Temperature dependence of bulk and shear viscosities from lattice $SU(3)$–gluodynamics

N. Astrakhantsev, V. Braguta, A. Kotov

based on arXiv:1701.02266, JHEP 1704 (2017) 101
& new results

ITEP Lattice

ICNFP 2017

17-29 August 2017, Crete

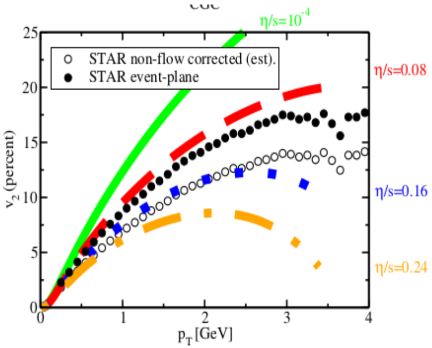
Outline

- Introduction
- Details of the calculation
- Shear viscosity
  - Fitting of the data
  - Backus-Gilbert method
- Bulk viscosity
  - Middle point method
  - Backus-Gilbert method
- Conclusion

Relativistic Hydrodynamics

- $T^{\mu\nu} = (e+p)u^\mu u^\nu + pg^{\mu\nu} + (\eta \nabla^{\langle\mu} u^{\nu\rangle} + \zeta \Delta^{\mu\nu}\nabla_\alpha u^\alpha) + ...$
  $\nabla^\alpha = \Delta^{\alpha\nu}\partial_\nu, \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$
  $\nabla^{\langle\mu} u^{\nu\rangle} = \nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha$
- EOM $\partial_\mu T^{\mu\nu} = 0$
- Non-relativistic limit $(u^\mu = (1, \vec{v}))$
  - *Continuity equation:* $\partial_t \rho + \rho(\vec{\partial}\vec{v}) + \vec{v}\vec{\partial}\rho = 0$
  - *Navier–Stokes equation:* $\frac{\partial v^i}{\partial t} + v^k \frac{\partial v^i}{\partial x^k} = -\frac{1}{\rho}\frac{\partial p}{\partial x^i} - \frac{1}{\rho}\frac{\partial \Pi^{ki}}{\partial x^k}$
  - *Viscous stress tensor:* $\Pi^{ik} = -\eta\left(\frac{\partial v^i}{\partial x^k} + \frac{\partial v^k}{\partial x^i} - \frac{2}{3}\delta^{ik}\frac{\partial v^l}{\partial x^l}\right) - \zeta\delta^{ik}\frac{\partial v^l}{\partial x^l}$
- η — shear viscosity, ζ — bulk viscosity

Relativistic hydrodynamics & QGP



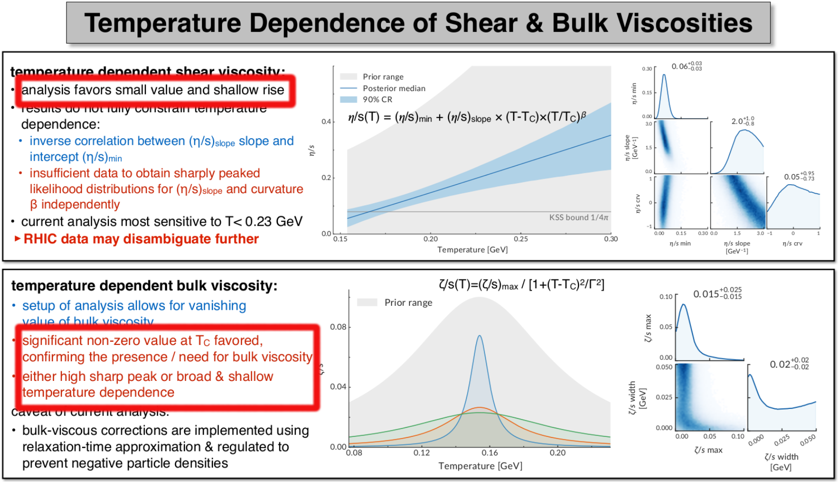
- Elliptic flow from STAR experiment (Nucl. Phys. A 757, 102 (2005))

$$\frac{dN}{d\phi} \sim (1 + 2v_1 \cos(\phi) + 2v_2 \cos^2(\phi)), \phi\texttt{-scattering angle}$$

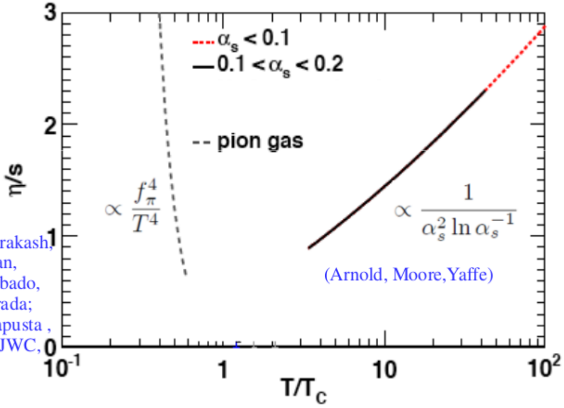
- Quark-gluon plasma is close to ideal liquid ($\frac{\eta}{s} = (1-3)\frac{1}{4\pi}$)

M. Luzum and P. Romatschke, Phys. Rev. C 78, 034915 (2008)

QM2017 (talk by S.Bass)



Shear viscosity in two limits



(Prakash, Prakash, Venugopalan, Welke; Dobado, Llanes-Estrada; Csernai, Kapusta, McLerran; JWC, Nakano)

Our goal

First-principle determination of shear and bulk viscosities!

Lattice calculation of shear & bulk viscocity

The first step:

Measurement of the correlation functions:

$$C_{sh}(t) = \int d^3\vec{x} \langle T_{12}(t, \vec{x}) T_{12}(0) \rangle$$

$$C_b(t) = \int d^3\vec{x} \langle T_{\mu\mu}(t, \vec{x}) T_{\nu\nu}(0) \rangle$$

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The second step (analytical continuation):

Calculation of the spectral function $\rho(\omega)$:

$$C(t) = \int_0^\infty d\omega \rho(\omega) \frac{\cosh\left(\frac{\omega}{2T} - \omega t\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

$$\eta = \pi \lim_{\omega \to 0} \frac{\rho_{sh}(\omega)}{\omega}$$

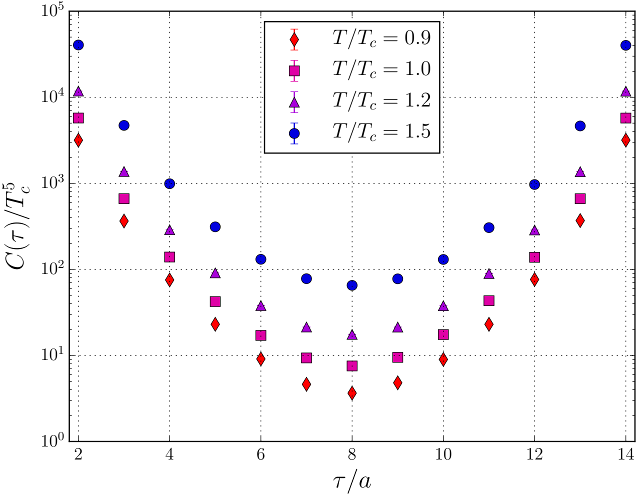
$$\zeta = \frac{\pi}{9} \lim_{\omega \to 0} \frac{\rho_b(\omega)}{\omega}$$

Details of the calculation

- $SU(3)$–gluodynamics
- Two-level algorithm (only for gluodynamics)
- Lattice size $32^3 \times 16$
- Temperatures $T/T_c = 0.9, 0.925, 0.95, 1.0, 1.2, 1.35, 1.5$
- Accuracy $\sim 2-3\%$ at $t = \frac{1}{2T}$
- For $\langle T_{12}(x)T_{12}(y)\rangle \sim (\langle T_{11}(x)T_{11}(y)\rangle - \langle T_{11}(x)T_{22}(y)\rangle)$
- Clover discretization for the $\hat{F}_{\mu\nu}$
- Renormalization of EMT: F. Karsch, Nucl.Phys. B205 (1982) 285

Shear viscosity

Correlation functions (shear viscosity)



Spectral function

$$C_{sh}(t) = \int_0^\infty d\omega \rho_{sh}(\omega) \frac{\cosh\left(\frac{\omega}{2T} - \omega t\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Properties of the spectral function:

- $\rho(\omega) \geq 0, \ \rho(-\omega) = -\rho(\omega)$
- Asymptotic freedom: $\rho(\omega)|_{\omega\to\infty}^{NLO} = \frac{1}{10}\frac{d_A}{(4\pi)^2}\omega^4\left(1 - \frac{5N_c\alpha_s}{9\pi}\right)$

  $\sim$ 90% of the total contribution $t = 1/(2T)$
- Hydrodynamics: $\rho(\omega)|_{\omega\to 0} = \frac{\eta}{\pi}\omega$

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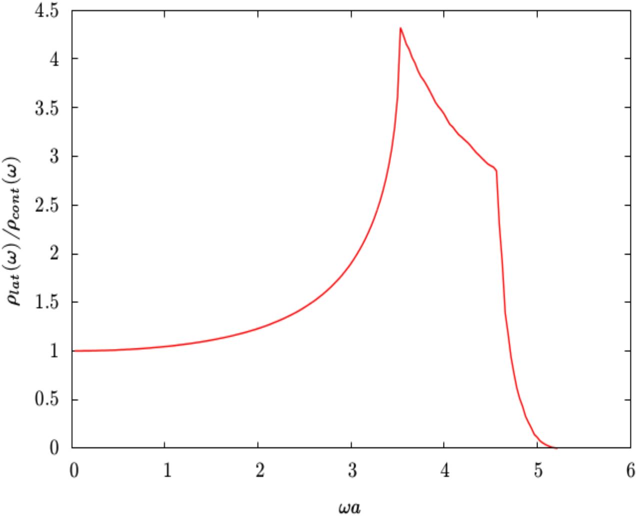
Ansatz for the spectral function (QCD sum rules motivation)

$$\rho(\omega) = \frac{\eta}{\pi}\omega\theta(\omega_0 - \omega) + A\rho_{lat}(\omega)\theta(\omega - \omega_0)$$

Lattice spectral function



Properties of the spectral function

- Hydrodynamical approximation works well up to $\omega < \pi T \sim 1$ GeV (H.B. Meyer, arXiv:0809.5202)

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- Hydrodynamical approximation works well up to $\omega < \pi T \sim 1$ GeV (H.B. Meyer, arXiv:0809.5202)
- Asymptotic freedom works well from $\omega > 3$ GeV
- Poor knowledge of the spectral function in the region $\omega \in (1, 3)$ GeV
  $\Rightarrow$ Main source of uncertainty in the fitting procedure

Backus-Gilbert method for the spectral function

- Problem: find $\rho(\omega)$ from the integral equation

$$C(x_i) = \int\limits_0^{\infty} d\omega \rho(\omega) K(x_i,\omega), \quad K(x_i,\omega) = \frac{\cosh\left(\frac{\omega}{2T} - \omega x_i\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

- Define an estimator $\tilde{\rho}(\bar{\omega})$ ($\delta(\bar{\omega},\omega)$ - resolution function):

$$\tilde{\rho}(\bar{\omega}) = \int_0^{\infty} d\omega \hat{\delta}(\bar{\omega},\omega)\rho(\omega)$$

- Let us expand $\delta(\bar{\omega},\omega)$ as

$$\delta(\bar{\omega},\omega) = \sum_i b_i(\bar{\omega}) K(x_i,\omega) \quad \tilde{\rho}(\bar{\omega}) = \sum_i b_i(\bar{\omega}) C(x_i)$$

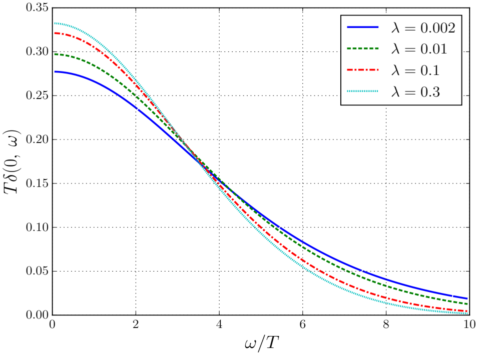
- Goal: minimize the width of the resolution function

$$b_i(\bar{\omega}) = \frac{\sum_j W_{ij}^{-1} R_j}{\sum_{ij} R_i W_{ij}^{-1} R_j},$$
$$W_{ij} = \int d\omega K(x_i,\omega)(\omega-\bar{\omega})^2 K(x_j,\omega), R_i = \int d\omega K(x_i,\omega)$$

- Regularization by the covariance matrix S_{ij}:

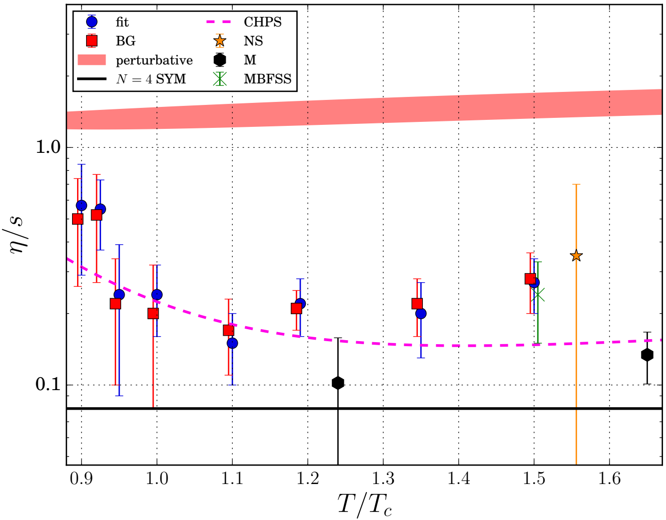
$$W_{ij} \to \lambda W_{ij} + (1-\lambda) S_{ij}, \quad 0 < \lambda < 1$$

Resolution function $\delta(0, \omega)$ $(T/T_c = 1.35)$



- Width of the resolution function $\omega/T \sim 4$
- Hydrodynamical approximation works up to $\omega/T < \pi$
- Problem: large contribution from ultraviolet tail ($\sim 50\%$)
- Solution: UV contribution can be subtracted as we know UV part from the fitting procedure quite well

Results

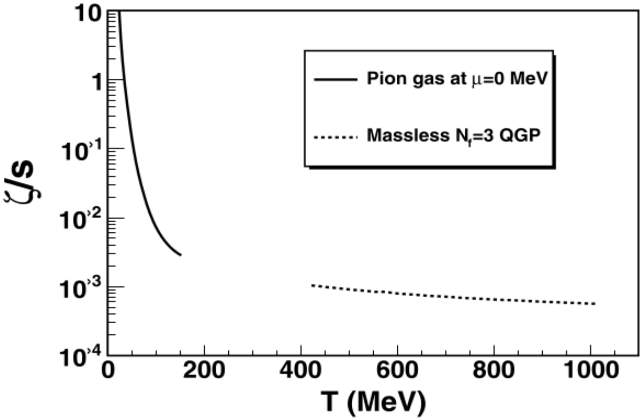


fit
BG
perturbative
$N = 4$ SYM
CHPS
NS
M
MBFSS
1.0
0.1
η/s
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
T/T_c

Bulk viscosity

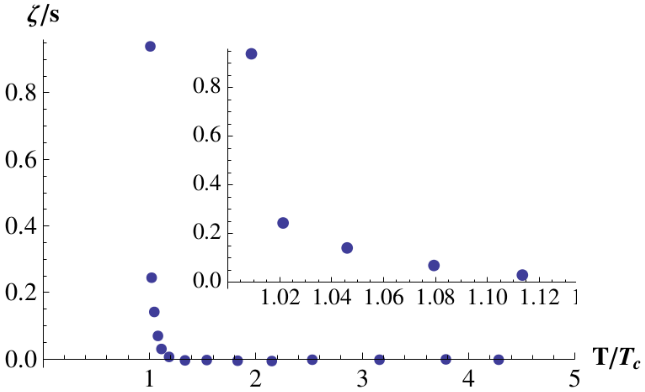
Preliminary results

Bulk viscosity in two limits



- **CHPT:** A. Dobado, F.J. Llanes-Estrada, J.M. Torres-Rincon, Physics Letters B 702 (2011) 43
- **Perturbative QCD:** P. Arnold, C. Dogan, G. Moore , Physical Review D 74, 085021 (2006)

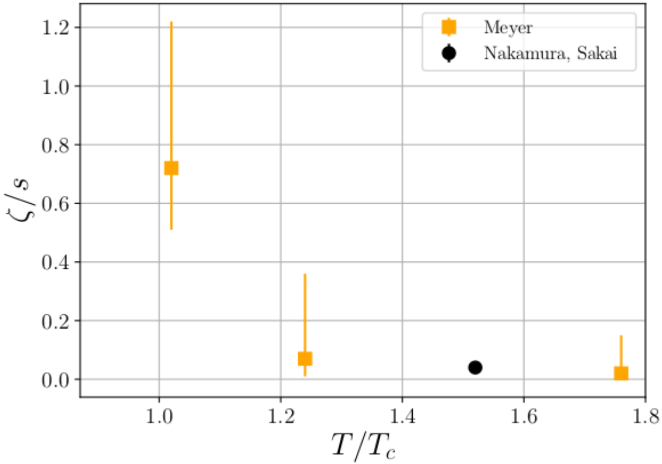
Low energy theorems of QCD



- $\zeta = \frac{1}{9\omega_0}\left(T^5 \frac{\partial}{\partial T}\frac{e-3p}{T^4} + 16\epsilon_v\right)$

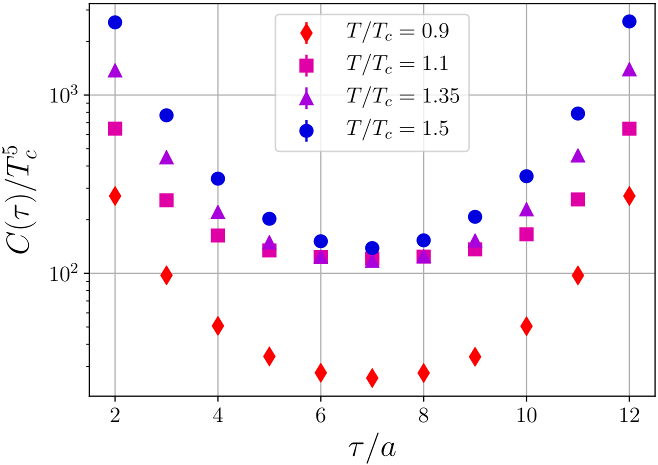
  D. Kharzeev, K. Tuchin, JHEP 0809 (2008) 093,
  D. Kharzeev, F. Karsch, K. Tuchin, Phys.Lett. B663 (2008) 217

Previous lattice works ($SU(3)$–gluodynamics)

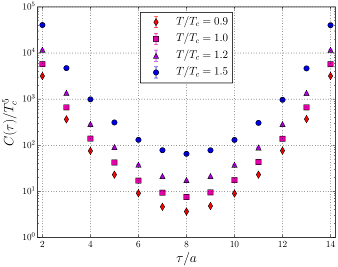


- A. Nakamura, S. Sakai Phys. Rev. Lett. 94, 072305 (2005)
- H. B. Meyer, Phys.Rev.Lett. 100 (2008) 162001

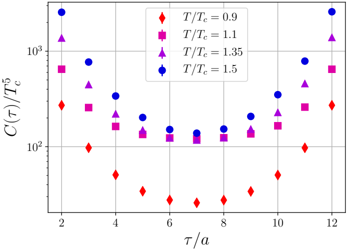
Correlation functions (bulk viscosity)



Correlation functions (shear & bulk viscosity)



shear



bulk

Spectral function

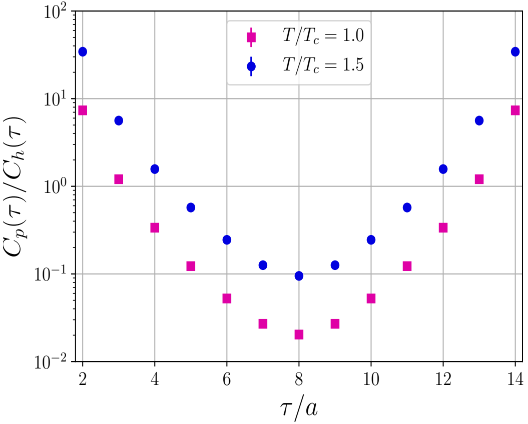
$$C_b(t) = \int_0^\infty d\omega \rho_b(\omega) \frac{\cosh\left(\frac{\omega}{2T} - \omega t\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Properties of the spectral function:

- $\rho(\omega) \geq 0,\ \rho(-\omega) = -\rho(\omega)$
- Asymptotic freedom: $\rho(\omega)|_{\omega\to\infty}^{NLO} = d_A \left(\frac{11\alpha_s}{(4\pi)^2}\right)^2 \omega^4$

  compare with shear channel $\sim d_A \frac{1}{10(4\pi)^2} \omega^4$
- Hydrodynamics: $\rho(\omega)|_{\omega\to 0} = \frac{9}{\pi}\zeta\omega$

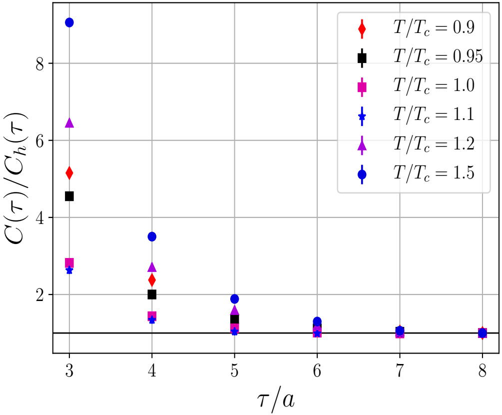
Perturbative part vs hydrodynamical part



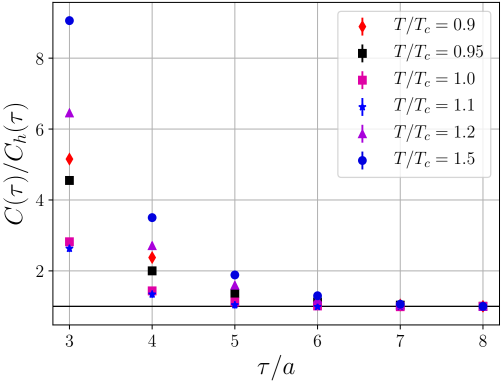
- In the region $\tau/a \sim \frac{\beta}{2}$ hydrodynamics is dominant
- In the region $\tau/a \sim$ few perturbative contribution is dominant

Hydrodynamical approximation

$$C_h(\tau) = \int_0^\infty d\omega \rho_h(\omega) \frac{\cosh\left(\frac{\omega}{2T} - \omega\tau\right)}{\sinh\left(\frac{\omega}{2T}\right)}, \quad \rho_h(\omega) = \frac{9}{\pi}\zeta\omega\theta(\omega_0 - \omega)$$



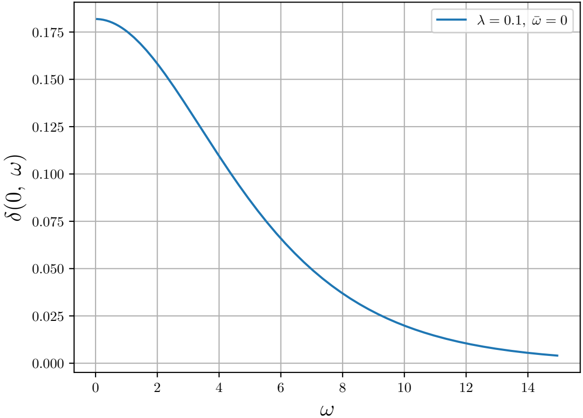
Middle point estimation of bulk viscosity



- In the vicinity of the phase transition hydrodynamics works very well!
- $C_h\left(\frac{\beta}{2}\right) = \frac{9}{\pi}\zeta \int_0^{\omega_0} d\omega \frac{\omega}{\sinh\left(\frac{\omega}{2T}\right)}$.

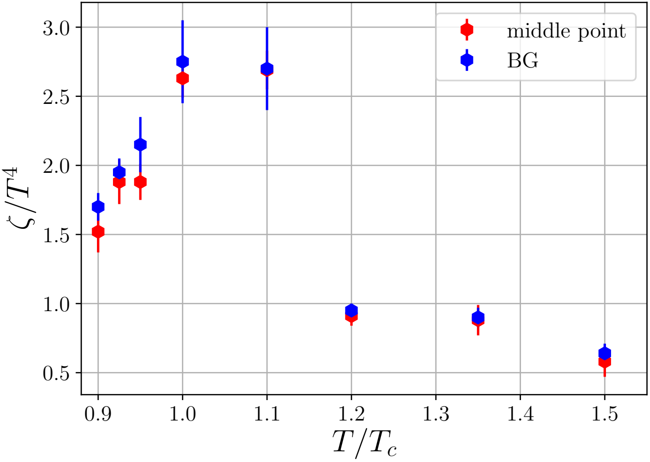
Backus-Gilbert method

Resolution function $\delta(0,\omega)$ $(T/T_c = 1.5,\ \lambda = 0.1)$

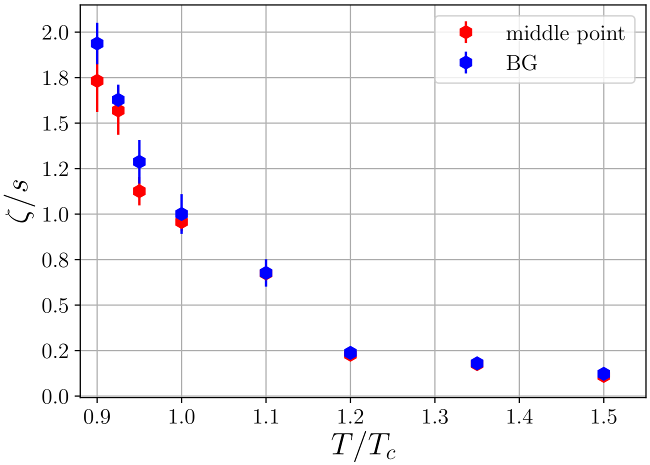


- Width of the resolution function $\omega/T \sim 5$

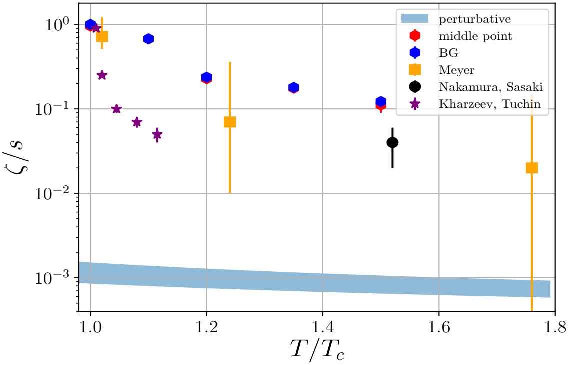
Bulk viscosity ζ/T^4 vs T (preliminary!)



Bulk viscosity ζ/s vs T (preliminary!)

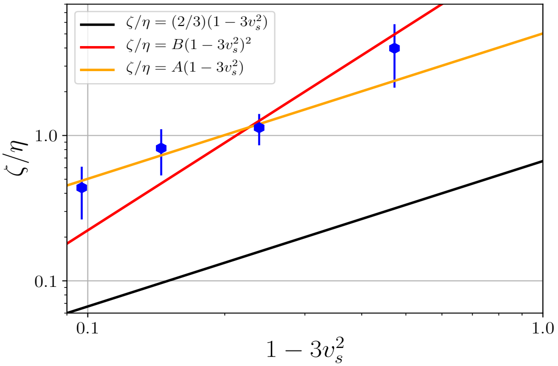


Comparison with other approaches



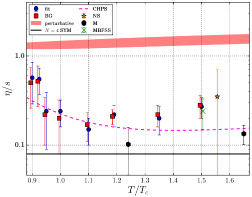
- Agreement with other lattice studies
- Large deviation from perturbative results

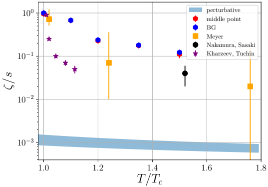
Is QGP weakly or strongly coupled?

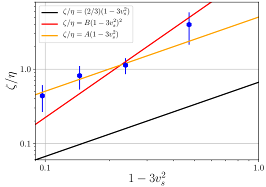


- Weakly coupled system $\zeta/\eta \sim (1-3v_s^2)^2$ $(\chi^2/dof \sim 4)$
- Strongly coupled system $\zeta/\eta \sim (1-3v_s^2)$ $(\chi^2/dof \sim 1)$
- $\zeta/\eta \geq \frac{2}{3}(1-3v_s^2)$ (A. Buchel, Physics Letters B663, 286 (2008))

Results and Conclusions







- We calculated η/s & ζ/s for set of temperatures $T/T_c \in (0.9, 1.5)$
- Agreement with previous lattice results
- Large deviation from perturbative calculation
- QGP reveals the properties of strongly coupled system