Infrared Quantum Information

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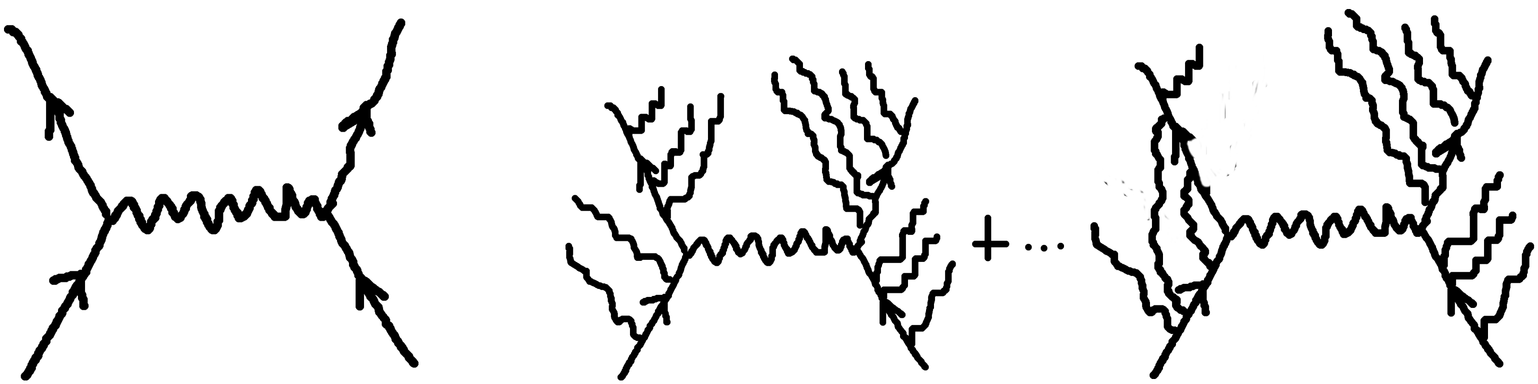
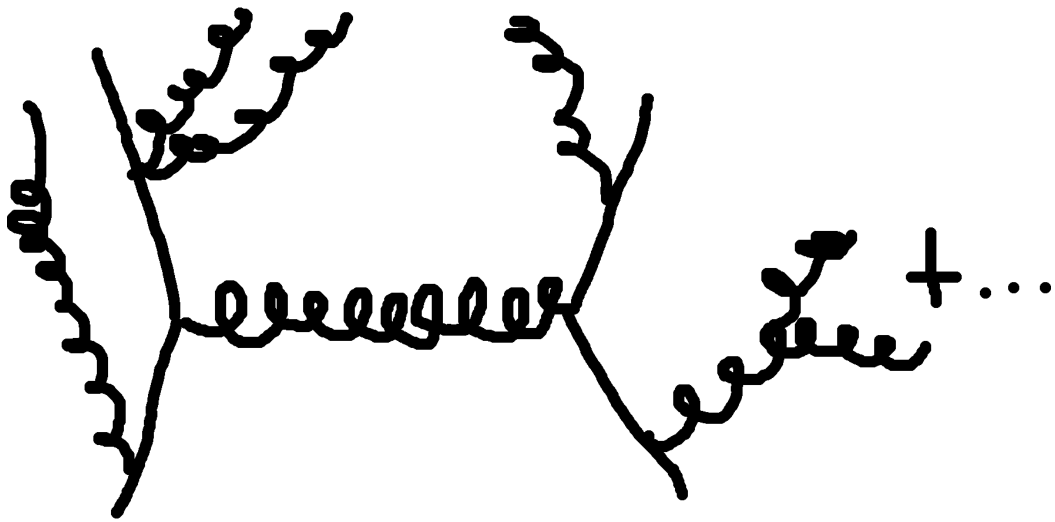
Central to the solution of the infrared problem in quantum electrodynamics is the fact that a scattering experiment with charged particles is accompanied by the production of an infinite number of arbitrarily soft photons $(k^\mu = (|\vec{k}|, \vec{k})\ ,\ k \sim 0)$



F. Bloch and A. Nordsieck, Phys. Rev. 52, 54 (1937)
D. R. Yennie, S. C. Frautschi, H. Suura, Ann. Phys. 13, 379 (1961)
soft photon theorems

The same phenomenon occurs in perturbative quantum gravity where *soft gravitons* are produced.

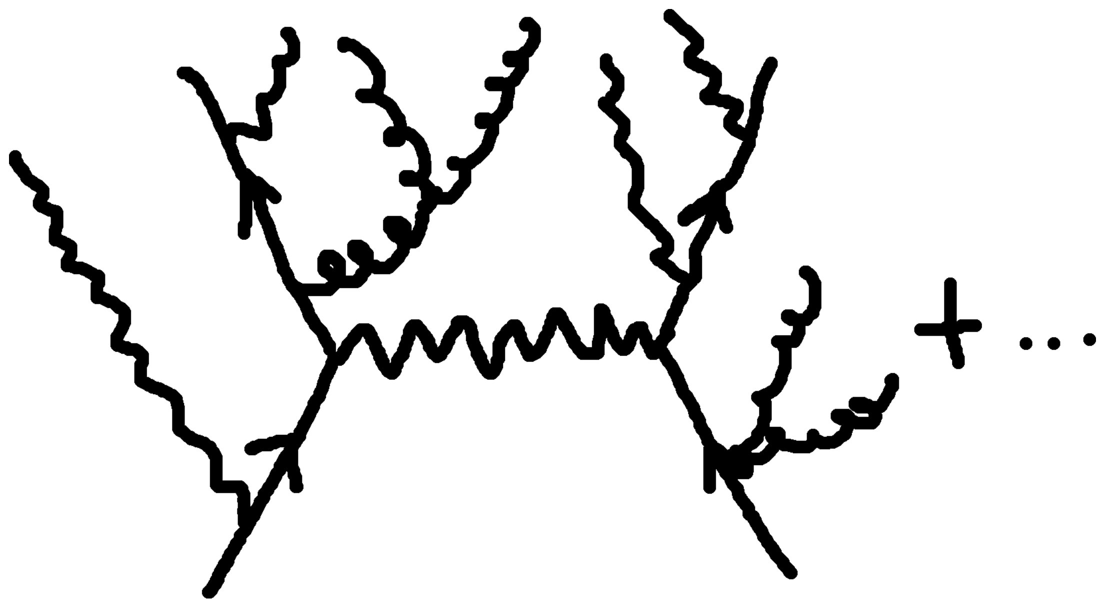
$$k^\mu = (|\vec{k}|, \vec{k}) \ , \ k \sim 0$$



soft graviton theorem

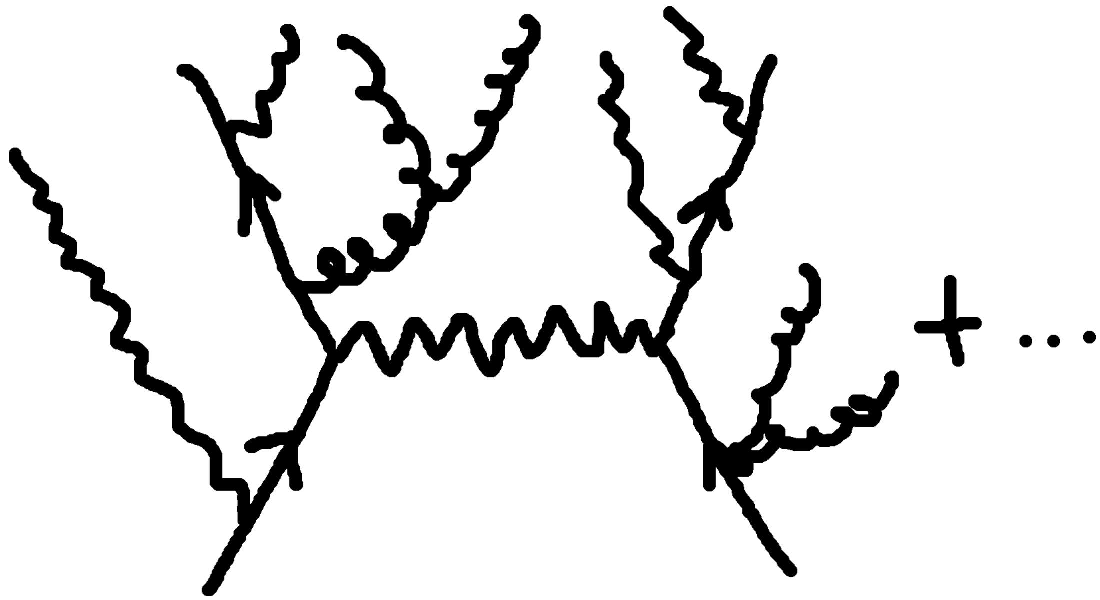
S. Weinberg, Phys. Rev. 140, B516 (1965)

The *soft photons* and *soft gravitons* which escape detection have polarizations and directions of propagation.



The *soft photons* and *soft gravitons* which escape detection have polarizations, directions of propagation.

How much information do they carry away?



D.Carney, L.Chaurette, D.Neuenfeld, GWS, arXiv:1706.03782

G.Grignani and GWS, Phys. Lett. B 772 (2017) 699

Information loss due to entanglement:

Composite system of two qubits: $|\ \rangle_1 \otimes |\ \rangle_2$

If subsystem $|\ \rangle_2$ becomes inaccessible to us, how much information about $|\ \rangle_1$ do we lose?

Unentangled state $|\psi\rangle = |\uparrow\rangle_1 \otimes |\downarrow\rangle_2$

Entangled state $|\psi\rangle = \frac{1}{\sqrt{2}}\,|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 + \frac{1}{\sqrt{2}}\,|\downarrow\rangle_1 \otimes |\uparrow\rangle_2$

Reduced density matrix $\rho = \mathrm{Tr}_2|\psi\rangle\langle\psi|$

Unentangled state $\rightarrow \rho = |\uparrow\rangle_1\langle\uparrow| \ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Entangled state $\rightarrow \rho = \frac{1}{2}\,\{|\uparrow\rangle_1\langle\uparrow| + |\downarrow\rangle_1\langle\downarrow|\} \ = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$

Entanglement entropy $S = -\mathrm{Tr}\rho\ln\rho$

Unentangled state $S = 0$ *Entangled state* $S = 2\ln 2$

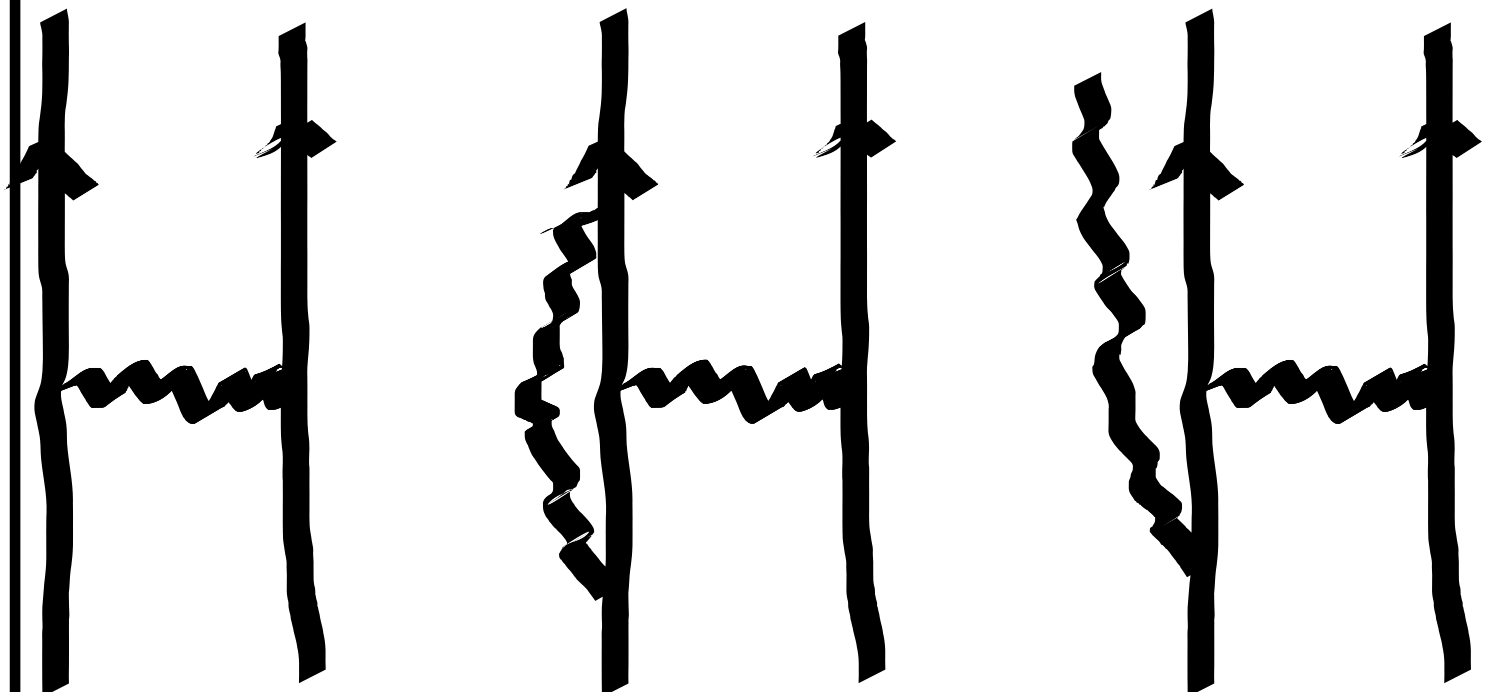
We shall compute the entropy of entanglement of the "hard" particles and the "soft" radiation composed of very long wavelength photons and gravitons in a generic scattering experiment.

We shall find that they are highly entangled.

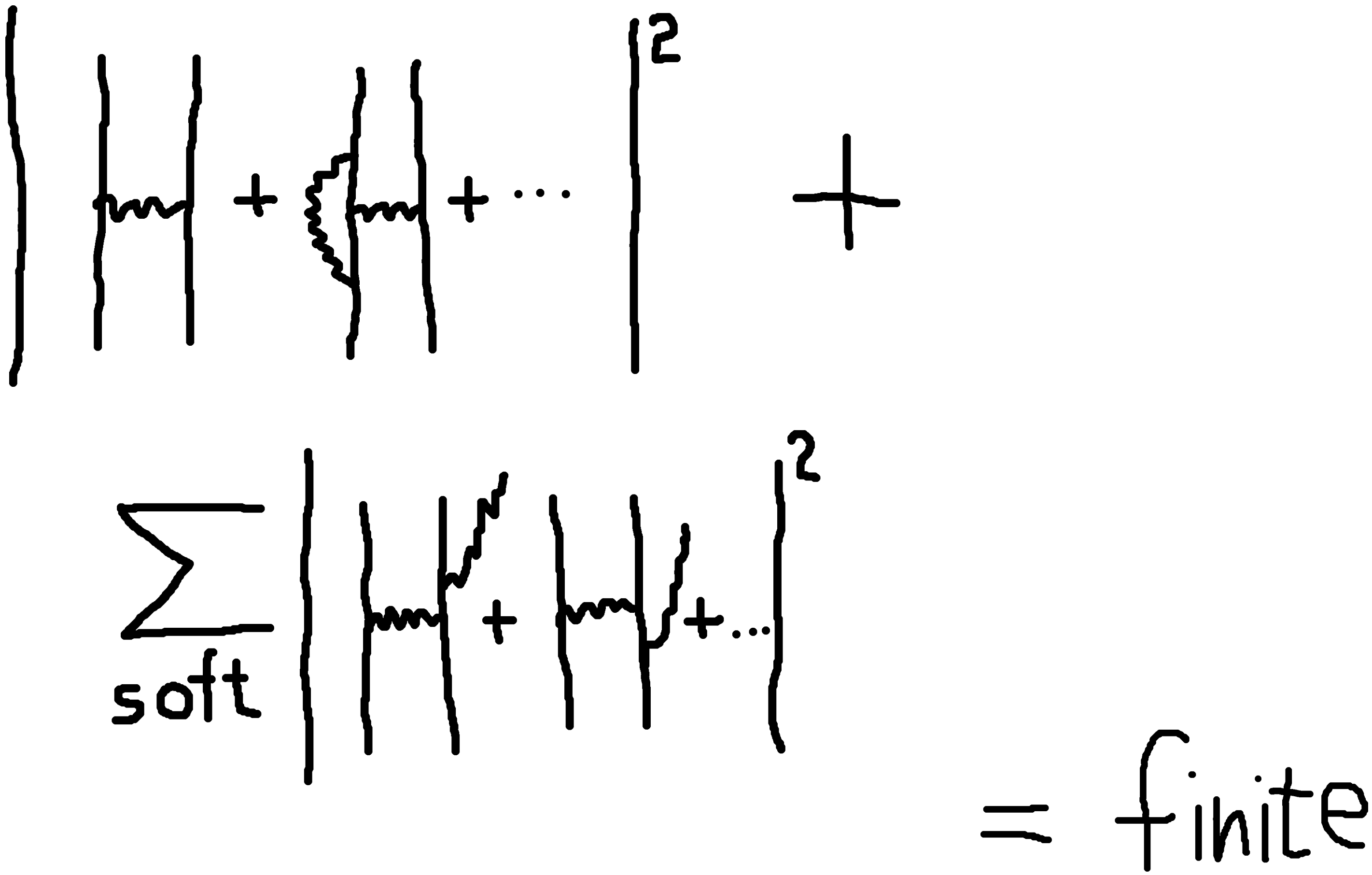
The result is decoherence of the final state.

D.Carney, L.Chaurette, D.Neuenfeld, GWS, arXiv:1706.03782

In **quantum electrodynamics**, the computation of the S-matrix for the scattering of charged particles encounters infrared divergences



Inclusive probability



Cancellation of infrared divergences is guaranteed by unitarity of infrared cutoff S-matrix.

Soft photon theorem (oversimplified)

$$1 - \left(\frac{e^2}{4\pi}\gamma(p_i)\ln\frac{\Lambda}{m_{\rm ph.}}\right) + \frac{1}{2!}\left(\frac{e^2}{4\pi}\gamma(p_i)\ln\frac{\Lambda}{m_{\rm ph.}}\right)^2 + \ldots = \left(\frac{m_{\rm ph.}}{\Lambda}\right)^{\frac{e^2}{4\pi}\gamma(p_i)}$$

$m_{\rm ph}(\to 0) << \Lambda << p_i, m_{\rm el}$

*Amplitude for process which does not emit photons vanishes.

*Amplitude for process which emits a finite number of photons also vanishes.

*Same for gravitons

Outgoing density matrix

Scattering event

$$|\alpha> \;\; \to \;\; S^{\dagger}|\alpha> = \sum_{\beta\gamma} |\beta\gamma>< \beta\gamma|S^{\dagger}|\alpha>$$

The states $|\beta\gamma>_{\rm in}$ contain both hard (β) and soft (γ) particles.
Form the out-going density matrix

$$\rho = {\rm Tr}_{\rm soft} S^{\dagger}|\alpha>< \alpha|S$$

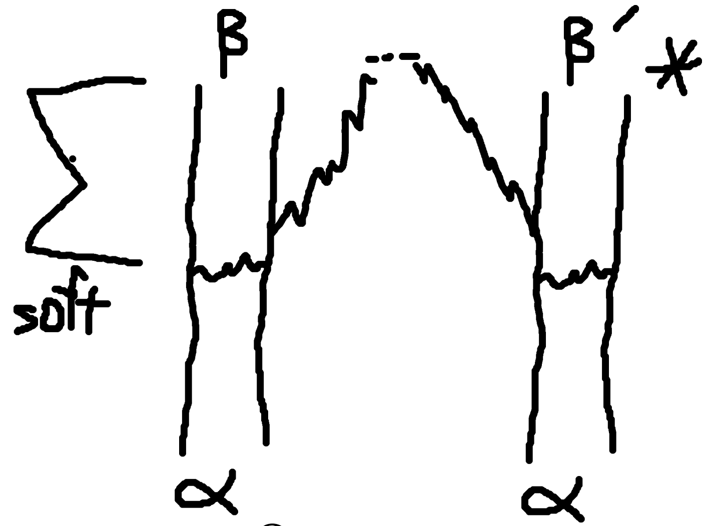
$$\rho_{\beta\beta'} =$$

$$= \sum_{\hat{\gamma}} < \hat{\gamma}| \boxed{\sum_{\beta\gamma} |\beta\gamma>< \beta\gamma|S^{\dagger}|\alpha> \sum_{\beta'\gamma'} < \alpha|S^{\dagger}|\beta'\gamma'>< \beta'\gamma'|} |\hat{\gamma}>$$

Diagonal elements are probabilities of outcomes of scattering experiments.
Off-diagonal elements are correlations between outcomes.

Entanglement entropy $S = -\mathrm{Tr}\ \rho\ \ln \rho\ \equiv\ -\sum_i \rho_i \ln \rho_i$

$$\rho = |\beta >< \beta'| \qquad\qquad +$$



Eigenvalues 1 and 0 perturbed by $\sim \left[\frac{e^2}{4\pi}\right]^3 \ln \frac{\Lambda}{m_{\mathrm{ph}}}$

ρ_i logarithmically infrared divergent at order e^6

The entangement entropy is infrared divergent

$$S \sim -\left[\frac{e^2}{4\pi}\right]^3 \ln \frac{\Lambda}{m_{\rm ph}} * \ln\left[\left[\frac{e^2}{4\pi}\right]^3 \ln \frac{\Lambda}{m_{\rm ph}}\right]$$
$$-\left[\frac{e^2}{4\pi}\right]^2 \left[\frac{\tilde{\Lambda}^2}{M_{\rm Pl}^2}\right] \ln \frac{\tilde{\Lambda}}{m_{\rm grav}} * \ln\left[\left[\frac{e^2}{4\pi}\right]^2 \left[\frac{\tilde{\Lambda}^2}{M_{\rm Pl}^2}\right] \ln \frac{\tilde{\Lambda}}{m_{\rm grav}}\right]$$

Big logarithm = "amplification" of small quantum corrections!!

Observable quantum gravity???

But, such infrared effects are non-perturbative.
We must do better.

Soft photon theorems.

Soft graviton theorems.

The density matrix: (D.Carney, L.Chaurette, D.Neuenfeld, GWS, arXiv:1706.03782) We can use soft photon and soft graviton theorems to show

$$\rho_{\beta\beta'} = S_{\beta\alpha}^{\Lambda_1\tilde{\Lambda}_1} S_{\beta'\alpha}^{\Lambda_1\tilde{\Lambda}_1 *} \; \mathcal{F}\left[\Lambda_i, \tilde{\Lambda}_i, E_T, m_{\rm ph}, m_{\rm grav}\right]$$

where

$$\mathcal{F} = \left(\frac{\Lambda_2}{m_{\rm ph}}\right)^{\tilde{A}_\alpha^{\beta\beta'}} \left(\frac{m_{\rm ph}}{\Lambda_1}\right)^{\frac{A_\alpha^\beta + A_\alpha^{\beta'}}{2}} \left(\frac{\tilde{\Lambda}_2}{m_{\rm grav}}\right)^{\tilde{B}_\alpha^{\beta\beta'}} \left(\frac{m_{\rm grav}}{\tilde{\Lambda}_1}\right)^{\frac{B_\alpha^\beta + B_\alpha^{\beta'}}{2}} \times$$

$$\times f\left(\frac{\Lambda_2}{E_T}, \tilde{A}_\alpha^{\beta\beta'}\right) f\left(\frac{\tilde{\Lambda}_2}{E_T}, \tilde{B}_\alpha^{\beta\beta'}\right)$$

$$\sim m_{\rm ph}^{\Delta A} m_{\rm grav}^{\Delta B} \quad , \quad \Delta A,\ \Delta B \geq 0$$

RESULT: $\rho_{\beta\beta'} \neq 0$ **as** $m_{\rm ph} \to 0$ **and** $m_{\rm grav} \to 0$ **only if** $\Delta A = 0$ **and** $\Delta B = 0$

density matrix element $\sim m_{\rm ph}^{\Delta A} m_{\rm grav}^{\Delta B}$

inequalities saturated and density matrix element nonzero only when

i)$\Delta A = 0$ if the set of out-going single particle currents

$$\left\{ e_1 \frac{p_{1\mu}}{2\omega_{p_1}}, e_2 \frac{p_{2\mu}}{2\omega_{p_2}}, \ldots \right\} \textbf{ equals } \left\{ e_1' \frac{p'_{1\mu}}{2\omega_{p'_1}}, e_2' \frac{p_{2\mu}{}'}{2\omega_{p'_2}}, \ldots \right\}$$

ii)$\Delta B = 0$ if the outgoing single particle stress-energies

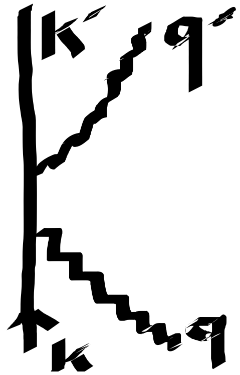
$$\left\{ \frac{p_{1\mu}p_{1\nu} - g_{\mu\nu}p_1^2}{2\omega_{p_1}}, \frac{p_{2\mu}p_{2\nu} - g_{\mu\nu}p_2^2}{2\omega_{p_2}}, \ldots \right\} \textbf{ equals}$$

$$\left\{ \frac{p'_{1\mu}p'_{1\nu} - g_{\mu\nu}p_1^{2\,\prime}}{2\omega_{p'_1}}, \frac{p'_{2\mu}p'_{2\nu} - g_{\mu\nu}p_2^{2\,\prime}}{2\omega_{p'_2}}, \ldots \right\}$$

A matrix element of the density matrix $\rho_{\beta\beta'}$ is nonzero if the distributions of charge currents and stress-energy currents in the states $|\beta>$ and $|\beta'>$ are identical.

Decoherence!

Example: Compton scattering in QED



$$\rho_{k'_1,q'_1;k'_2,q'_2} \sim (m_{\rm ph})^{\frac{e^2}{4\pi^2}\left[\frac{1}{2\beta}\ln\frac{1+\beta}{1-\beta}-1\right]} \ , \ \beta = \text{relative electron velocity}$$

Exponent ≥ 0. Exponent $= 0$ *only when* $\beta = 0$

As $m_{\rm ph.} \to 0$, $\rho_{k',q';k',q'} \neq 0$ but $\rho_{k'_1,q'_1;k'_2\neq k'_1,q'_2} \to 0$.

Implication:

Probability of $|k, q> \to |k'q'>$ as in QFT textbooks

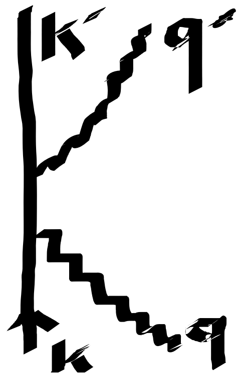
But $|kq> \to \frac{1}{\sqrt{2}}\{|k'_1q'_1> + |k'_2q'_2>\}$ is $\frac{1}{2}\left(P_{kq\to k'_1q'_1} + P_{kq\to k'_2q'_2}\right)$

Decoherence of out-state!

Conclusions

- The solution of the infrared problem in quantum electrodynamics and in perturbative quantum gravity leads to a fundamental decoherence of final states in scattering experiments.
- Could such a decoherence coming from quantum gravity be observable?
- What if the photon/graviton has a small mass?
- Applications to Hawking evaporation of black holes with "soft hair".

What if the photon has a mass?



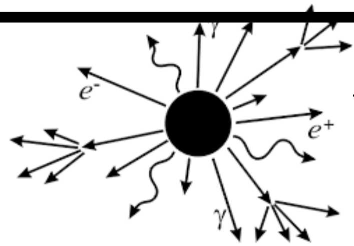
$$\rho_{k'_1,q'_1;k'_2,q'_2} \sim (m_{\rm ph.})^{\frac{e^2}{4\pi^2}\left[\frac{1}{2\beta}\ln\frac{1+\beta}{1-\beta}-1\right]}$$

$$\sim (m_{\rm ph.})^{\frac{e^2}{8\pi^2}\beta^2} \quad (\beta << 1) \quad , \quad \sim (m_{\rm ph.})^{\frac{e^2}{8\pi^2}\ln\frac{2}{1-\beta}} \quad (\beta \to 1)$$

Experimental bound on photon mass $m_{\rm ph} < 10^{-32} m_{\rm el.}$

$$\sim e^{-0.04\beta^2} \quad (\beta << 1) \quad , \quad \sim \left(\frac{1-\beta}{2}\right)^{0.04} \quad (\beta \to 1)$$

Gravity is even more weakly coupled.



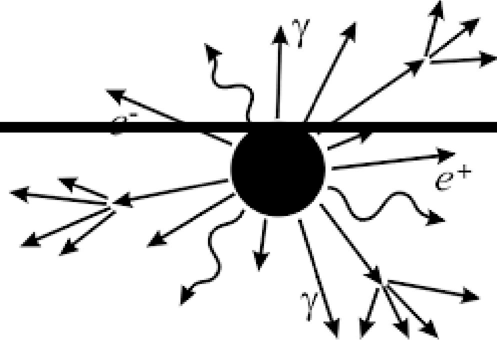
nformation paradox

In a theory of quantum gravity, the collision of two high-energy particles (i.e. gravitons) could produce a black hole which would the evaporate by emitting Hawking radiation.

Pure quantum state of two incoming particles evolves to thermal state of Hawking radiation.

$$|\alpha><\alpha| \;\rightarrow\; \sum_E e^{-\beta_H E}\;|E><E|$$

In a unitary quantum mechanical theory, a pure state should not evolve to a mixed state.



Strominger's idea (**A.Strominger, arXiv:1706.07143**):
soft gravitons purify the Hawking radiation

$$|\Psi >= \sum_E |E, \mathrm{soft} >$$

$$\rho = \mathrm{Tr}_{\mathrm{soft}} |\Psi >< \Psi| = \sum_E e^{-\beta_H E} \, |E >< E|$$