J/ψ and Υ Production Physics

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The NRQCD Factorization Approach in Quarkonium Production

Factorization Conjecture

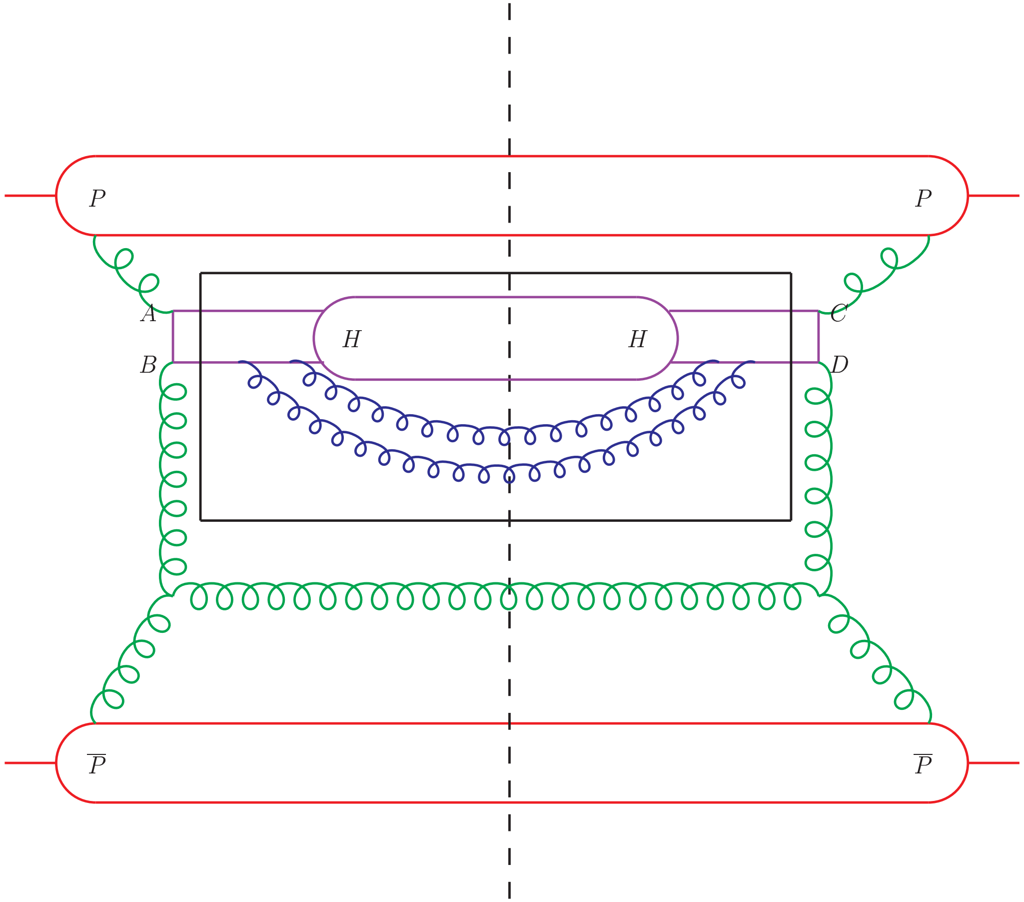
(GTB, Braaten, Lepage (1995))

- The inclusive cross section for producing a quarkonium at large momentum transfer (p_T) can be written as sum of products of short-distance coefficients with NRQCD matrix elements.

$$\sigma(H) = \sum_n F_n(\Lambda) \langle 0 | \mathcal{O}_n^H(\Lambda) | 0 \rangle .$$

- The matrix elements $\langle 0 | \mathcal{O}_n^H(\Lambda) | 0 \rangle$ give the probability for the nonperturbative evolution of a $Q\bar{Q}$ pair with certain color, spin, orbital-angular-momentum quantum numbers into a quarkonium.
- The short-distance coefficients $F_n(\Lambda)$ are essentially the process-dependent partonic cross sections to make a $Q\bar{Q}$ pair convolved with the parton distributions.

- The subdiagram inside the box corresponds to an NRQCD matrix element.
- The subdiagram outside the box corresponds to a short-distance coefficient.



- The points $A(C)$ and $B(D)$ are within $\sim 1/m$ of each other.
  - Kinematics implies that the virtual Q is off shell by order m.
- The points $A(B)$ and $C(D)$ are within $1/p_T$ of each other.
  - The part of the diagram outside the box is insensitive to changes of momentum flow from $A(B)$ to $C(D)$ of order p_T.

- The short-distance coefficients can be calculated in perturbative QCD as an expansion in powers of α_s.
- The operator matrix elements are universal (process independent).
  - The matrix elements have a known scaling with the heavy-quark velocity v.
  - $v^2 \approx 0.3$ for charmonium; $v^2 \approx 0.1$ for bottomonium.
- The NRQCD factorization formula for production is a double expansion in powers of α_s and v.
- A key feature of NRQCD factorization:
  Quarkonium production can occur through color-octet, as well as color-singlet, $Q\bar{Q}$ states.
- If we drop all of the color-octet contributions and retain only the leading color-singlet contribution, then we have the color-singlet model (CSM).
  - Inconsistent for P-wave production: IR divergent.
- Only the color-singlet production matrix elements are simply related to the decay matrix elements.
- In most calculations, quarkonium decay rates are used to fix the color-singlet matrix elements.

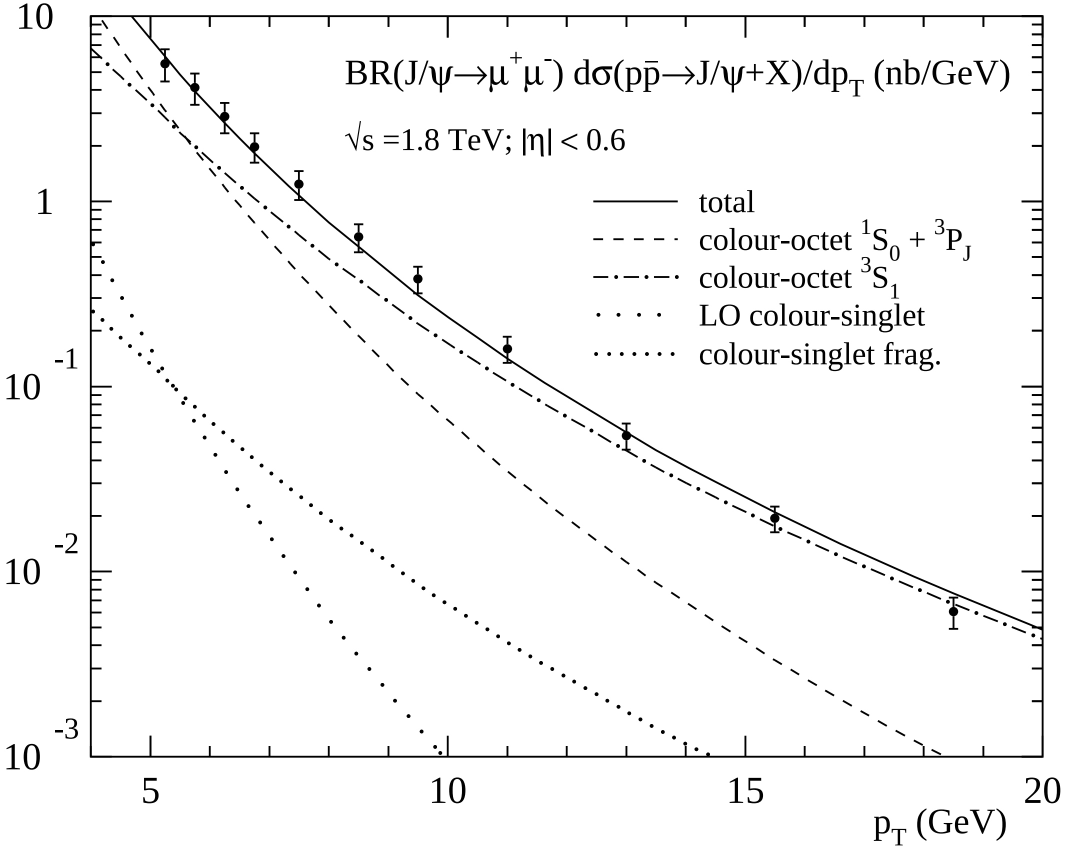
Status of a Proof of Factorization

- A proof is complicated because individual gluons can dress the basic production process in ways that apparently violate factorization.
- A proof of factorization would involve a demonstration that diagrams in each order in α_s can be re-organized so that
  - All soft singularities cancel or can be absorbed into NRQCD matrix elements,
  - All collinear singularities and spectator interactions can be absorbed into parton distributions.
- Nayak, Qiu, Sterman (2005, 2006):
  - The NRQCD matrix elements must be modified by the inclusion of eikonal lines to make them gauge invariant.
  - Factorization holds at two-loop order, but it is not known if a key cancellation generalizes to higher orders.
- Factorization of the inclusive cross section beyond two-loop order is still an open question.
- An all-orders proof is essential because the α_s associated with soft gluons is not small.
- GTB, Garcia i Tormo, Lee (2008): A proof of factorization now exists for the simpler case of exclusive quarkonium production.

Comparisons of NRQCD Factorization with Experiment

Quarkonium Production at the Tevatron

Production Cross Section



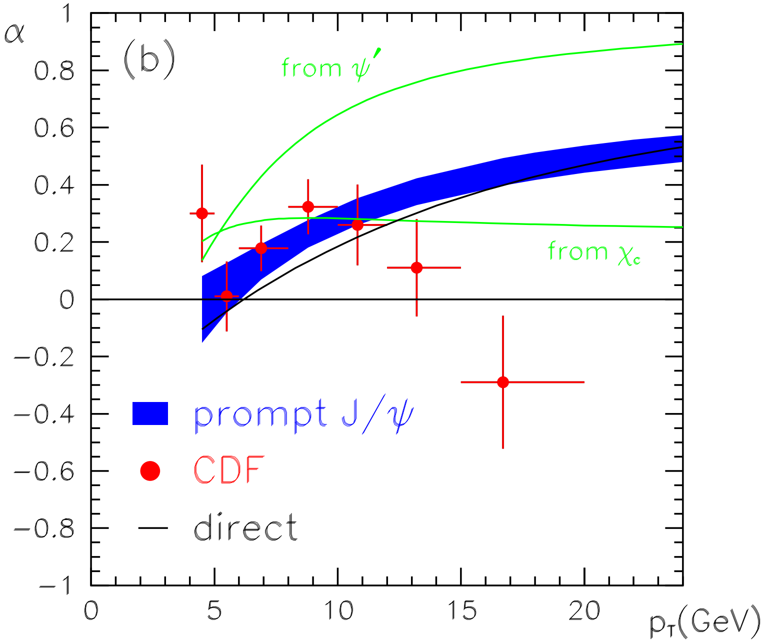
- Data are more than an order of magnitude larger than the predictions of the color-singlet model.
- p_T distributions are consistent with NRQCD, but not with the LO color-singlet model.
- Color-octet matrix elements are determined from fits to the data.
- Satisfactory fits can be obtained for J/ψ, $\psi(2S)$, χ_c, $\Upsilon(1S)$ production.
- Use color-octet matrix elements from these fits to predict quarkonium production in other processes (test universality).

Polarization

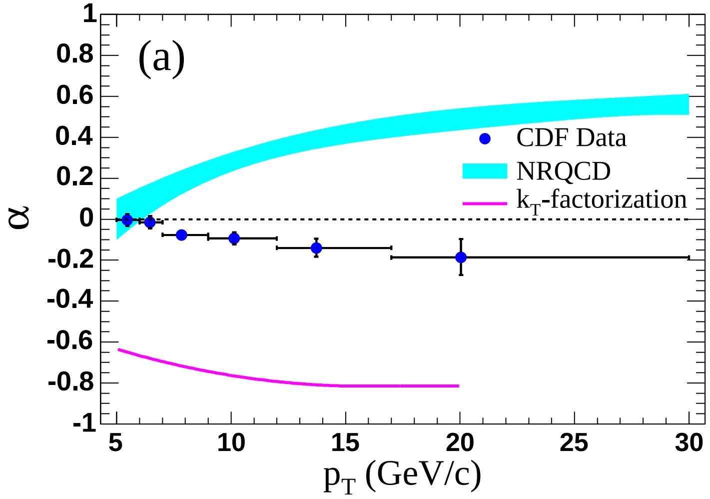
- Touted as a “smoking gun” for the color-octet mechanism.
- In LO quarkonium production at large p_T ($p_T \gtrsim 4m_c$ for J/ψ), gluon fragmentation via the color-octet mechanism dominates ($\langle \mathcal{O}_8(^3S_1) \rangle$).
- At large p_T, the gluon is nearly on mass shell, and, so, is transversely polarized.
- In color-octet gluon fragmentation, most of the gluon’s polarization is transferred to the quarkonium. (Cho, Wise (1994))
  - Spin-flip interactions are suppressed as v^2.
  - Verified in a lattice calculation of decay matrix elements (GTB, Lee, Sinclair (2005)).
- Radiative corrections dilute this (Beneke, Rothstein (1995); Beneke, Krämer (1996)).

J/ψ Polarization

Run I:



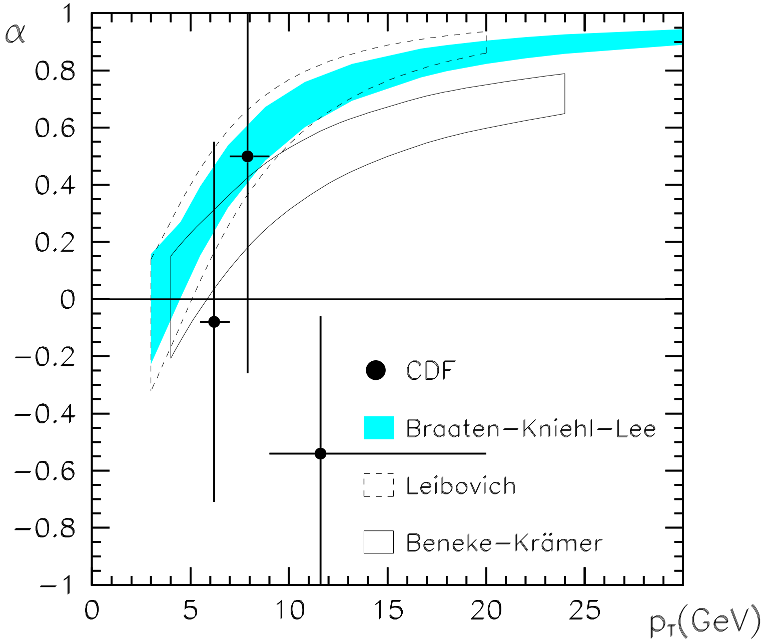
Run II:



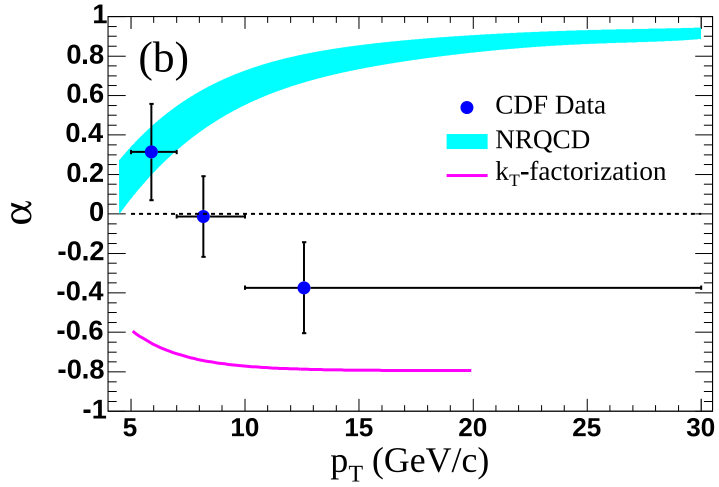
- $d\sigma/d(\cos\theta) \propto 1 + \alpha\cos^2\theta$.
  - $\alpha = 1$ is completely transverse;
  - $\alpha = -1$ is completely longitudinal.
- NRQCD prediction from Braaten, Kniehl, Lee (1999).
  - Feeddown from χ_c states is about 30% of the J/ψ sample and dilutes the polarization.
  - Feeddown from $\psi(2S)$ is about 10% of the J/ψ sample and is largely transversely polarized.
- Run I results are marginally compatible with the NRQCD prediction.
- Run II results are inconsistent with the NRQCD prediction.
- Also, inconsistent with Run I results.
  CDF was unable to track down the source of the Run I-Run II discrepancy.

$\psi(2S)$ Polarization

Run: I



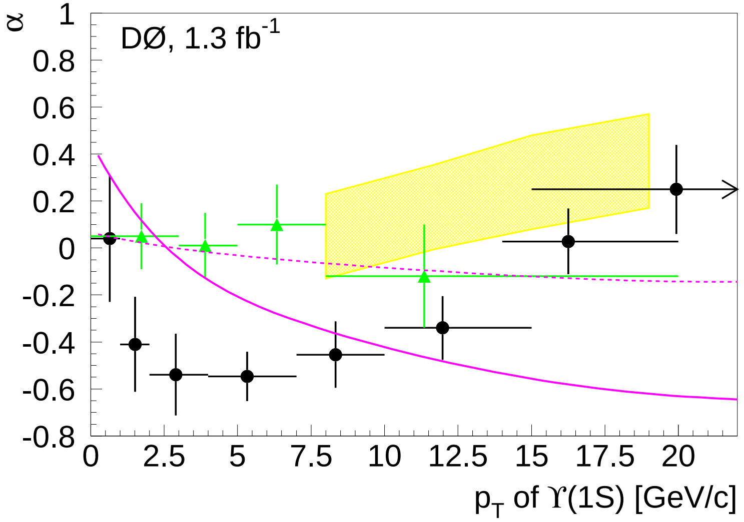
Run: II



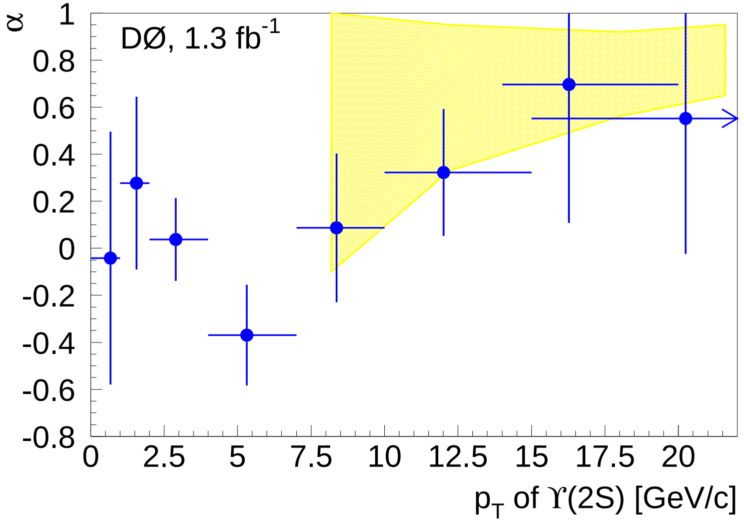
- The Run II data are incompatible with the NRQCD prediction.

Υ Polarization

$\Upsilon(1S)$ Polarization:



$\Upsilon(2S)$ Polarization:

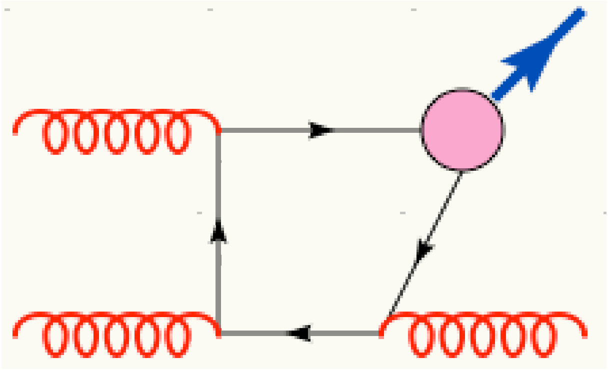


- In the $\Upsilon(1S)$ case, the D0 results (black) are incompatible with the CDF results (green).
- The CDF results are compatible with the NRQCD prediction (yellow).
- The D0 results are marginally incompatible with the NRQCD prediction.
- The curves are the limiting cases of the k_T-factorization prediction.
- In the $\Upsilon(2S)$ case, the theoretical and experimental error bars are too large to make a stringent test.

Recent Theoretical Developments

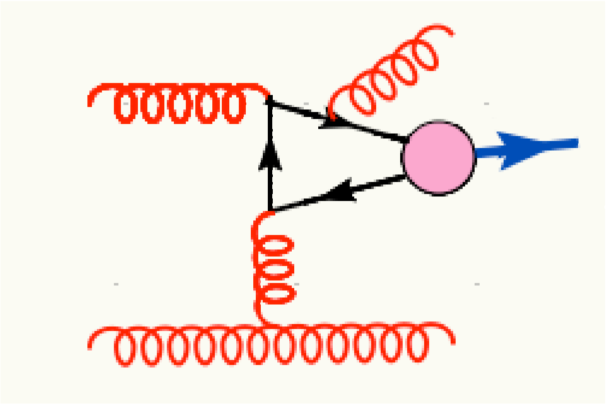
- Campbell, Maltoni, Tramontano(2007); Artoisenet, Lansberg, Maltoni (2007): Higher-order corrections to color-singlet quarkonium production at the Tevatron are unexpectedly large.
- At high p_T, higher powers of α_s can be offset by a less rapid fall-off with p_T.

LO:

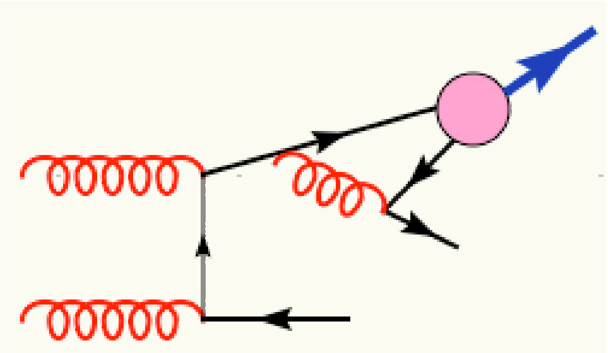


$$\sim \alpha_s^3 \frac{(2m_c)^4}{p_T^8}$$

NLO:

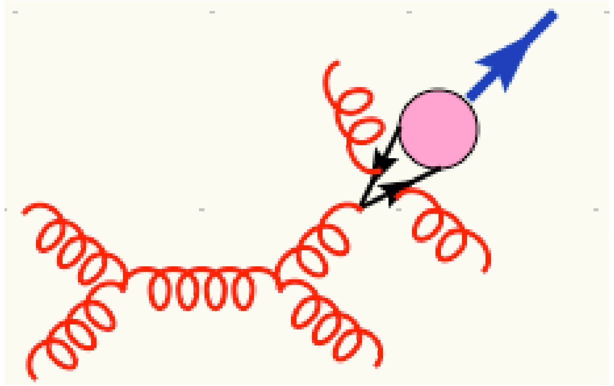


$$\sim \alpha_s^4 \frac{(2m_c)^2}{p_T^6}$$



$$\sim \alpha_s^4 \frac{1}{p_T^4}$$

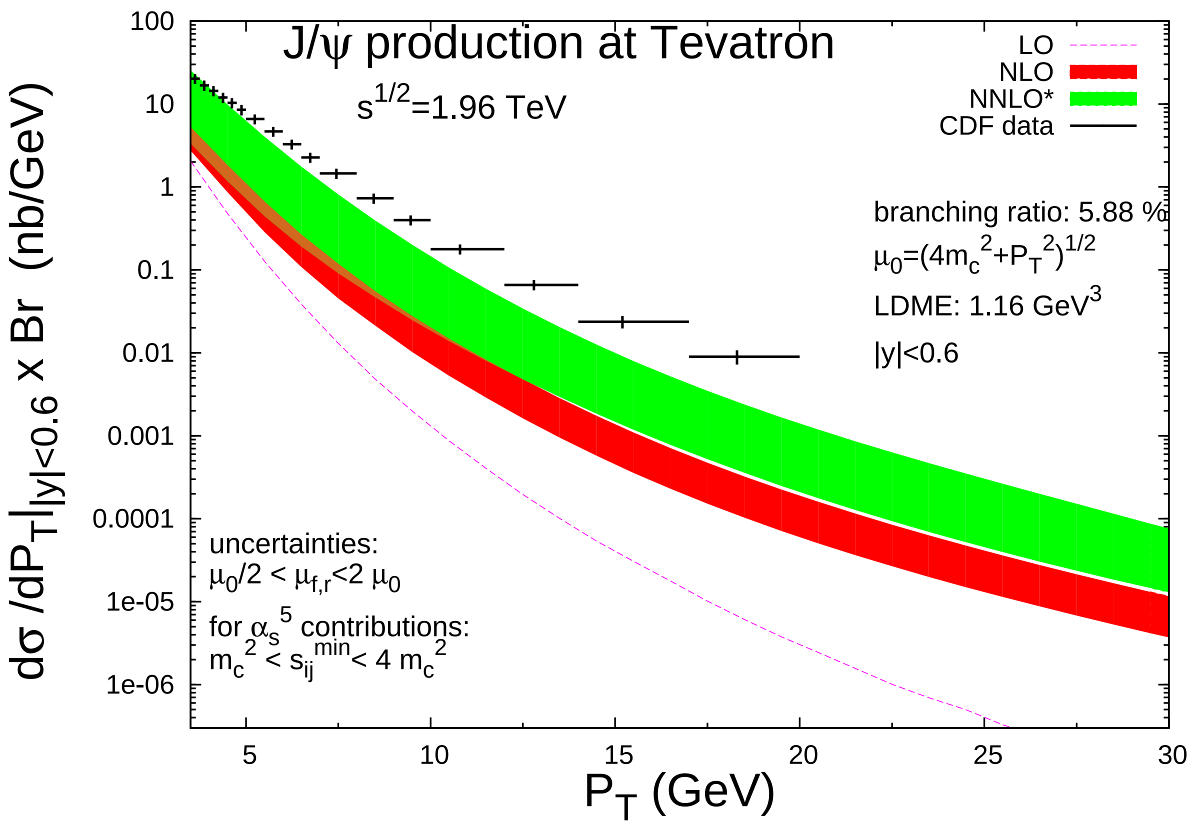
NNLO:



$$\sim \alpha_s^5 \frac{1}{p_T^4}$$

New Results for J/ψ Production

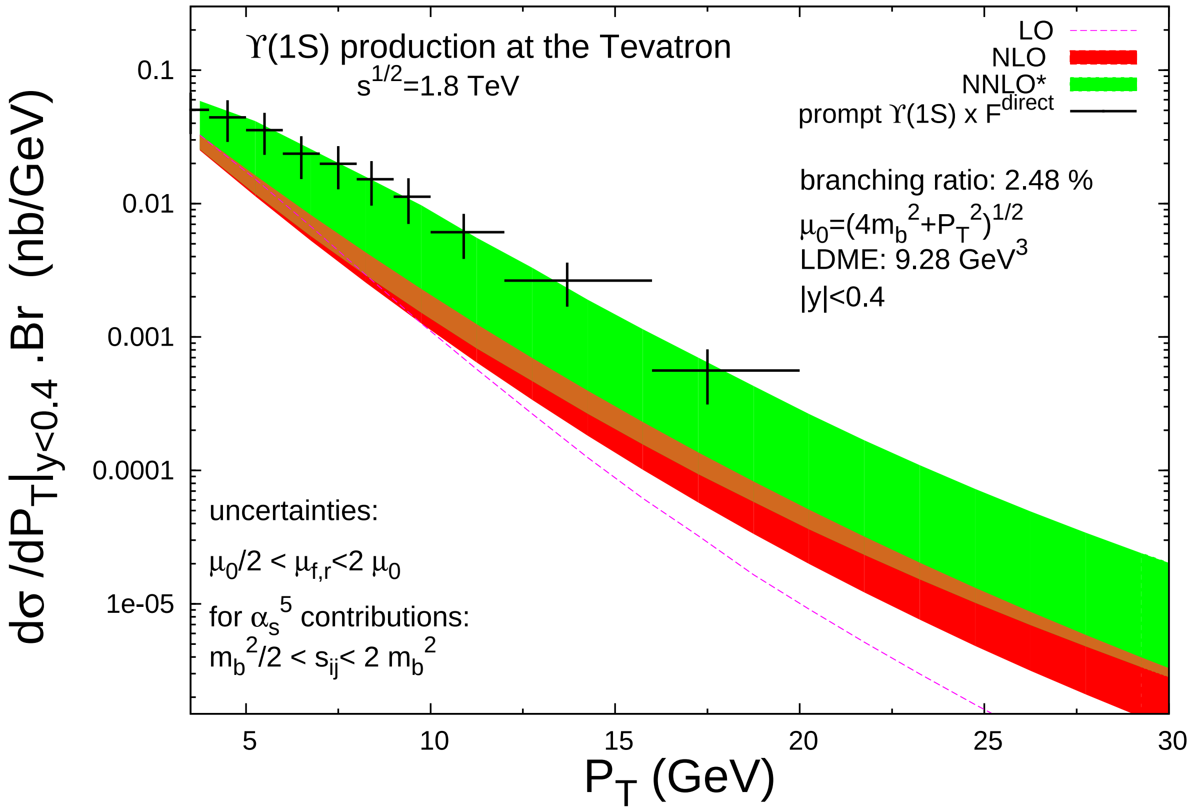
- Color-singlet contribution:



- Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (in progress)
- The NNLO* calculation is an estimate based on real-emission contributions only.
- There is still room for a color-octet contribution, but its size may be reduced from previous estimates. Affects the matrix elements used to compute all other processes.

- Color-octet contribution:
  NLO corrections are about 14% (Gong, Li, and Wang (2008)).

New Results for Color-Singlet Υ Production

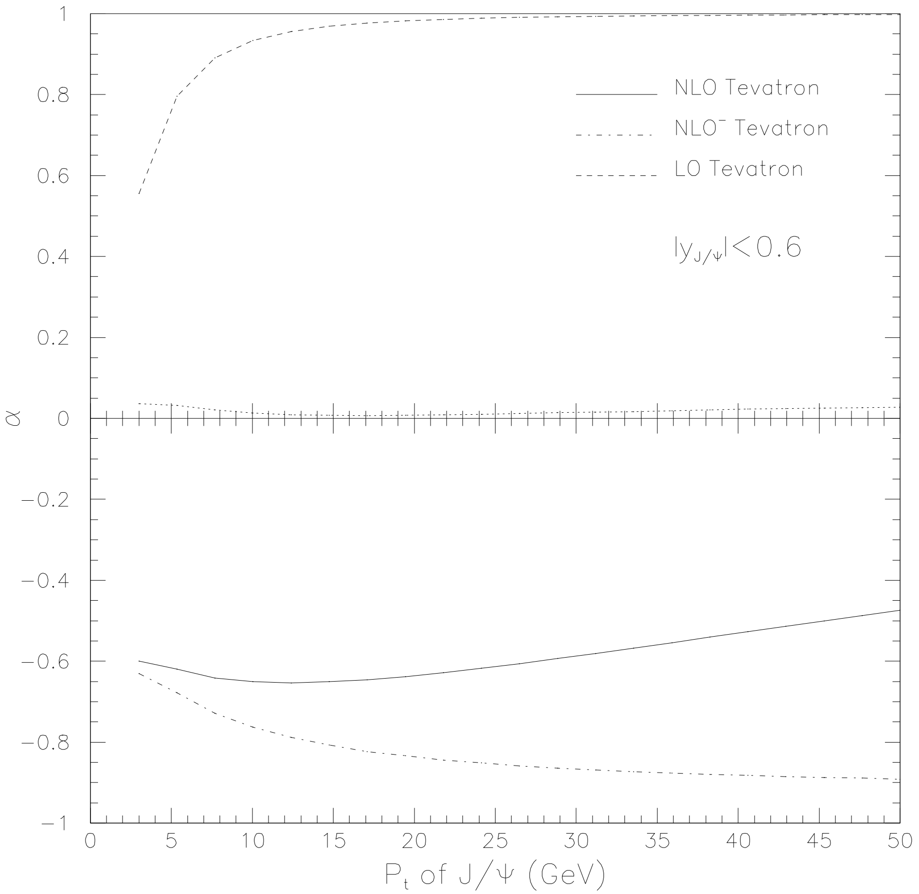


- Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (2008)
- NLO results confirmed by Gong and Wang (2007).

- There is almost no room for color-octet production.
- Consistent with the fact that color-octet production is suppressed as v^4. $v^2 \approx 0.3$ for charmonium; $v^2 \approx 0.1$ for bottomonium.

New Results for Polarization

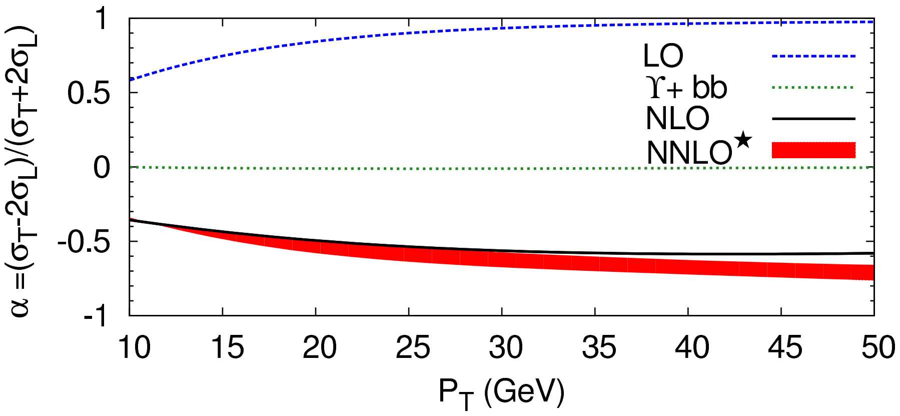
- Gong and Wang (2008) find that color-singlet J/ψ polarization at the Tevatron changes from transverse to longitudinal when NLO corrections are included.



- NLO$^-$ excludes $gg \to J/\psi c\bar{c}$.
- Unlabeled line is contribution of $gg \to J/\psi c\bar{c}$.

- Gong, Li, and Wang (2008): NLO corrections to the color-octet contribution to the J/ψ polarization have very little effect.

- Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (2008) find that color-singlet Υ polarization at the Tevatron changes from transverse to longitudinal when NLO and NNLO* corrections are included.



- NLO result confirmed by Gong and Wang (2008).

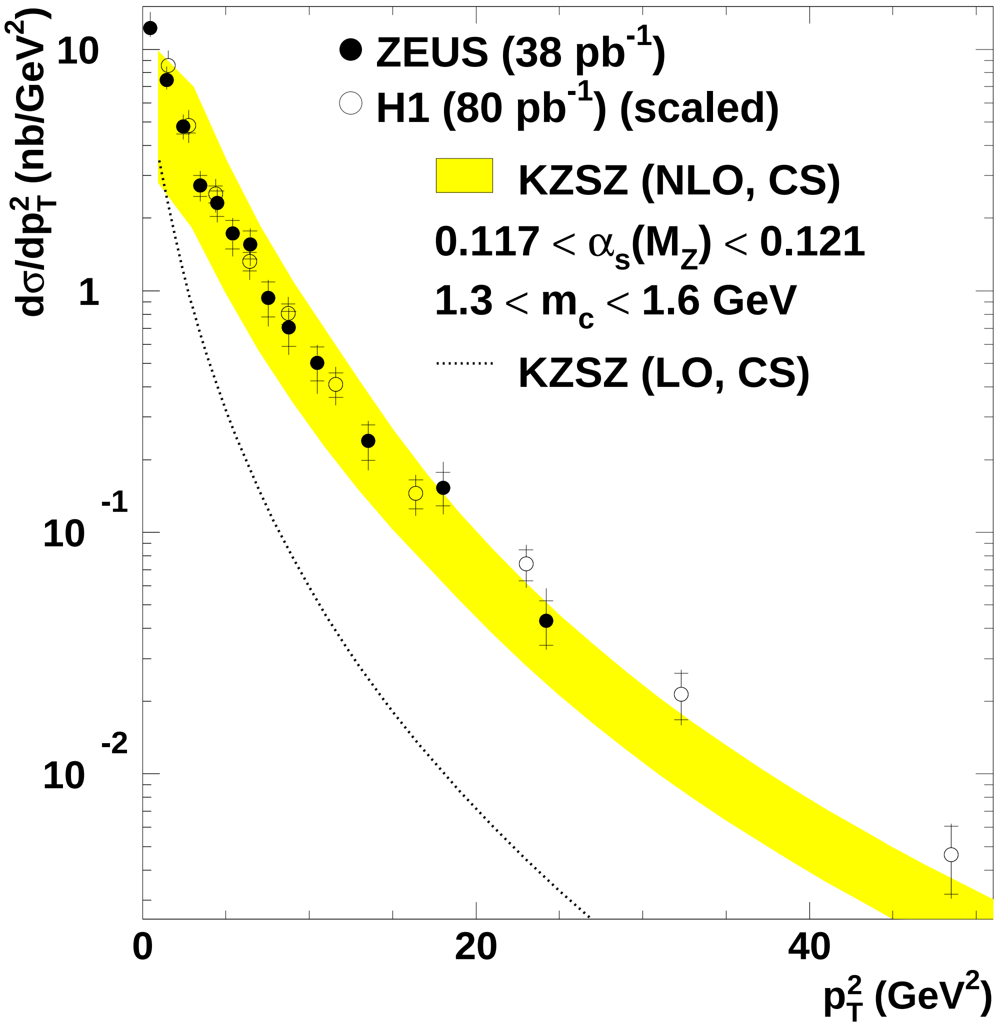
Discussion

- The NNLO* corrections greatly increase the color-singlet contributions to the J/ψ and Υ cross sections.
  - The J/ψ cross section still seems to be dominated by color-octet production.
  - However, the uncertainties in the NNLO* color-singlet cross section are large and are not completely reflected in the error bands.
- A reduced color-octet contribution plus significant longitudinal polarization from the color-singlet contribution could bring theory into agreement with the CDF data for J/ψ polarization.
- A substantial amount of color-octet production is probably still needed to account for the deviations of the J/ψ data from pure longitudinal polarization.
  - Interpretation of the Tevatron J/ψ polarization data is complicated by feeddown from $\psi(2S)$ and χ_{cJ}.
- A color-octet contribution is possibly needed to explain the Υ polarization data.
- GTB, Lee, Sinclair (2005): Lattice determinations of J/ψ color-octet decay matrix elements yield values that are about an order of magnitude lower than estimates from v-scaling rules.
  - Suggests that J/ψ color-octet production matrix elements from LO theory fits to the Tevatron data may be about an order of magnitude too large.

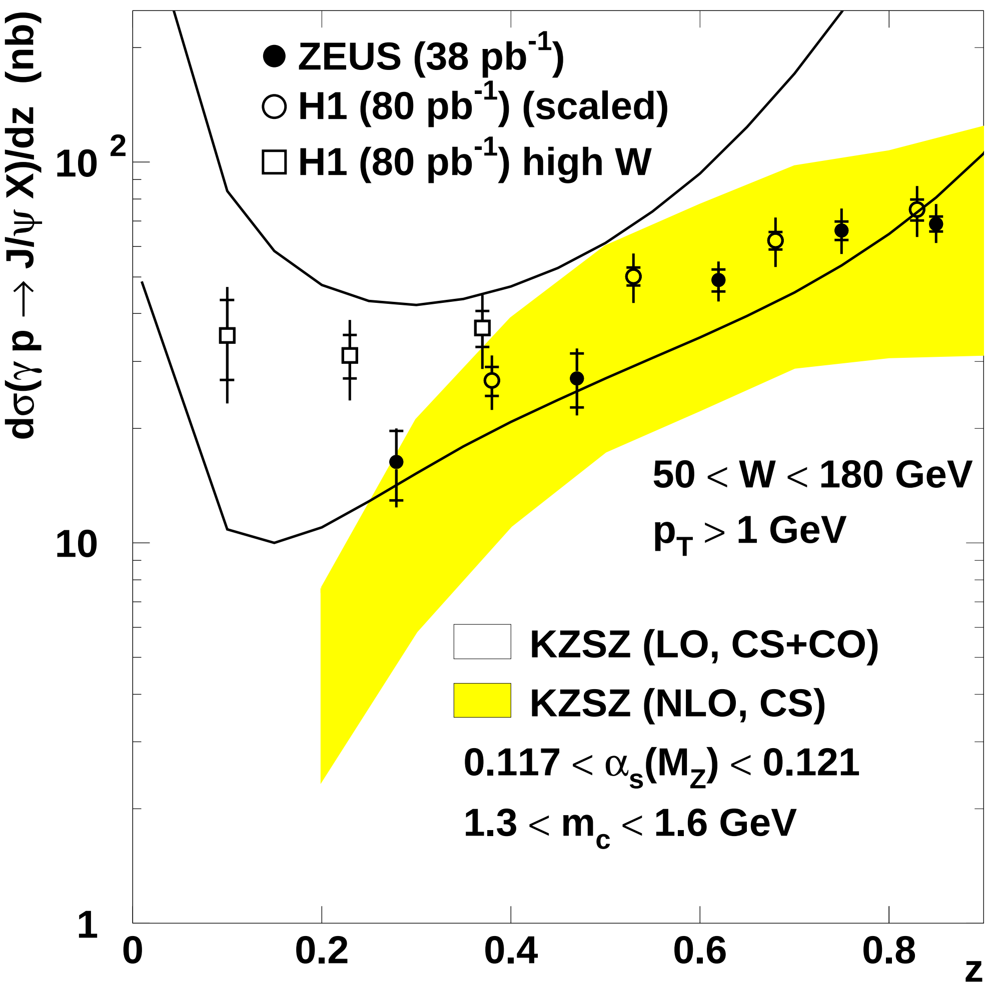
Inelastic Quarkonium Photoproduction at HERA

Production Cross Section

- It had been believed that NLO color-singlet calculations leave little room for a color-octet contribution.



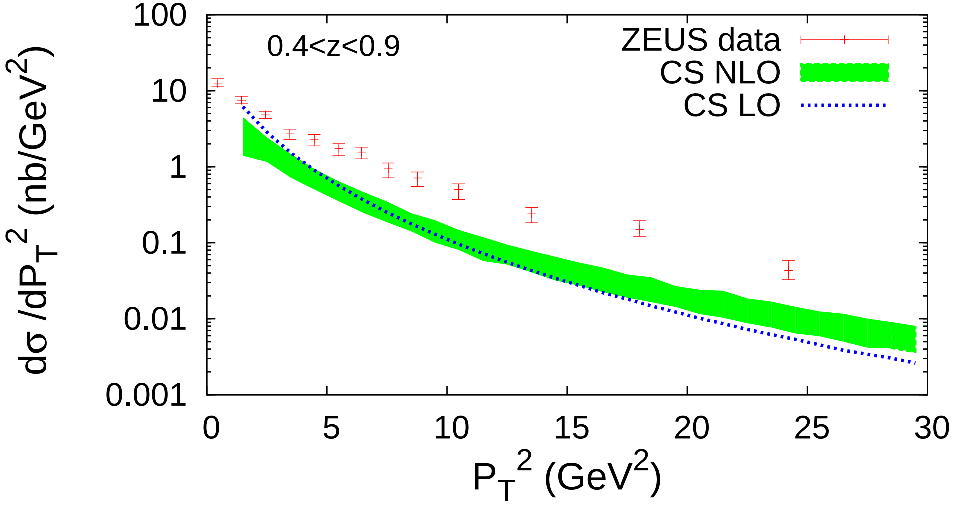
- NLO corrections increase the color-singlet contribution substantially. (Krämer, Zunft, Steegborn, Zerwas (1994); Krämer (1995))
- NLO corrections include $\gamma + g \to (c\bar{c}) + gg$, which is dominated by t-channel gluon exchange.
- For large p_T, this process goes as $\alpha_s^3 m_c^2/p_T^6$, instead of $\alpha_s^2 m_c^4/p_T^8$.

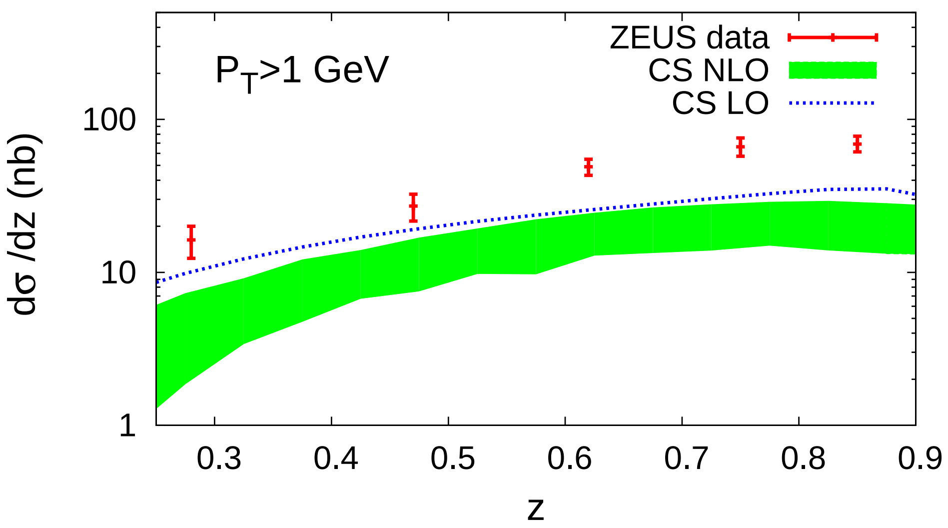


- LO NRQCD calculations by Cacciari, Krämer (1996); Amundson, Fleming, Maksymyk (1996); Ko, Lee, Song (1996); Kniehl, Krämer (1997).
- NLO color-singlet calculations by (Krämer, Zunft, Steegborn, Zerwas (1994); Krämer (1995))

Recent Theoretical Developments

- Artoisenet, Campbell, Maltoni, Tramontano (2009): A new calculation of NLO color-singlet contribution
  - Confirms the analytic results of previous calculations.
  - But a more reasonable choice of renormalization/factorization scale ($\sqrt{4m_c^2 + p_T^2}$ instead of $m_c/\sqrt{2}$) yields much smaller numerical results for cross sections.

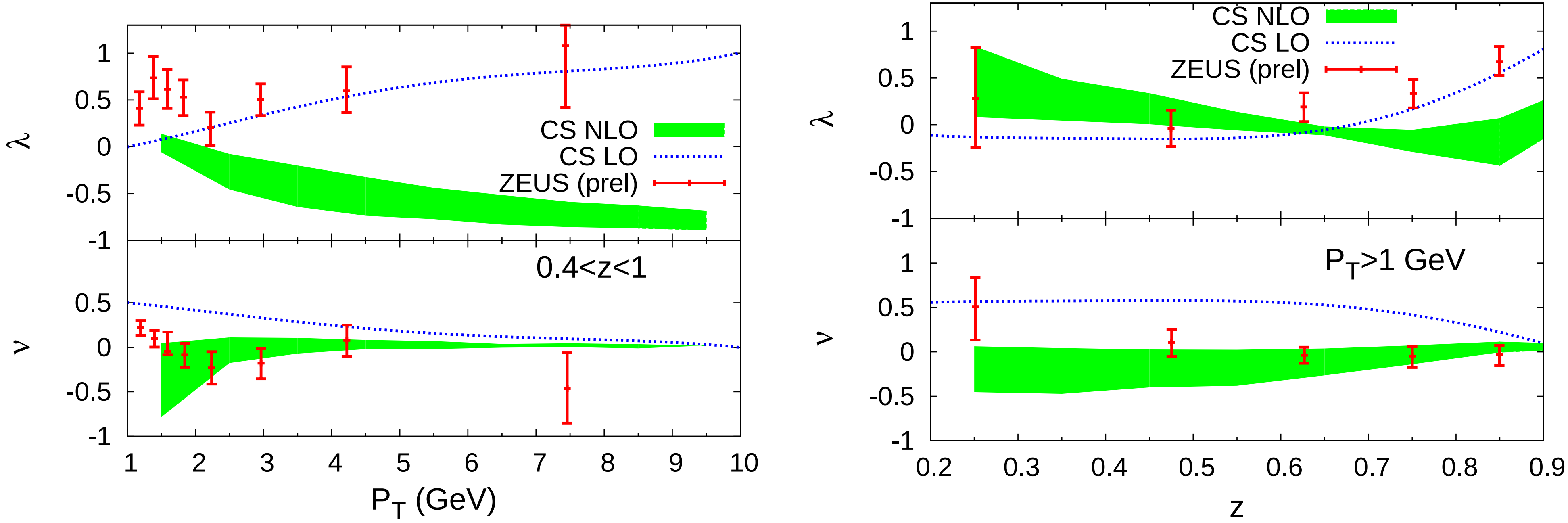




- Leaves room for a color-octet contribution.
- There is no longer an obvious conflict between the Tevatron and HERA data.

Polarization

- NLO calculations have a significant effect on the color-singlet polarization predictions.
  - LO calculation by Beneke, Krämer, Vanttinen (1997).
  - NLO calculation by Artoisenet, Campbell, Maltoni, Tramontano (2009).



- $\frac{d\Gamma(J/\psi\to l^+l^-)}{d\Omega} \propto 1 + \lambda\cos^2\theta + \mu\sin(2\theta)\cos\phi + \frac{\nu}{2}\sin^2\theta\cos(2\phi)$
- The NLO color-singlet contribution alone cannot explain the data for λ at large p_T or large z.
- Would a color-octet contribution bring theory into agreement with data?

What Can We Learn at 10 TeV with 10 pb^{-1} of Data?

The Key Physics Issues

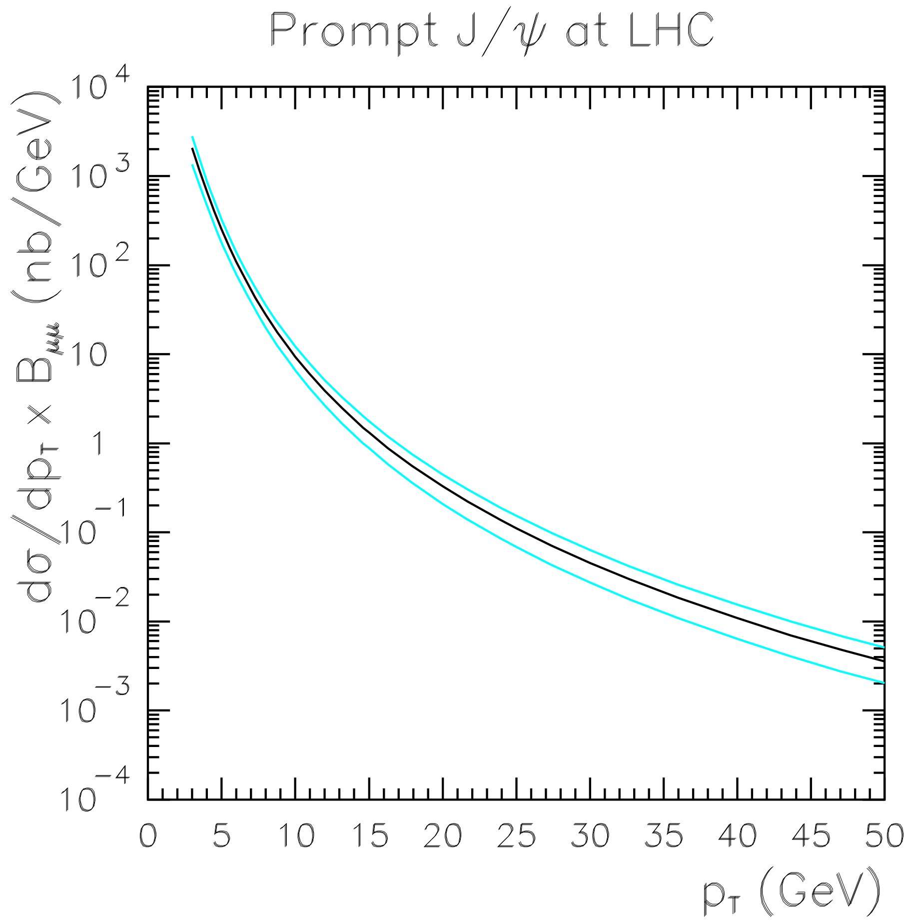
- The main physics goal is to understand which mechanisms are important in quarkonium production.
- Measurements of $d\sigma/p_T$ and the polarization parameter α should give important information.
- If we assume that the production mechanisms are described within the NRQCD factorization formalism, then
  - $d\sigma/p_T$ can, in principle, allow us to determine the color-octet matrix elements, once we have subtracted the color-singlet rate.
  - α can give a check of the relative proportions of the color-singlet and color-octet production rates.
- It is highly desirable to measure the production rates and polarization for J/ψ and Υ in direct production (no feeddown).
  - The theoretical calculations for feeddown are incomplete and would introduce new uncertainties.

- Unfortunately, the color-singlet rate is very uncertain, largely because of renormalization scale uncertainties.
  - For the new production mechanisms that first appear at NNLO, the NNLO cross section is the Born cross section.
  - Scale uncertainties are magnified by the appearance of five powers of α_s in the NNLO cross section.
- Measurements at different values of $\sqrt{s}$ allow us to take ratios of cross sections.
  - The largest theoretical uncertainties should tend to cancel in the ratio.
  - The ratio may still have some discriminating power with regard to the production mechanisms.
  - In any case, such ratios will provide another test of our theoretical ideas.
- Factorization theorems for quarkonium production, if correct, likely hold only for $p_T >> m_Q$.
  - The higher-p_T reach of the LHC is therefore very important, especially for the bottomonia.
- High statistics, high-p_T studies of Υ production and polarization give us a window into a second quarkonium system at smaller v than the J/ψ.
  - The NRQCD velocity expansion is on more solid footing at smaller v.

What Production Rates Can We Expect at 10 TeV?

J/ψ Production

- Estimate using LO color-singlet plus color-octet production rates.
  Calculation at 10 TeV courtesy of Jungil Lee and Hee Sok Chung.



Uncertainties from

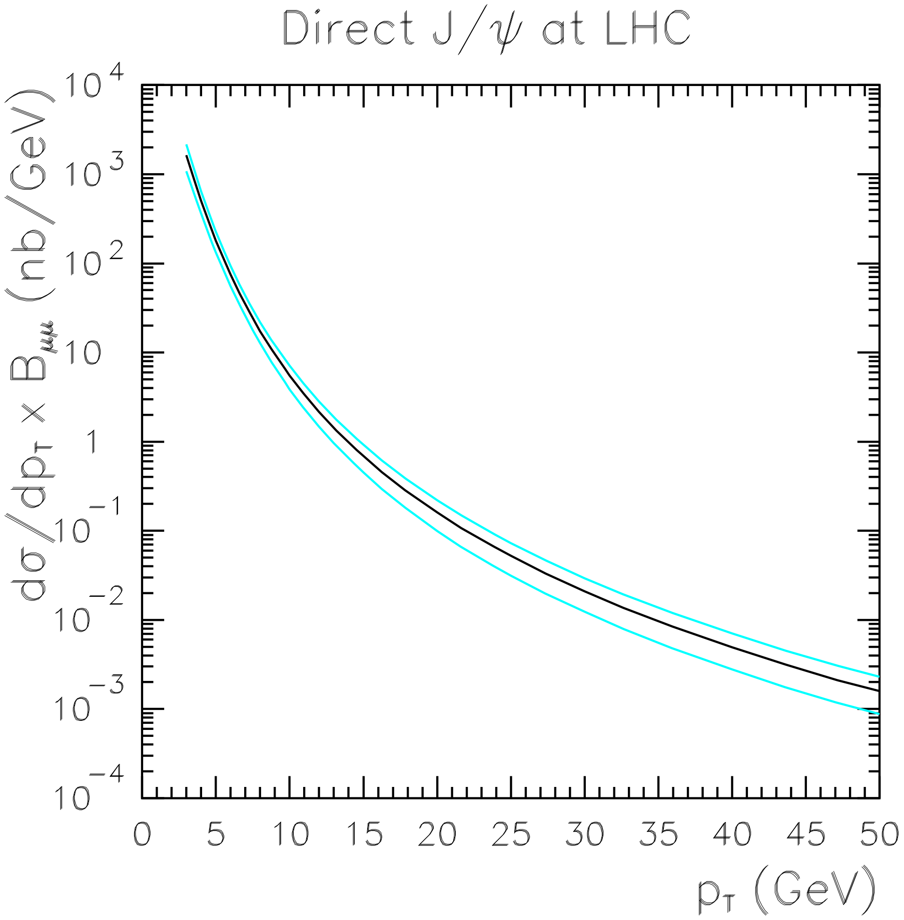
- fits of color-octet matrix elements to CDF data,
- choice of renorm./fact. scale,
- m_c,
- choice of parton distributions (CTEQ5L compared with MRST98LO [default]).

Caveats

- Does not include effects of NLO and NNLO color-singlet corrections.
  Could affect the relative proportions of color-singlet and color-octet contributions.
  But they scale similarly with $\sqrt{s}$.
- Does not include NLO color-octet corrections (small) or DGLAP evolution of final-state partons (about -27% at $p_T = 20$ TeV).

- At $p_T = 10$ GeV, the 10 TeV prompt rate is about 29 times the Tevatron prompt rate.
  At $p_T = 20$ GeV, the 10 TeV prompt rate is about 38 times the Tevatron prompt rate.
- The CDF Run II cross-section measurement is based on about 40 pb^{-1} of data.
  The CDF Run I polarization measurement is based on about 110 pb^{-1} of data.
  The CDF Run II polarization measurement is based on about 800 pb^{-1} of data.
- At $p_T = 20$ GeV with 10 pb^{-1} of luminosity, the LHC data samples should be about 10, 3.5, and 0.5 times the CDF data samples.
- It should be possible to make competitive cross-section and polarization measurements and perhaps even to extend the p_T range of the cross-section measurement to $p_T = 25$ GeV or so.
- It is worth looking at $d\sigma/dp_T$ and polarization for the $\psi(2S)$.
  The rate is lower, but complications from feeddown are absent.

- Estimate the direct J/ψ rate from the LO color-singlet plus color-octet contributions.



- Calculation at $\sqrt{s} = 10$ TeV courtesy of Jungil Lee and Hee Sok Chung.
- The direct rate is about 50% of the prompt J/ψ rate at large p_T.
- At $p_T = 20$ GeV, $d\sigma/dp_T \times B_{\mu\mu} = 0.11$ pb/GeV.

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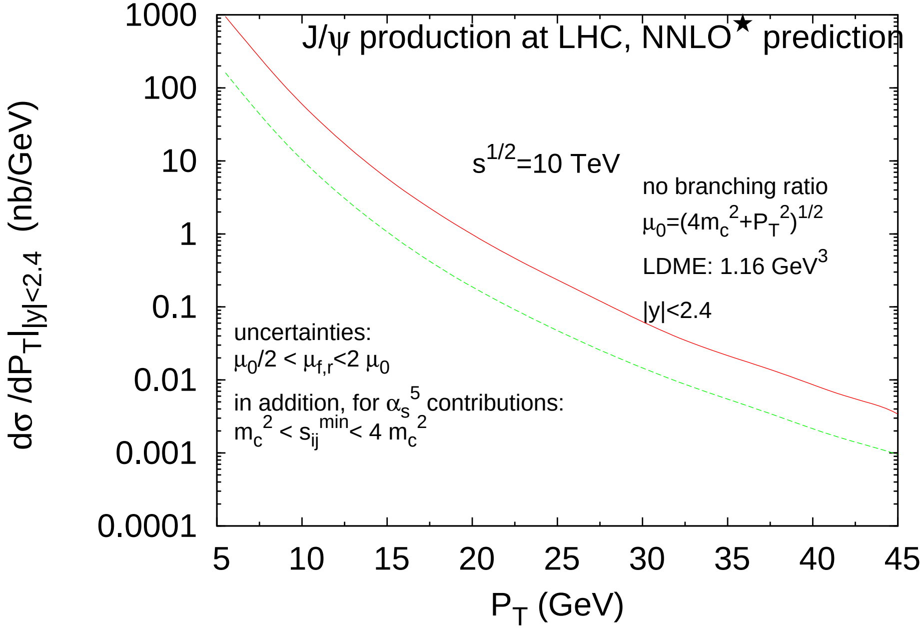
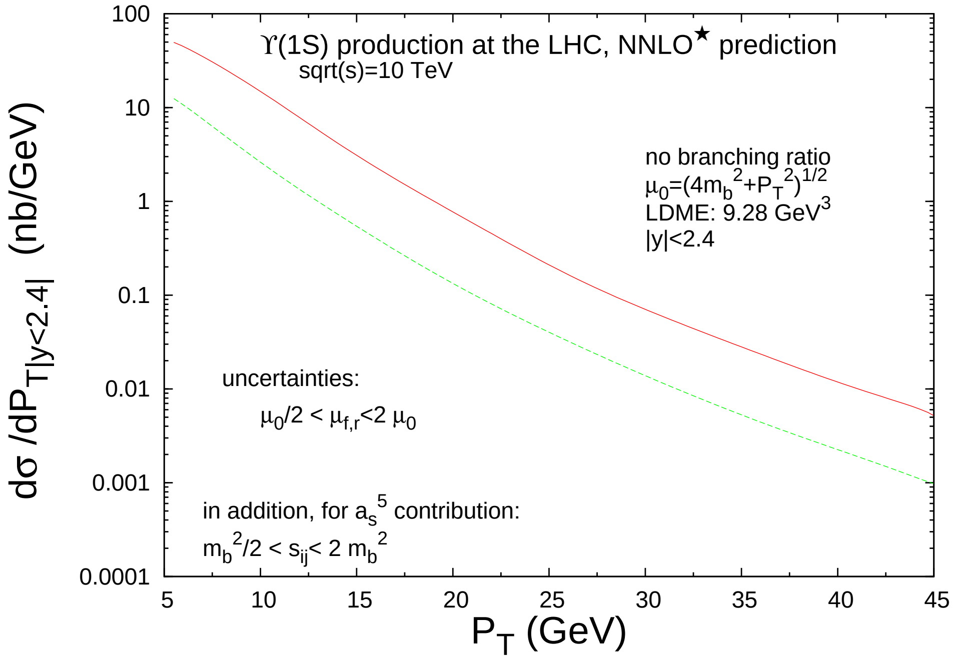
- The NNLO color-singlet J/ψ direct production rate is smaller than the estimate based on the LO color-singlet plus color-octet contributions.



- Calculation at 10 TeV courtesy of Pierre Artoisenet.
- Based on the calculation of Artoisenet, Campbell, Maltoni, Tramontano (2008).
- Note that $B_{\mu\mu} \approx 0.0593$ is not included as a factor in this plot.
- At $p_T = 20$ GeV, $d\sigma/dp_T \times B_{\mu\mu} =$0.11–0.47 pb/GeV.
- At $p_T = 20$ TeV, this is about 10–40% of the direct production rate from LO color-singlet plus color-octet contributions.

$\Upsilon(1S)$ Production Rates

- Use the NNLO* color-singlet Υ production rate to estimate the direct Υ production rate.



- Calculation at 10 TeV courtesy of Pierre Artoisenet.
- Based on the calculation of Artoisenet, Campbell, Maltoni, Tramontano (2008).
- Note that $B_{\mu\mu} \approx 0.0248$ is not included as a factor in this plot.

- Probably underestimates the direct rate.
  (The LO color-octet Υ production rate is expected to be comparable to the NNLO* color-singlet production rate.)
- The ratio of the direct rate at $\sqrt{s} = 10$ TeV to the direct rate at $\sqrt{s} = 1.96$ TeV is probably more reliable.
  - At $p_T = 10$ GeV, the ratio of direct rates is about 50.
  - At $p_T = 20$ GeV, the ratio of direct rates is about 77.

- The CDF Run I cross-section and polarization measurements are based on about 77 pb^{-1} of data.
  The D0 Run II cross-section measurement is based on about 159 pb^{-1} of data.
  The D0 Run II polarization measurement is based on about 1.3 fb^{-1} of data.
- At $p_T = 20$ GeV with 10 pb^{-1} of luminosity, the LHC data samples should be about 10 times the CDF Run I data samples and 5 and 0.6 times the D0 Run II data samples.
- At $\sqrt{s} = 14$ TeV and $p_T = 20$ GeV, the direct Υ production rate is expected to be about 50% of the prompt Υ production rate (Krämer (2001)).
- It should be possible to make competitive cross-section and polarization measurements and perhaps even to extend the p_T range of the cross-section measurement.

Ratios of Cross Sections at Different Values of $\sqrt{s}$

- We may be able to disentangle the color-singlet and color-octet contributions by taking ratios of cross sections at different values of $\sqrt{s}$.
  - The largest theoretical uncertainties cancel.
  - In a given experiment, some systematic uncertainties may tend to cancel as well.
- R is the ratio of the direct production rate at $\sqrt{s} = 10$ TeV to the direct production rate at $\sqrt{s} = 1.96$ TeV.
  - R_1 is the color-singlet ratio of rates.
  - R_8 is the color-octet ratio of rates.
  - R^{Exp} is the experimental ratio of cross sections.
  - r^X is the ratio of the color-octet contribution to the color-singlet contribution at experiment X.
- If R_1 and R_8 are well separated, then we can use R^{Exp} to determine r^X.

$$R^{\mathrm{Exp}} = \frac{\sigma_1^{\mathrm{LHC}} + \sigma_8^{\mathrm{LHC}}}{\sigma_1^{\mathrm{Tev}} + \sigma_8^{\mathrm{Tev}}} = \frac{R_1 + r^{\mathrm{Tev}} R_8}{1 + r^{\mathrm{Tev}}}, \qquad \text{where} \qquad r^{\mathrm{Tev}} = \frac{\sigma_8^{\mathrm{Tev}}}{\sigma_1^{\mathrm{Tev}}} = \frac{R_1}{R_8} r^{\mathrm{LHC}}.$$

- Then $r^{\mathrm{Tev}} = (R^{\mathrm{Exp}} - R_1)/(R_8 - R^{\mathrm{Exp}})$ can be used to make predictions for the polarization.

Ratios of J/ψ Production Rates

- At $p_T = 10$ GeV
  Ratio of LO color-octet contributions: $R_8 \approx 27$
  Ratio of NNLO* color-singlet contributions: $R_1 \approx 22\text{–}32$
- At $p_T = 20$ GeV
  Ratio of LO color-octet contributions: $R_8 \approx 38$
  Ratio of NNLO* color-singlet contributions: $R_1 \approx 42\text{–}50$
- It is not clear if R_1 and R_8 are sufficiently well separated to compute r^{Tev}.
- But measurement of R^{Exp} is still an important test of theory.
- More theoretical work is needed to pin these numbers down and to assess uncertainties.

Ratios of $\Upsilon(1S)$ Production Rates

- At $p_T = 10$ GeV
  Ratio of color-octet contributions: $R_8 =??$
  Ratio of color-singlet contributions: $R_1 \approx 47$–52
- At $p_T = 20$ GeV
  Ratio of color-octet contributions: $R_8 =??$
  Ratio of color-singlet contributions: $R_1 \approx 66$–87
- More theoretical work is needed to compute the color-octet ratio and to pin down the color-singlet ratio more precisely.

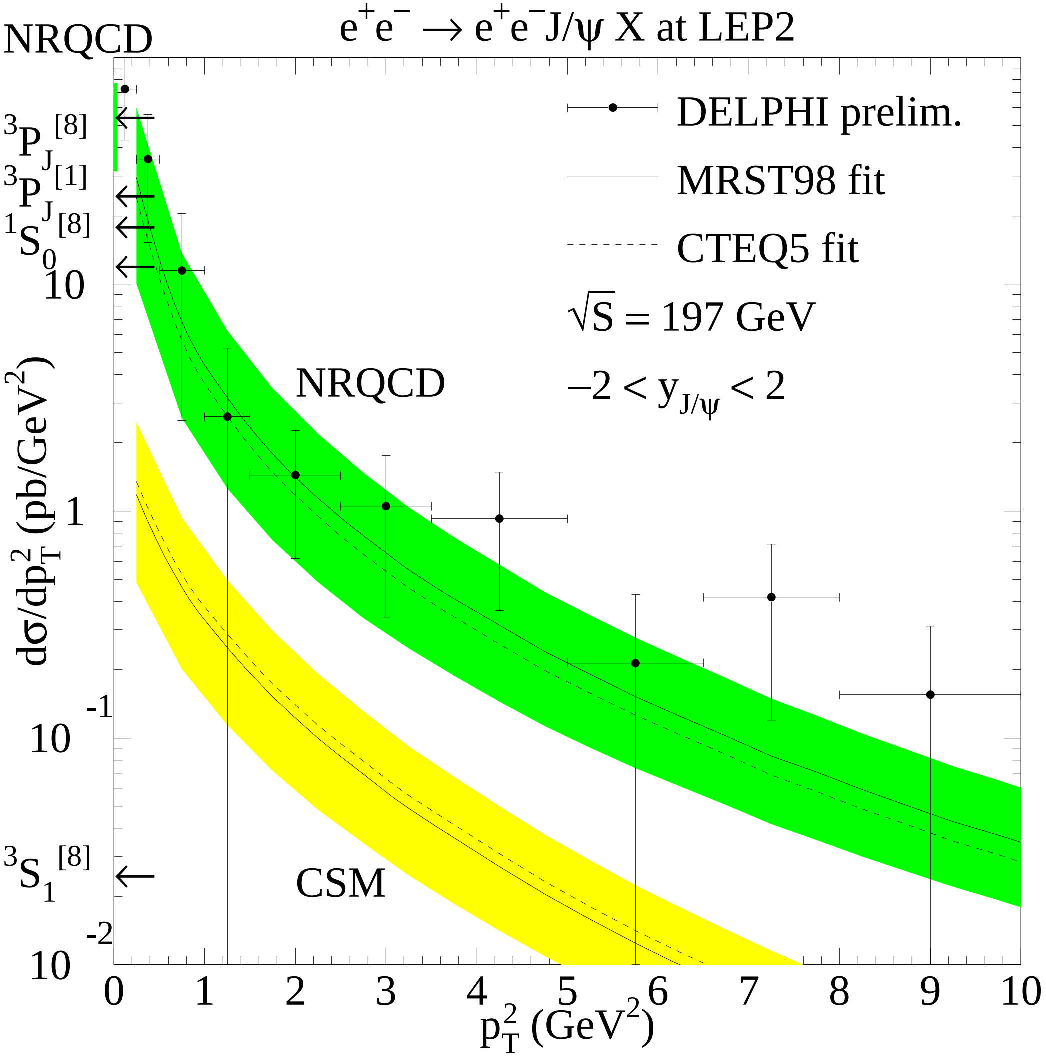
Summary

- The NRQCD factorization approach provides a systematic method for calculating quarkonium production (and decay) rates as a double expansion in powers of α_s and v.
- The NRQCD factorization conjecture for inclusive production rates has not yet been proven.
- NRQCD factorization predicts that quarkonia can be produced through both color-singlet and color-octet $Q\bar{Q}$ states.
- Color-octet matrix elements extracted from the Tevatron data in LO lead to disagreements with the polarization data at the Tevatron and in photoproduction at HERA.
- NLO and NNLO corrections to the color-singlet contributions can change them by orders of magnitude.
  - May reduce the sizes of color-octet matrix elements from the Tevatron fits.
  - The ratio of color-singlet to color-octet contributions may be much larger than previously supposed.
  - This may lead to a consistent picture for $d\sigma/p_T$ and polarization.

- There is a large increase in $d\sigma/p_T$ in going from $\sqrt{s} = 1.96$ TeV to $\sqrt{s} = 10$ TeV.
  - Should allow one to make meaningful measurements of $d\sigma/dp_T$ and polarization, even with low integrated luminosities.
  - It may be worthwhile to look at $d\sigma/dp_T$ and polarization for $\psi(2S)$.
    Complications from feeddown are absent.
  - These measurements could provide important clues as to the quarkonium production mechanism (color-singlet vs. color-octet).
  - The potential higher-p_T reach of the LHC may be very important in sorting out theoretical issues.
- Ratios of cross sections at different values of $\sqrt{s}$ have reduced theoretical uncertainties and provide important tests of the theoretical understanding of production mechanisms.

Backup Slides

$\gamma\gamma \to J/\psi + X$ at LEP

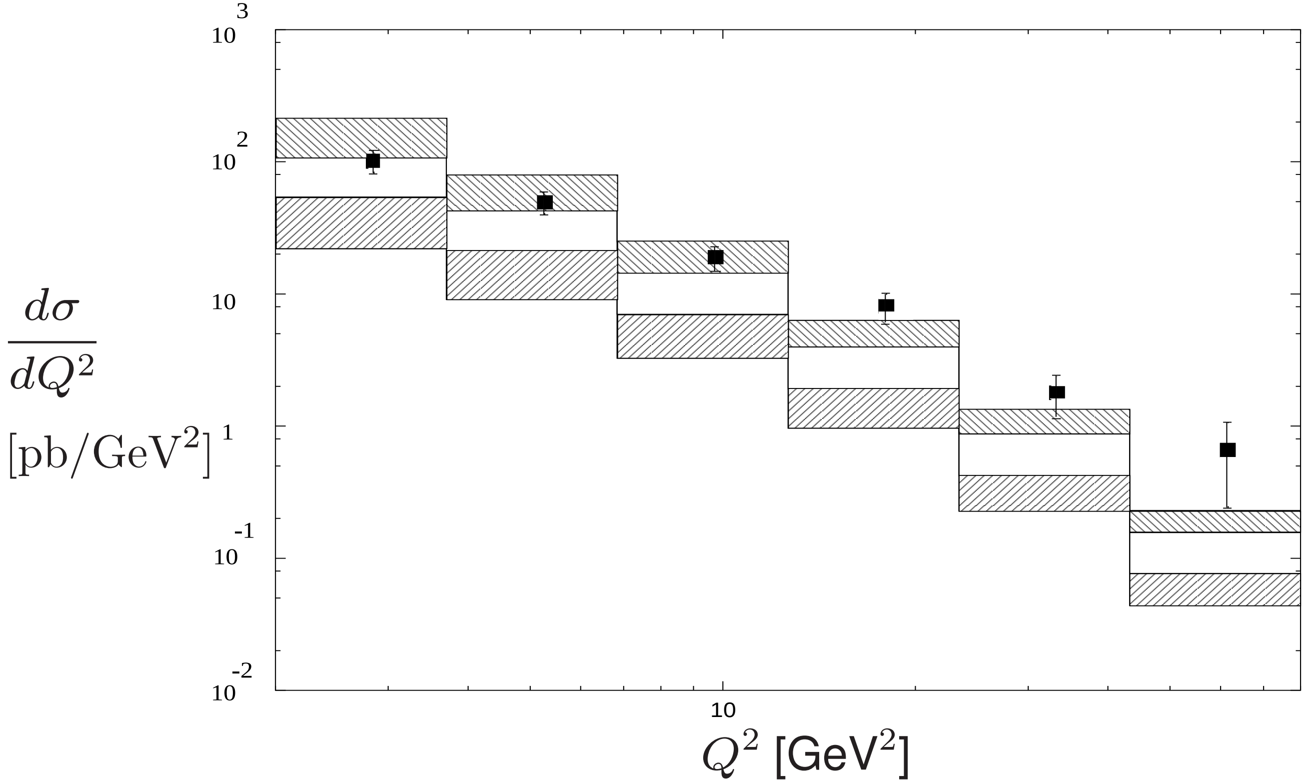


- Comparison of theory (Klasen, Kniehl, Mihaila, Steinhauser (2001)) with Delphi data clearly favors NRQCD over the color-singlet model.
- Theory uses Braaten-Kniehl-Lee (1999) matrix elements from Tevatron data and MRST98LO (solid) and CTEQ5L (dashed) PDF's.
- Theoretical uncertainties from
  - Renormalization and factorization scales (varied by a factor 2),
  - NRQCD color-octet matrix elements,
  - Different linear combination of matrix elements than in Tevatron cross sections.

J/ψ Production in DIS at HERA

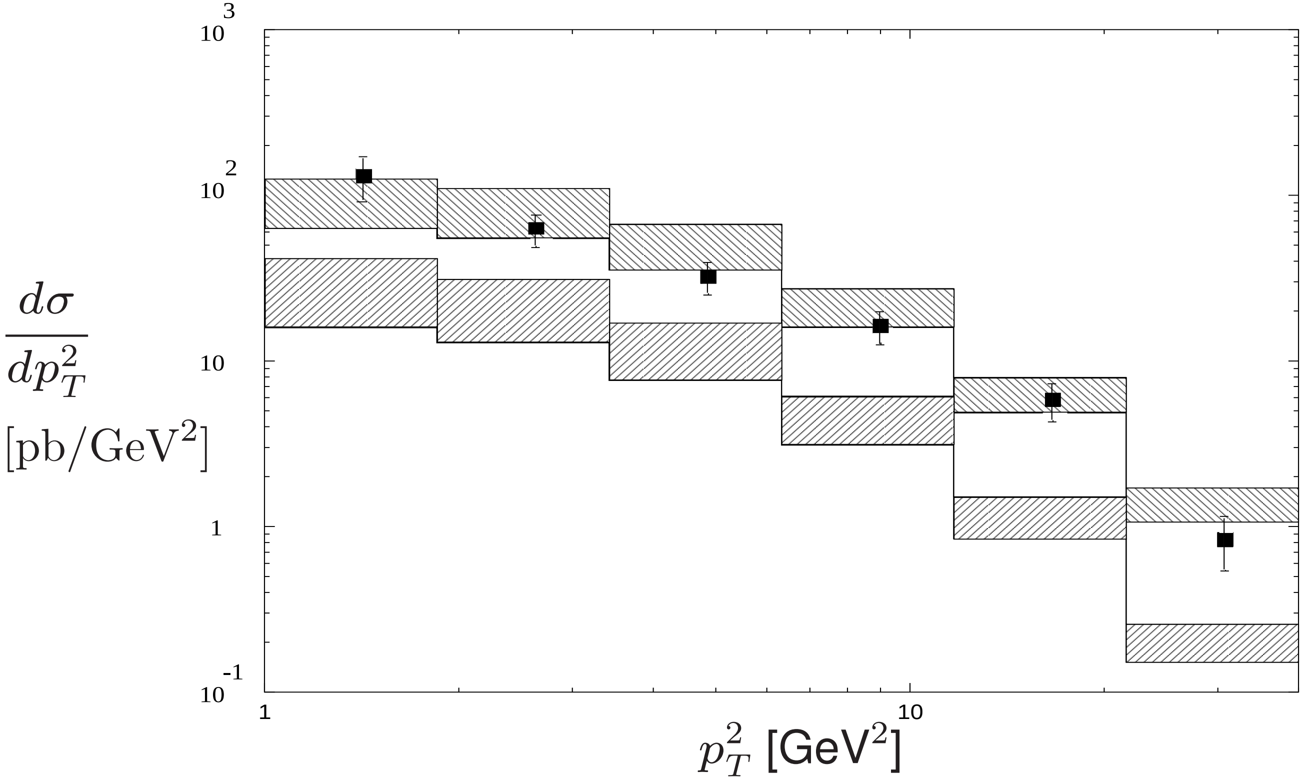
- The NRQCD (Kniehl, Zwirner (2001)) prediction uses Braaten-Kniehl-Lee (1999) matrix elements extracted from the Tevatron data and MRST98LO and CTEQ5L PDF's.
- Theoretical uncertainties from
  - PDF's,
  - Renormalization and factorization scales (varied by a factor 2),
  - NRQCD color-octet matrix elements,
  - Different linear combination of matrix elements than in Tevatron cross sections.

- The H1 data plotted as a function of Q^2 favor the NRQCD prediction over the color-singlet-model prediction. (Q is the virtual-photon momentum.)



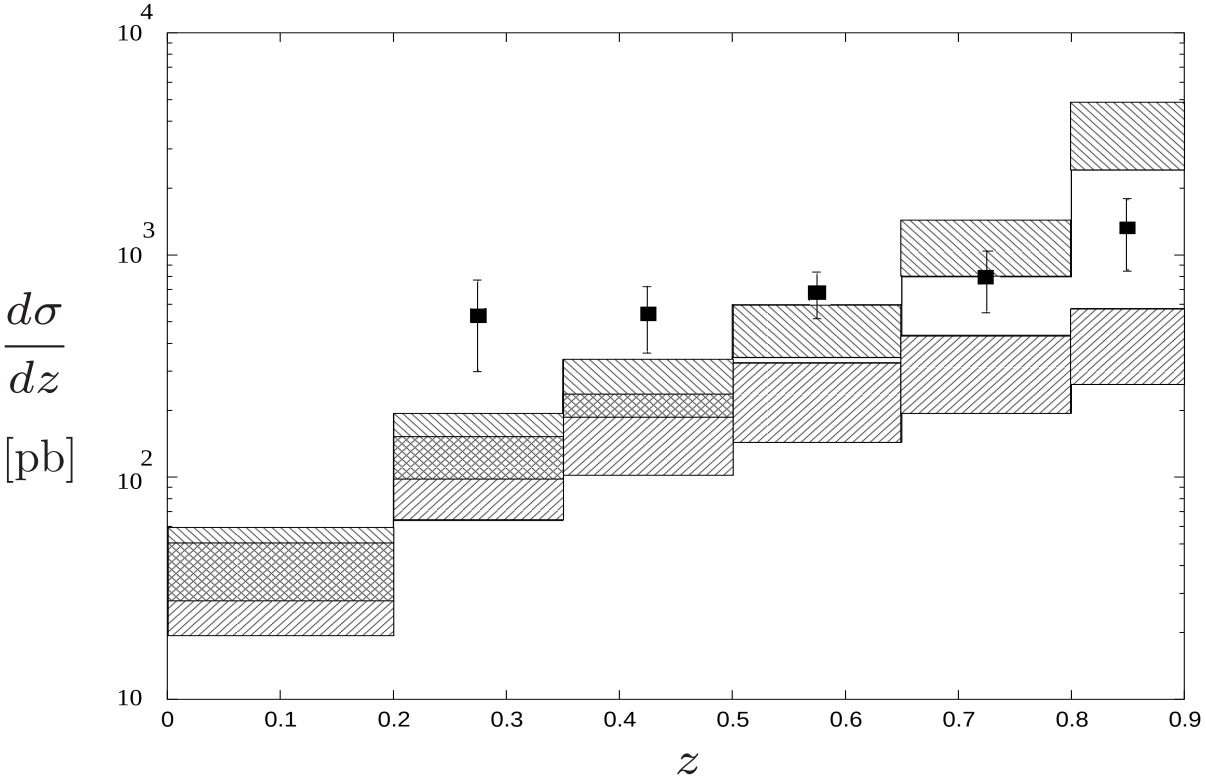
H1 data vs. leading-order NRQCD (upper) and Color-Singlet Model (lower).

- The H1 data plotted as a function of P_T^2 favor the NRQCD prediction over the color-singlet-model prediction. p_T is the transverse momentum of the J/ψ.



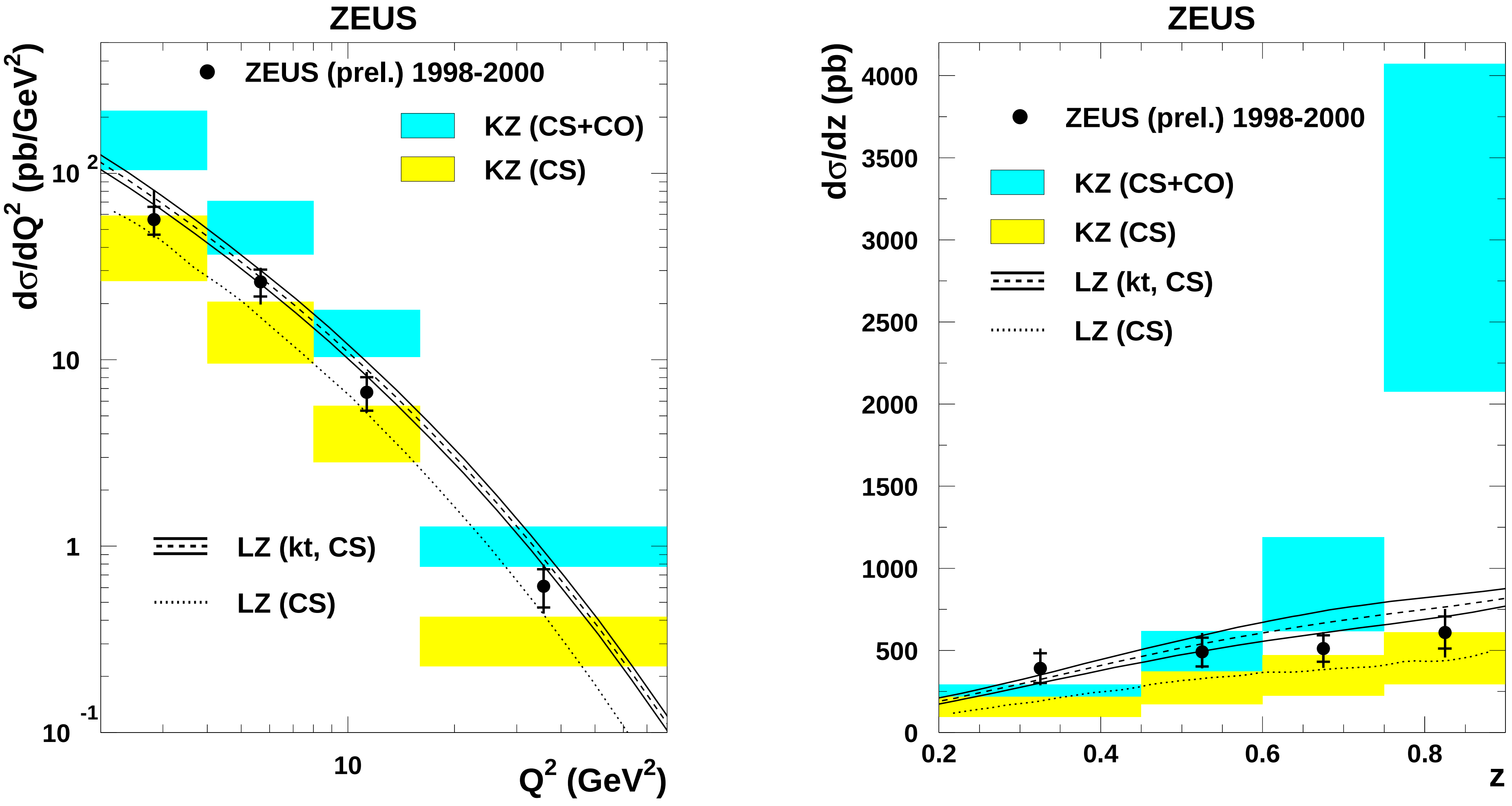
H1 data vs. leading-order NRQCD (upper) and Color-Singlet Model (lower).

- The H1 data plotted as a function of z do not agree well with either the NRQCD prediction or the color-singlet-model prediction. (z is the energy fraction of the J/ψ.)
- The data do not show the expected color-octet rise at $z = 1$.
  Resummations of the α_s and v expansions are needed.



H1 data vs. leading-order NRQCD (upper) and Color-Singlet Model (lower).

- The ZEUS data are systematically lower than the H1 data and agree less well with the NRQCD prediction (but have larger error bars).
- The data plotted as a function of z do not show the expected color-octet rise at $z = 1$.



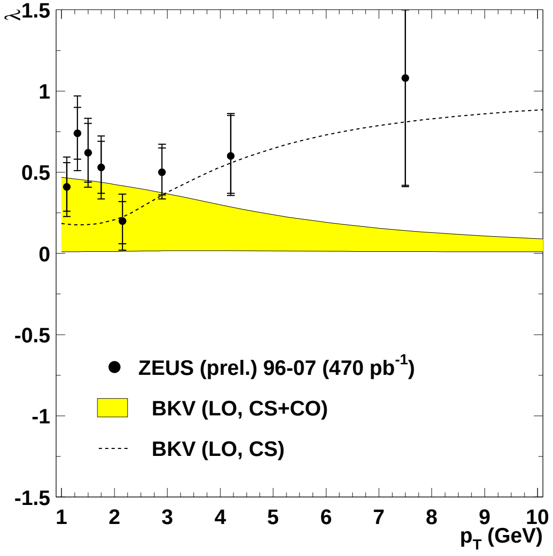
Curves from A.V. Lipatov and N.P. Zotov.

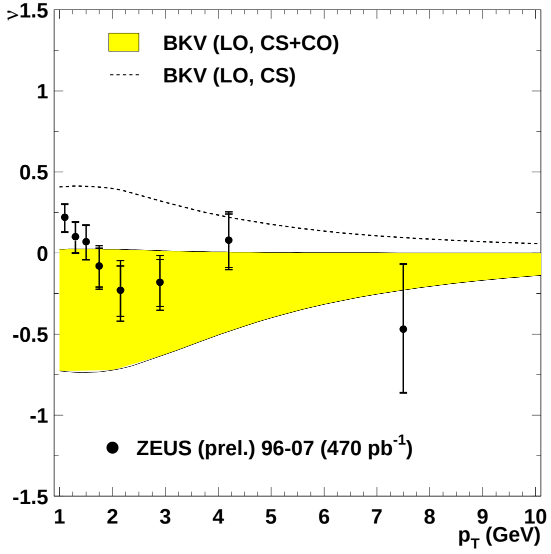
Is the Assumption of 100% Polarization Transfer Valid?

- Existing calculations assume that 100% of the $Q\bar{Q}$ polarization is transferred to the quarkonium.
- Spin-flip corrections are suppressed only by v^2, not v^4, relative to the non-flip part.
  (GTB, Braaten, Lepage)
- It could happen that the spin-flip corrections are anomalously large.
- Do the velocity-scaling rules need to be modified?
  (Brambilla, Pineda, Soto, Vairo; Fleming, Rothstein, Leibovich)
- A lattice calculation of color-octet decay matrix elements indicates that spin-flip processes are indeed suppressed by a factor v^2 or smaller (GTB, Lee, Sinclair).

Polarization in Inelastic J/ψ Photoproduction at HERA

$$\frac{d\Gamma(J/\psi\to l^+l^-)}{d\Omega} \propto 1 + \lambda\cos^2\theta + \mu\sin(2\theta)\cos\phi + \frac{\nu}{2}\sin^2\theta\cos(2\phi)$$





- NRQCD calculation by Beneke, Krämer, Vanttinen (1997) using Beneke-Krämer (1996) matrix elements from fits to the Tevatron data.
- θ and ϕ are the polar and azimuthal angles of the l^+ 3-momentum with respect to a coordinate system that is defined in the J/ψ rest frame.
- The data for λ at high p_T slightly favor the color-singlet prediction.
- The data for ν at high p_T slightly favor the color-singlet+color-octet prediction.

Exclusive Double Charmonium Production at Belle and BABAR

$$e^+e^- \longrightarrow J/\psi + \eta_c$$

- Experiment
  Belle (2004): $\sigma[e^+e^- \longrightarrow J/\psi + \eta_c] \times B_{>2} = 25.6 \pm 2.8 \pm 3.4$ fb.
  BABAR (2005): $\sigma[e^+e^- \longrightarrow J/\psi + \eta_c] \times B_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1}$ fb.

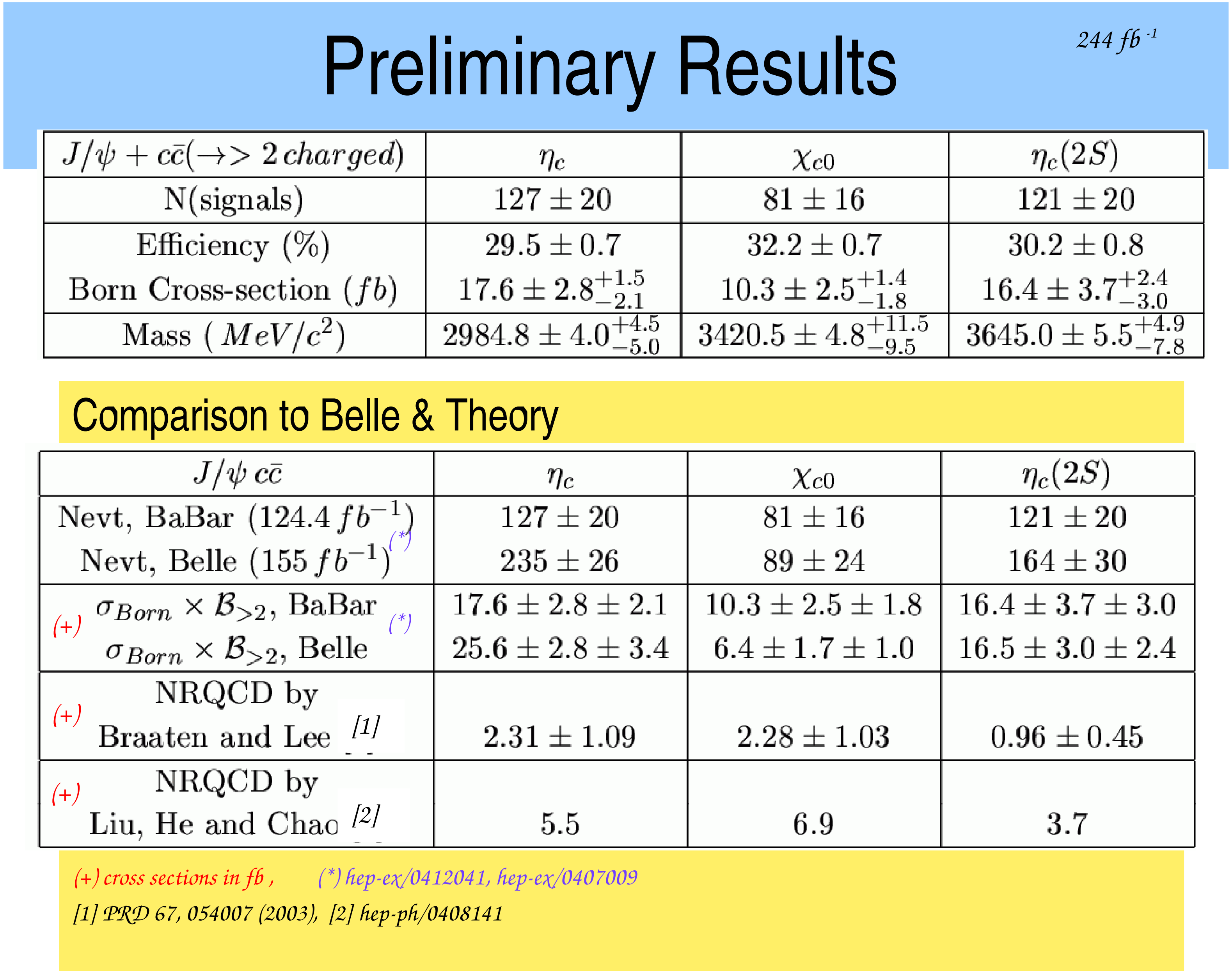
- NRQCD at LO in α_s and v
  Braaten, Lee (2003): $\sigma[e^+e^- \longrightarrow J/\psi + \eta_c] = 3.78 \pm 1.26$ fb.
  Liu, He, Chao (2003): $\sigma[e^+e^- \longrightarrow J/\psi + \eta_c] = 5.5$ fb.
  The two calculations employ different choices of m_c, NRQCD matrix elements, and α_s.
  Braaten and Lee include QED effects.

- Exclusive process: the color-octet contribution is suppressed as v^4.

- The color-singlet matrix elements are determined from $\eta_c \rightarrow \gamma\gamma$ and $J/\psi \rightarrow e^+e^-$.

- An important step in resolving the discrepancy:
  A calculation of corrections at NLO in α_s by Zhang, Gao, and Chao (2005) shows that the K factor is approximately 1.96.
  - Confirmed by Gong and Wang (2007).
  - Not enough to bring theory into agreement with experiment.
- There are similar discrepancies between theory and experiment for $\sigma[e^+e^- \longrightarrow J/\psi + \chi_{c0}]$ and $\sigma[e^+e^- \longrightarrow J/\psi + \eta_c(2S)]$.

- Discrepancy in exclusive production of J/ψ plus χ_{c0}, $\eta_c(2S)$ at the B factories.



Preliminary Results

$244\ fb^{-1}$

$J/\psi + c\bar{c}(\to > 2\,charged)$	η_c	χ_{c0}	$\eta_c(2S)$
N(signals)	127 ± 20	81 ± 16	121 ± 20
Efficiency (%)	29.5 ± 0.7	32.2 ± 0.7	30.2 ± 0.8
Born Cross-section (fb)	$17.6 \pm 2.8^{+1.5}_{-2.1}$	$10.3 \pm 2.5^{+1.4}_{-1.8}$	$16.4 \pm 3.7^{+2.4}_{-3.0}$
Mass (MeV/c^2)	$2984.8 \pm 4.0^{+4.5}_{-5.0}$	$3420.5 \pm 4.8^{+11.5}_{-9.5}$	$3645.0 \pm 5.5^{+4.9}_{-7.8}$

Comparison to Belle & Theory

$J/\psi\ c\bar{c}$	η_c	χ_{c0}	$\eta_c(2S)$
Nevt, BaBar ($124.4\,fb^{-1}$)	127 ± 20	81 ± 16	121 ± 20
Nevt, Belle ($155\,fb^{-1}$) (*)	235 ± 26	89 ± 24	164 ± 30
(+) $\sigma_{Born} \times \mathcal{B}_{>2}$, BaBar	$17.6 \pm 2.8 \pm 2.1$	$10.3 \pm 2.5 \pm 1.8$	$16.4 \pm 3.7 \pm 3.0$
$\sigma_{Born} \times \mathcal{B}_{>2}$, Belle (*)	$25.6 \pm 2.8 \pm 3.4$	$6.4 \pm 1.7 \pm 1.0$	$16.5 \pm 3.0 \pm 2.4$
(+) NRQCD by Braaten and Lee [1]	2.31 ± 1.09	2.28 ± 1.03	0.96 ± 0.45
(+) NRQCD by Liu, He and Chao [2]	5.5	6.9	3.7

(+) cross sections in fb , *(*) hep-ex/0412041, hep-ex/0407009*

[1] PRD 67, 054007 (2003), [2] hep-ph/0408141

From J. Coleman Moriond talk.

Including Relativistic and α_s Corrections to $e^+e^- \longrightarrow J/\psi + \eta_c$

- Relativistic corrections $\sigma[e^+e^- \longrightarrow J/\psi + \eta_c]$ can come from two sources:
  - Direct corrections to the process $e^+e^- \longrightarrow J/\psi + \eta_c$ itself,
  - Indirect corrections that enter through the matrix element of leading order in v. Appear when $\Gamma[J/\psi \rightarrow e^+e^-]$ is used to determine the matrix element because of relativistic corrections to the theoretical expression for $\Gamma[J/\psi \rightarrow e^+e^-]$.
- Relativistic corrections depend on matrix elements of higher order in v.
- GTB, Kang, Lee (2006): Determine matrix elements of higher order in v by making use of a potential model.
  - If the static $Q\bar{Q}$ potential is known exactly, then the uncertainty is of relative order v^2.
  - First determination of these matrix elements with small enough uncertainties to be useful.
- GTB, Chung, Kang, Kim, Lee, Yu (2006): Corrections at NLO in α_s plus relativistic corrections may bring theory into agreement with experiment.
- Confirmed by He, Fan, Chao (2007).
- GTB, Chung, Kang, Lee (2007): New determination of the matrix elements of LO and NLO in v.

- New Calculation of $\sigma[e^+e^- \to J/\psi + \eta_c]$
  (GTB, Chung, Kang, Lee, Yu (2007))
  - Makes use of the matrix elements from GTB, Chung, Kang, Lee (2007).
  - Resums a class of relativistic corrections.
    Includes all corrections that arise from the potential-model $Q\bar{Q}$-Fock-state wave function, up to the UV cutoff of NRQCD.
  - Uses the results of Zhang, Gao, and Chao (2005) for the corrections of NLO in α_s.
  - Includes the interference between the relativistic corrections and the corrections of NLO in α_s.
  - Includes a detailed error analysis

$$\sigma_{\rm tot} = 17.6^{+0.8+5.3+0.7+3.9+0.7+2.8+1.6+1.4+1.9+1.32+1.89}_{-0.9-3.7-0.7-3.0-0.7-2.9-1.5-1.1-2.0-1.32-1.89}\ {\rm fb} = 17.6^{+8.1}_{-6.7}\ {\rm fb}$$

- Uncertainty in the NRQCD factorization formula: $\sim m_H^2/(s/4) \approx 34\%$.

- σ_{tot} consists of

5.4 fb	Leading order in α_s and v (including indir. rel. corr., but without QED contribution)
1.0 fb	QED contribution
2.9 fb	Direct relativistic corrections
6.9 fb	Corrections of NLO in α_s
1.4 fb	Interference between rel. corr. and corr. of NLO in α_s
17.6 fb	Total

- Indirect relativistic corrections are about 31% per quarkonium.
- Result for $\sigma[e^+e^- \to J/\psi + \eta_c]$ confirmed, within uncertainties, by He, Fan, Chao (2007).

- Theory and experiment agree within uncertainties:
  - Theory: $\sigma[e^+e^- \to J/\psi + \eta_c] = 17.6^{+8.1}_{-6.7}\,\mathrm{fb}$
  - Belle: $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 25.6 \pm 2.8 \pm 3.4$ fb.
  - BABAR: $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1}$ fb.
- *Caveat:* $B_{>2}$ is not known.
  - Could be as small as 0.5–0.6.
  - Even so, the error bars of theory and the BABAR experiment overlap.
- Zhang, Ma, Chao (2008): In the cases of $\sigma[e^+e^- \to J/\psi(\psi(2S)) + \chi_{c0}]$, large K factors (~ 2.8) may bring theory into agreement with experiment.
- See the talk by Victor Braguta in Parallel Session C for a discussion of the light-cone approach to exclusive double-charmonium production.

Inclusive Double $c\bar{c}$ Production at Belle

- Belle:

$$\begin{aligned}&\sigma(e^+e^- \to J/\psi + c\bar{c} + X)/\sigma(e^+e^- \to J/\psi + X)\\ &\quad = 0.82 \pm 0.15 \pm 0.14\\ &\quad > 0.48 \text{ (90\% confidence level)}\end{aligned}$$

- pQCD plus color-singlet model (Cho, Leibovich (1996); Baek, Ko, Lee, Song (1997); Yuan, Qiao, Chao (1997)):

$$\sigma(e^+e^- \to J/\psi + c\bar{c} + X)/\sigma(e^+e^- \to J/\psi + X) \approx 0.1.$$

- The Numerator: Experiment and theory also disagree.
  - Belle (2002): $\sigma(e^+e^- \to J/\psi + c\bar{c} + X) = 0.87^{+0.21}_{-0.19} \pm 0.17$ pb.
  - Leading-Order Theory (Color-Singlet): $\sigma(e^+e^- \to J/\psi + c\bar{c} + X) =$ 0.10–0.27 pb.
    Large renormalization-scale dependence.
- New NLO Calculation of the Numerator
  (Zhang and Chao (2007))
  - Find a K factor of about 1.8.
  - Taking into account QED corrections, two-photon processes, feeddown from $\psi(2S)$ (the largest effect) and χ_{cJ}, and color-octet corrections, they obtain
    $\sigma(e^+e^- \to J/\psi + c\bar{c} + X) = 0.53^{+0.59}_{-0.23}$ pb. ($\mu = \sqrt{s}/2$)
    The uncertainties come from m_c.
  - Resolves the discrepancy between theory and experiment, but the theoretical uncertainties are large.
- He, Fan, Chao (2007): Direct relativistic corrections to the numerator are only about $+31\%$.
- Nayak, Qiu, and Sterman: there could be a nonperturbative enhancement to production of $J/\psi + c\bar{c}$ when the c or the $\bar{c}$ is co-moving with the J/ψ.
  This effect can't be calculated reliably in perturbation theory.
  Its size must be determined experimentally.

- New Calculation of the Denominator
  (Zhang, Ma, Chao (2008))
  $\sigma(e^+e^- \to J/\psi^{(8)} + g) = 0.586$ pb (LO, $\mu = 2m_c$)
  $\sigma(e^+e^- \to J/\psi^{(8)} + g) = 1.19$ pb (NLO, $\mu = 2m_c$).
  - At the B-factory energy, this color-octet process dominates in LO.
  - NLO result yields $\sigma(e^+e^- \to J/\psi + c\bar{c} + X)/\sigma(e^+e^- \to J/\psi + X) = 0.31^{+0.18}_{-0.11}\%$.
  - The color-singlet processes should also be calculated in NLO.
- The Belle result for $\sigma(e^+e^- \to J/\psi + c\bar{c} + X)/\sigma(e^+e^- \to J/\psi + X)$ still poses a significant challenge to existing theory.
- It is important for BABAR to check the Belle results for inclusive double $c\bar{c}$ production.