J/$\psi$ and $\Upsilon$ Production Physics

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• Summary
The NRQCD Factorization Approach in Quarkonium Production

Factorization Conjecture
(GTB, Braaten, Lepage (1995))

- The inclusive cross section for producing a quarkonium at large momentum transfer ($p_T$) can be written as sum of products of short-distance coefficients with NRQCD matrix elements.

\[ \sigma(H) = \sum_n F_n(\Lambda) \langle 0 | O_n^H(\Lambda) | 0 \rangle. \]

- The matrix elements $\langle 0 | O_n^H(\Lambda) | 0 \rangle$ give the probability for the nonperturbative evolution of a $Q\bar{Q}$ pair with certain color, spin, orbital-angular-momentum quantum numbers into a quarkonium.

- The short-distance coefficients $F_n(\Lambda)$ are essentially the process-dependent partonic cross sections to make a $Q\bar{Q}$ pair convolved with the parton distributions.
• The subdiagram inside the box corresponds to an NRQCD matrix element.

• The subdiagram outside the box corresponds to a short-distance coefficient.

• The points $A(C)$ and $B(D)$ are within $\sim 1/m$ of each other.
  
  – Kinematics implies that the virtual $Q$ is off shell by order $m$.

• The points $A(B)$ and $C(D)$ are within $1/p_T$ of each other.
  
  – The part of the diagram outside the box is insensitive to changes of momentum flow from $A(B)$ to $C(D)$ of order $p_T$. 
• The short-distance coefficients can be calculated in perturbative QCD as an expansion in powers of $\alpha_s$.

• The operator matrix elements are universal (process independent).
  – The matrix elements have a known scaling with the heavy-quark velocity $v$.
  – $v^2 \approx 0.3$ for charmonium; $v^2 \approx 0.1$ for bottomonium.

• The NRQCD factorization formula for production is a double expansion in powers of $\alpha_s$ and $v$.

• A key feature of NRQCD factorization:
  Quarkonium production can occur through color-octet, as well as color-singlet, $Q\bar{Q}$ states.

• If we drop all of the color-octet contributions and retain only the leading color-singlet contribution, then we have the color-singlet model (CSM).
  – Inconsistent for $P$-wave production: IR divergent.

• Only the color-singlet production matrix elements are simply related to the decay matrix elements.

• In most calculations, quarkonium decay rates are used to fix the color-singlet matrix elements.
Status of a Proof of Factorization

- A proof is complicated because individual gluons can dress the basic production process in ways that apparently violate factorization.

- A proof of factorization would involve a demonstration that diagrams in each order in $\alpha_s$ can be re-organized so that
  - All soft singularities cancel or can be absorbed into NRQCD matrix elements,
  - All collinear singularities and spectator interactions can be absorbed into parton distributions.

- Nayak, Qiu, Sterman (2005, 2006):
  - The NRQCD matrix elements must be modified by the inclusion of eikonal lines to make them gauge invariant.
  - Factorization holds at two-loop order, but it is not known if a key cancellation generalizes to higher orders.

- Factorization of the inclusive cross section beyond two-loop order is still an open question.

- An all-orders proof is essential because the $\alpha_s$ associated with soft gluons is not small.

Comparisons of NRQCD Factorization with Experiment

Quarkonium Production at the Tevatron

Production Cross Section

- Data are more than an order of magnitude larger than the predictions of the color-singlet model.
- $p_T$ distributions are consistent with NRQCD, but not with the LO color-singlet model.
- Color-octet matrix elements are determined from fits to the data.
- Satisfactory fits can be obtained for $J/\psi$, $\psi(2S)$, $\chi_c$, $\Upsilon(1S)$ production.
- Use color-octet matrix elements from these fits to predict quarkonium production in other processes (test universality).
Polarization

• Touted as a “smoking gun” for the color-octet mechanism.

• In LO quarkonium production at large $p_T$ ($p_T \gtrsim 4m_c$ for $J/\psi$), gluon fragmentation via the color-octet mechanism dominates ($\langle O_8(3S_1) \rangle$).

• At large $p_T$, the gluon is nearly on mass shell, and, so, is transversely polarized.

• In color-octet gluon fragmentation, most of the gluon’s polarization is transferred to the quarkonium. (Cho, Wise (1994))
  
  – Spin-flip interactions are suppressed as $v^2$.
  – Verified in a lattice calculation of decay matrix elements (GTB, Lee, Sinclair (2005)).

• Radiative corrections dilute this (Beneke, Rothstein (1995); Beneke, Krämer (1996)).
$J/\psi$ Polarization

Run I:

- $d\sigma/d(\cos \theta) \propto 1 + \alpha \cos^2 \theta$.
  - $\alpha = 1$ is completely transverse;
  - $\alpha = -1$ is completely longitudinal.

- NRQCD prediction from Braaten, Kniehl, Lee (1999).
  - Feeddown from $\chi_c$ states is about 30% of the $J/\psi$ sample and dilutes the polarization.
  - Feeddown from $\psi(2S)$ is about 10% of the $J/\psi$ sample and is largely transversely polarized.

- Run I results are marginally compatible with the NRQCD prediction.

- Run II results are inconsistent with the NRQCD prediction.

- Also, inconsistent with Run I results.
  CDF was unable to track down the source of the Run I-Run II discrepancy.

Run II:
\( \psi(2S) \) Polarization

Run: I

The Run II data are incompatible with the NRQCD prediction.
In the $\Upsilon(1S)$ case, the D0 results (black) are incompatible with the CDF results (green).

- The CDF results are compatible with the NRQCD prediction (yellow).
- The D0 results are marginally incompatible with the NRQCD prediction.
- The curves are the limiting cases of the $k_T$-factorization prediction.

In the $\Upsilon(2S)$ case, the theoretical and experimental error bars are too large to make a stringent test.
**Recent Theoretical Developments**

- Campbell, Maltoni, Tramontano (2007); Artoisenet, Lansberg, Maltoni (2007): Higher-order corrections to color-singlet quarkonium production at the Tevatron are unexpectedly large.

- At high $p_T$, higher powers of $\alpha_s$ can be offset by a less rapid fall-off with $p_T$.

\[ \sim \alpha_s^3 \left( \frac{2m_c}{p_T} \right)^4 \]
NLO:

\[ \sim \alpha_s^4 \frac{(2m_c)^2}{p_T^6} \]

NNLO:

\[ \sim \alpha_s^4 \frac{1}{p_T^4} \]

\[ \sim \alpha_s^5 \frac{1}{p_T^4} \]
New Results for $J/\psi$ Production

- **Color-singlet contribution:**

![Graph showing $J/\psi$ production at Tevatron](image)

- **Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (in progress)**

- **The NNLO* calculation is an estimate based on real-emission contributions only.**

- **There is still room for a color-octet contribution, but its size may be reduced from previous estimates.** Affects the matrix elements used to compute all other processes.

- **Color-octet contribution:**

  NLO corrections are about 14% (Gong, Li, and Wang (2008)).
New Results for Color-Singlet $\Upsilon$ Production

- Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (2008)

- There is almost no room for color-octet production.
- Consistent with the fact that color-octet production is suppressed as $v^4$.
  
  $v^2 \approx 0.3$ for charmonium; $v^2 \approx 0.1$ for bottomonium.

$\Upsilon(1S)$ production at the Tevatron $s^{1/2}=1.8$ TeV

branching ratio: 2.48 %

$\mu_0=(4m_b^2+P_T^2)^{1/2}$

LDME: 9.28 GeV$^3$

$|y|<0.4$

uncertainties:

$\mu_0/2 < \mu_{r,s} < 2 \mu_0$

for $\alpha_s^5$ contributions:

$m_b^2/2 < s_{ij} < 2 m_b^2$

$P_T$ (GeV)

$d \sigma / dP_T |_{y<0.4} . Br$ (nb/GeV)

$\Upsilon(1S) \times F_{\text{direct}}$
New Results for Polarization

- Gong and Wang (2008) find that color-singlet $J/\psi$ polarization at the Tevatron changes from transverse to longitudinal when NLO corrections are included.

- NLO$^-$ excludes $gg \rightarrow J/\psi c\bar{c}$.

- Unlabeled line is contribution of $gg \rightarrow J/\psi c\bar{c}$.

- Gong, Li, and Wang (2008): NLO corrections to the color-octet contribution to the $J/\psi$ polarization have very little effect.
• Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (2008) find that color-singlet $\Upsilon$ polarization at the Tevatron changes from transverse to longitudinal when NLO and NNLO* corrections are included.

• NLO result confirmed by Gong and Wang (2008).
Discussion

- The NNLO* corrections greatly increase the color-singlet contributions to the $J/\psi$ and $\Upsilon$ cross sections.
  - The $J/\psi$ cross section still seems to be dominated by color-octet production.
  - However, the uncertainties in the NNLO* color-singlet cross section are large and are not completely reflected in the error bands.

- A reduced color-octet contribution plus significant longitudinal polarization from the color-singlet contribution could bring theory into agreement with the CDF data for $J/\psi$ polarization.

- A substantial amount of color-octet production is probably still needed to account for the deviations of the $J/\psi$ data from pure longitudinal polarization.
  - Interpretation of the Tevatron $J/\psi$ polarization data is complicated by feeddown from $\psi(2S)$ and $\chi_{cJ}$.

- A color-octet contribution is possibly needed to explain the $\Upsilon$ polarization data.

- GTB, Lee, Sinclair (2005): Lattice determinations of $J/\psi$ color-octet decay matrix elements yield values that are about an order of magnitude lower than estimates from $v$-scaling rules.
  - Suggests that $J/\psi$ color-octet production matrix elements from LO theory fits to the Tevatron data may be about an order of magnitude too large.
It had been believed that NLO color-singlet calculations leave little room for a color-octet contribution. NLO corrections increase the color-singlet contribution substantially. (Krämer, Zunft, Steegborn, Zerwas (1994); Krämer (1995))

NLO corrections include $\gamma + g \rightarrow (c\bar{c}) + gg$, which is dominated by $t$-channel gluon exchange.

For large $p_T$, this process goes as $\alpha_s^3 m_c^2 / p_T^6$, instead of $\alpha_s^2 m_c^4 / p_T^8$. 
\( \frac{d\sigma(p + g \rightarrow J/\psi + X)}{dz} (\text{nb}) \)

- KZSZ (LO, CS+CO)
- KZSZ (NLO, CS)

\[ 0.117 < a_s(M_Z) < 0.121 \]

\[ 1.3 < m_c < 1.6 \text{ GeV} \]

- ZEUS (38 pb\(^{-1}\))
- H1 (80 pb\(^{-1}\)) (scaled)
- H1 (80 pb\(^{-1}\)) high \( W \)

\[ 50 < W < 180 \text{ GeV} \]
\[ p_T > 1 \text{ GeV} \]


- NLO color-singlet calculations by (Krämer, Zunft, Steegborn, Zerwas (1994); Krämer (1995))
Recent Theoretical Developments

- Artoisenet, Campbell, Maltoni, Tramontano (2009): A new calculation of NLO color-singlet contribution
  - Confirms the analytic results of previous calculations.
  - But a more reasonable choice of renormalization/factorization scale $(\sqrt{4m_c^2 + p_T^2}$ instead of $m_c/\sqrt{2})$ yields much smaller numerical results for cross sections.

- Leaves room for a color-octet contribution.
- There is no longer an obvious conflict between the Tevatron and HERA data.
• NLO calculations have a significant effect on the color-singlet polarization predictions.
  – NLO calculation by Artoisenet, Campbell, Maltoni, Tramontano (2009).

\[
\frac{d\Gamma(J/\psi \rightarrow l^+l^-)}{d\Omega} \propto 1 + \lambda \cos^2 \theta + \mu \sin(2\theta) \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos(2\phi)
\]

• The NLO color-singlet contribution alone cannot explain the data for \( \lambda \) at large \( p_T \) or large \( z \).
• Would a color-octet contribution bring theory into agreement with data?
What Can We Learn at 10 TeV with 10 pb$^{-1}$ of Data?

The Key Physics Issues

- The main physics goal is to understand which mechanisms are important in quarkonium production.

- Measurements of $d\sigma/p_T$ and the polarization parameter $\alpha$ should give important information.

- If we assume that the production mechanisms are described within the NRQCD factorization formalism, then
  - $d\sigma/p_T$ can, in principle, allow us to determine the color-octet matrix elements, once we have subtracted the color-singlet rate.
  - $\alpha$ can give a check of the relative proportions of the color-singlet and color-octet production rates.

- It is highly desirable to measure the production rates and polarization for $J/\psi$ and $\Upsilon$ in direct production (no feeddown).
  - The theoretical calculations for feeddown are incomplete and would introduce new uncertainties.
• Unfortunately, the color-singlet rate is very uncertain, largely because of renormalization scale uncertainties.
  – For the new production mechanisms that first appear at NNLO, the NNLO cross section is the Born cross section.
  – Scale uncertainties are magnified by the appearance of five powers of $\alpha_s$ in the NNLO cross section.
• Measurements at different values of $\sqrt{s}$ allow us to take ratios of cross sections.
  – The largest theoretical uncertainties should tend to cancel in the ratio.
  – The ratio may still have some discriminating power with regard to the production mechanisms.
  – In any case, such ratios will provide another test of our theoretical ideas.
• Factorization theorems for quarkonium production, if correct, likely hold only for $p_T >> m_Q$.
  – The higher-$p_T$ reach of the LHC is therefore very important, especially for the bottomonia.
• High statistics, high-$p_T$ studies of $\Upsilon$ production and polarization give us a window into a second quarkonium system at smaller $v$ than the $J/\psi$.
  – The NRQCD velocity expansion is on more solid footing at smaller $v$.
What Production Rates Can We Expect at 10 TeV?

$J/\psi$ Production

- Estimate using LO color-singlet plus color-octet production rates. Calculation at 10 TeV courtesy of Jungil Lee and Hee Sok Chung.

Uncertainties from

- fits of color-octet matrix elements to CDF data,
- choice of renorm./fact. scale,
- $m_c$,
- choice of parton distributions (CTEQ5L compared with MRST98LO [default]).

Caveats

- Does not include effects of NLO and NNLO color-singlet corrections. Could affect the relative proportions of color-singlet and color-octet contributions. But they scale similarly with $\sqrt{s}$.
- Does not include NLO color-octet corrections (small) or DGLAP evolution of final-state partons (about $-27\%$ at $p_T = 20$ TeV).
At $p_T = 10$ GeV, the 10 TeV prompt rate is about 29 times the Tevatron prompt rate. At $p_T = 20$ GeV, the 10 TeV prompt rate is about 38 times the Tevatron prompt rate.

- The CDF Run II cross-section measurement is based on about $40 \text{ pb}^{-1}$ of data. The CDF Run I polarization measurement is based on about $110 \text{ pb}^{-1}$ of data. The CDF Run II polarization measurement is based on about $800 \text{ pb}^{-1}$ of data.

- At $p_T = 20$ GeV with $10 \text{ pb}^{-1}$ of luminosity, the LHC data samples should be about 10, 3.5, and 0.5 times the CDF data samples.

- It should be possible to make competitive cross-section and polarization measurements and perhaps even to extend the $p_T$ range of the cross-section measurement to $p_T = 25$ GeV or so.

- It is worth looking at $d\sigma/dp_T$ and polarization for the $\psi(2S)$. The rate is lower, but complications from feeddown are absent.
• Estimate the direct $J/\psi$ rate from the LO color-singlet plus color-octet contributions.

Calculation at $\sqrt{s} = 10$ TeV courtesy of Jungil Lee and Hee Sok Chung.

• The direct rate is about 50% of the prompt $J/\psi$ rate at large $p_T$.

• At $p_T = 20$ GeV,
  $d\sigma / dp_T \times B_{\mu\mu} = 0.11$ pb/GeV.
• The NNLO color-singlet $J/\psi$ direct production rate is smaller than the estimate based on the LO color-singlet plus color-octet contributions.

- Calculation at 10 TeV courtesy of Pierre Artoisenet.
- Based on the calculation of Artoisenet, Campbell, Maltoni, Tramontano (2008).
- Note that $B_{\mu\mu} \approx 0.0593$ is not included as a factor in this plot.
- At $p_T = 20$ GeV,
  $$d\sigma/dp_T \times B_{\mu\mu} = 0.11-0.47 \text{ pb/GeV}.$$ 
- At $p_T = 20$ TeV, this is about 10–40% of the direct production rate from LO color-singlet plus color-octet contributions.
**ϒ(1S) Production Rates**

- Use the NNLO* color-singlet $ϒ$ production rate to estimate the direct $ϒ$ production rate.

- **Calculation at 10 TeV courtesy of** Pierre Artoisenet.

- **Based on the calculation of** Artoisenet, Campbell, Maltoni, Tramontano (2008).

- **Note that** $B_{μμ} ≈ 0.0248$ is not included as a factor in this plot.

- Probably underestimates the direct rate.
  (The LO color-octet $ϒ$ production rate is expected to be comparable to the NNLO* color-singlet production rate.)

- The ratio of the direct rate at $\sqrt{s} = 10$ TeV to the direct rate at $\sqrt{s} = 1.96$ TeV is probably more reliable.
  - At $p_T = 10$ GeV, the ratio of direct rates is about 50.
  - At $p_T = 20$ GeV, the ratio of direct rates is about 77.
• The CDF Run I cross-section and polarization measurements are based on about 77 pb$^{-1}$ of data.
  The D0 Run II cross-section measurement is based on about 159 pb$^{-1}$ of data.
  The D0 Run II polarization measurement is based on about 1.3 fb$^{-1}$ of data.

• At $p_T = 20$ GeV with 10 pb$^{-1}$ of luminosity, the LHC data samples should be about 10 times the CDF Run I data samples and 5 and 0.6 times the D0 Run II data samples.

• At $\sqrt{s} = 14$ TeV and $p_T = 20$ GeV, the direct $\Upsilon$ production rate is expected to be about 50% of the prompt $\Upsilon$ production rate (Krämer (2001)).

• It should be possible to make competitive cross-section and polarization measurements and perhaps even to extend the $p_T$ range of the cross-section measurement.
Ratios of Cross Sections at Different Values of \( \sqrt{s} \)

- We may be able to disentangle the color-singlet and color-octet contributions by taking ratios of cross sections at different values of \( \sqrt{s} \).
  - The largest theoretical uncertainties cancel.
  - In a given experiment, some systematic uncertainties may tend to cancel as well.

- \( R \) is the ratio of the direct production rate at \( \sqrt{s} = 10 \) TeV to the direct production rate at \( \sqrt{s} = 1.96 \) TeV.
  - \( R_1 \) is the color-singlet ratio of rates.
  - \( R_8 \) is the color-octet ratio of rates.
  - \( R^{\text{Exp}} \) is the experimental ratio of cross sections.
  - \( r^X \) is the ratio of the color-octet contribution to the color-singlet contribution at experiment \( X \).

- If \( R_1 \) and \( R_8 \) are well separated, then we can use \( R^{\text{Exp}} \) to determine \( r^X \).

\[
R^{\text{Exp}} = \frac{\sigma_1^{\text{LHC}} + \sigma_8^{\text{LHC}}}{\sigma_1^{\text{TeV}} + \sigma_8^{\text{TeV}}} = \frac{R_1 + r^{\text{TeV}} R_8}{1 + r^{\text{TeV}}}, \quad \text{where} \quad r^{\text{TeV}} = \frac{\sigma_8^{\text{TeV}}}{\sigma_1^{\text{TeV}}} = \frac{R_1}{R_8} r^{\text{LHC}}.
\]

- Then \( r^{\text{Tev}} = (R^{\text{Exp}} - R_1)/(R_8 - R^{\text{Exp}}) \) can be used to make predictions for the polarization.
Ratios of $J/\psi$ Production Rates

- At $p_T = 10$ GeV
  - Ratio of LO color-octet contributions: $R_8 \approx 27$
  - Ratio of NNLO* color-singlet contributions: $R_1 \approx 22–32$

- At $p_T = 20$ GeV
  - Ratio of LO color-octet contributions: $R_8 \approx 38$
  - Ratio of NNLO* color-singlet contributions: $R_1 \approx 42–50$

- It is not clear if $R_1$ and $R_8$ are sufficiently well separated to compute $r^{\text{Tev}}$.

- But measurement of $R^{\text{Exp}}$ is still an important test of theory.

- More theoretical work is needed to pin these numbers down and to assess uncertainties.
Ratios of $\Upsilon(1S)$ Production Rates

- At $p_T = 10$ GeV
  - Ratio of color-octet contributions: $R_8 = ??$
  - Ratio of color-singlet contributions: $R_1 \approx 47-52$

- At $p_T = 20$ GeV
  - Ratio of color-octet contributions: $R_8 = ??$
  - Ratio of color-singlet contributions: $R_1 \approx 66-87$

- More theoretical work is needed to compute the color-octet ratio and to pin down the color-singlet ratio more precisely.
Summary

- The NRQCD factorization approach provides a systematic method for calculating quarkonium production (and decay) rates as a double expansion in powers of $\alpha_s$ and $v$.
- The NRQCD factorization conjecture for inclusive production rates has not yet been proven.
- NRQCD factorization predicts that quarkonia can be produced through both color-singlet and color-octet $Q\bar{Q}$ states.
- Color-octet matrix elements extracted from the Tevatron data in LO lead to disagreements with the polarization data at the Tevatron and in photoproduction at HERA.
- NLO and NNLO corrections to the color-singlet contributions can change them by orders of magnitude.
  - May reduce the sizes of color-octet matrix elements from the Tevatron fits.
  - The ratio of color-singlet to color-octet contributions may be much larger than previously supposed.
  - This may lead to a consistent picture for $d\sigma/p_T$ and polarization.
There is a large increase in $d\sigma/dp_T$ in going from $\sqrt{s} = 1.96$ TeV to $\sqrt{s} = 10$ TeV.

- Should allow one to make meaningful measurements of $d\sigma/dp_T$ and polarization, even with low integrated luminosities.
- It may be worthwhile to look at $d\sigma/dp_T$ and polarization for $\psi(2S)$. Complications from feeddown are absent.
- These measurements could provide important clues as to the quarkonium production mechanism (color-singlet vs. color-octet).
- The potential higher-$p_T$ reach of the LHC may be very important in sorting out theoretical issues.

Ratios of cross sections at different values of $\sqrt{s}$ have reduced theoretical uncertainties and provide important tests of the theoretical understanding of production mechanisms.
Backup Slides
\[ \gamma \gamma \rightarrow J/\psi + X \text{ at LEP} \]

- **Comparison of theory** (Klasen, Kniehl, Mihaila, Steinhauser (2001)) with Delphi data clearly favors NRQCD over the color-singlet model.

- **Theory uses** Braaten-Kniehl-Lee (1999) matrix elements from Tevatron data and MRST98LO (solid) and CTEQ5L (dashed) PDF's.

- **Theoretical uncertainties from**
  - Renormalization and factorization scales (varied by a factor 2),
  - NRQCD color-octet matrix elements,
  - Different linear combination of matrix elements than in Tevatron cross sections.
The NRQCD (Kniehl, Zwirner (2001)) prediction uses Braaten-Kniehl-Lee (1999) matrix elements extracted from the Tevatron data and MRST98LO and CTEQ5L PDF’s.

Theoretical uncertainties from

- PDF’s,
- Renormalization and factorization scales (varied by a factor 2),
- NRQCD color-octet matrix elements,
- Different linear combination of matrix elements than in Tevatron cross sections.
The H1 data plotted as a function of $Q^2$ favor the NRQCD prediction over the color-singlet-model prediction. ($Q$ is the virtual-photon momentum.)

H1 data vs. leading-order NRQCD (upper) and Color-Singlet Model (lower).
• The H1 data plotted as a function of $P_T^2$ favor the NRQCD prediction over the color-singlet-model prediction. $p_T$ is the transverse momentum of the $J/\psi$. 

H1 data vs. leading-order NRQCD (upper) and Color-Singlet Model (lower).
• The H1 data plotted as a function of $z$ do not agree well with either the NRQCD prediction or the color-singlet-model prediction. ($z$ is the energy fraction of the $J/\psi$.)

• The data do not show the expected color-octet rise at $z = 1$. Resummations of the $\alpha_s$ and $\nu$ expansions are needed.

H1 data vs. leading-order NRQCD (upper) and Color-Singlet Model (lower).
• The ZEUS data are systematically lower than the H1 data and agree less well with the NRQCD prediction (but have larger error bars).

• The data plotted as a function of $z$ do not show the expected color-octet rise at $z = 1$.

Curves from A.V. Lipatov and N.P. Zotov.
Is the Assumption of 100% Polarization Transfer Valid?

- Existing calculations assume that 100% of the $Q\bar{Q}$ polarization is transferred to the quarkonium.

- Spin-flip corrections are suppressed only by $v^2$, not $v^4$, relative to the non-flip part.  
  (GTB, Braaten, Lepage)

- It could happen that the spin-flip corrections are anomalously large.

- Do the velocity-scaling rules need to be modified?  
  (Brambilla, Pineda, Soto, Vairo; Fleming, Rothstein, Leibovich)

- A lattice calculation of color-octet decay matrix elements indicates that spin-flip processes are indeed suppressed by a factor $v^2$ or smaller (GTB, Lee, Sinclair).  

Polarization in Inelastic $J/\psi$ Photoproduction at HERA

\[
\frac{d\Gamma(J/\psi \rightarrow l^+l^-)}{d\Omega} \propto 1 + \lambda \cos^2 \theta + \mu \sin(2\theta) \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos(2\phi)
\]

- $\theta$ and $\phi$ are the polar and azimuthal angles of the $l^+$ 3-momentum with respect to a coordinate system that is defined in the $J/\psi$ rest frame.
- The data for $\lambda$ at high $p_T$ slightly favor the color-singlet prediction.
- The data for $\nu$ at high $p_T$ slightly favor the color-singlet + color-octet prediction.
Exclusive Double Charmonium Production at Belle and BABAR

\[ e^+e^- \rightarrow J/\psi + \eta_c \]

- **Experiment**
  - **Belle (2004):** \( \sigma [e^+e^- \rightarrow J/\psi + \eta_c] \times B_{>2} = 25.6 \pm 2.8 \pm 3.4 \text{ fb.} \)
  - **BABAR (2005):** \( \sigma [e^+e^- \rightarrow J/\psi + \eta_c] \times B_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1} \text{ fb.} \)

- **NRQCD at LO in \( \alpha_s \) and \( v \)**
  - **Braaten, Lee (2003):** \( \sigma [e^+e^- \rightarrow J/\psi + \eta_c] = 3.78 \pm 1.26 \text{ fb.} \)
  - **Liu, He, Chao (2003):** \( \sigma [e^+e^- \rightarrow J/\psi + \eta_c] = 5.5 \text{ fb.} \)

  The two calculations employ different choices of \( m_c \), NRQCD matrix elements, and \( \alpha_s \).

  Braaten and Lee include QED effects.

- **Exclusive process:** the color-octet contribution is suppressed as \( v^4 \).

- The color-singlet matrix elements are determined from \( \eta_c \rightarrow \gamma\gamma \) and \( J/\psi \rightarrow e^+e^- \).
• An important step in resolving the discrepancy:

A calculation of corrections at NLO in $\alpha_s$ by Zhang, Gao, and Chao (2005) shows that the $K$ factor is approximately 1.96.

  – Not enough to bring theory into agreement with experiment.

• There are similar discrepancies between theory and experiment for $\sigma[e^+e^- \rightarrow J/\psi + \chi_{c0}]$ and $\sigma[e^+e^- \rightarrow J/\psi + \eta_c(2S)]$. 
Discrepancy in exclusive production of $J/\psi$ plus $\chi_{c0}$, $\eta_c(2S)$ at the $B$ factories.

### Preliminary Results

<table>
<thead>
<tr>
<th>$J/\psi + c\bar{c} (\rightarrow 2\text{ charged})$</th>
<th>$\eta_c$</th>
<th>$\chi_{c0}$</th>
<th>$\eta_c(2S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N(signals)</td>
<td>127 ± 20</td>
<td>81 ± 16</td>
<td>121 ± 20</td>
</tr>
<tr>
<td>Efficiency (%)</td>
<td>29.5 ± 0.7</td>
<td>32.2 ± 0.7</td>
<td>30.2 ± 0.8</td>
</tr>
<tr>
<td>Born Cross-section (fb)</td>
<td>17.6 ± 2.8$^{+1.5}_{-2.1}$</td>
<td>10.3 ± 2.5$^{+1.4}_{-1.8}$</td>
<td>16.4 ± 3.7$^{+2.4}_{-3.0}$</td>
</tr>
<tr>
<td>Mass (MeV/$c^2$)</td>
<td>2984.8 ± 4.0$^{+4.5}_{-5.0}$</td>
<td>3420.5 ± 4.8$^{+11.5}_{-9.5}$</td>
<td>3645.0 ± 5.5$^{+4.9}_{-7.8}$</td>
</tr>
</tbody>
</table>

### Comparison to Belle & Theory

<table>
<thead>
<tr>
<th>$J/\psi c\bar{c}$</th>
<th>$\eta_c$</th>
<th>$\chi_{c0}$</th>
<th>$\eta_c(2S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nevt, BaBar (124.4 fb$^{-1}$)</td>
<td>127 ± 20</td>
<td>81 ± 16</td>
<td>121 ± 20</td>
</tr>
<tr>
<td>Nevt, Belle (155 fb$^{-1}$)$^{(*)}$</td>
<td>235 ± 26</td>
<td>89 ± 24</td>
<td>164 ± 30</td>
</tr>
<tr>
<td>$\sigma_{Born} \times B_{&gt;2}, \text{ BaBar}$</td>
<td>$^{(*)}$</td>
<td>$^{(*)}$</td>
<td>$^{(*)}$</td>
</tr>
<tr>
<td>$\sigma_{Born} \times B_{&gt;2}, \text{ Belle}$</td>
<td>$^{(*)}$</td>
<td>$^{(*)}$</td>
<td>$^{(*)}$</td>
</tr>
<tr>
<td>$^{(+)}$ NRQCD by Braaten and Lee $^{[1]}$</td>
<td>2.31 ± 1.09</td>
<td>2.28 ± 1.03</td>
<td>0.96 ± 0.45</td>
</tr>
<tr>
<td>$^{(+)}$ NRQCD by Liu, He and Chao $^{[2]}$</td>
<td>5.5</td>
<td>6.9</td>
<td>3.7</td>
</tr>
</tbody>
</table>

$^{(+)}$ cross sections in fb , $^{(*)}$ hep-ex/0412041, hep-ex/0407009

$^{[1]}$ PRD 67, 054007 (2003), $^{[2]}$ hep-ph/0408141

From J. Coleman Moriond talk.
Including Relativistic and $\alpha_s$ Corrections to $e^+e^- \rightarrow J/\psi + \eta_c$

- Relativistic corrections $\sigma[e^+e^- \rightarrow J/\psi + \eta_c]$ can come from two sources:
  - Direct corrections to the process $e^+e^- \rightarrow J/\psi + \eta_c$ itself,
  - Indirect corrections that enter through the matrix element of leading order in $v$.
    Appear when $\Gamma[J/\psi \rightarrow e^+e^-]$ is used to determine the matrix element because of relativistic corrections to the theoretical expression for $\Gamma[J/\psi \rightarrow e^+e^-]$.

- Relativistic corrections depend on matrix elements of higher order in $v$.

  - If the static $Q\bar{Q}$ potential is known exactly, then the uncertainty is of relative order $v^2$.
  - First determination of these matrix elements with small enough uncertainties to be useful.

- GTB, Chung, Kang, Kim, Lee, Yu (2006): Corrections at NLO in $\alpha_s$ plus relativistic corrections may bring theory into agreement with experiment.

- Confirmed by He, Fan, Chao (2007).


New Calculation of $\sigma[e^+e^- \rightarrow J/\psi + \eta_c]$
(GTB, Chung, Kang, Lee, Yu (2007))

- Resums a class of relativistic corrections.
  Includes all corrections that arise from the potential-model $Q\bar{Q}$-Fock-state wave function, up to the UV cutoff of NRQCD.
- Uses the results of Zhang, Gao, and Chao (2005) for the corrections of NLO in $\alpha_s$.
- Includes the interference between the relativistic corrections and the corrections of NLO in $\alpha_s$.
- Includes a detailed error analysis

$\sigma_{tot} = 17.6^{+0.8+5.3+0.7+3.9+0.7+2.8+1.6+1.4+1.9+1.32+1.89}_{-0.9-3.7-0.7-3.0-0.7-2.9-1.5-1.1-2.0-1.32-1.89} \text{fb} = 17.6^{+8.1}_{-6.7} \text{fb}$

Uncertainty in the NRQCD factorization formula: $\sim m_H^2/(s/4) \approx 34\%$. 
- $\sigma_{\text{tot}}$ consists of

<table>
<thead>
<tr>
<th>Term</th>
<th>Value (fb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leading order in $\alpha_s$ and $v$ (including indir. rel. corr., but without QED contribution)</td>
<td>5.4</td>
</tr>
<tr>
<td>QED contribution</td>
<td>1.0</td>
</tr>
<tr>
<td>Direct relativistic corrections</td>
<td>2.9</td>
</tr>
<tr>
<td>Corrections of NLO in $\alpha_s$</td>
<td>6.9</td>
</tr>
<tr>
<td>Interference between rel. corr. and corr. of NLO in $\alpha_s$</td>
<td>1.4</td>
</tr>
<tr>
<td>Total</td>
<td>17.6</td>
</tr>
</tbody>
</table>

- Indirect relativistic corrections are about 31% per quarkonium.

- Result for $\sigma[e^+e^- \rightarrow J/\psi + \eta_c]$ confirmed, within uncertainties, by He, Fan, Chao (2007).
• Theory and experiment agree within uncertainties:
  – Theory: \( \sigma[e^+e^- \to J/\psi + \eta_c] = 17.6^{+8.1}_{-6.7} \text{ fb} \)
  – Belle: \( \sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 25.6 \pm 2.8 \pm 3.4 \text{ fb} \)
  – BABAR: \( \sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1} \text{ fb} \)

• **Caveat:** \( B_{>2} \) is not known.
  – Could be as small as 0.5–0.6.
  – Even so, the error bars of theory and the BABAR experiment overlap.

• Zhang, Ma, Chao (2008): In the cases of \( \sigma[e^+e^- \to J/\psi(\psi(2S)) + \chi_{c0}] \), large \( K \) factors (\( \sim 2.8 \)) may bring theory into agreement with experiment.

• See the talk by Victor Braguta in Parallel Session C for a discussion of the light-cone approach to exclusive double-charmonium production.
Inclusive Double $c\bar{c}$ Production at Belle

- **Belle:**

\[
\frac{\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X)}{\sigma(e^+e^- \rightarrow J/\psi + X)} = 0.82 \pm 0.15 \pm 0.14
\]

\[
> 0.48 \text{ (90\% confidence level)}
\]

- **pQCD plus color-singlet model** (Cho, Leibovich (1996); Baek, Ko, Lee, Song (1997); Yuan, Qiao, Chao (1997)):

\[
\frac{\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X)}{\sigma(e^+e^- \rightarrow J/\psi + X)} \approx 0.1.
\]
The Numerator: Experiment and theory also disagree.

- Belle (2002): \( \sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X) = 0.87^{+0.21}_{-0.19} \pm 0.17 \text{ pb.} \)
- Leading-Order Theory (Color-Singlet): \( \sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X) = 0.10–0.27 \text{ pb.} \)
  
  Large renormalization-scale dependence.

New NLO Calculation of the Numerator
(Zhang and Chao (2007))

- Find a \( K \) factor of about 1.8.
- Taking into account QED corrections, two-photon processes, feeddown from \( \psi(2S) \) (the largest effect) and \( \chi_{cJ} \), and color-octet corrections, they obtain
  \( \sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X) = 0.53^{+0.59}_{-0.23} \text{ pb.} \) (\( \mu = \sqrt{s}/2 \))
  The uncertainties come from \( m_c \).
- Resolves the discrepancy between theory and experiment, but the theoretical uncertainties are large.

He, Fan, Chao (2007): Direct relativistic corrections to the numerator are only about +31%.

Nayak, Qiu, and Sterman: there could be a nonperturbative enhancement to production of \( J/\psi + c\bar{c} \) when the \( c \) or the \( \bar{c} \) is co-moving with the \( J/\psi \).
This effect can’t be calculated reliably in perturbation theory. Its size must be determined experimentally.
• New Calculation of the Denominator
  (Zhang, Ma, Chao (2008))
  \[\sigma(e^+e^- \rightarrow J/\psi^{(8)} + g) = 0.586 \text{ pb (LO, } \mu = 2m_c)\]
  \[\sigma(e^+e^- \rightarrow J/\psi^{(8)} + g) = 1.19 \text{ pb (NLO, } \mu = 2m_c)\].

  – At the \(B\)-factory energy, this color-octet process dominates in LO.
  – NLO result yields \(\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X)/\sigma(e^+e^- \rightarrow J/\psi + X) = 0.31^{+0.18}_{-0.11}\%\).
  – The color-singlet processes should also be calculated in NLO.

• The Belle result for \(\sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X)/\sigma(e^+e^- \rightarrow J/\psi + X)\) still poses a significant challenge to existing theory.

• It is important for BABAR to check the Belle results for inclusive double \(c\bar{c}\) production.