Entanglement Entropy in Yang-Mils theory: Lattice measurements vs a simple qualitatve picture

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Theory

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Measurements

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Entanglement in Four-Dimensional SU(3) Gauge Theory
Etsuko Itou, Keitaro Nagata, Yoshiyuki Nakagawa, Atsushi Nakamura, V.I. Zakharov

Basic notions and definitions

Two regions of space, $\mathcal{A}$ and $\mathcal{B}$
On $\mathcal{A} + \mathcal{B}$ lives a pure state

Total Hamiltonian $\mathcal{H} = \mathcal{H}_A \times \mathcal{H}_B$

In our case, pure state=3d vacuum, $\mathcal{A}$ segment of length l in one spatial direction and the whole space in other directions (a slab) $\mathcal{B}$ is the rest of 3d

Since the basic jbject is defined on 3d space
lattice can be operational

Entaglement entropy, replica trick

Going to the reduced density matrix

$$\rho_{\mathcal{A}} \ = \ Tr_{\mathcal{B}}\rho_{tot}$$

lose information on the region $\mathcal{B}$ Define

$$S_{\mathcal{A}} \ = \ \lim_{n\to 1}\Big(-\frac{\partial}{\partial n} ln(Tr\rho_{\mathcal{A}}^{n}\Big)$$

Replace ln ρ by $\partial_\alpha \rho^\alpha$ in the limit $\alpha \to 1$ and use
$\partial_\alpha \rho^\alpha \approx (\rho^\alpha - \rho)/\Delta\alpha$
Next step is most “difficult”: repace Hamiltonian picture by path integral.   For integer α

$$Tr\rho^{\alpha} \ = \ \frac{Z(l,\alpha)}{Z^{\alpha}}$$

where $Z(\alpha = 1) \ = \ $ (*partition function*) and $Z(l,\alpha)$ is periodic in time direction with period $\beta \cdot \alpha$

Boundary conditions

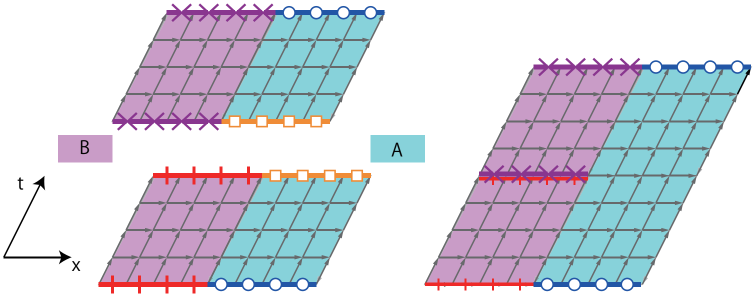

Fig. 1 from Itou et al.

Entropic function

We need one more input to define precisely what is measured on the lattice
The entaglement entropy possesses UV divergence because of d.o.f living on the boundary:

$$|\partial\mathcal{A}|/a^{d-1}$$

where a is the lattice spacing, $|\partial\mathcal{A}|$ is the the area of the boundary surface between the two regions

entropic function:

$$C(l) \;=\; \frac{l^3}{|\partial\mathcal{A}|}\frac{\partial S_{\mathcal{A}}}{\partial l}$$

Measurements

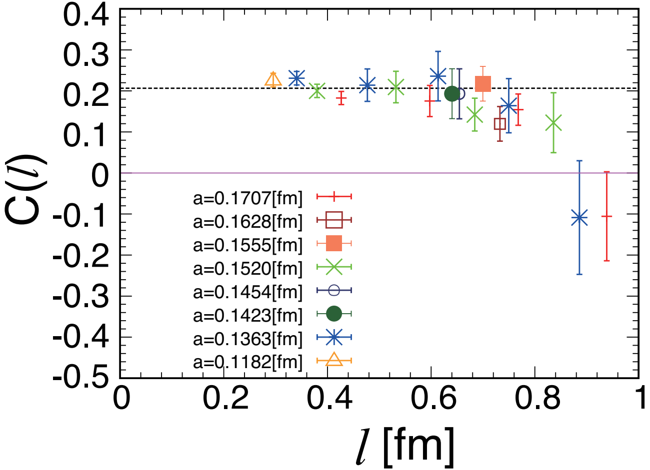

Fig. 2 from Itou et al.

Orientation in the data

Three regions:

- Asymptotic freedom, exchang by free gluons

$$0 < \ l \ \leq 0.7 fm$$

- Transition region. Evidence for fast but smooth change.

$$0.7 fm \ < \ l \ \leq 0.9 fm$$

  Earlier, a phase transition was indicated (in SU(2) case)

- Confinement region, Vanishing $C(l)$ (within error bars)

$$l \ > \ 0.9 fm$$

Data vs expectations. Qualitatively

Theorists are delighted:

- Qualitatively, similar to $V_{\bar{Q}Q}$ potential . Very simple picture
- Quantum phase transion was predicted at $l \sim \Lambda_{QCD}^{-1}$ Transition from

$$N_{d.o.f} \sim N_c^2 \text{ to } N_{d.o.f.} \sim N_c^0$$

Murugan, Klebanov, Kutasov

Data vs expectations, more quantitatively

- Free gluon exchange js calculable:

$$C(l)_{free} \approx 0.16, \; while \; C(l)_{measured} \approx 0.20$$

- Hagedorn transition(?)

$$C(l)_{Hagedorn} \sim \int dmm^{\beta} \exp(\beta_H - 2l \cdot m)$$

The factor of 2 is not confirmed. Rather, evidence that both the thermal phase transition and the quantum phase transition (in l) is not of a Hagedorn type

Holography. Witten's model

The model is in the same universality class as large-N_c YM theories in the far-infrared region. Gross features:

- There is an extra dimension, such that $z \to 0$ corresponds to ordinary space, while $z \to$ (*horizon*) corresponds to IR).
- Periodic Euclidean time $\tau \sim \tau + \beta_\tau(z)$ where the periodocity, β_τ depends on an extra coordinate z.
- There is another periodic coordinate $\sigma \sim \sigma + \beta_\sigma(z)$. Wrapping around σ counts the topological charge associated with the corresponding stringy state
- At the deconfining phase transtion, $T = T_{cr}$ the geometries in the $(\tau + z)$ and $(\sigma + z)$ coordinates are interchanged. Cylinder into a cigar-shape and vice versa

Crucial test of holography?

Relates correlation length revealed in the entanglenebt entropy (about $l_{cr} \sim 0.7cm$) and string length at which the confining potentil breaks down, near the phase transition

$$l_{entropy} = l_{confinement}$$

Probably data not much in disagreement but further efforts are needed

Conclusions

- A simple picture for the entropic function has emerged
- Qualitatively supports the model which were put forward
- Quantitatively, data seem to disagree with the Hagedorn picture
- A crucial test of holography seems possible