hamiltonial Dynamics

$$\frac{df}{dt} = \frac{\partial f}{\partial t} - \{H, f\}_p \Rightarrow \frac{df}{dt} = 0, \ \frac{dS_B}{dt} = 0$$

no entropy production

$$S_B = -\kappa \int f \ln f \, dpdq$$

$f(p,q)$: pdf in phase space

Bogoliubov–Born–Green–Kirkwood–Yvon Hierarchy (BBGKY)

$$\frac{\partial f(1)}{\partial t} = \underbrace{\{H, f(1)\}_p}_{drift} + \underbrace{C_2}_{diffusion} \Rightarrow \frac{df(1)}{dt} = C_2 \neq 0, \ \frac{dS_B}{dt} \geq 0$$

C_2 **: two point correlations**

C_n **: n-point correlations**

$$\vdots$$

$$\frac{\partial f(n)}{\partial t} = \underbrace{\{H, f(n)\}_p}_{drift} + \underbrace{C_n}_{diffusion} \Rightarrow \frac{df(n)}{dt} = C_n \neq 0, \ \frac{dS_B}{dt} \geq 0$$

$$S_B = -\kappa \int f(n) \ln f(n) \times dp(n)dq(n)$$

Fractional Extension of BBGKY Hierarchy

$$\frac{\partial}{\partial t} \Rightarrow \frac{\partial^\alpha}{\partial t^\alpha}, \ \{\frac{\partial}{\partial q}, \frac{\partial}{\partial p}\} \Rightarrow \{\frac{\partial^\beta}{\partial q^\beta}, \frac{\partial^\beta}{\partial p^\beta}\},$$

C_n *: n-point correlations*
non Gaussian statistics

Entropy Principles the basic tool for Unification

Entropy Principle ⟷ **Thermodynamics : Z,F many point correlations**

Quantum Field Theory ⟷ **n-point Green Function correlations**

Thermodynamical Correlations Cn in (d+1) dimensional space $\overset{\tau=-it}{\longleftrightarrow}$ **QFT n-point Green Functions in d-dimensional space**

$X_E = (ix_0, \mathbf{x})$:*Wick rotation*

$$Z(J) \approx W_E(J)$$

Statistical Mechanics (d+1) space ~ Euclidean QFT in (d) space

QFT

$$G^N(x_1,\dots,x_n) = \langle 0|T[\varphi(x_1),\dots,\varphi(x_n)]|0\rangle \;,\quad G^N(x_1,\dots,x_n) = (\frac{\hbar}{i})^n \frac{\delta^n W(J)}{\delta J(x_1)\dots\delta J(x_n)}$$

$$W_E = N\int D\varphi \exp[-i\hbar^{-1}S_E(\varphi)] = Z(J):$$ Generating Functional

Thermodynamics

$$Z(J)=\sum_i e^{-E_i/\kappa T} \quad (\textit{partition function})$$

$$C_n(x_1,\dots,x_n) = \frac{1}{Z}\frac{\delta^n Z(J)}{\delta J(x_1)\dots\delta J(x_n)} \quad (\textit{correlations})$$

QFT-Thermodynamics

t =-iτ

$$\{QFT, W_E(J)\} \rightarrow \{Z(J),\ Thermodynamics\}$$

QFT as self-organization process of Subquantum Thermodynamics

Hooft, Beck, Parisi , Stochastic – Chaotic Quantization

$\vec{\varphi}(\mathbf{x},t)$: **random field ,** $$\frac{\partial}{\partial t}\vec{\varphi}(\mathbf{x},t) = -\Gamma\frac{\delta H}{\delta\vec{\varphi}} + \vec{N}$$ **(Random Field Langevin equation)**

, $\vec{N}$: **ucnorrelated noise** $< \vec{N}_i(\mathbf{x},t)\vec{N}_j(\mathbf{x}',t') > \delta_{ij}\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')$

Random Field Fokker-Planck Equation

$$\frac{1}{\Gamma}\frac{\partial}{\partial t}P(\vec{\varphi},t) = \frac{\delta}{\delta\vec{\varphi}}\cdot\{\frac{\delta H(\vec{\varphi})}{\delta\vec{\varphi}}P(\vec{\varphi},t) + \frac{\delta}{\delta\vec{\varphi}}[\Gamma P(\vec{\varphi},t)]\}$$

Equillibrium Solution : $P(\vec{\varphi},t) \sim \exp[-\frac{1}{\Gamma}H(\vec{\varphi})]$ **(stationary state)**

Random Field FPE → Quantum Field Schroendinger Equation

(*Thermodynamics*) (*Quantum Field Theory)*

QFT as self-organization process of Subquantum Thermodynamics

Random Field FPE (*Thermodynamics*) → Quantum Field Schroendinger Equation (*Quantum Field Theory)*

Quantum Hamiltonian Operator

$$\hat{H} = \hat{p}^2[H(\hat{\psi}) - i\Gamma(\hat{\psi})] \quad \textbf{where} \quad [\hat{p},\hat{\psi}] = i, [\hat{p},\hat{p}] = [\hat{\psi},\hat{\psi}] = 0$$

QFT Schroendinger Equation

Time Evolution Operator $\hat{U}(t,t_0)$ **:** $i\dfrac{d}{dt}\hat{U}(t,t_0) = \hat{H}\,\hat{U}(t,t_0)$

Schroendinger Field Equation : $P(\vec{\varphi}(\mathbf{x},t),t) = \hat{U}(t,t_0)\,P(\vec{\varphi}(\mathbf{x},t_0),t_0)$

$$\hat{U}(t,t_0) = \hat{T}\exp[-i\int_{t_0}^{t} H\,d\tau]$$

J. Binney et al., The Theory of Critical Phenomena: An Introduction to the Renormalization Group, Oxford Univ. Press, 1992.

Why Study Phase Transitions ?

First phase changes are immensely influential in every corner of the Universe-indeed it is widely argued that the very existence of the observable Universe is attributable to a phase change in the state of some pre-existing vacuum, and that the disposition of matter in and around galaxies should be understood in terms of fluctuations associated with some such transition (see for example Kolb and Turner 1989).

Not only is the creation of long-range structure by short-range inter-molecular forces intriguing, but any example of scale-freedom is worthy of close examination since this phenomenon occurs in several physical systems that are inadequately understood-the clustering of galaxies (e.g., Peebles 1980), the distribution of earthquakes (e.g., Carlson and Langer 1989, Bak *et al.* 1988), turbulence in fluids and plasmas (e.g., Mandlebrot 1974), polymers (de Gennes 1972), snow flakes *(Ballet al.* 1989, Meakin and Tolman 1989)-to name but a few. In each case there is a wide range of scales over which some phenomenon varies as a power law of the scale, presumably because there is a gross mismatch between the largest and smallest scales in the problem.

J. Binney et al., The Theory of Critical Phenomena: An Introduction to the Renormalization Group, Oxford Univ. Press, 1992.

An elementary particle is represented by a structure of a certain physical size on the lattice. As the lattice is refined this structure should retain its physical size by covering more and more lattice sites. Hence, as the discrete model approaches the continuum limit of real quantum fields, the particle must be represented by correlations on the lattice of longer and longer range, and the field theory that gives rise to these correlations must be approaching what in statistical mechanics we would call a critical point.

Elementary particles as Thermodynamic self-organization process

Random Field in D-dimensions

$\mathbf{x} \in \Re^D$

n-point Corelations
n-point Green's Functions
n-point interactions

$$G(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N) = < \varphi(\mathbf{x}_1), \varphi(\mathbf{x}_2) \ldots \varphi(\mathbf{x}_N) >$$

$$= \frac{1}{Z} \int D\varphi \, \varphi(\mathbf{x}_1), \varphi(\mathbf{x}_2) \ldots \varphi(\mathbf{x}_N) \, \mathrm{e}^{-\int F(\varphi) \, d\mathbf{x}^D}$$

where $F(\varphi) = h(\varphi) - J\varphi$ **,** $h(\varphi) =$ **field energy density,** $J =$ **source field**

$$Z(J) = \int D\varphi \; e^{-\int F \, d\mathbf{x}^D}$$

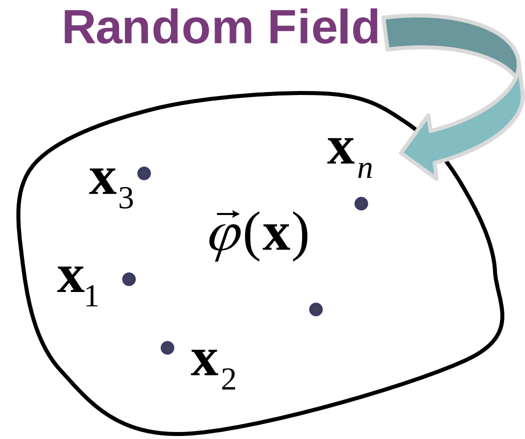
Random Field Partition Funnction

n-point correlation function

$$G^n(x_1, \mathrm{x}_2, \ldots, x_n) = \frac{1}{Z} \left. \frac{\delta^n Z(J)}{\delta J(x_1) \ldots \delta J(x_n)} \right|_{J=0}$$

n-point particle interactions
n-point correlations

Elementary particles as Thermodynamic self-organization process



Random Field

- Self-organization
- Phase Transitions
- Long-range Correlations

→ Interactive Particle System

Stochastic Process as macroscopic quantity

Diffusion Process

$$\frac{\partial}{\partial t} f(\vec{x},t) = D_0 \nabla^2 f(\vec{x},t)$$, velocity random variable $v = \Delta x / \Delta t$

Statistical-Thermodynamical Uncertainty Principle

$$\boxed{\Delta x \ \Delta t \geq D_0}$$

Quanticity and Stochasticity

StochasticProcesses ⟷ Quantum Processes

C. Beck, Chaotic strings and standard model parameters, Physica D: Nonlinear Phenomena, 171, 72-106, 2002.

A fundamental problem of particle physics is the fact that there are about 25 free fundamental constants which are not understood on a theoretical basis. These constants are essentially the values of the three coupling constants, the quark and lepton masses, the W and Higgs boson mass, and various mass mixing angles. An explanation of the observed numerical values is ultimately expected to come from a larger theory that embeds the standard model. Prime candidates for this are superstring and M theory. However, so far the predictive power of these and other theories is not large enough to allow for precise numerical predictions.

We will report on a numerical observation that may shed more light on this problem. We have found that there is a simple class of 1+1 -dimensional strongly self-interacting discrete field theories (called 'chaotic strings' in the following) that have a remarkable property. The expectation of the vacuum energy of these strings is minimized for string couplings that numerically coincide with running standard model couplings $\alpha(E)$, the energy E being given by the masses of the known quarks, leptons, and gauge bosons.

C. Beck, Chaotic strings and standard model parameters, Physica D: Nonlinear Phenomena, 171, 72-106, 2002.

Chaotic strings can thus be used to provide theoretical arguments why certain standard model parameters are realized in nature, others are not. We may assume that the *a priori* free parameters evolve to the local minima of the effective potentials generated by the chaotic strings. Out of the many possible vacua, chaotic strings may select the physically relevant vacuum of superstring theories.

The dynamics of the chaotic strings is discrete in both space and time and exhibits strongest possible chaotic behaviour. It can be regarded as a dynamics of vacuum fluctuations that can be used to 2nd-quantize other fields, for example ordinary standard model fields, or ordinary strings, by dynamically generating the noise of the Parisi-Wu approach of stochastic quantization on a very small scale. Mathematically, chaotic strings are coupled map lattices of diffusively coupled Tchebyscheff maps TN of order N. It turns out that there are six different relevant chaotic string theories - similar to the six components that make up M-theory in the moduli space of superstring theory.

T.S. Biro et al., Chaotic Quantization of Classical Gauge Fields, Foundations of Physics Letters, 14(5), 2001.

We argue that the *quantized* non-Abelian gauge theory can be obtained as the infrared limit of the corresponding *classical* gauge theory in a higher dimension. We show how the transformation from classical to quantum field theory emerges, and calculate Planck's constant from quantities defined in the underlying classical gauge theory.

Although much progress has been made in recent years, the question, how gravitation and quantum mechanics should be combined into one consistent unified theory of fundamental interactions, is still open. Superstring theory, which describes four-dimensional space-time as the low-energy limit of a ten- or eleven-dimensional theory (“M-theory”), may provide the correct answer, but the precise form and content of the theory is not yet entirely clear. It is therefore legitimate to raise the question whether the fundamental description of nature at the Planck scale is really quantum mechanical, or whether the underlying theory could be a classical extension of general relativity. This questions was initially raised by 't Hooft, who has argued that quantum mechanics can logically arise as low-energy limit of a microscopically deterministic, dissipative theory

T.S. Biro et al., Chaotic Quantization of Classical Gauge Fields, Foundations of Physics Letters, 14(5), 2001.

We here show that in some cases, specifically for non-Abelian gauge fields, the functional integral of the three-dimensional Euclidean *quantum* field theory arises naturally as the long-distance limit of the corresponding *classical* gauge theory defined in (3+1)-dimensional Minkowski space. Because of the general nature of the mechanism underlying this transformation, for which we have coined the term *chaotic quantization*, it is expected to work equally well in other dimensions. For example, the four-dimensional Euclidean quantum gauge theory arises as the infrared limit of the (4+1)-dimensional classical gauge theory. We emphasize that the dimensional reduction is not caused by compactification; the classical field theory does not exhibit periodicity either in real or imaginary time.

Stochastic Process as macroscopic quanticity

Quanticity and Stochasticity

StochasticProcesses ⟷ Quantum Processes

Fokker-Planck Equation

$$\frac{\partial}{\partial t} f(\vec{x},t) = \nabla_\chi^2 f(\vec{x},t) - V(\vec{x}) f(\vec{x},t)$$

(Feynman-Kac Formuma)

$t \rightarrow -i\,t$ ⬇ *Wick Rotation*

$$i\frac{\partial}{\partial t} f(\vec{x},t) = -\nabla_\chi^2 f(\vec{x},t) + V(\vec{x}) f(\vec{x},t)$$

(Schroendinger Equation)

Quantum Theory ~ Subquantum Stochastic Processes

<u>Significant Analogy</u>

Entropy

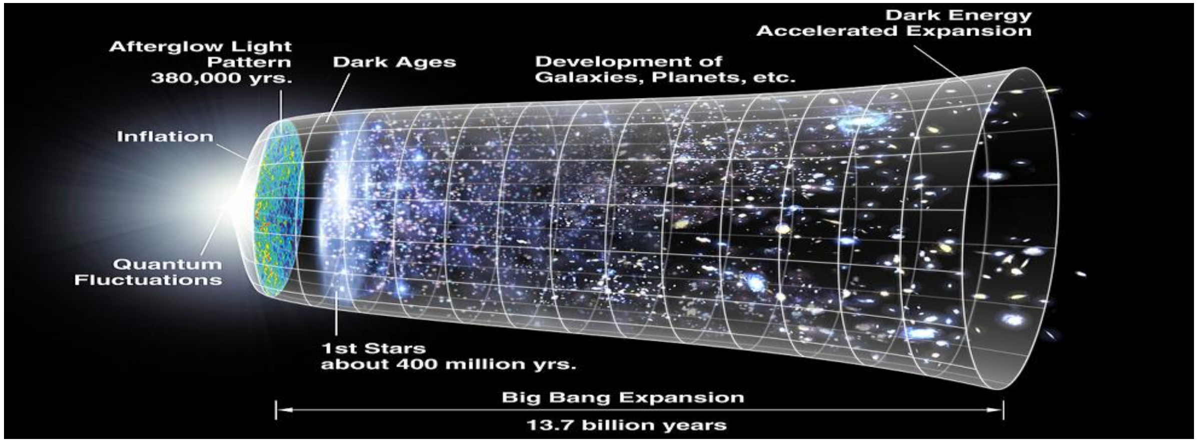
$$S = \sum_i p_i \ln p_i$$

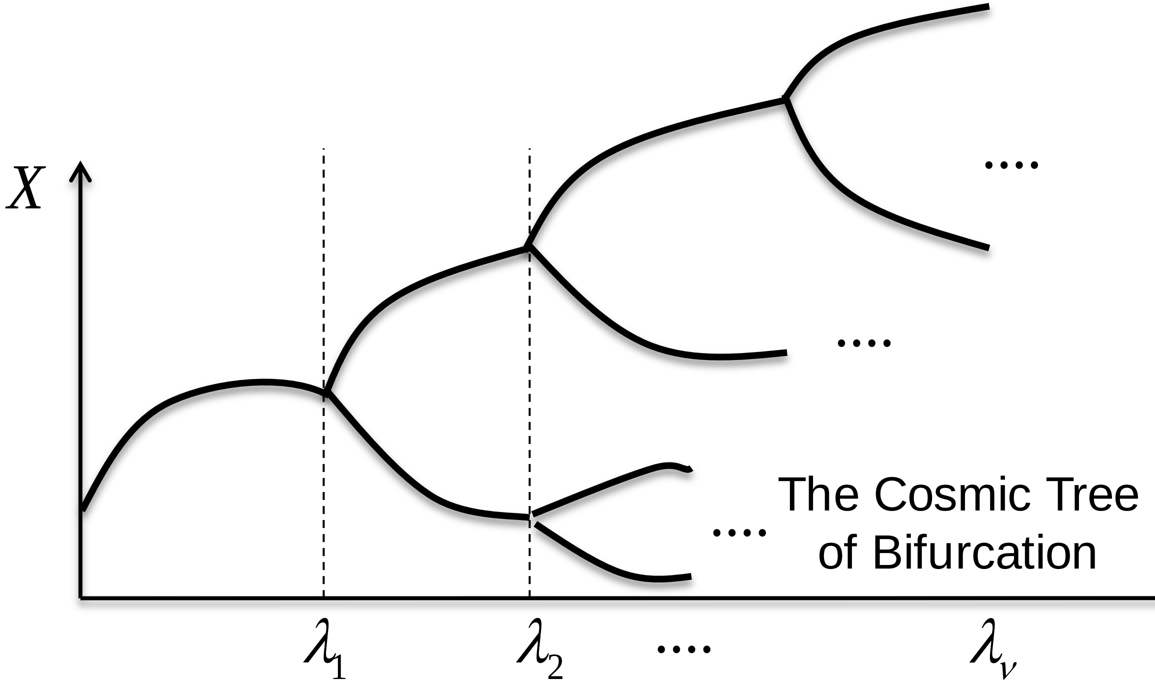
⟷

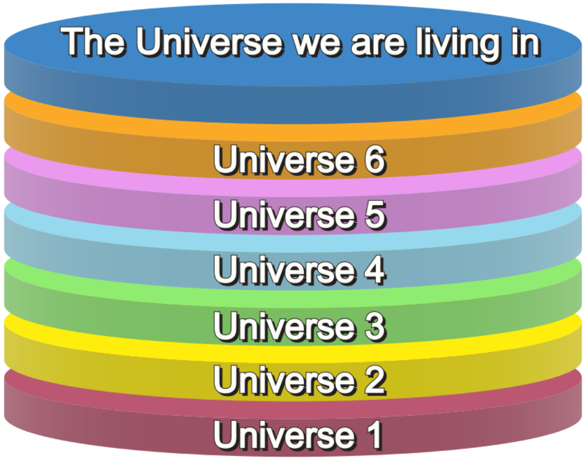
$$|\psi\rangle = \int |x\rangle dx \,\langle x|\psi\rangle = \sum_i |\varepsilon_i\rangle\langle\varepsilon_i|\psi\rangle$$

all microstates ⟶ **quantum eigenstates participate**

Cosmic Thermodynamics – Cosmic Novelty







The Cosmif Fractal
Entropic non-Equillibrium Cosmos

Symmetry Breaking-Cosmic self-organization
Cosmic Renormalization

- Cosmic Fractal self-similarity
- Long Range Space-Time Correlations
- Cosmic Evolution
- Reduction of Dimensionality
- Symmetry Breaking
- Multiplicative Structuring
- Non-Linear Evolution
- Entropy and Information Production
- Goedel non-Computable non-Algoritmic Dynamics

- Cosmic Self-Organization
- Multi-level Renormalization
- Fixed Points
- Bifurcation Points
- Multiscale Cohenrence – Cooperations
- Entropy Principle Cronos Principle
- Contraction of Fractal Space-Time
- Scale Relativity
- Scale Unification
- Fractal Laws
- Dissipative Structures
- Symmetries

THANK YOU
for your attention