Status of higher order QCD computations

(full focus on calculations relevant for the LHC)

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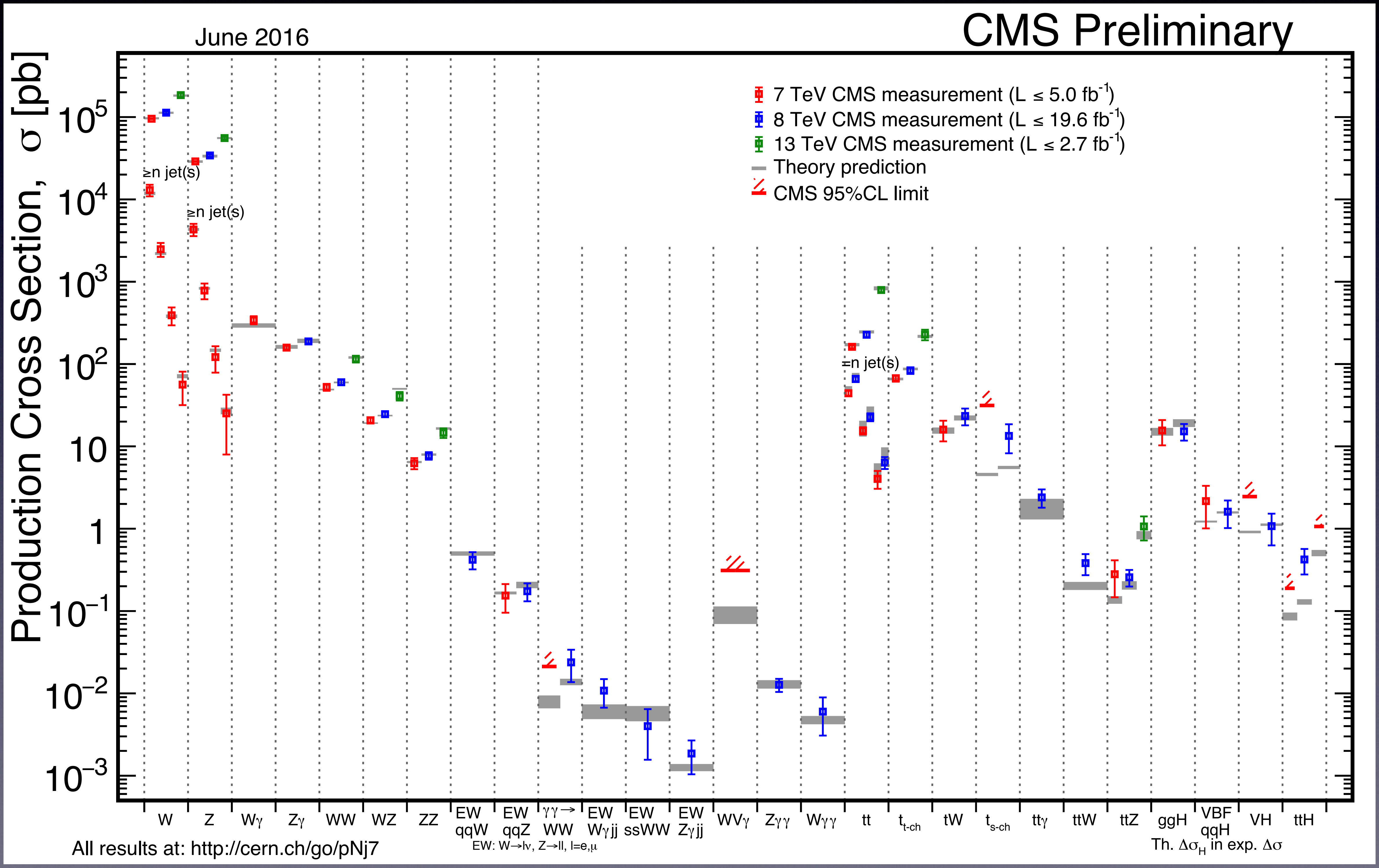
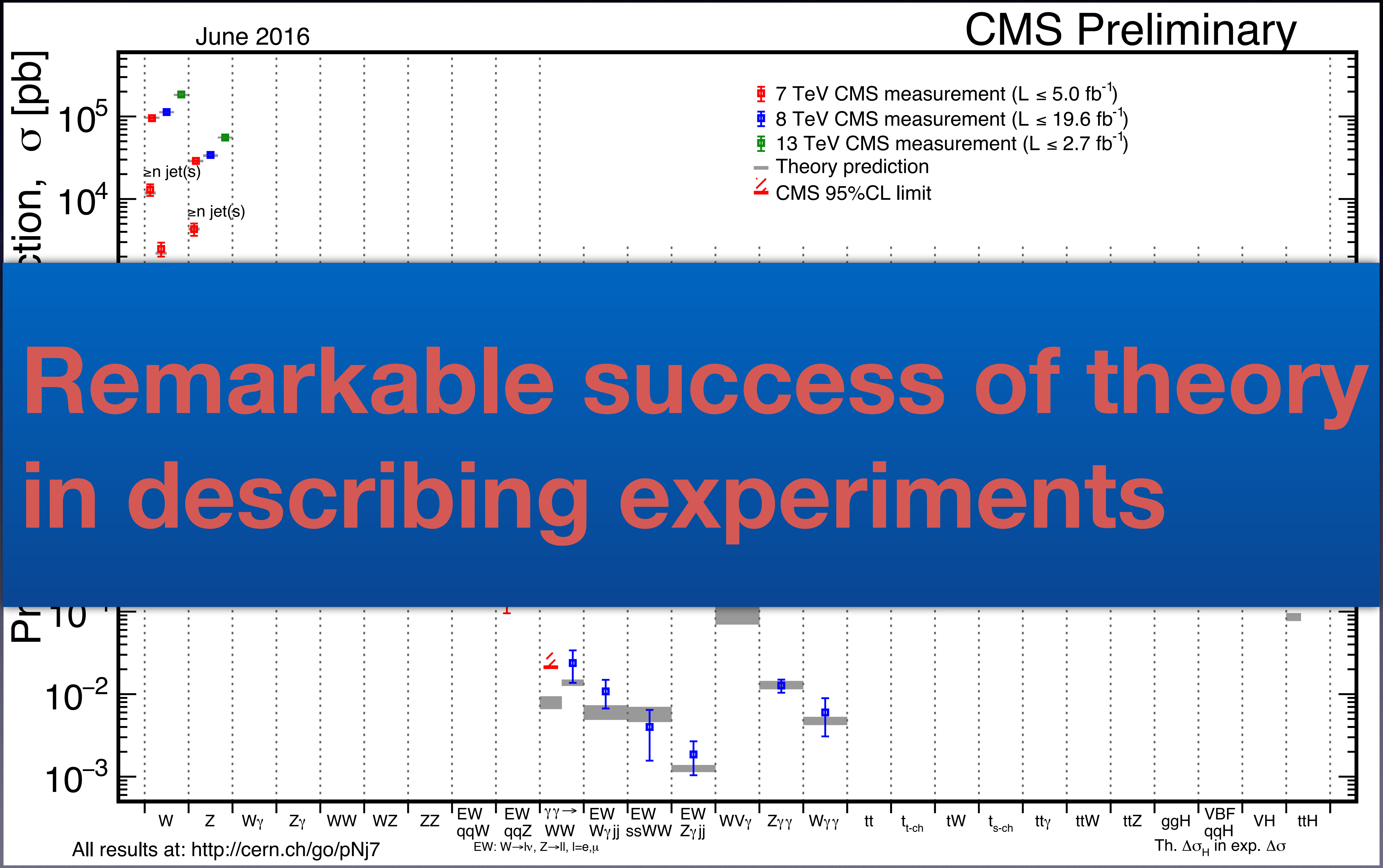
CERN, University of Oxford & ERC

Precision theory for precision measurements at the LHC and future colliders
Quy-Nhon, 26th September 2016

The poster boy plot



The poster boy plot



Remarkable success of theory in describing experiments

Precision, precision, precision …

- after a first glance at Run II data: no direct evidence for New Physics
- indirect searches will play a prominent role in the coming years: crucial to stress-test the Standard Model (in particular the still poorly explored Higgs sector) and establish possible deviations from the SM

In this game, precision is crucial to maximise sensitivity

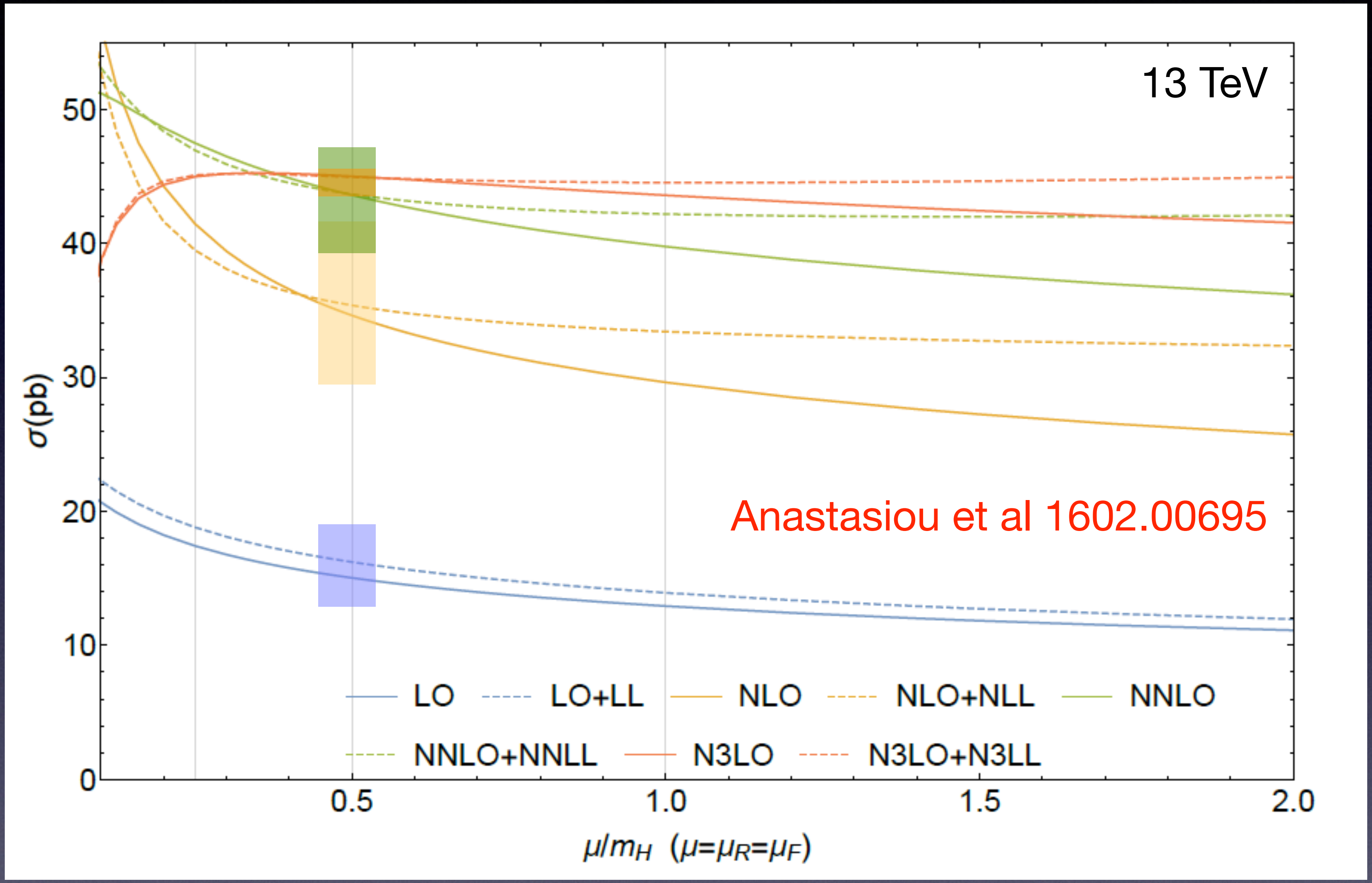
N3LO

Two LHC processes known at N3LO

Gluon fusion Higgs production (in the large m_t effective theory)

Vector boson fusion Higgs production (in the structure function approximation, i.e. double DIS process)

N^3LO Higgs production



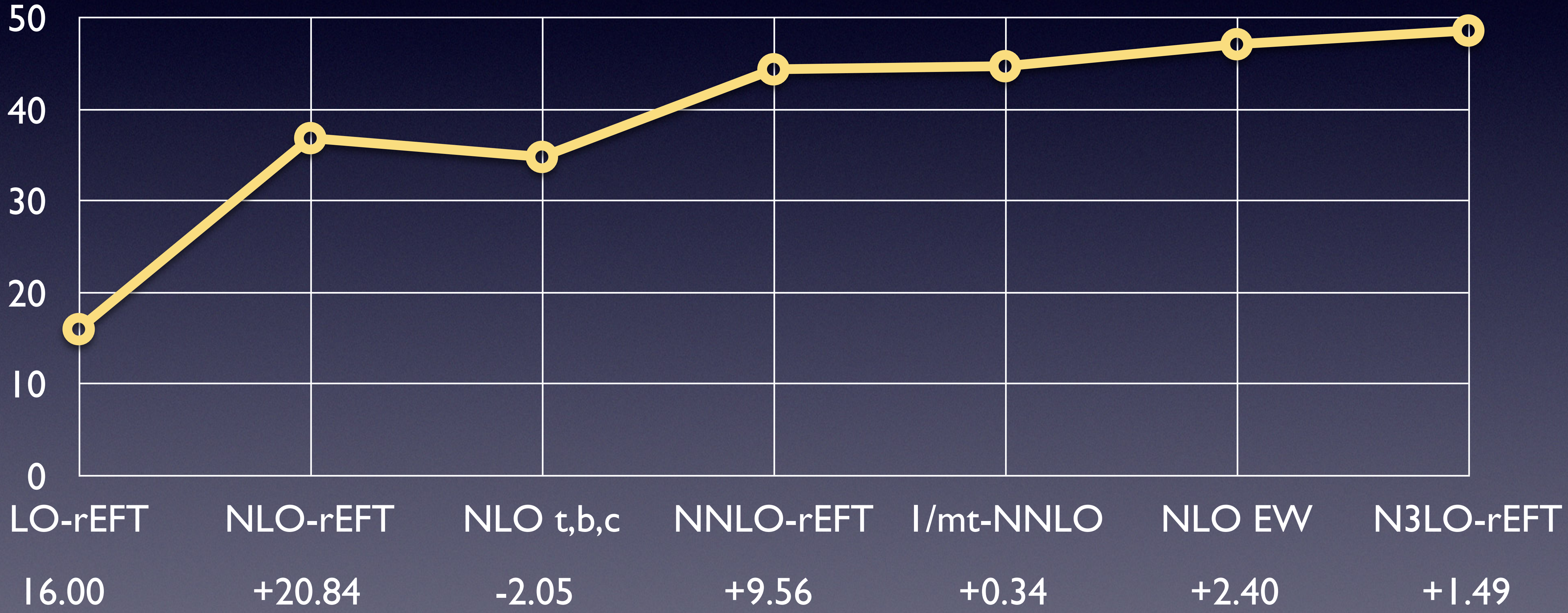
- result also matched to resummed calculation (essentially no impact on central value at preferred scale $m_H/2$)
- N^3LO stabilizes the perturbative expansion (N^3LO band contained in NNLO band, while NNLO was not in the NLO band)

N^3LO Higgs production

Anastasiou et al 1602.00695

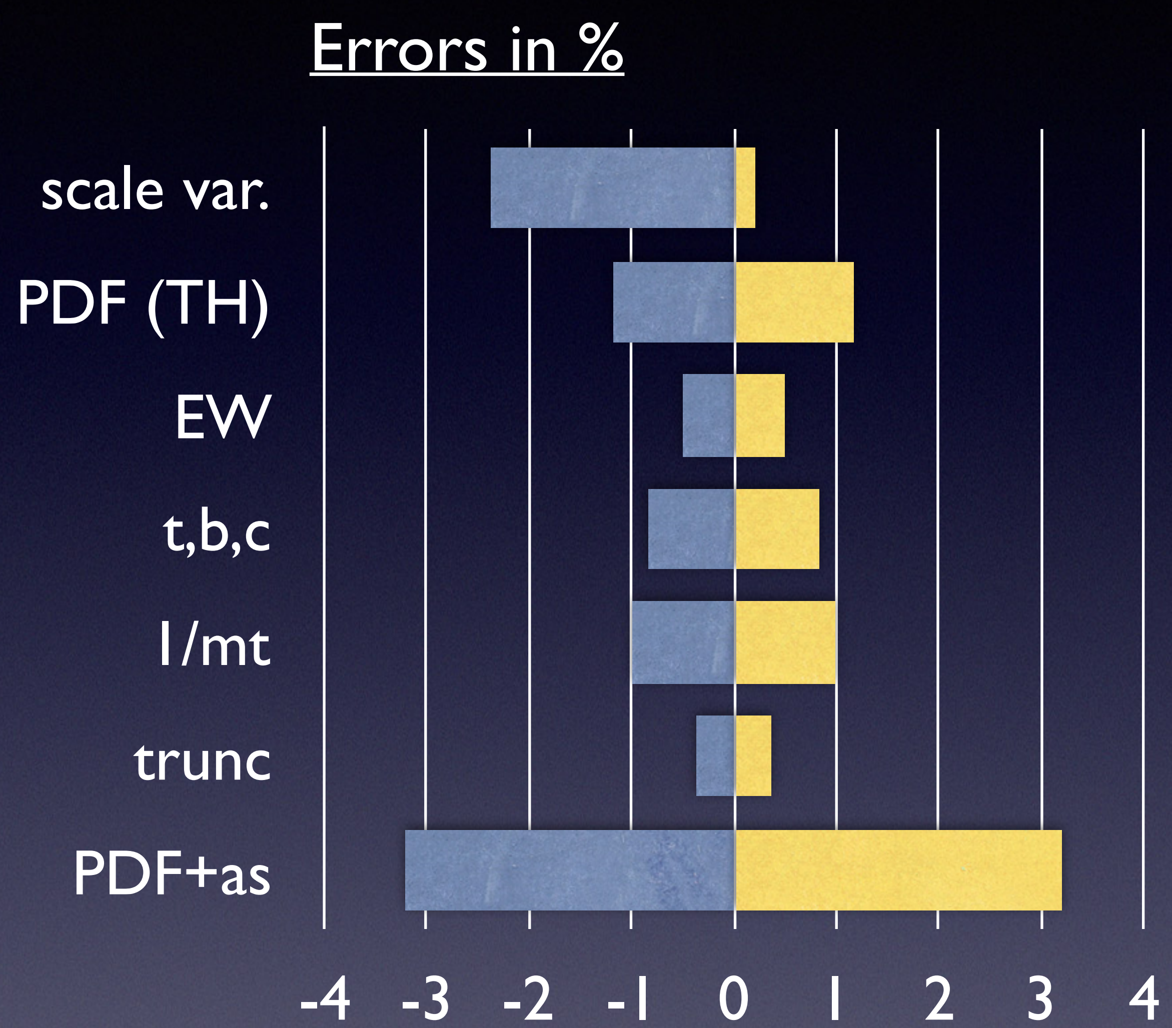
At this level of accuracy, many other effects must be accounted for

LHC 13 TeV: cross section in [pb] = 48.58 pb



rEFT = EFT (i.e. heavy-top approximation) but rescaled by (exact Born) / (EFT Born) ≈ 1.07

Error budget from 1602.00695



Most debated points in the Higgs Cross Section working group (HXSWG)

- include or not a resummation?
- 3 or 7 point scale variation? symmetrize scale var. error?
- alternative estimate of (bottom,charm) effects
- quadratic vs linear combination of errors

Total theory error: add all 6 theory errors linearly and keep the (PDF+α_s) error separate (to be added quadratically)

$$\sigma = 48.58\mathrm{pb}^{+2.22\mathrm{pb}(4.56\%)}_{-3.27\mathrm{pb}(-6.72\%)}\,\mathrm{theory} \pm 1.56\mathrm{pb}(3.2\%)(\mathrm{PDF} + \alpha_\mathrm{s})$$

The new HXSWG recommendation

Discussion resulted in a new recommendation of the HSXWG for 4th Yellow Report: use the pure fixed order result from 1602.00695 for the central value, and take it's uncertainty interpreted as

100% flat

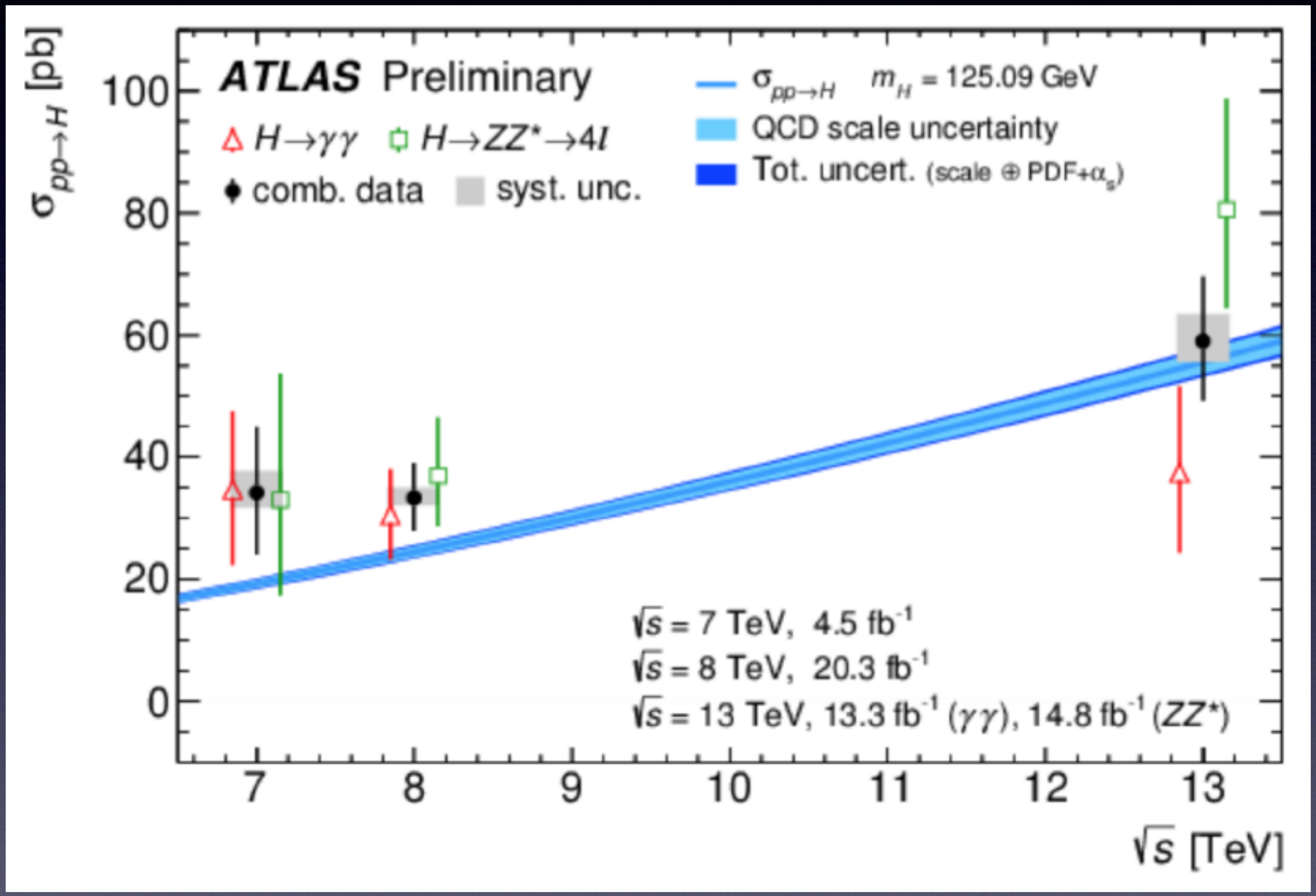
68% gaussian

$$\sigma = 48.58\text{pb}^{+2.22\text{pb}(4.56\%)}_{-3.27\text{pb}(-6.72\%)}\,\text{theory} \pm 1.56\text{pb}(3.2\%)(\text{PDF}+\alpha_\text{s})$$

$$\Delta_\text{th} = 3.9\%$$

If it is highly preferred to have only gaussian theory uncertainties then transform to gaussian one (symmetrize and divide by $\sqrt{3}$)

Data vs theory



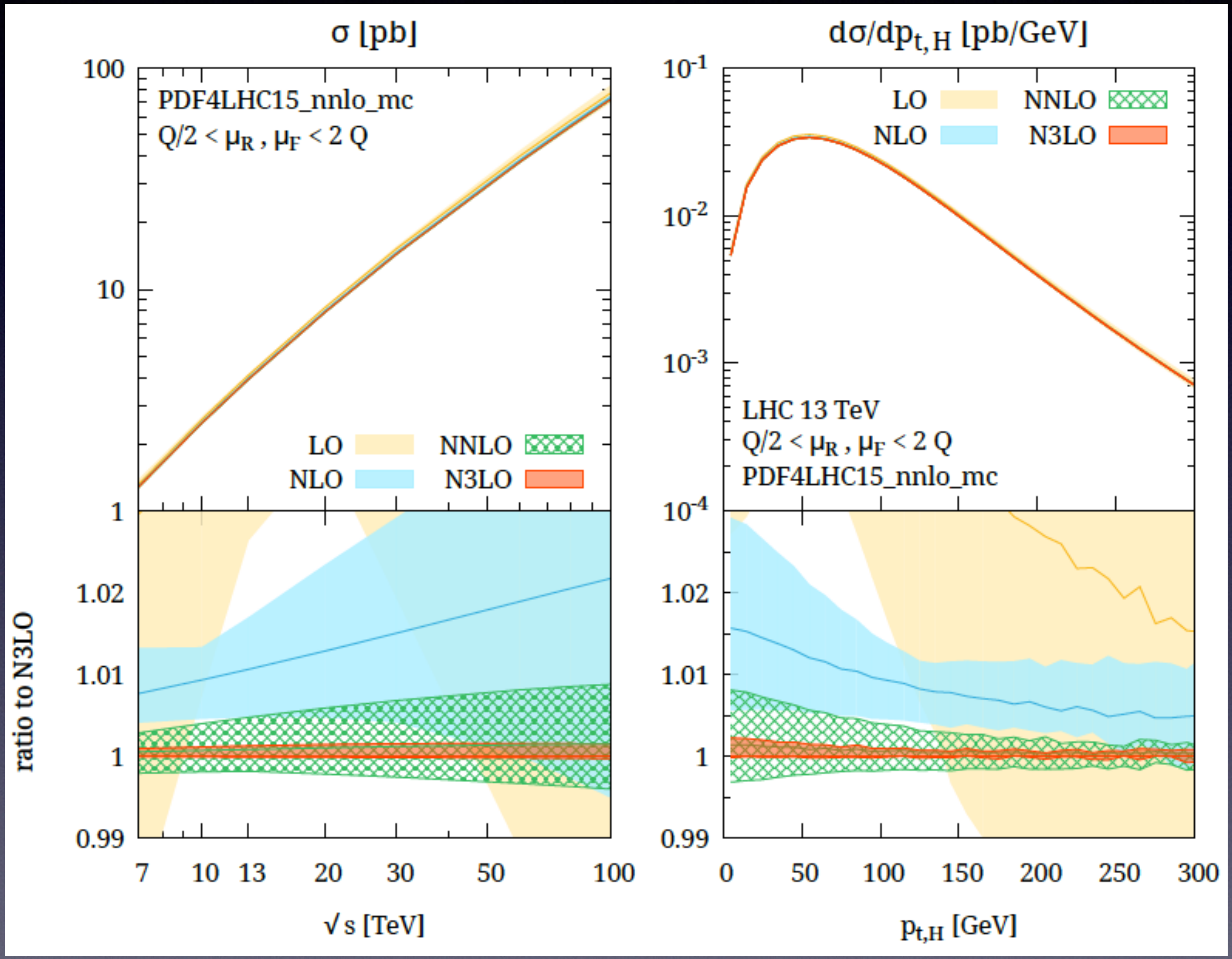
"... EXP precision is very far away (TH went ahead 15 years of EXP?), but it would be better to have numbers with best precision."

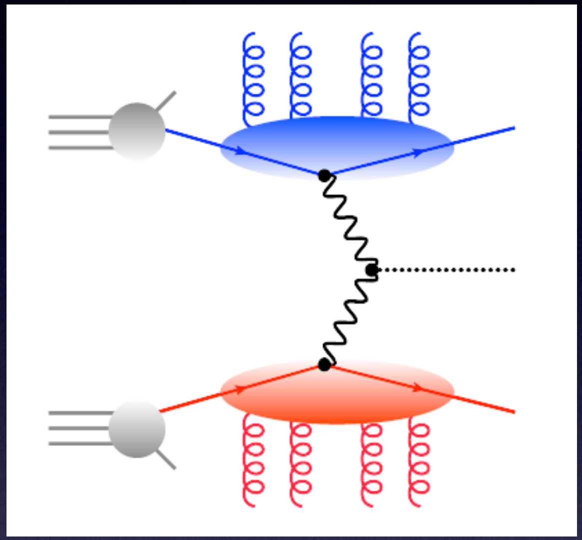
[email by Reisaburo Tanaka to the ggF conveners]

Next challenge: extend N3LO accuracy to differential distributions (hard but within reach?)

… and inclusive VBFH at N³LO

Dreyer & Karlberg 1606.00840





Again, NNLO was outside the NLO uncertainty band, while N3LO band (with sensible scale) is fully contained in the NNLO band

Impact of NNLO PDF in N3LO results

N3LO PDFs currently not available. Calculations use NNLO PDFs.
Error estimated as (1606.02695)

$$\Delta_{\text{PDF(A)}} = \frac{1}{2}\left|\frac{\sigma^{\text{NNLO}}_{\text{NNLO-PDF}} - \sigma^{\text{NNLO}}_{\text{NLO-PDF}}}{\sigma^{\text{NNLO}}_{\text{NNLO-PDF}}}\right|$$

Expect smaller effect at one order higher

Estimate effect from one order lower

Impact of NNLO PDF in N3LO result

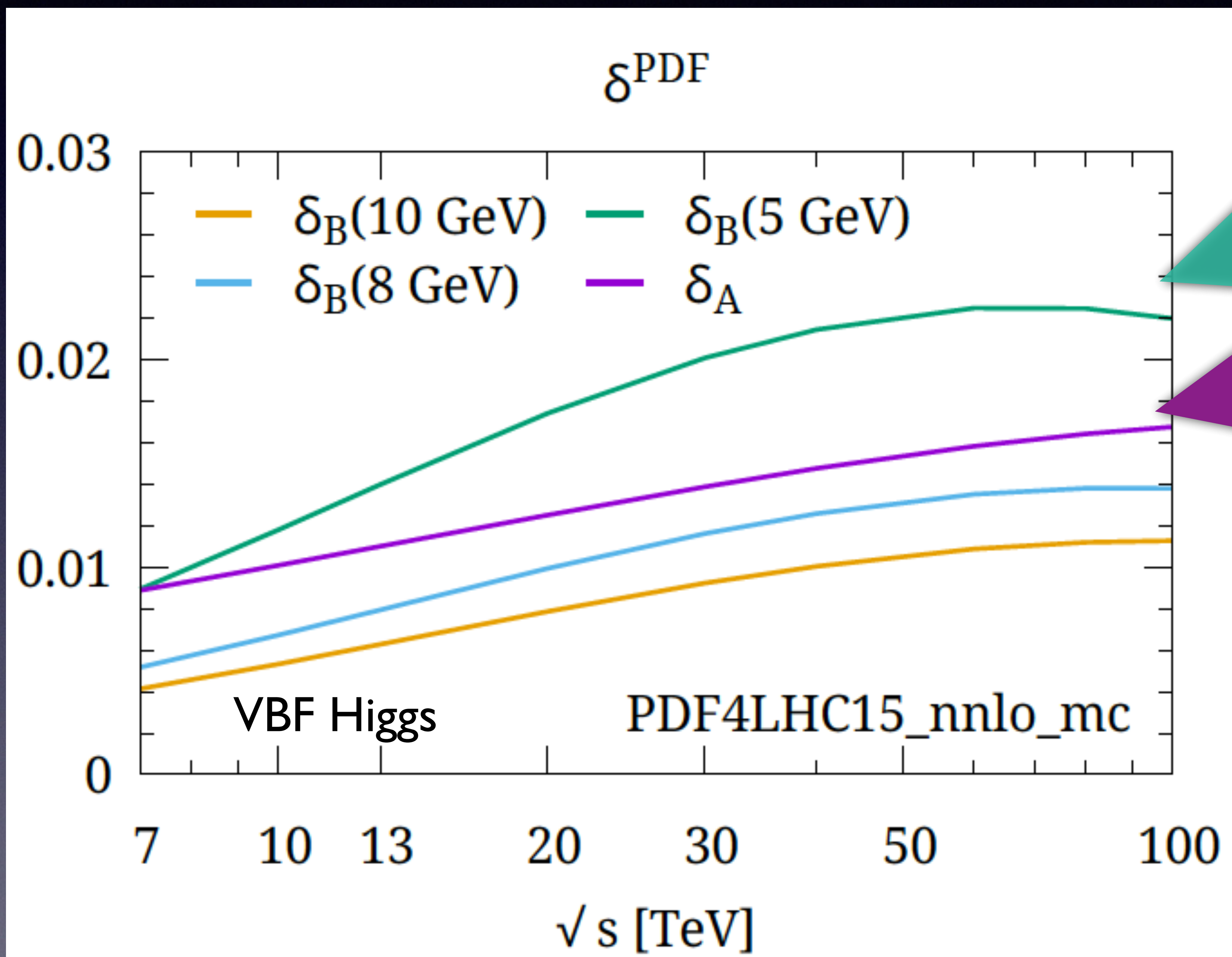
Alternative estimate (from 1606.00840): rescale parton distributions using the F_2 structure function at N3LO at some scale Q_0

$$f^{\mathrm{N3LO,approx}}(x,Q) = f^{\mathrm{NNLO}}(x,Q)\frac{F_2^{\mathrm{NNLO}}(x,Q_0)}{F_2^{\mathrm{N3LO}}(x,Q_0)}$$

Re-evaluate the cross-section using approximate higher-order PDFs

$$\Delta_{\mathrm{PDF(B)}} = \left|\frac{\sigma^{\mathrm{N3LO}} - \sigma^{\mathrm{N3LO\text{-}approx}}}{\sigma^{\mathrm{N3LO}}}\right|$$

Impact of NNLO PDF in N3LO result



$\Delta_{PDF(B)}$

$\Delta_{PDF(A)}$

Impact of N3LO coefficient functions not negligible; missing N3LO terms in the evolution smaller impact

NNLO

NNLO is one of the most active areas in QCD now

After pioneering calculations for Higgs and Drell Yan more than 10 years ago, recently many $2 \to 2$ processes computed at NNLO

NNLO most important in three different situations

Benchmark processes measured with highest accuracy

- $Z \to ll$
- $W \to l\nu$
- Z+ jet
- …

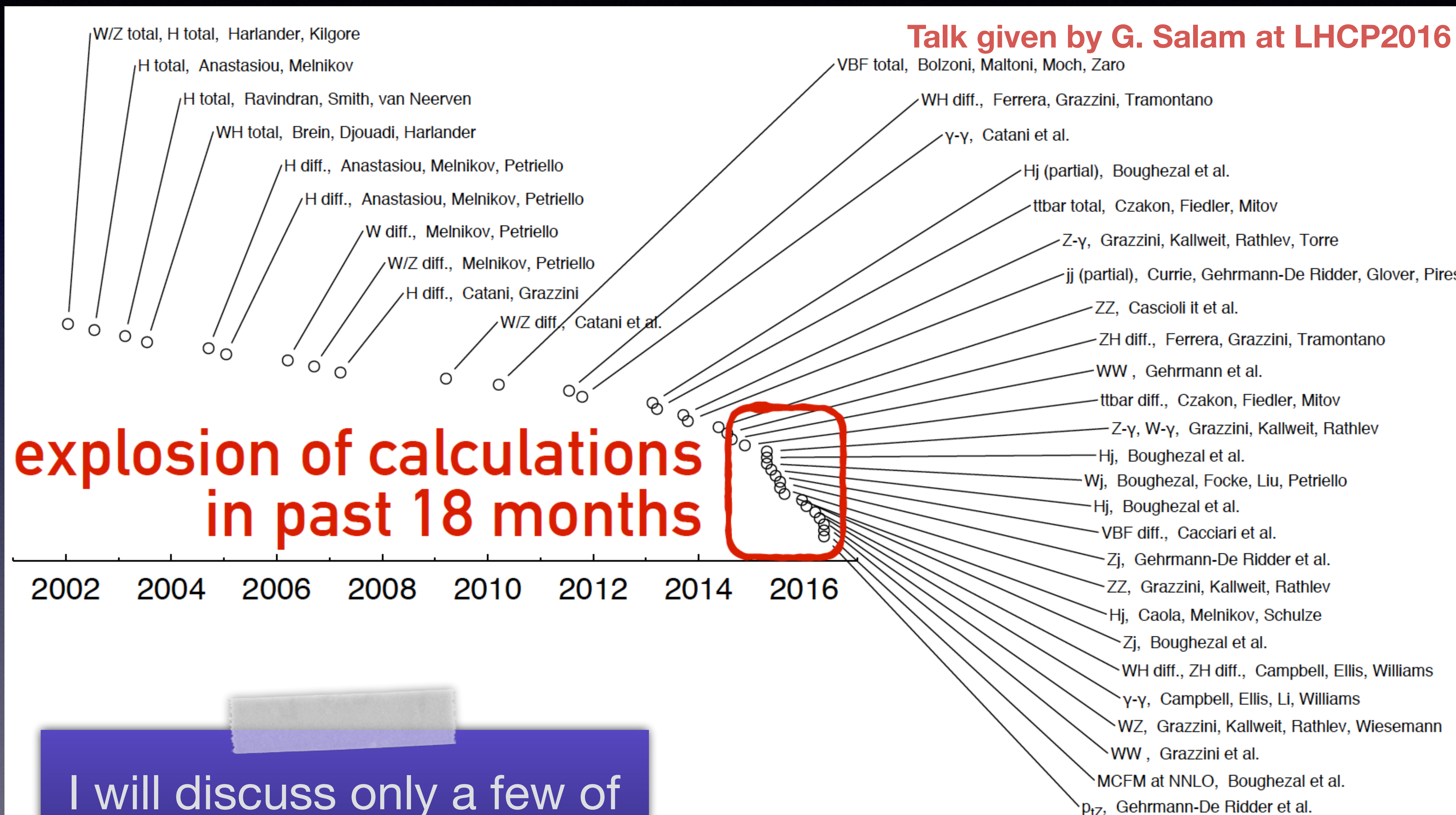
Input to PDF fits + background to Higgs studies

- diboson
- boson + jet
- top-pairs
- …

Very large NLO corrections (moderate precision requires NNLO)

- Higgs
- Higgs+ jet
- …

NNLO

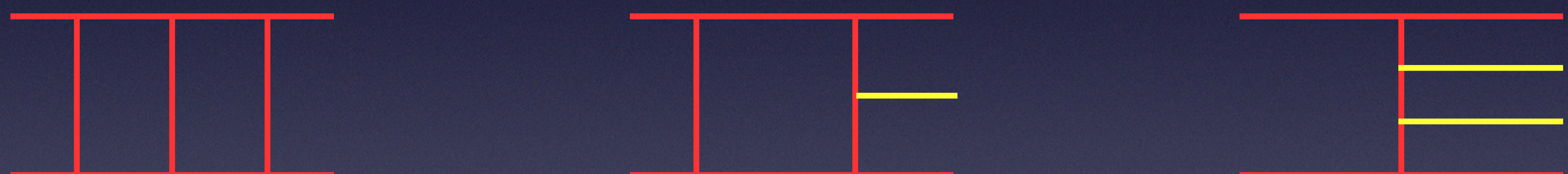


I will discuss only a few of these results (obviously)

Two main difficulties at NNLO

calculation of two-loop master integrals (when many scales are involved)

methods to cancel (overlapping) divergences before integration



$$\int d\Phi_n 2\mathrm{Re}|\mathcal{M}^{2-loop}\mathcal{M}^{tree}| \qquad \int d\Phi_n d\Phi_1 2Re|\mathcal{M}_{n+1}^{\mathrm{one-loop}}\mathcal{M}_{n+1}^{\mathrm{tree}}| \qquad \int d\Phi_n d\Phi_2 |\mathcal{M}^{\mathrm{tree}}|_{n+2}^2$$

$$\int d\Phi_n \left\{ \left(a_4 \frac{1}{\epsilon^4} + a_3 \frac{1}{\epsilon^3} + \ldots + a_0 \right) - \left(a_4 \frac{1}{\epsilon^4} + a_3 \frac{1}{\epsilon^3} + \ldots + b_0) \right) \right\}$$

Cancelation manifest after phase space integration, but to have fully differential results must achieve cancelation before integration

1. Cancelation of divergences

Two strategies

Slicing methods:

partition the phase space with a (small) slicing parameter so that divergences are all below the slicing cut. In the divergent region use an approximate expression, neglecting finite terms, above use the exact (finite) integrand

(need to test independent of slicing parameter)

Subtraction methods:

since IR singularities of amplitudes are knows, add and subtract counterterms so as to make integrals finite. "Easy" at NLO, but complicated at NNLO due to the more intricate structure of (overlapping) singularities

(possible to use local subtraction terms)

Practical realisations

Slicing methods:

- q_T subtraction Catani, Grazzini
- N-jettiness subtraction Boughezal, Focke, Liu, Petriello; Gaunt, Stahlhofen, Tackmann, Walsh

Subtraction methods:

- Sector decomposition Anastasiou, Melnikov, Petriello; Binoth, Heinrich
- Antenna subtraction Kosower; Gehrmann, Gehrmann De Ridder, Glover
- Sector Improved residue subtraction Czakon; Boughezal, Melnikov, Petriello
- Colourful subtraction Del Duca, Somogyi, Trocsanyi
- Projection to Born Cacciari, Dreyer, Karlberg, Salam, GZ

Practical realisations

In principle, the problem of cancelation of singularities solved in theory in a generic way

In practise, methods applied for 2 → 2 processes. Require long runs on large computer farms (plus, possibly, a way to deal with outliers/spikes)

NB: the attitude “Today we have big farms, so why care?” is not acceptable. The phenomenology that one gets out of a calculation scales as inverse power of the computation time

2. Two-loop integrals

- Rather than brute-force calculation, master integrals in many cases computed solving differential equations (DE)

Kotikov 1991; Remiddi 1997; Henn 2013; Papadopoulos 2014

- Method "straightforward" when only polylogarithmic functions are involved

e.g. method even pushed to 3-loop 4-point functions in N=4 SYM Henn & Mistberger 1608.00850
or to 2-loop planar 5-point functions Gehrmann, Henn, Lo Presti 1511.05409 …

- Internal masses complicate the problem considerably: elliptic functions appear
- First four-point multi scale problem computed analytically in terms of elliptic functions (use proper parametrisation of integrals, optimal basis choice, DE method in terms of elliptic iterated integrals …)

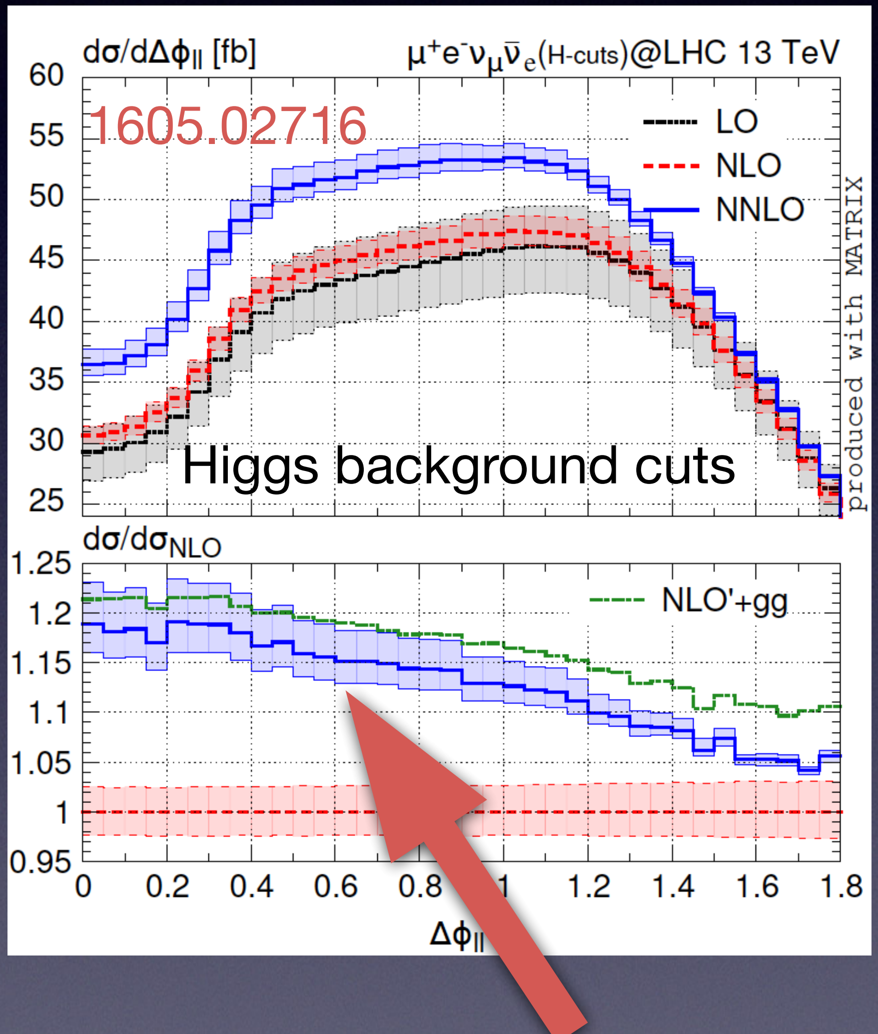
Two-loop planar results for H ⟶ 3 with full mass dependence, Bonciani et al 1609.06685

For processes with internal masses still conceptual bottleneck. Internal masses necessary for Higgs physics at high p_t

Recent NNLO: WW/WZ

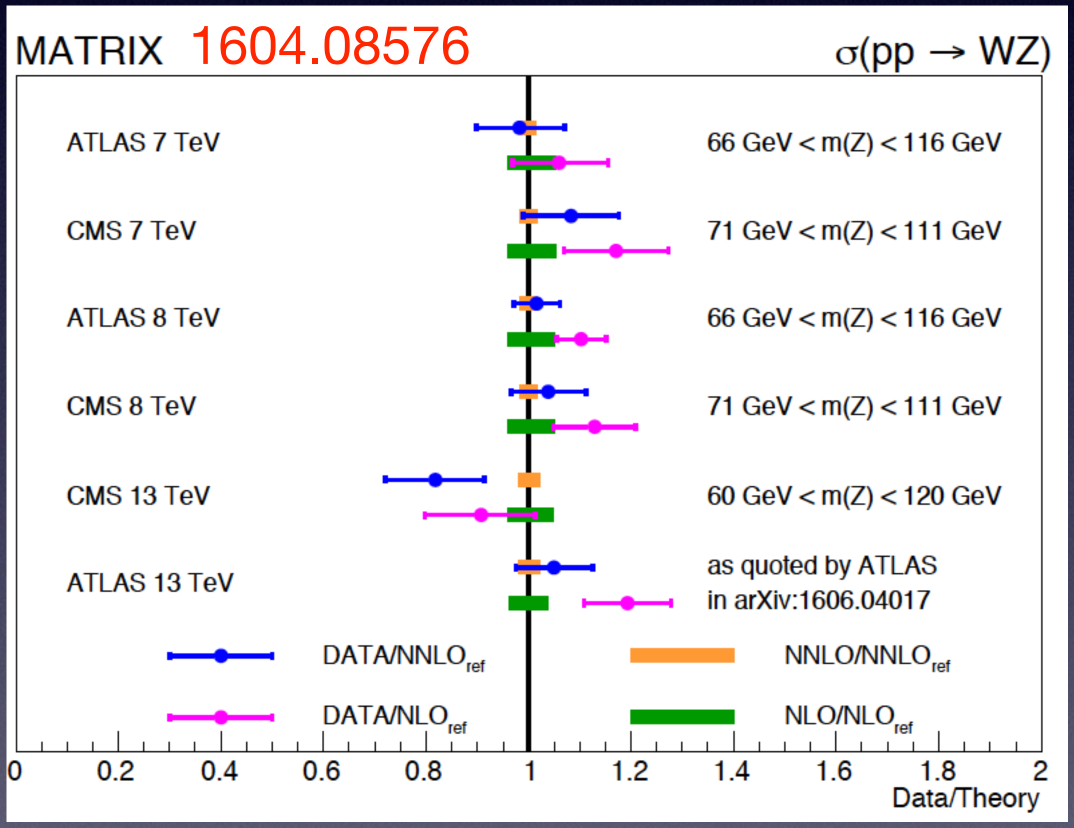
All VV processes now know to NNLO

Catani et al '11; Grazzini et al '14; Cascioli et al '15; Gehrmann et al. '15; Grazzini et al '15-'16;



considerable shape change wrt NLO

LHC data prefers NNLO

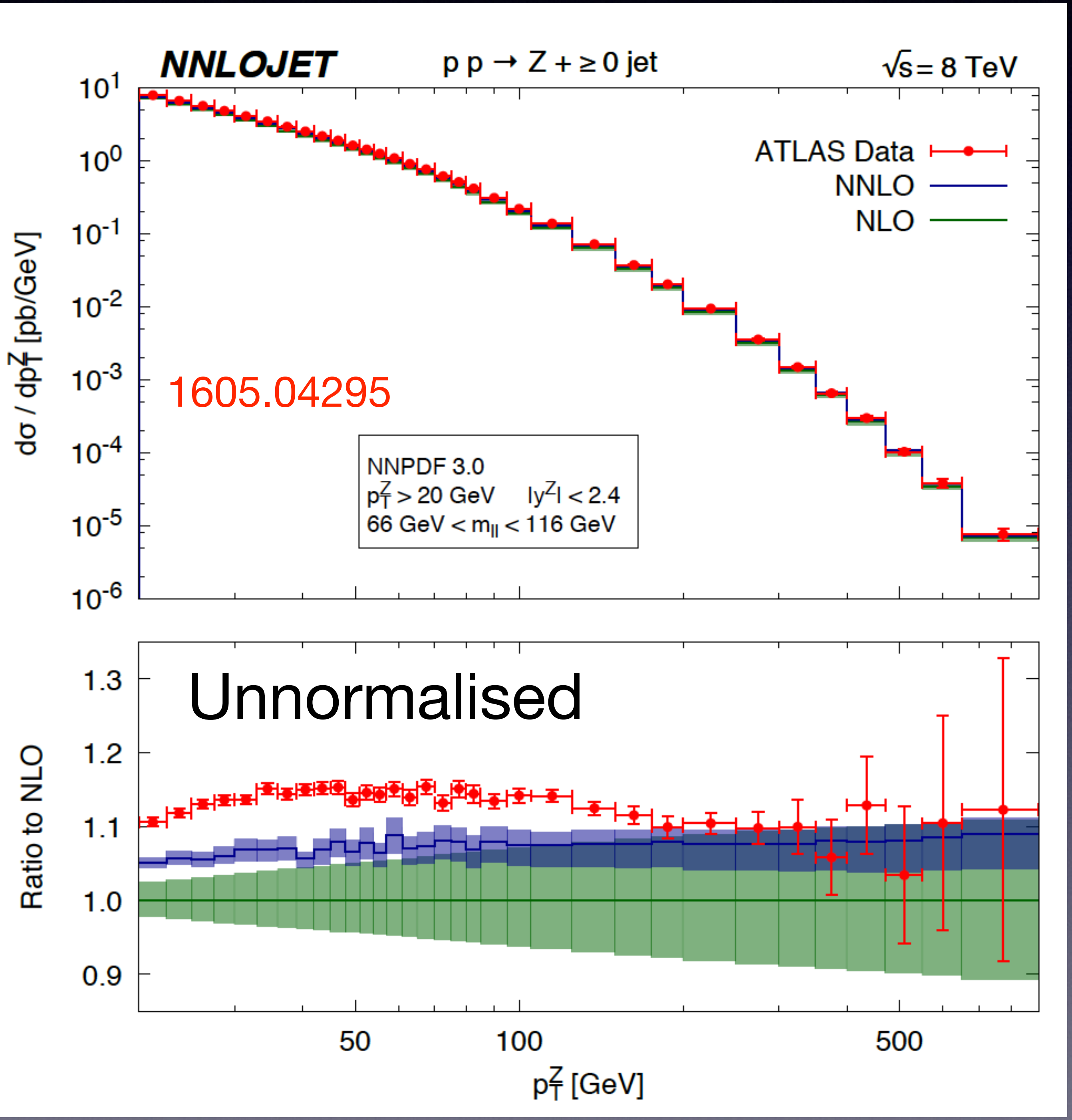


Color singlet production processes now available in MCFM @ NNLO

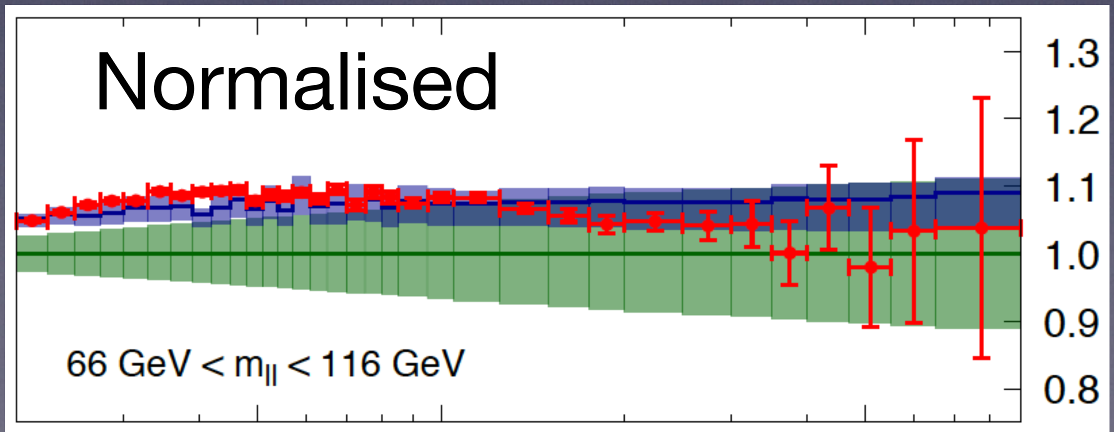
Campbell et al '16; Boughezal et al '16

Recent NNLO: Zj

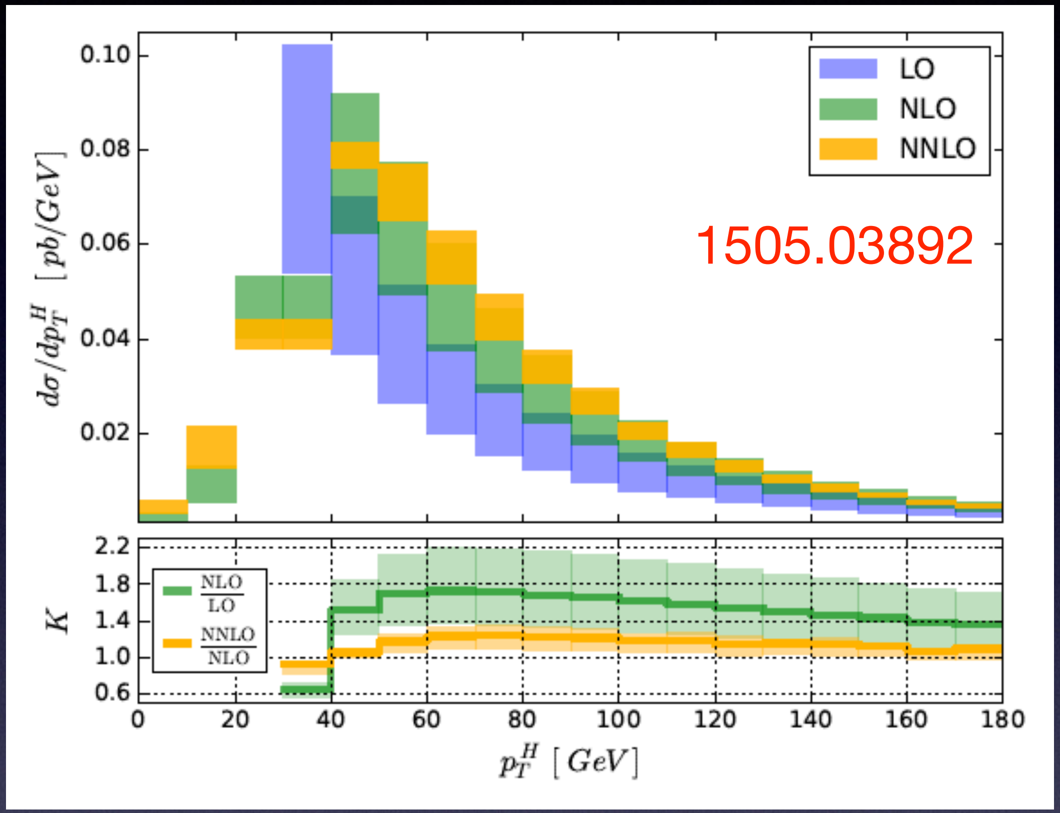
Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan ’16
Boughezal, Liu, Petriello ’16
Boughezal, Ellis, Focke, Giele, Liu, Petriello ’15

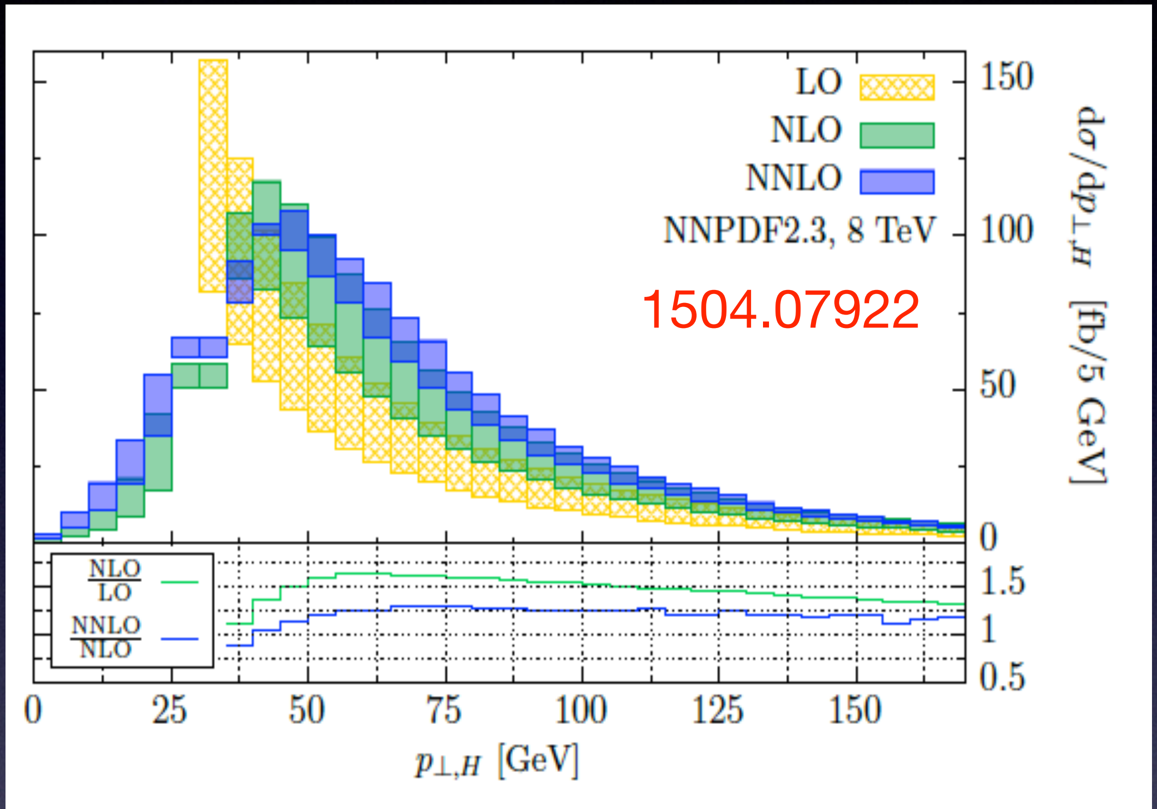


- inclusion of NNLO does not fully resolve tension between data and theory
- better agreement in normalised distribution
- remember 2-3% luminosity error on data



H + 1jet at NNLO



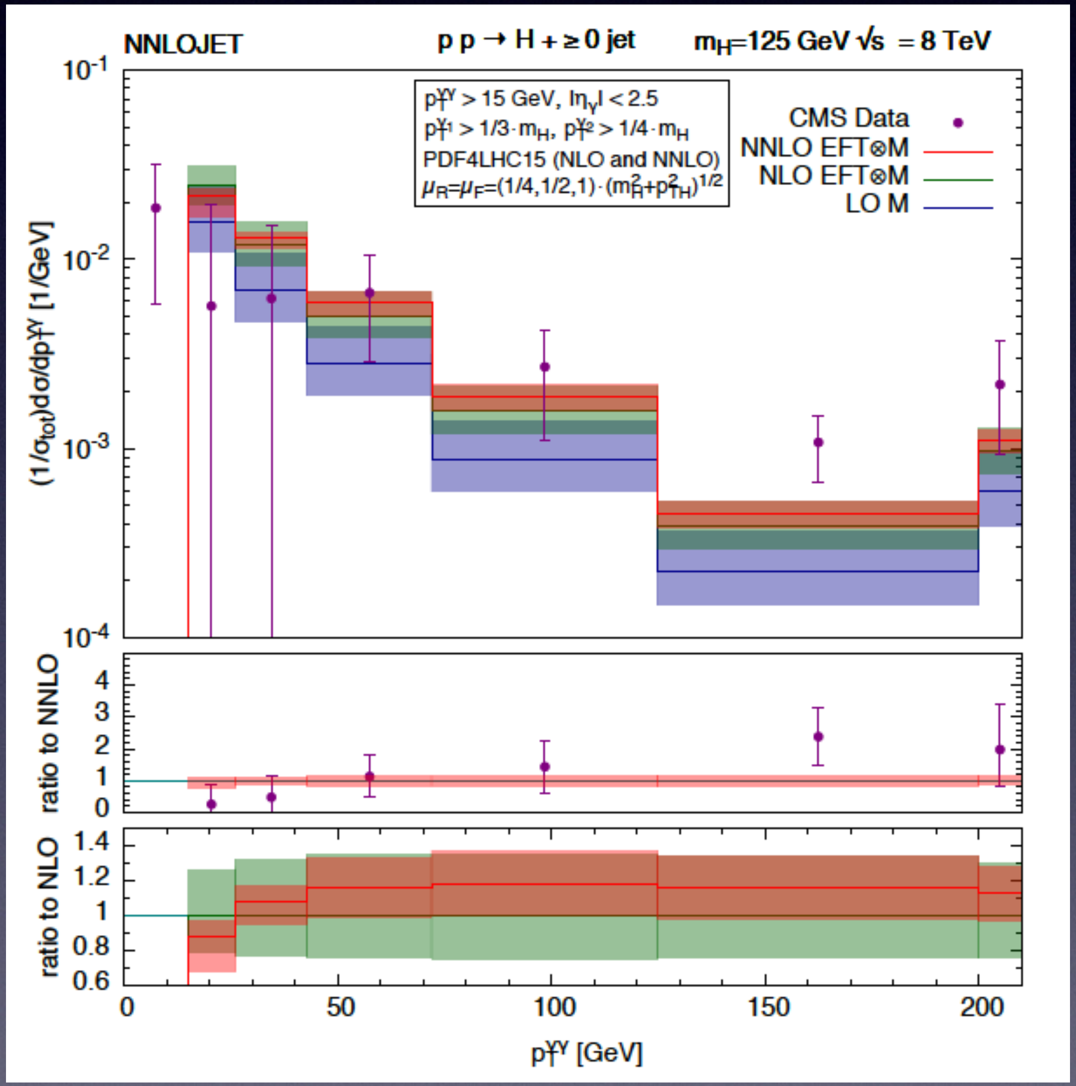


- useful comparison between independent calculations
- sizable K-factor (≈1.15-1.20) and shape changes
- reduction of theory error (still about 10-15%)

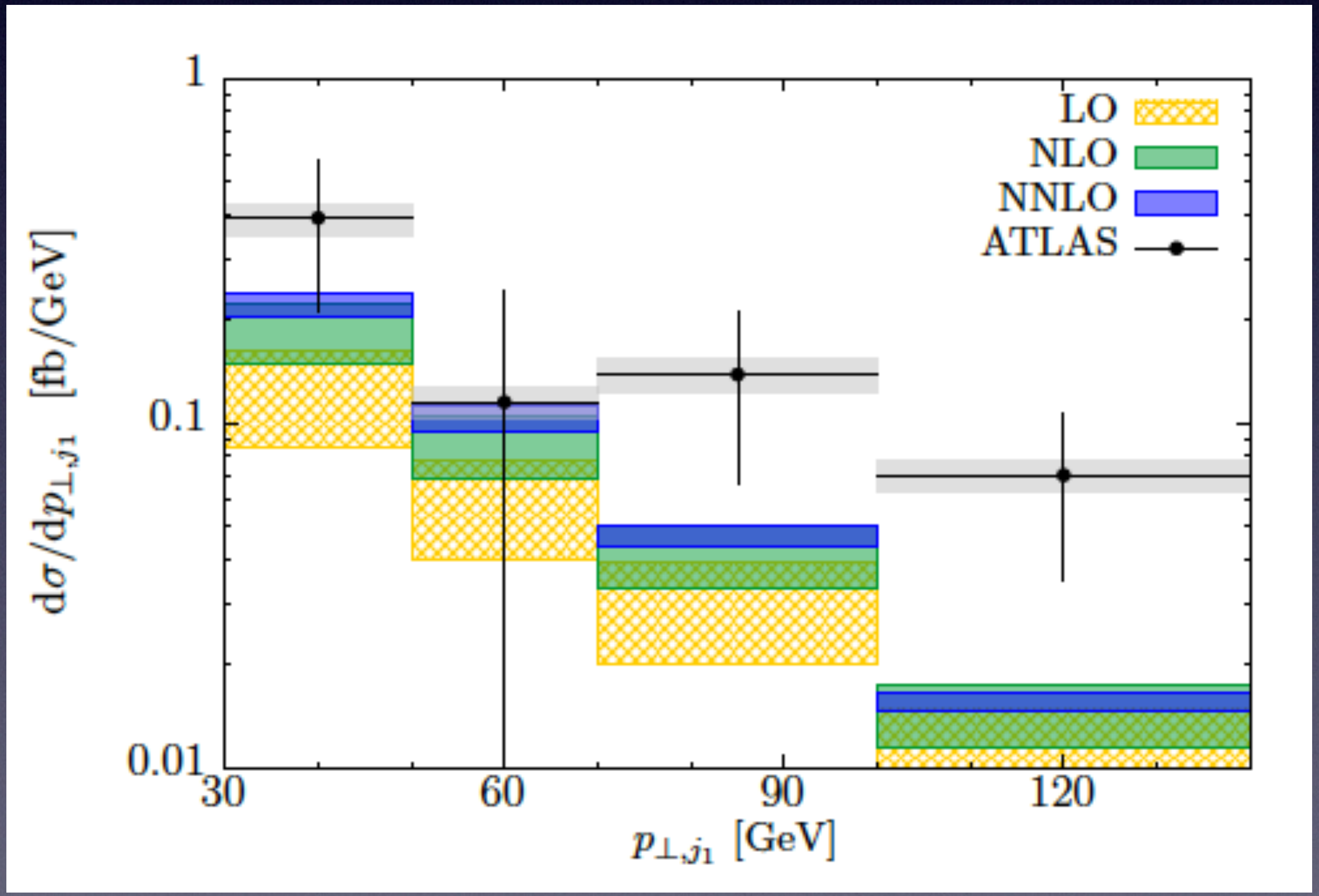
Boughezal, Caola, Melnikov, Petriello, Schulze ’15
Boughezal, Focke, Giele, Liu, Petriello ’15
Chen, Gehrmann, Glover, Jacquier ’15

H + 1jet at NNLO

Decays of Higgs to bosons also included. Fiducial cross-sections compared to ATLAS and CMS data



Caola, Melnikov, Schulze 1508.02684

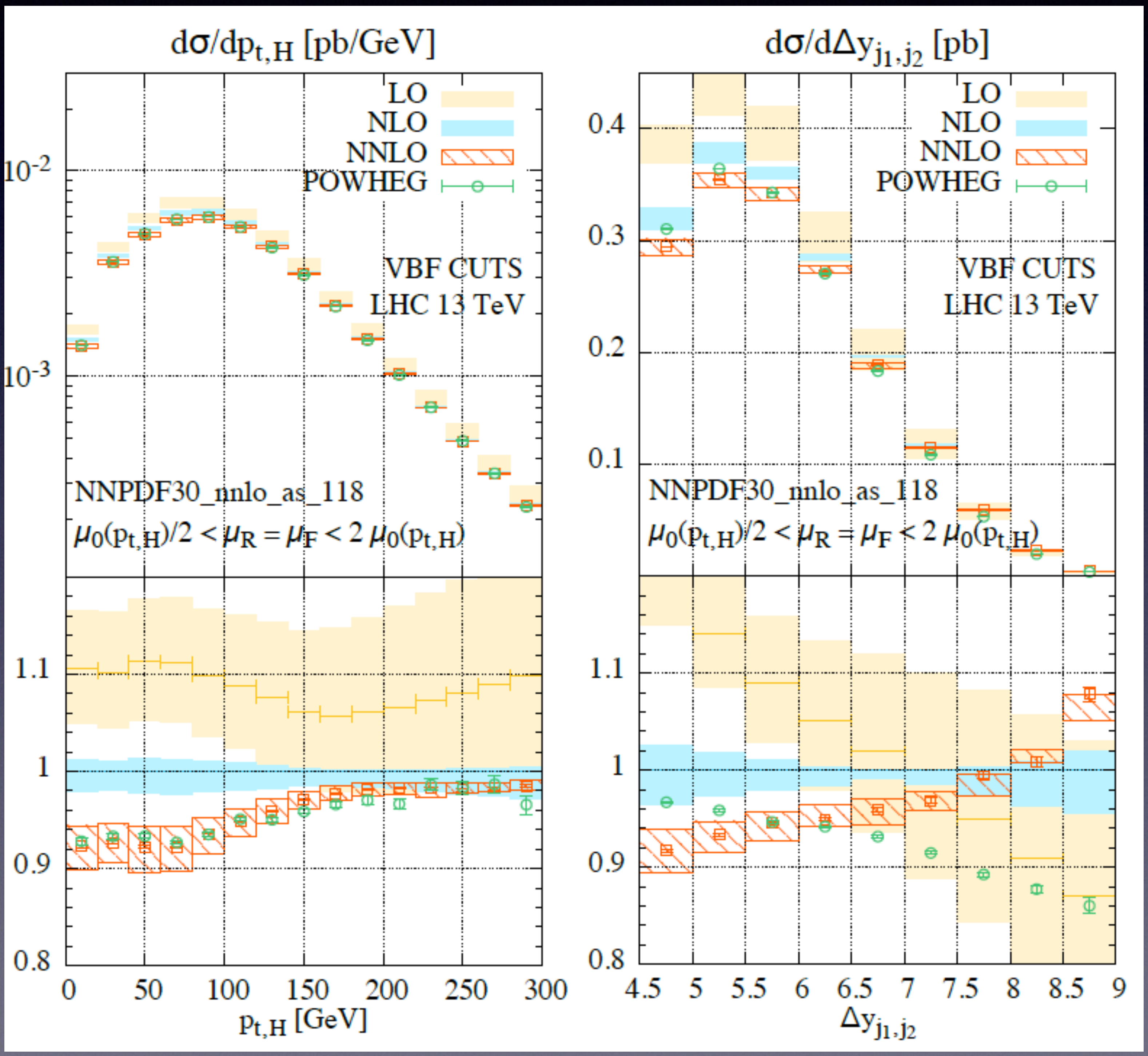


Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier 1607.08817

Less good agreement on unnormalised distributions

Fully differential VBFH at NNLO

Cacciari, Dreyer, Karlberg, Salam, GZ 1506.02660



- Allows to study realistic observables, with realistic cuts
- NNLO corrections much larger (10%) than expected (NNLO just 1% in the inclusive case)
- Important for coupling measurements

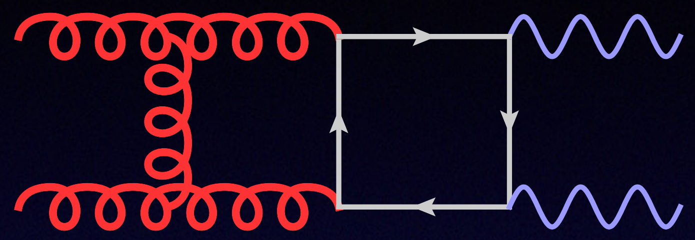
NLO calculations

Thanks to a number of breakthrough ideas developed in the last 10 years the problem of NLO calculations is now considered solved
Various tools developed: Blackhat+Sherpa, GoSam+Sherpa, Helac-NLO, Madgraph5_aMC@NLO, NJet+Sherpa, OpenLoops+Sherpa, Samurai, Recola ...

- Practical limitation: high-multiplicity processes still difficult because of numerical instabilities, need long run-time on clusters to obtain stable results (edge: 5-6 particles in the final state, depending on the process)
- Today focus on
  - ➡ automation of NLO electroweak corrections (necessary to match accuracy of NNLO)
  - ➡ automation of NLO for BSM signals
  - ➡ loop-induced processes: formally higher-order, but enhanced by gluon PDF

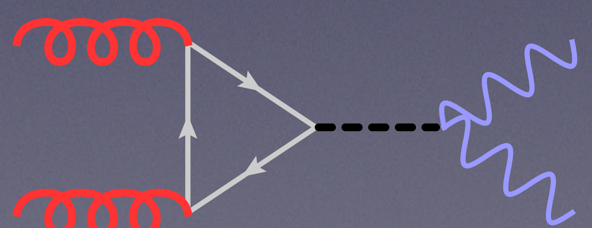
NLO gg → VV

- important contribution from gg → VV
- recently NLO corrections to gg → VV computed K ~ 1.6-1.8 (caveat: treatment of 3rd generation incomplete)
- ZZ: the result lies outside the NNLO uncertainty bands quoted
- WW: massive interference term between signal and background known to LO only (approx. as geometric average of K-factors)

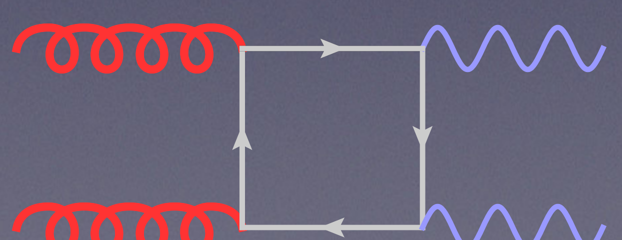


Caola et al ’15; Caola et al ’16; Campbell, Ellis, Czakon, Kirchner ’16

expect more progress, relevant for constraints on the Higgs width



K_{signal}



$K_{\text{back.}}$

$$\text{Int.} \propto \sqrt{K_{\text{signal}} K_{\text{back.}}}$$

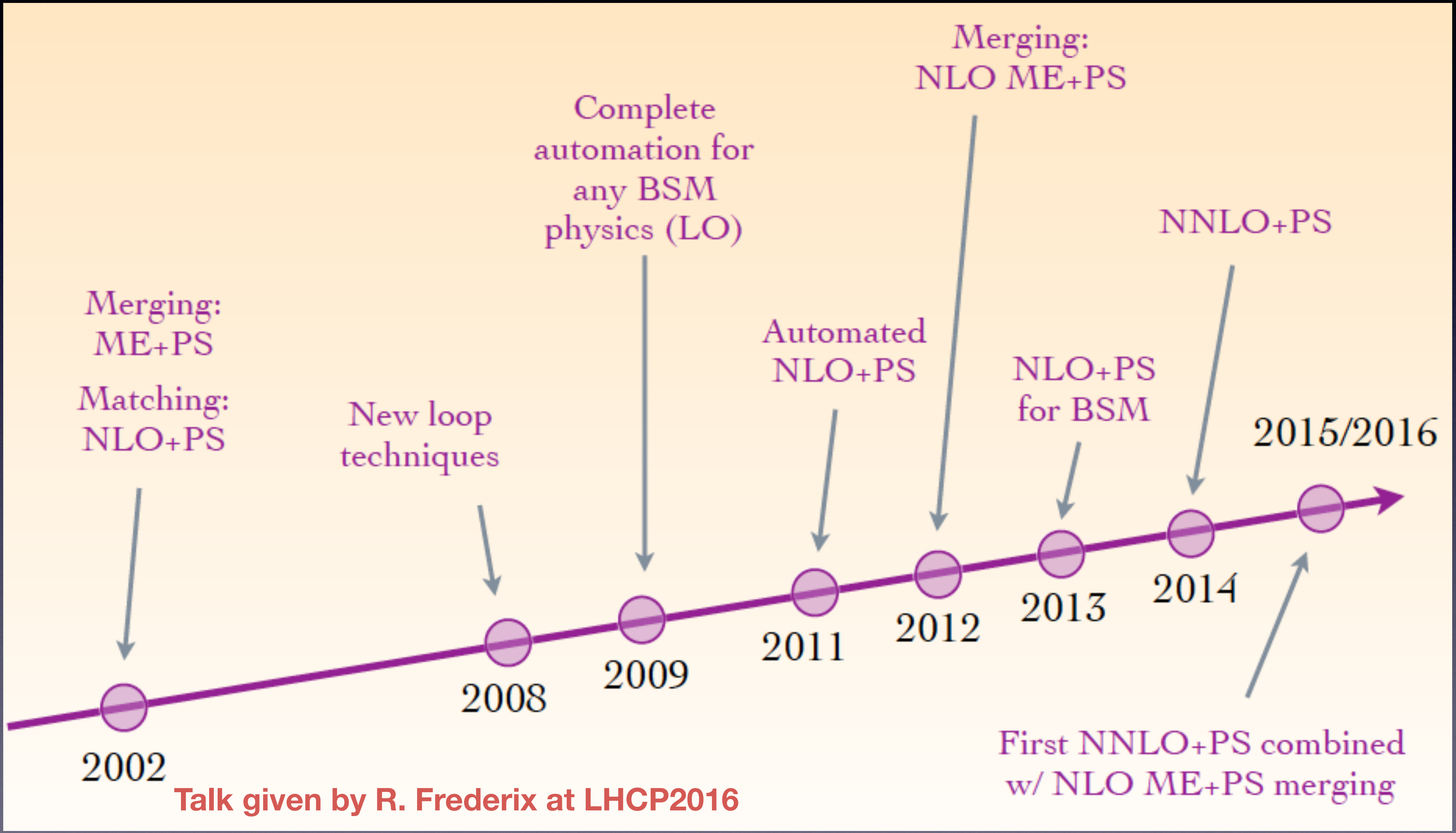
Merging fixed-oder and all-orders

Q: but isn't fixed-order (FO) good enough? doesn't NLO or NNLO do such a great job now?
A: In many cases NO!

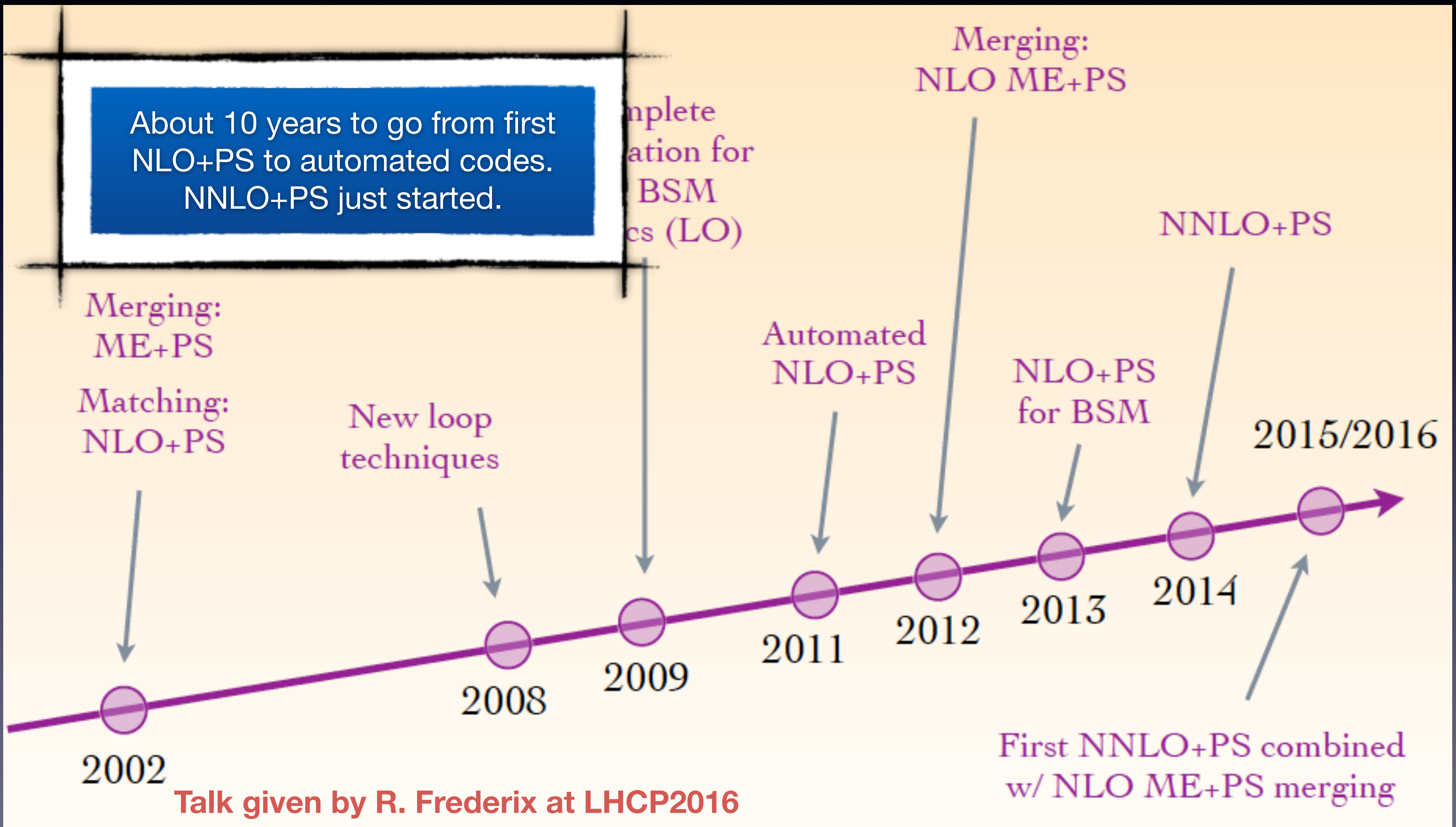
- LHC processes are intrinsically multi-scale problems. Kinematical cuts often force a hierarchy between scales (jet-veto, small p_t or large p_t wrt masses involved, soft radiation, threshold effects ...)
- in exclusive regions of phase space the error estimate of FO is unreliable. FO should be supplemented with accurate analytic resummation (more handles to estimate error)
- parton showers have formally a lower logarithmic accuracy but the clever choices done in the shower evolution embody more then thirty years of experience of a whole community and are widely used for detector simulations

➡ matching/merging means getting the best out of two worlds

Merging NxLO and parton shower



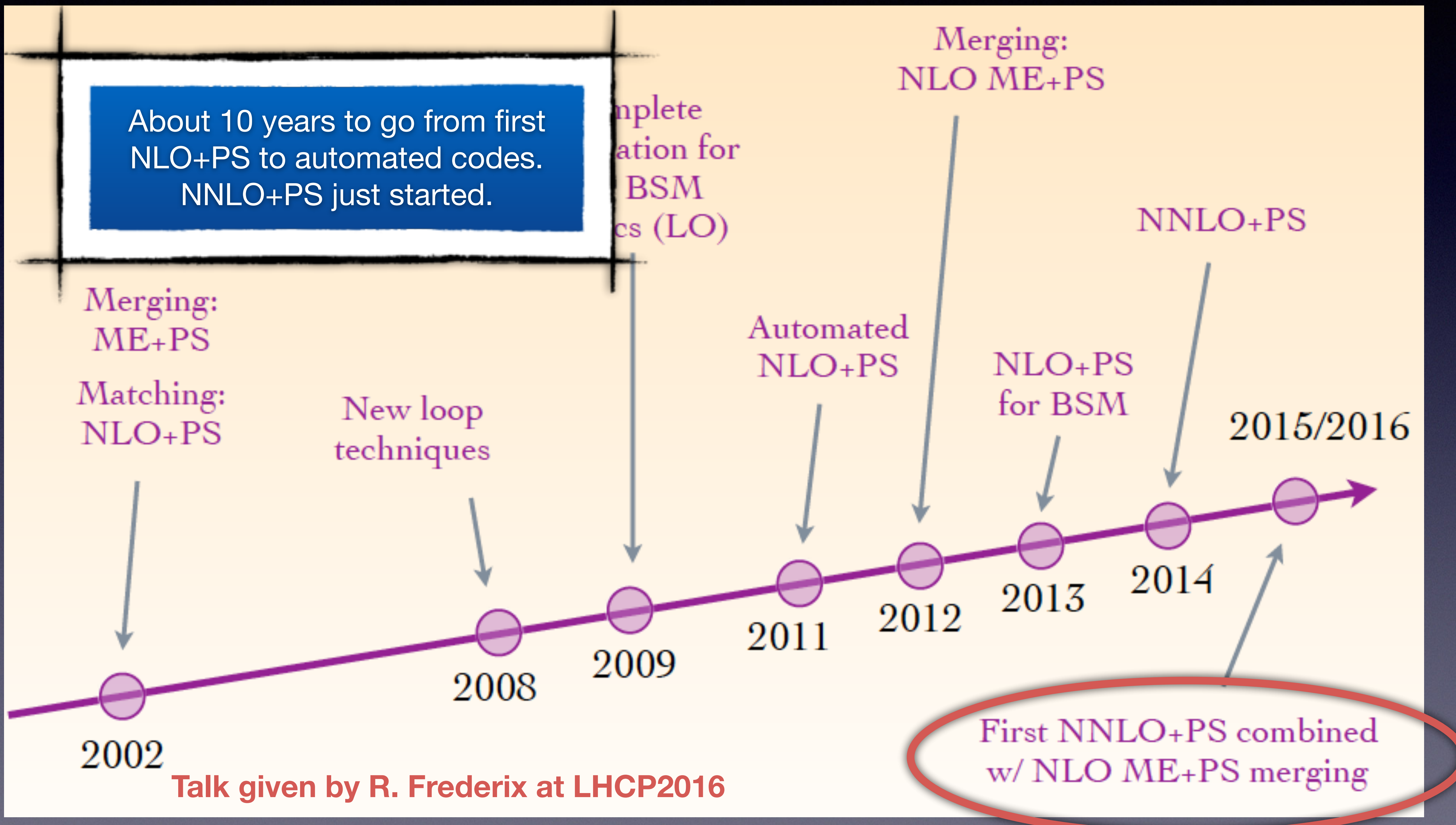
Merging NxLO and parton shower



About 10 years to go from first NLO+PS to automated codes. NNLO+PS just started.

Talk given by R. Frederix at LHCP2016

Merging NxLO and parton shower



Talk given by R. Frederix at LHCP2016

MiNLO’ merging

Reminder:

Hamilton & Frederix 1512.02663

- MiNLO Sudakov applied to X+m-jets ensures cross-sections are finite in the X+(m-1)-jet phase space
- NLO accuracy for X+(m-1)-jet can be achieved exploiting analytically known NNLL resummation

New approach:

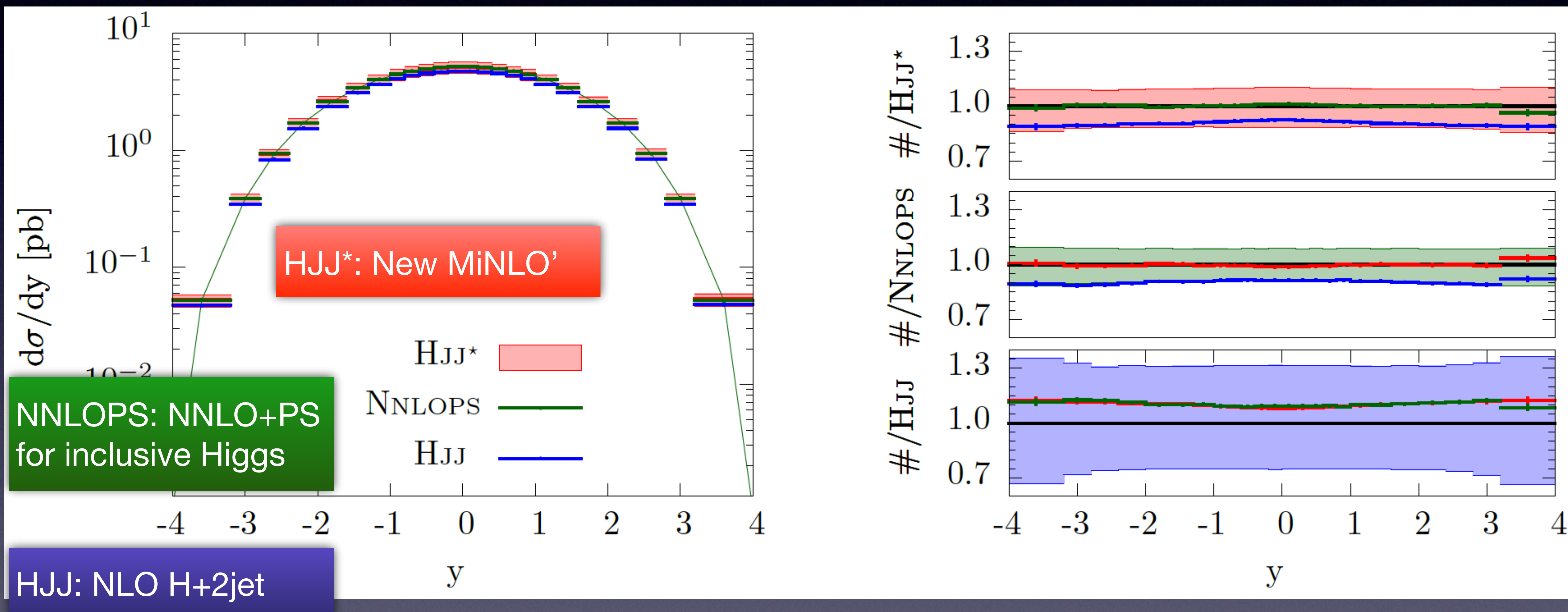
- numerically derive missing higher-order terms in the MiNLO Sudakov by enforcing unitarity, differentially in the (n-1) jet phase space
- advantage: method general and independent of process, can combine different multiplicities and different levels of accuracy

First application:

- merging for H(NNLOPS), H+1jet (NLOPS) and H+2jets (NLOPS)

MiNLO’ merging

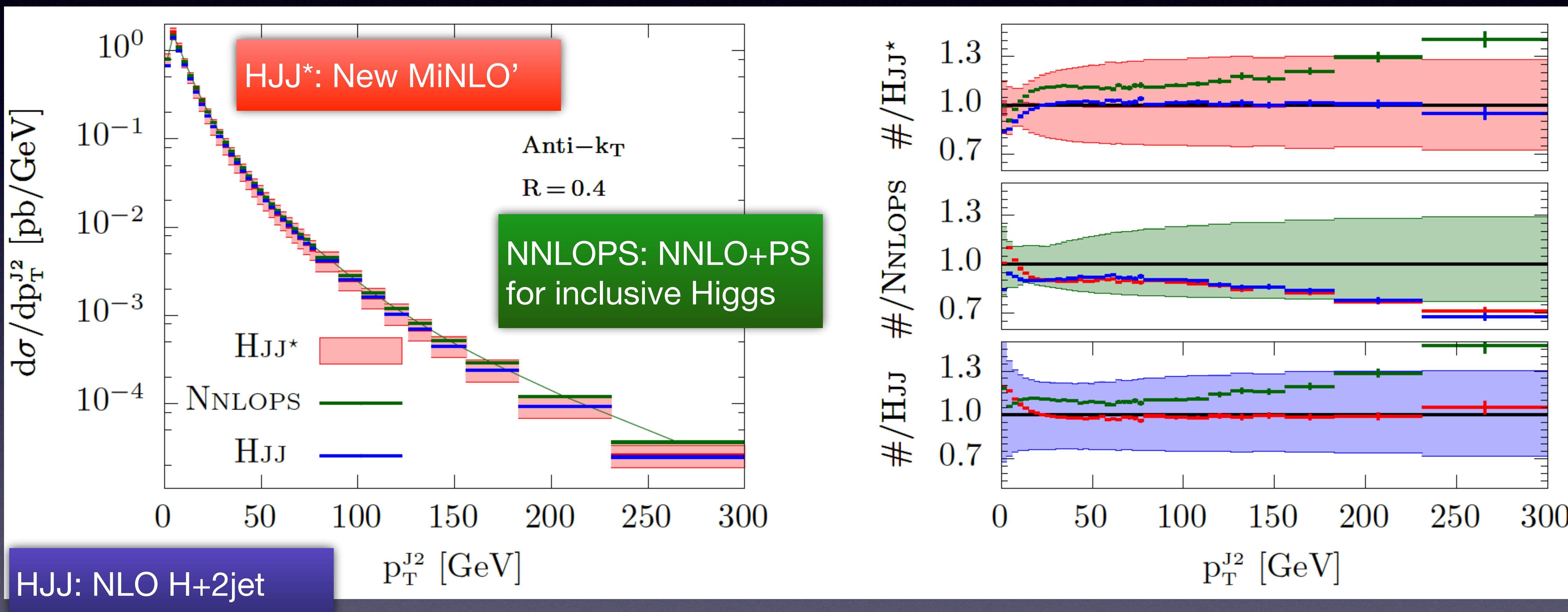
Hamilton & Frederix 1512.02663



HJJ* agrees with NNLOPS
in 0-jet phase space

MiNLO' merging

Hamilton & Frederix 1512.02663

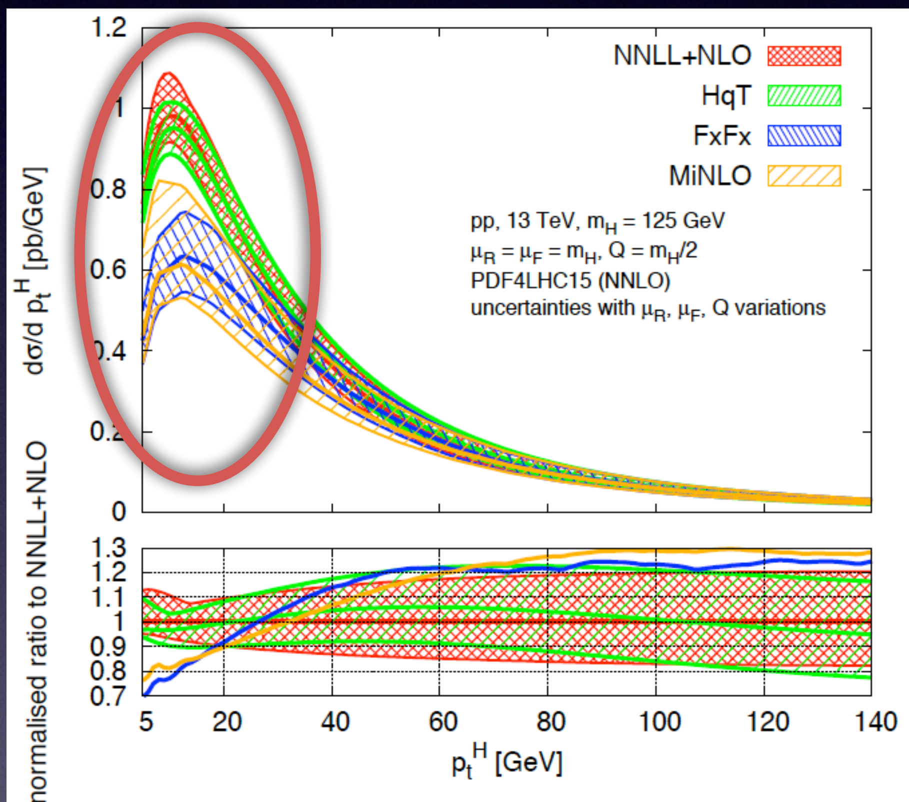


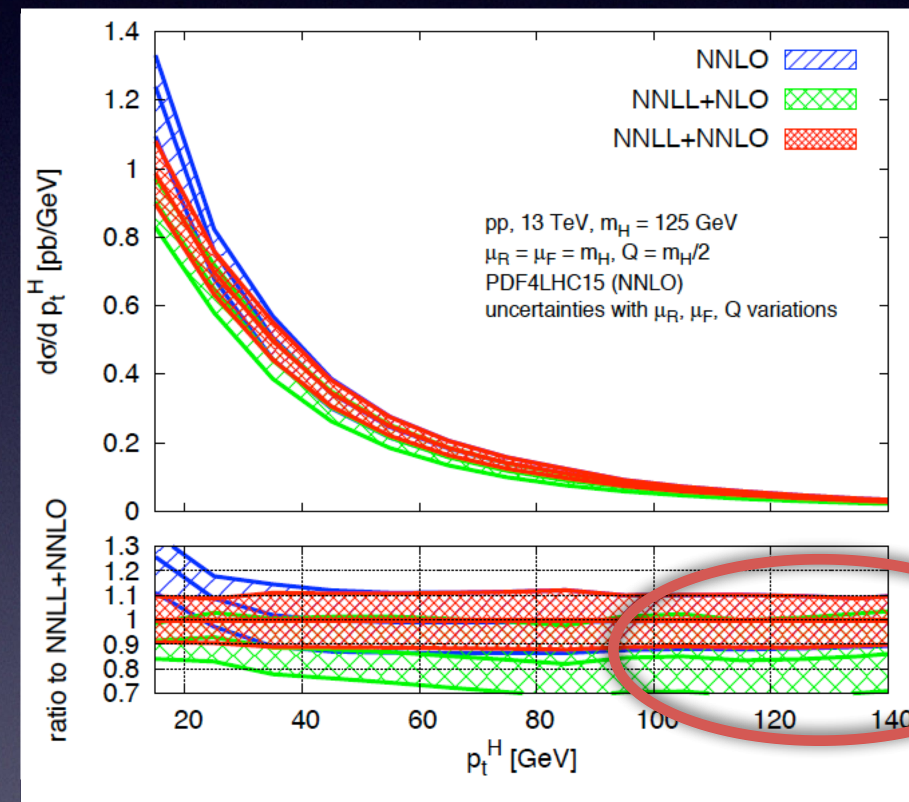
HJJ* agrees with HJJ
in 2-jet phase space

NNLO + NNLL Higgs p_t spectrum

New method to resum Higgs transverse momentum directly in momentum space (rather than in impact parameter space)

Monni, Re, Torrielli 1604.02191

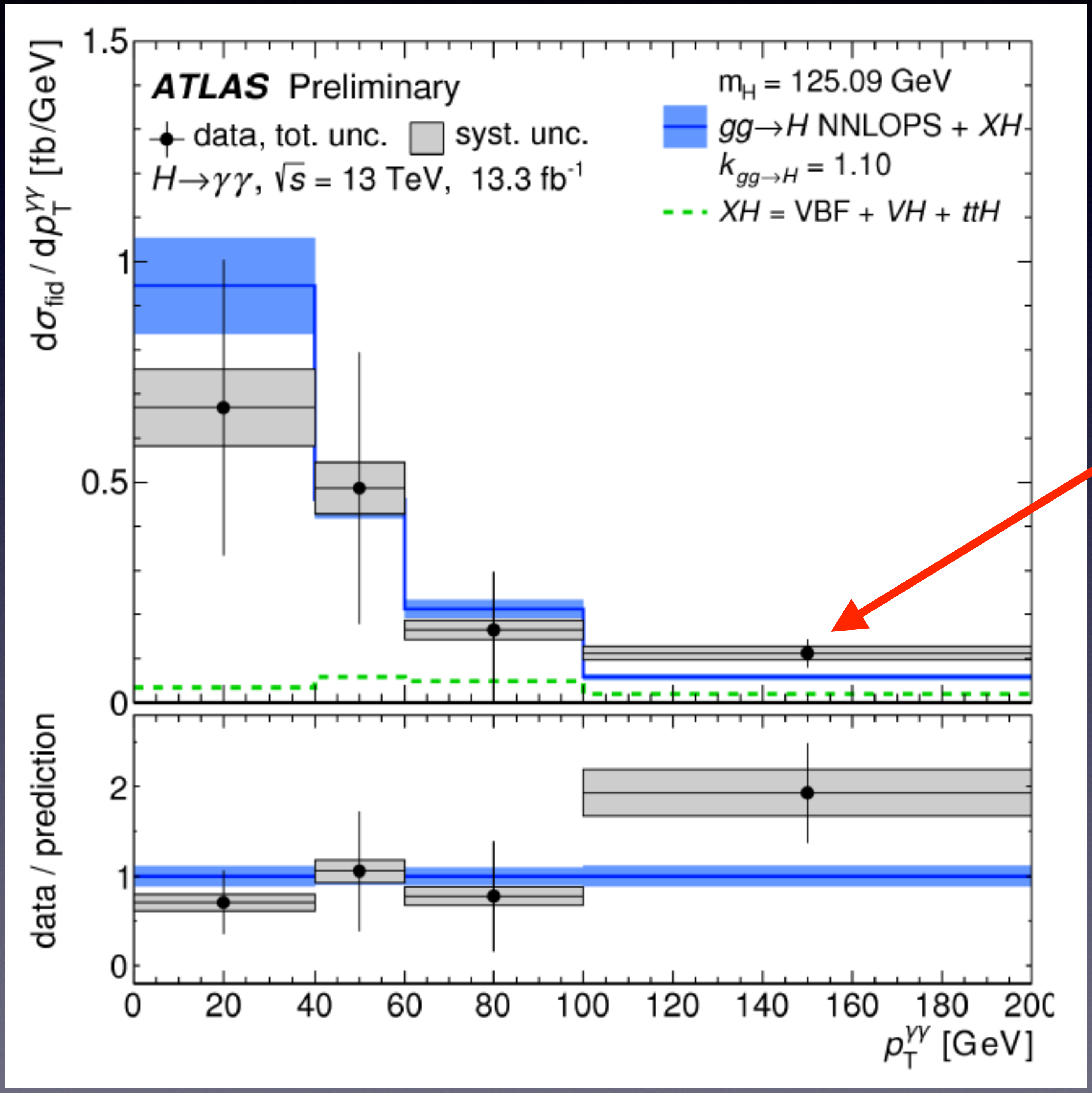




- good agreement with previous NNLL+NLO (HqT)
- less good agreement with other NLO+PS simulations

- improvement over HqT with NNLO corrections at high p_t
- resummation: sizable impact below 25 GeV

Measurement of Higgs pt



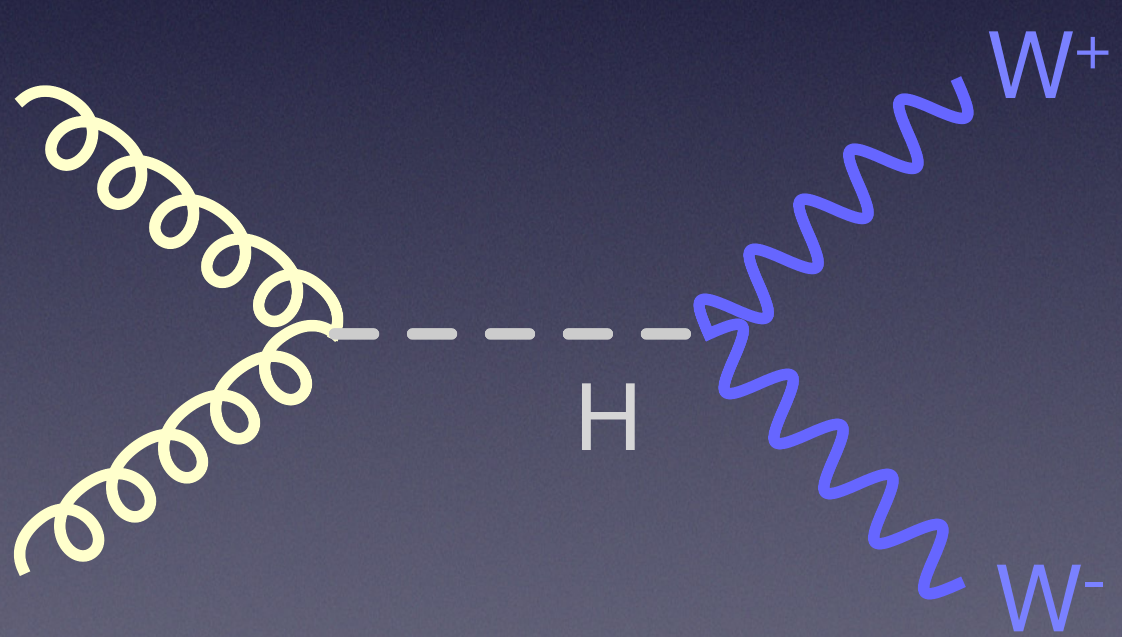
Harder spectrum (as in Run I), but compared to NNLOPS, misses NNLO correction at high transverse momentum

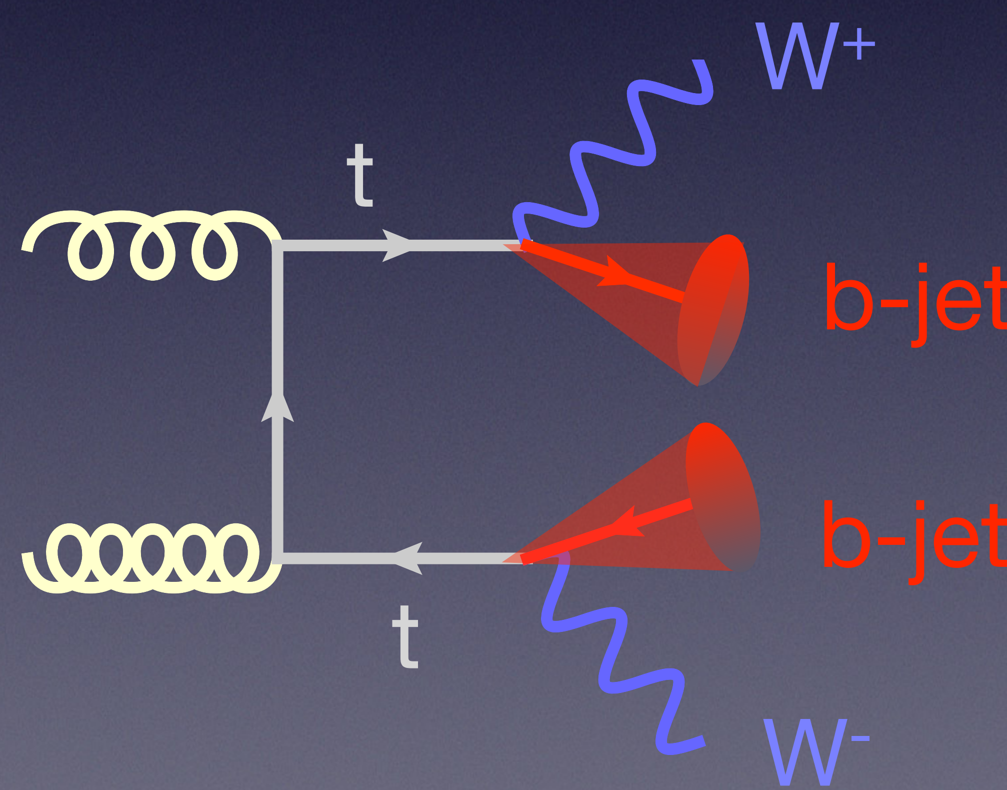
Room for improvement

The zero-jet cross-section

In H $\rightarrow$ WW and H $\rightarrow \tau\tau$, zero-jet cross section particularly important as it is nearly free of (difficult) top-antitop background

(aim is accurate extraction of HWW and H$\tau\tau$ couplings)

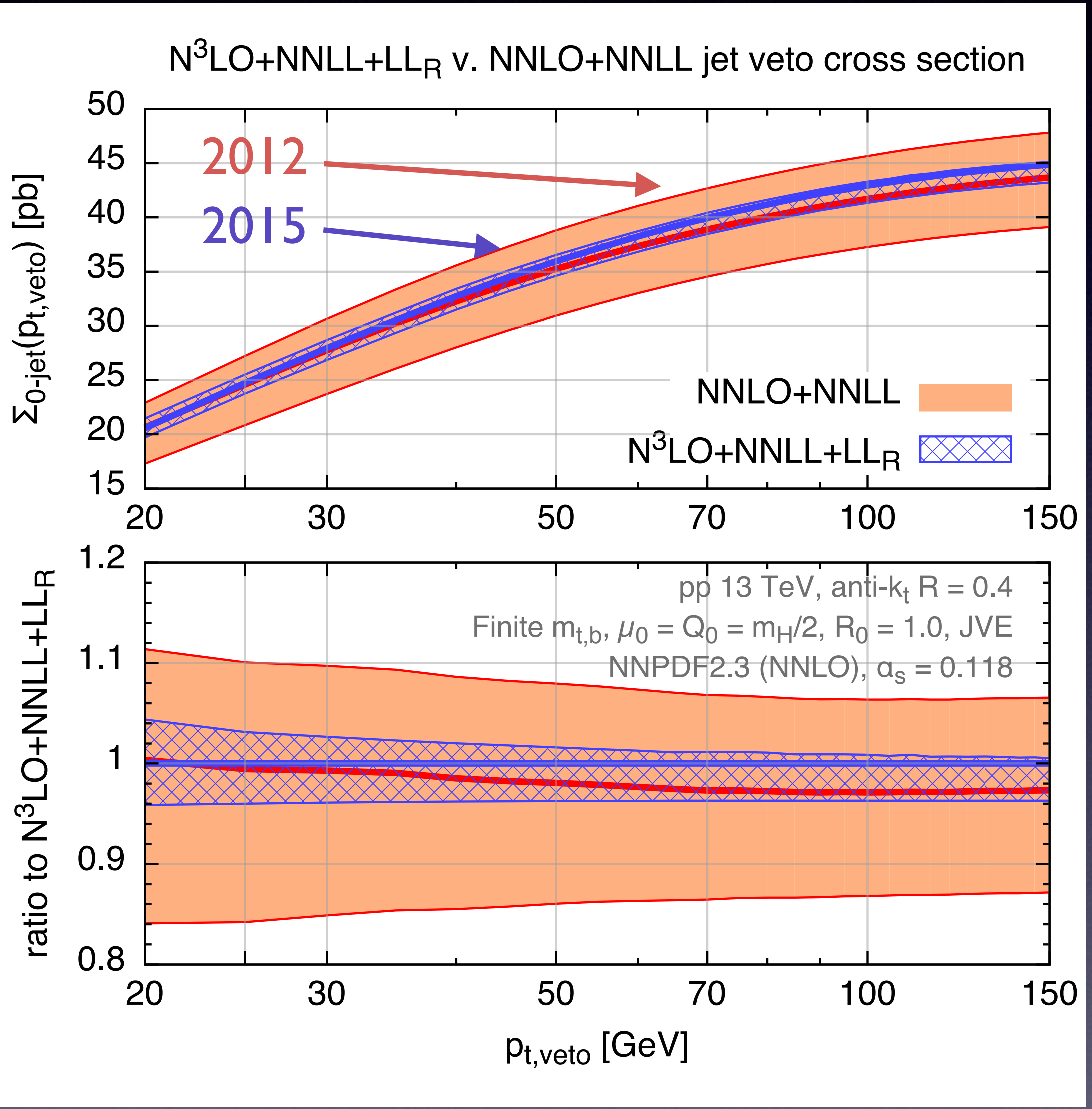




Improved jet-veto

Recently jet-veto predictions updated to include

- ✓ N^3LO corrections to inclusive cross-section Anastasiou et al 1503.06056
- ✓ NNLO corrections to H + 1 jet Caola et al 1504.07922
- ✓ mass corrections Banfi et al 1308.4634
- ✓ resummation of logarithms of (small) jet-radius Dreyer et al 1411.5182

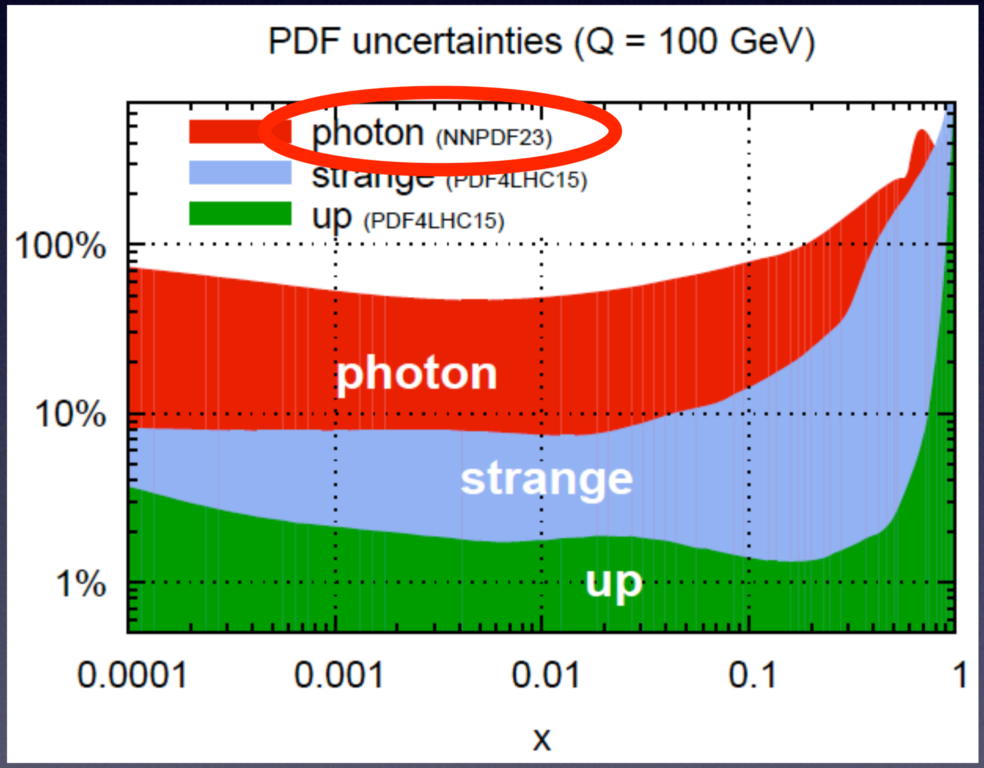


Few percent theory error (considerable reduction in the last years)

Banfi, Caola, Dreyer, Monni, Salam, GZ, Dulat 1511.02886

The photon PDF

Interest in photon PDF spurred by 750 GeV di-photon resonance, but also important for precision physics in general (electro-weak corrections, Higgs, Drell Yan, di-bosons …)



- valence quarks known to few percent
- others quarks to 10% over a large x-range
- data driven photon PDFs have O(100%) uncertainty (model dependent PDFs have much small uncertainties, vast literature)

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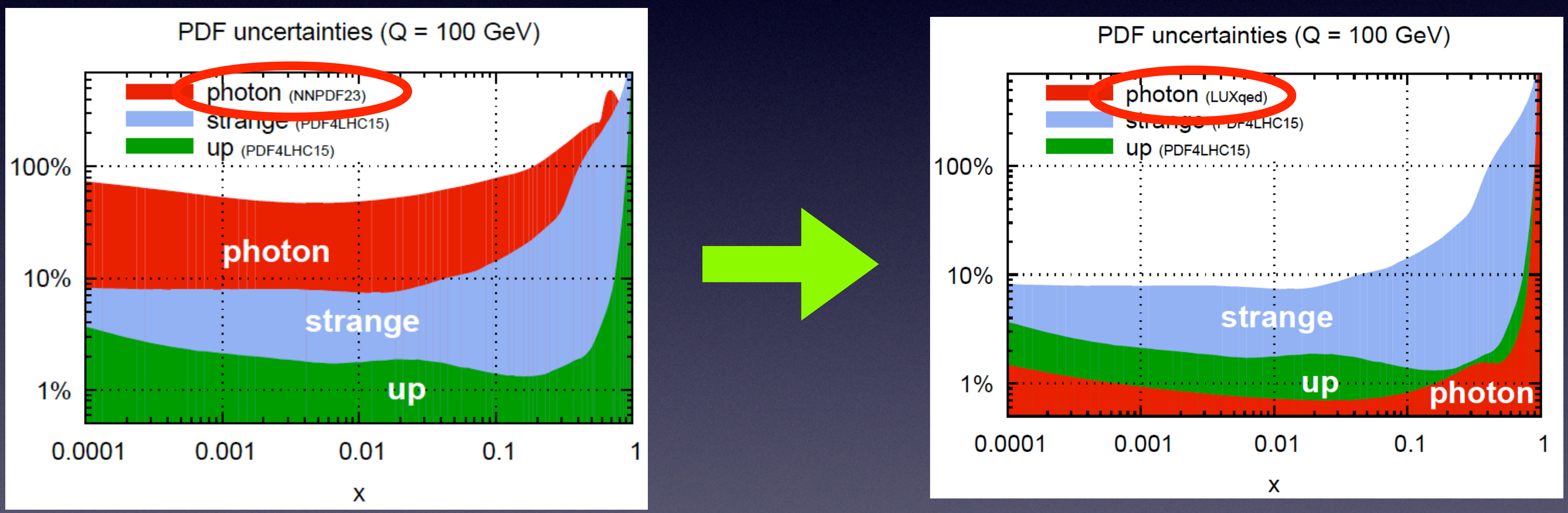

A. Manohar, P. Nason, G. Salam, GZ 1607.04266

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The LUX photon PDF determination

Take a hypothetical (BSM) flavour-changing heavy-neutral lepton production process, and calculate the cross section in two ways

- using proton structure functions (F_2 and F_L)
- using photon parton distribution function

Imposing an equality between the two expression gives a model-independent, data driven determination on the photon PDF

$$x f_{\gamma/p}(x,\mu^2) = \frac{1}{2\pi\alpha(\mu^2)} \int_x^1 \frac{dz}{z} \left\{ \int_{Q^2_{\min}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2(Q^2) \left[\left(2 - 2z + z^2 + \frac{2x^2 m_p^2}{Q^2} \right) F_2(x/z, Q^2) - z^2 F_L\left(\frac{x}{z}, Q^2\right) \right] - \alpha^2(\mu^2) z^2 F_2\left(\frac{x}{z}, \mu^2\right) \right\}$$

Photon PDF determination relies on high precision DIS data

Impact on associated production

Cross section for associated HW(→ lν) production at 13 TeV

Cross section without photon	91.2 ±1.8 fb
Photon induced with NNPDF2.3	$6.0\ ^{+4.4}_{-2.9}$ fb
Photon induced with LUXqed	4.4 ± 0.1 fb

The error on the photon induced contribution goes from being the dominant one to being negligible

Impact of photon PDF on VBF:

tential enhancement effects. Note that the whole photon-induced cross-section contribution σ_γ is treated as uncertainty here, because the PDF uncertainty of σ_γ is estimated to be 100% with the NNPDF2.3QED PDF set. At present, this source, which is about 1.5%, dominates the EW uncertainty of the integrated VBF cross section

HXSWG 4th report https://cds.cern.ch/record/2150771/

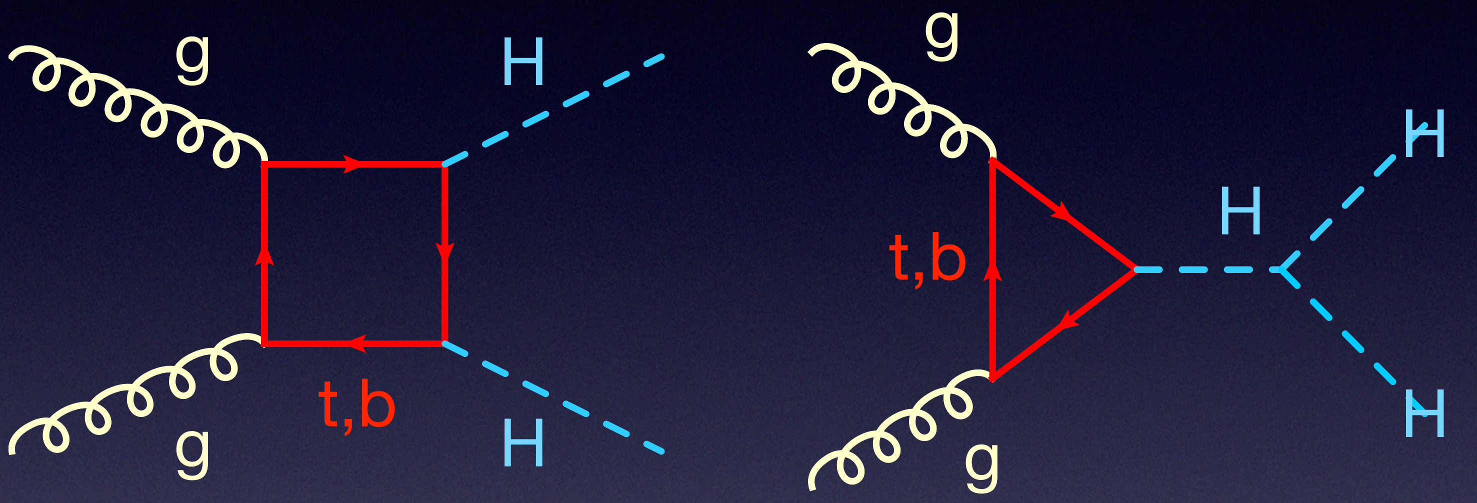
Included now in LHAPDF: (LUXqed_plus_PDF4LHC15_nnlo_100)

The Higgs self-coupling

Suitable process: Higgs pair production but sensitivity limited due to box terms

$$V(H) = \frac{1}{2} m_H^2 H^2 + \lambda_3 v H^3 + \frac{1}{4} \lambda_4 H^4$$

SM: $\lambda_3 = \lambda_4 = \frac{m_H^2}{2v^2}$



Cross-section at 13 TeV: ~ 40 fb)
(compare to ~ 40 pb for single Higgs production)

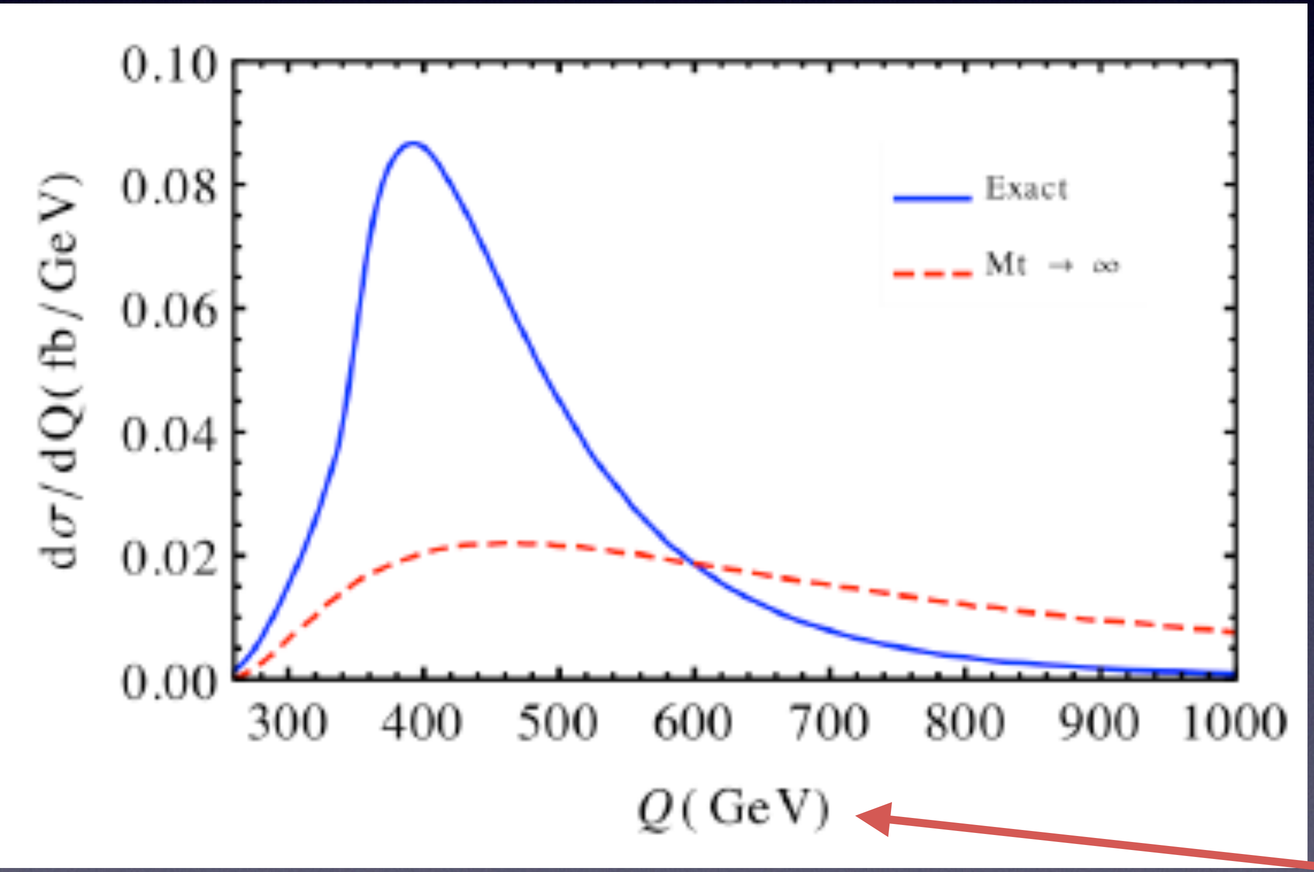
Additionally high price paid for both Higgs bosons to decay (hence hadronic decays also studied)

Current Run 2 bound of 30 × SM (was 70 in Run 1) imply that trilinear Higgs coupling can deviate from SM value by a factor of about 11

State-of-the-art predictions for HH

As for single Higgs production use large m_t effective theory (EFT):

Does it work at leading order?



- EFT approximation works less well than for single Higgs (no surprise)
- still EFT widely used (after rescaling by the correct Born)

invariant mass of HH

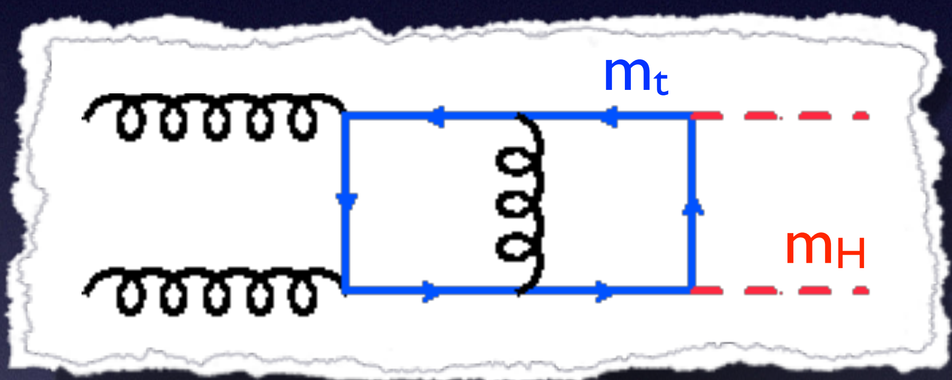
Recently fully differential NNLO calculation of HH in pure EFT

De Florian et al. 1606.09519

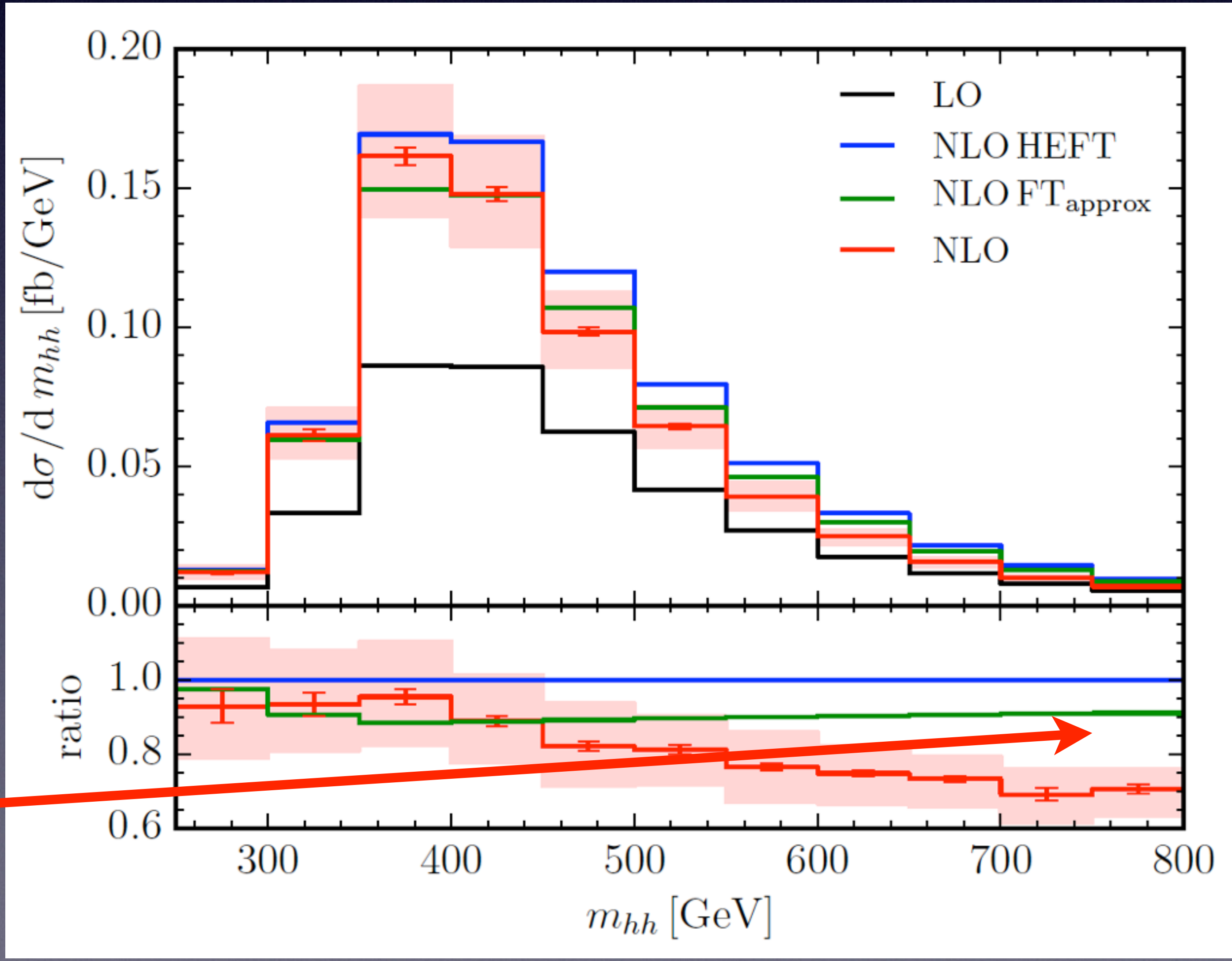
State-of-the-art predictions for HH

Exact NLO calculation of mass-effects performed recently

Borowka et al. 1604.06447



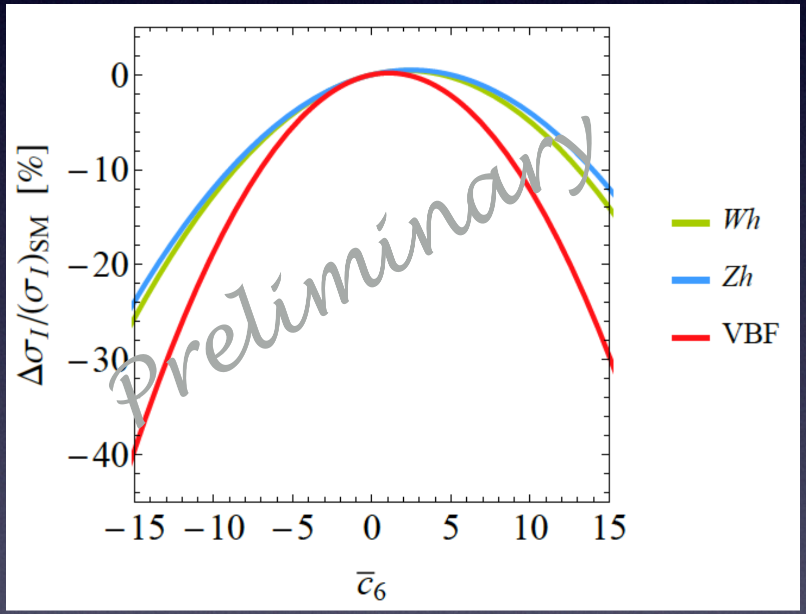
not known analytically, but computed numerically

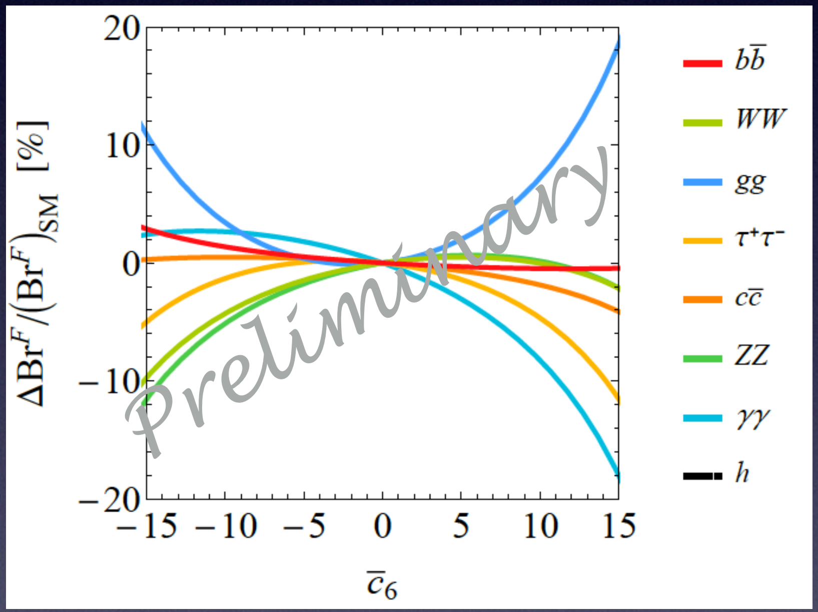


Large effects high m_{HH}, shape change missed by EFT (not a real surprise)

Exploiting precision: probe λ$_3$ in single H

Exploit NNLO determination of VH and VBFH (including Higgs decays) to probe λ$_3$ indirectly. Work in EFT framework and assume that only non-vanishing coefficient is c_6





$$\mathcal{L}_{\rm EFT} = \sum_k \frac{c_k}{v^2}\mathcal{O}_k$$

$$\mathcal{O}_6 = -\lambda_3^{\rm SM}(H^\dagger H)^3$$

$$\lambda_3 = \lambda_3^{\rm SM}(1+c_6)$$

Bizon, Gorbahn, Haisch, GZ
1609.xxxxx

From Run I only ATLAS and CMS data one obtains $c_6 \in [-15;16]$

See also:

- probe λ$_3$ through gg → H and H → γγ Gorbahn and Haisch 1607.03773
- sensitivity to λ$_3$ in main H production (ggF, VBF, WH, ZH, tth) and decay modes (γγ, ZZ, WW, ff, gg) using a coupling modifier De Grassi, Giardino, Maltoni, Pagani 1607.04251

Conclusions

- Many open questions for the LHC Run II to explore: precision crucial role to enhance sensitivity
- Precision calculations are making giant steps: first N^3LO results, NNLO $2 \to 2$ done, NLO fully automated, NLO+PS and merging. I presented only a personal selection of topics and examples
- Overall picture
  - residual uncertainties at the level of the few percent for cross-sections (larger for distributions)
  - lots of attention paid to robust estimate of theory uncertainty: perturbative QCD uncertainty often already not the dominant theory error, other effects must be included (EW corrections, PDF and α_s uncertainties, non-perturbative effects, corrections to large-m_t effective theory in gluon-fusion production ...)
- Progress in theory and experiment go truly hand in hand