New Evidence for the Dominance of Collective Flow in Pb-Pb Collisions

Claude A. Pruneau
Wayne State University
for the ALICE Collaboration

6th International Conference on New Frontiers in Physics (ICNFP 2017)
August 2017, Kolymbari, Crete, Grece
The Multiple Facets of Correlation Functions

- Majority of papers published in Heavy-Ion physics involve some form of correlation measurement.
  - Flow
  - HBT: System size/geometry
  - Jet Modification
  - QGP Properties

- What else can we learn from correlation measurements?

And much more …

Figure from Data Analysis Techniques for Physical Scientists, C.P., Cambridge University Press, to appear in print Fall 2017.
A comprehensive guide to data analysis techniques for physical scientists, providing a valuable resource for advanced undergraduate and graduate students, as well as seasoned researchers. The book begins with an extensive discussion of the foundational concepts and methods of probability and statistics under both the frequentist and Bayesian interpretations of probability. It next presents basic concepts and techniques used for measurements of particle production cross-sections, correlation functions, and particle identification. Much attention is devoted to notions of statistical and systematic errors, beginning with intuitive discussions and progressively introducing the more formal concepts of confidence intervals, credible range, and hypothesis testing. The book also includes an in-depth discussion of the methods used to unfold or correct data for instrumental effects associated with measurement and process noise as well as particle and event losses, before ending with a presentation of elementary Monte Carlo techniques.

CLAUDE PRUNEAU is a Professor of Physics at the Wayne State University Faculty, from where he received the 2006 Excellence in Teaching Presidential award. He is also a member of the ALICE collaboration, and conducts an active research program in the study of the Quark Gluon Plasma produced in relativistic heavy ion collisions at the CERN Large Hadron Collider. He has worked as a Research Fellow at both Atomic Energy for Canada Limited and McGill University, and is a member of the American Physical Society, Canadian Association of Physicists and the Union of Concerned Scientists.

Cover image courtesy of CERN.
Correlation Function Measurements

- **Correlation functions** are based on **n-cumulants**
- Cumulants are calculated starting w/ **n-particle densities**

\[
\rho_1(\vec{p}_1) = \rho_1(\phi_1, \eta_1, p_{T,1}) \\
\rho_2(\vec{p}_1, \vec{p}_2) = \rho_2(\phi_1, \eta_1, p_{T,1}, \phi_2, \eta_2, p_{T,2}) \\
\vdots \\
\rho_n(\vec{p}_1, \ldots, \vec{p}_n) = \rho_n(\phi_1, \eta_1, p_{T,1}, \ldots, \phi_n, \eta_n, p_{T,n})
\]

- **n-Particle densities** can be
  - **fully differential**
  - **partially integrated** *(some coordinates marginalized)*
    - **Examples:**
      \[
      \rho_1(\phi_1) = \int_{\Omega_1} \rho_1(\phi_1, \eta_1, p_{T,1}) d\eta_1 d\phi_1 \\
      \rho_2(\eta_1, \eta_2) = \int_{\Omega_1, \Omega_2} \rho_2(\phi_1, \eta_1, p_{T,1}, \phi_2, \eta_2, p_{T,2}) d\phi_1 d\eta_1 d\phi_2 d\eta_2 \\
      \]
  - **fully integrated** — all coordinates marginalized.

\[
\langle N \rangle = \int_{\Omega_1} \rho_1(\phi_1, \eta_1, p_{T,1}) d\eta_1 d\phi_1 \\
\langle N_1 N_2 \rangle = \int_{\Omega_1, \Omega_2} \rho_2(\phi_1, \eta_1, p_{T,1}, \phi_2, \eta_2, p_{T,2}) d\phi_1 d\phi_2 d\eta_1 d\eta_2 \quad \text{if} \quad \Omega_1 \neq \Omega_2 \\
\langle N(N-1) \rangle = \int_{\Omega_1, \Omega_2} \rho_2(\phi_1, \eta_1, p_{T,1}, \phi_2, \eta_2, p_{T,2}) d\phi_1 d\phi_2 d\eta_1 d\eta_2 \quad \text{if} \quad \Omega_1 = \Omega_2 \\
\langle N(N-1) \cdots (N-n+1) \rangle = \int_{\Omega} \rho_n(\phi_1, \eta_1, p_{T,1}, \ldots, \phi_n, \eta_n, p_{T,n}) d\phi_1 d\eta_1 d\phi_2 d\eta_2 \cdots d\phi_n d\eta_n \quad \text{if} \quad \Omega_1 = \Omega_2 = \ldots = \Omega_n
\]

Very many ways to marginalize the densities to bring focus on specific aspects of correlations.
Correlation Function Measurements (2)

- n-Cumulants
  - Densities

\[
\rho_1(\eta_1) = \int d\phi_1 dp_{T,1} \rho_1(\phi_1, \eta_1, p_{T,1}) \\
\rho_2(\eta_1, \eta_2) = \int d\phi_1 dp_{T,1} \int d\phi_2 dp_{T,2} \rho_2(\phi_1, \eta_1, p_{T,1}, \phi_2, \eta_2, p_{T,2})
\]

Cumulants

\[
C_2(\eta_1, \eta_2) \equiv \rho_2(\eta_1, \eta_2) - \rho_1(\eta_1)\rho_1(\eta_2)
\]

Normalized Cumulants

\[
R_2(\eta_1, \eta_2) \equiv \frac{\rho_2(\eta_1, \eta_2) - \rho_1(\eta_1)\rho_1(\eta_2)}{\rho_1(\eta_1)\rho_1(\eta_2)}
\]
**This talk’s focus : 2-Particle Correlations**

**Densities:**

\[ \rho_1(\vec{p}_1) = \rho_1(\phi_1, \eta_1, p_{T,1}) \]
\[ \rho_2(\vec{p}_1, \vec{p}_2) = \rho_2(\phi_1, \eta_1, p_{T,1}, \phi_2, \eta_2, p_{T,2}) \]

**2-Cumulant:**

\[ C_2(\eta_1, \eta_2) = \rho_2(\eta_1, \eta_2) - \rho_1(\eta_1)\rho_1(\eta_2) \]

**Normalized 2-Cumulants:**

\[ R_2(\eta_1, \eta_2) = \frac{\rho_2(\eta_1, \eta_2) - \rho_1(\eta_1)\rho_1(\eta_2)}{\rho_1(\eta_1)\rho_1(\eta_2)} \]

**Transverse Momentum Correlator:**

\[ \langle \Delta p_T \Delta p_T \rangle_I = \frac{\int_{\text{accept}} \rho_2 \Delta p_{T,i} \Delta p_{T,j} d\eta_1 d\phi_1 dp_{T,1} d\eta_2 d\phi_2 dp_{T,2}}{\int_{\text{accept}} \rho_2 d\eta_1 d\phi_1 dp_{T,1} d\eta_2 d\phi_2 dp_{T,2}} \]

**Claude Pruneau**  
Wayne State University  
College of Liberal Arts & Sciences  
Department of Physics and Astronomy
Weighted Cumulants

M. Sharma & C.P., PRC 79, 024905 (2009)

• Transverse Momentum Correlator (Definition)

\[
\langle \Delta p_T \Delta p_T \rangle_I = \frac{\int_{\text{accept}} \rho_2 \Delta p_{T,i} \Delta p_{T,j} d\eta_1 d\phi_1 d\rho_{T,1} d\eta_2 d\phi_2 d\rho_{T,2}}{\int_{\text{accept}} \rho_2 d\eta_1 d\phi_1 d\rho_{T,1} d\eta_2 d\phi_2 d\rho_{T,2}}
\]

• Measured according to:

\[
\langle \Delta p_T \Delta p_T \rangle_I = \frac{\sum_{\alpha=1}^{N_{ev}} S^\prime \Delta p_T \Delta p_T}{\sum_{\alpha=1}^{N_{ev}} N_\alpha (N_\alpha - 1)} = \frac{\langle S^\prime \Delta p_T \Delta p_T \rangle}{\langle N(N - 1) \rangle}
\]

• where

\[S^\Delta p_T \Delta p_T = \sum_{i,j=1}^{N_\alpha} \Delta p_{T,i} \Delta p_{T,j}\]

\[\Delta p_T = p_{T,i} - \langle p_T \rangle\]

• Dimensionless pT Correlator:

\[P_2 = \frac{\langle \Delta p_T \Delta p_T \rangle}{\langle p_T \rangle^2}\]
Differential \( p_T \) correlations

M. Sharma & C.P., PRC 79, 024905 (2009)

\[
\langle \Delta p_T \Delta p_T \rangle \propto \int_{\text{accept}} \rho_2(\vec{p}_1, \vec{p}_2) \Delta p_T \Delta p_T \, dp_{T,1} \, dp_{T,2}
\]

- Not positive definite
- Sensitivity to …
  - \( p_T \) hardness of particles & correlations (e.g. HBT, jet, non-jet)
  - e-by-e temperature & average momentum fluctuations,
- collective flow
A Flow Ansatz

M. Sharma & C.P., PRC 79, 024905 (2009)

• Assume particle correlations only determined by their emission relative to the reaction plane (RP).

• Probability of Emission of one particle:

\[ P_1(\eta, \varphi, p|\Psi) = P_1(\eta, p) \left\{ 1 + 2 \sum_n v_n(\eta, p) \cos(n(\varphi - \Psi)) \right\} \]

• Joint Probability of Emission of two particles:

\[ P_2(\eta_1, \varphi_1, p_1, \eta_2, \varphi_2, p_2) = P_1(\eta_1, p_1)P_1(\eta_2, p_2) \times \left\{ 1 + 2 \sum_n v_n(\eta_1, p_1)v_n(\eta_2, p_2) \cos(n(\varphi_1 - \varphi_2)) \right\} \]
A Flow Ansatz (2)

Sharma & Pruneau, PRC 79, 024905 (2009)

• The pT\text{pT} correlator determined by:

\[ \langle \Delta p_T \Delta p_T \rangle(\eta_1, \varphi_1, \eta_2, \varphi_2) = \frac{2 \sum_n \left[ v_{n}^{\text{pT}}(\eta_1) - \langle p_T \rangle(\eta_1) v_{n}(\eta_1) \right] \left[ v_{n}^{\text{pT}}(\eta_2) - \langle p_T \rangle(\eta_2) v_{n}(\eta_2) \right] \cos\left[ n(\varphi_1 - \varphi_2) \right]}{1 + 2 \sum_n v_{n}(\eta_1) v_{n}(\eta_2) \cos\left[ n(\varphi_1 - \varphi_2) \right]} \]

= \frac{2 \sum_n \left[ v_{n}^{\text{pT}}(\eta_1) - \langle p_T \rangle(\eta_1) v_{n}(\eta_1) \right] \left[ v_{n}^{\text{pT}}(\eta_2) - \langle p_T \rangle(\eta_2) v_{n}(\eta_2) \right] \cos\left[ n(\varphi_1 - \varphi_2) \right]}{1 + 2 \sum_n v_{n}(\eta_1) v_{n}(\eta_2) \cos\left[ n(\varphi_1 - \varphi_2) \right]}

= \frac{2 \sum_n v_{n}^{\Delta p_T \Delta p_T}(\eta_1) v_{n}^{\Delta p_T \Delta p_T}(\eta_2) \cos\left[ n(\varphi_1 - \varphi_2) \right]}{2 \sum_n v_{n}(\eta_1) v_{n}(\eta_2) \cos\left[ n(\varphi_1 - \varphi_2) \right]}

• with

\[ v_{n}(\eta) = \frac{1}{P_n(\eta)} \int P_{n}(\eta, p_T) v_{n}(\eta, p_T) dp_T \]

\[ v_{n}^{\text{pT}}(\eta) = \frac{1}{P_n(\eta)} \int P_{n}(\eta, p_T) v_{n}(\eta, p_T) p_T dp_T \]

\[ \langle p_T \rangle = \frac{1}{P_n(\eta)} \int P_{n}(\eta, p_T) p_T dp_T \]

Test of Flow Dominance:
Measure Fourier coefficients of pT\text{pT} momentum correlator & compare to this ansatz with flow coefficients obtained by “standard methods”
Dimensionless pT Correlator

- Express correlator in terms of dimensionless quantity:

\[ P_2(\Delta \eta, \Delta \phi) = \frac{\langle \Delta p_T \Delta p_T \rangle}{\langle p_T \rangle^2} \]

\[ \nu_n[P_2] = \frac{\nu_n^{pT}}{\langle p_T \rangle} - \nu_n \]

“Regular” Flow Coefficient

\[ \nu_n^{pT} = \int \rho_1 \nu_n(p_T) p_T \, dp_T \]

“pT Weighted” Flow Coefficient
2-Particle Correlation Functions

- Experimental Program: Measure Correlation vs. ...
  - Collision system size, centrality, pT range, particle species, etc.
- For specific charged particle pair combinations
  - LS: Like-sign pairs
    \[ O^{(LS)} = \frac{1}{2}(O^{(++)} + O^{(-)}) \]
  - US: Unlike-sign pairs
    \[ O^{(US)} = \frac{1}{2}(O^{(+-)} + O^{(-+)}) \]
  - CI: Charge Independent
    \[ O^{(CI)} = \frac{1}{2}(O^{(LS)} + O^{(US)}) \]
  - CD: Charge Dependent
    \[ O^{(CD)} = \frac{1}{2}(O^{(US)} - O^{(LS)}) \]
R$_2$ & P$_2$ Measurements by ALICE

- Pb - Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV
- Charged particle tracks & Pairs (ITS+TPC):
  
  \[
  0.2 \leq p_T \leq 2.0; \quad |\eta| \leq 1.0; \quad |\Delta \eta| \leq 2.0
  \]
- Centrality Determination w/ V0 Detectors
- Track Quality Criteria:
  - DCA$_z$<3.2 cm, DCA$_{xy}$<2.4 cm;
  - >70 hits/track;
- Suppression of electrons with TPC track dE/dx
R₂ Correlations in Pb - Pb

P. Pujahari, C. Pruneau, et al, ALICE Collaboration, To be submitted to PRC

0.2 < pT < 2.0 GeV/c
P₂ Correlations in Pb - Pb

P. Pujahari, C. Pruneau, et al, ALICE Collaboration, To be submitted to PRC
P₂ vs R₂ Comparison


ALICE, Pb-Pb \( \sqrt{s_{NN}} = 2.76 \) TeV

\( \eta \Delta \phi \) Comparison

\( \eta \Delta \phi \) (rad)

(1)0-5%

(2)0-5%

(3)30-40%

(4)70-80%

Pb - Pb 0-5%

Very strong 3rd harmonic component

Looks Familiar?
Historical Context & Short Digression…

Adams et al., STAR Collaboration,
PRL 91 (2003) 072304

Disappearance of the away-side jet!

Adams et al, STAR Collaboration
PRL 97 (2006) 162301

Reappearance of the away-side jet!

After ZYAM subtraction…

Mark Horner, STAR Collaboration

Claude Pruneau
Wayne State University
College of Liberal Arts & Sciences
Department of Physics and Astronomy
Conical Emission Maybe?

E. Shuryak, B. Muller, H. Stocker + many others

Potential sensitivity to speed of sound in medium!!

Very Cool Concept!

Alternative Interpretation
Cherenkov radiation:
V. Koch, A. Majumder

What about flow?
How could we miss this?

We did not!
3-Particle Density
(3 views)

Combinatorial Terms

3-Cumulant
(2 views)

STAR Preliminary

C.P., STAR, QM2006 Shanghai
Anisotropic Flow

\[ P_r(\varphi_i | \psi) = 1 + 2 \sum_m v_m(i) \cos(m(\varphi_i - \psi)) \]

\[ C_{2,\text{Flow}}(\Delta \varphi_{ij}) = (2\pi)^{-2} \left\langle N_i N_j \right\rangle \left[ 1 - d_{ij} + 2 \sum_m v_m(i) v_m(j) \cos(m(\varphi_i - \psi)) \right] \]

\[ C_{3,\text{Flow}}(\Delta \varphi_{ij}, \Delta \varphi_{ik}) = (2\pi)^{-3} \left\langle N_i N_j N_k \right\rangle \left[ \Phi_3(\Delta \varphi_{ij}, \Delta \varphi_{ik}) + (1 - f_{ij}) \Phi_3(\Delta \varphi_{ik}) + (1 - f_{ik}) \Phi_3(\Delta \varphi_{ij}) + 1 - f_{ij} - f_{ik} + 2 g_{ijk} \right] \]

\[ d_{ij} = \frac{\left\langle N_i \right\rangle \left\langle N_j \right\rangle}{\left\langle N_i N_j \right\rangle}; \quad f_{ij} = \frac{\left\langle N_i N_j \right\rangle \left\langle N_k \right\rangle}{\left\langle N_i N_j N_k \right\rangle}; \quad g_{ij} = \frac{\left\langle N_i \right\rangle \left\langle N_j \right\rangle \left\langle N_k \right\rangle}{\left\langle N_i N_j N_k \right\rangle} \]

3-Cumulant

\[ d_{ij} = f_{ij} = g_{ij} = 1 \]

\[ v_4 > 0 \]

\[ \frac{v_2 v_4}{(1 - f_{ij}) v_2} = 0.5 \]

\[ \frac{v_2 v_4}{(1 - f_{ij}) v_2} = 2 \]

\[ f_{ij} < 1 \]

\[ v_4 = 0 \]
• B. Alver & G. Roland, PRC 81, 054905 (2010)
Collision-geometry fluctuations and triangular flow in heavy-ion collisions

• Underlying Assumptions
  • Initial Collisions Produce Long Range Correlations
  • Fast Thermalization >>> Medium
  • Hydrodynamic Evolution
    • Spatial Anisotropy >>> Momentum Anisotropy
  • Finite System Has Non-zero Odd Spatial Eccentricities

• Consequence:
  • Initial Geometry Fluctuations Produce Odd Harmonics
The “ridge” and the “valley”

ALICE Collaboration
PLB 708 (2012) 249–264

0-2% 2.76 TeV Pb-Pb Collisions

2 < $p_T^1$ < 2.5 GeV/c
1.5 < $p_T^2$ < 2 GeV/c
0.8 < |Δη| < 1.8

$\chi^2$/ndf = 33.3 / 35

Valley

Ridge
Fourier Decomposition of $R_2$ & $P_2$

- Examine Fourier decomposition of $P_2$ vs. $\Delta \eta$

$$O_2 = \left( b_0 + 2 \sum_{n=1}^{6} b_n \cos \left[ n (\varphi_1 - \varphi_2) \right] \right)$$

in slices of $\Delta \eta$

$$P_2(\Delta \eta, \Delta \phi) = \frac{\langle \Delta p_T \Delta p_T \rangle}{\langle p_T \rangle^2}$$

$$\nu_n [P_2] = \frac{\nu_n^{pT}}{\langle p_T \rangle} - \nu_n^{pT}$$

Flow coefficients from Scalar Product Method
$V_n[P_2], V_n[R_2]$ vs Pair Separation

**Vₙ[P₂]** vs. Collision Centrality


Very good agreement between P₂ Fourier coefficients and flow ansatz in 1 < Δη < 1.9.

Further evidence for the dominance (and existence) of flow in A+A collisions.
Summary/Conclusions

• $P_2$ correlation functions exhibit dip in 0-5% Pb - Pb collisions.
  • $P_2$ more sensitive than $R_2$ to third harmonic

• Fourier coefficients of $P_2$ …
  • “plateau earlier” than those of $R_2$.
  • factorize
  • have a magnitude in excellent agreement w/ expectation from flow ansatz.

• This work provides an additional test (and confirmation) of the strong dominance of flow in 2-particle correlations observed in Pb - Pb collisions (for pair separation in excess of 0.9 units of rapidity).

**Yes: Flow is real!**

**But: How large does the system have to be?**