History and status of PDF determination

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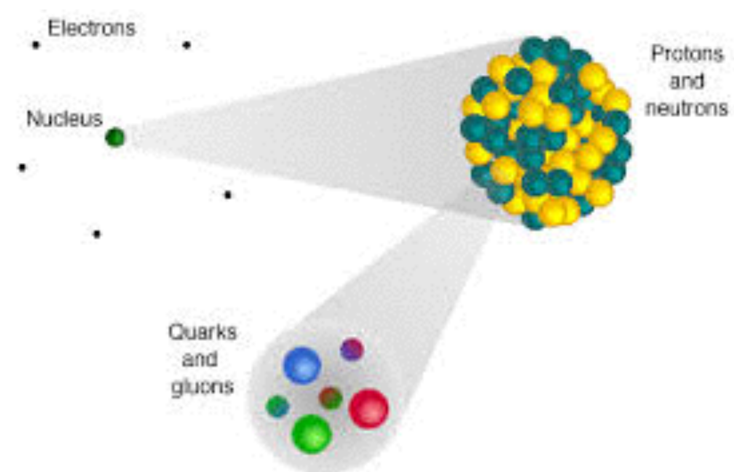
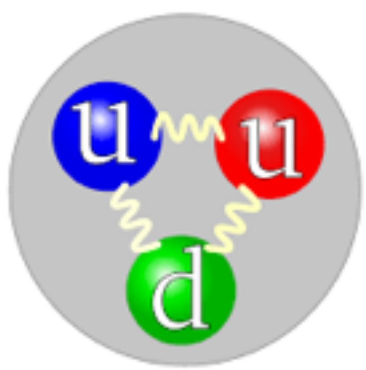
Plan

- Form Factor
- Deep Inelastic Scattering
  - Bjorken Scaling
  - Naive Parton Model
- QCD improved Parton Model
  - NLO Coefficient
  - DGLAP evolution
- NNLO and Beyond
- Higher twist, Heavy flavours
- Conclusions

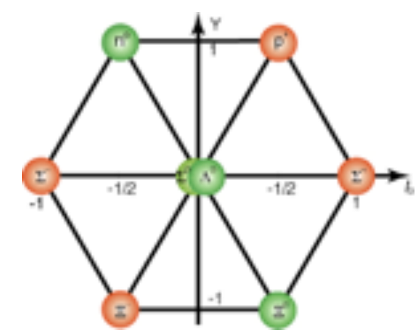
Structure of Matter

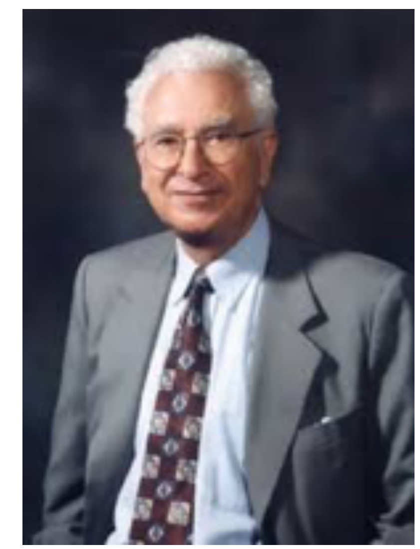
Atoms

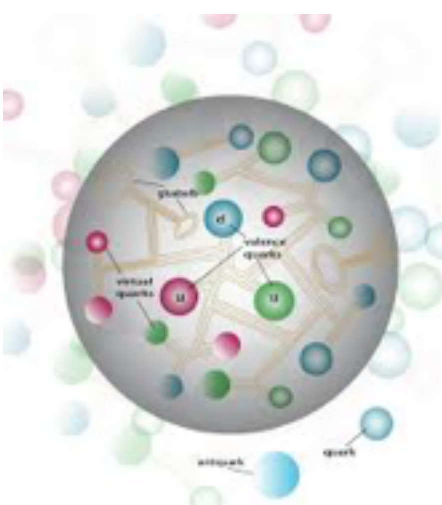




Quark Model







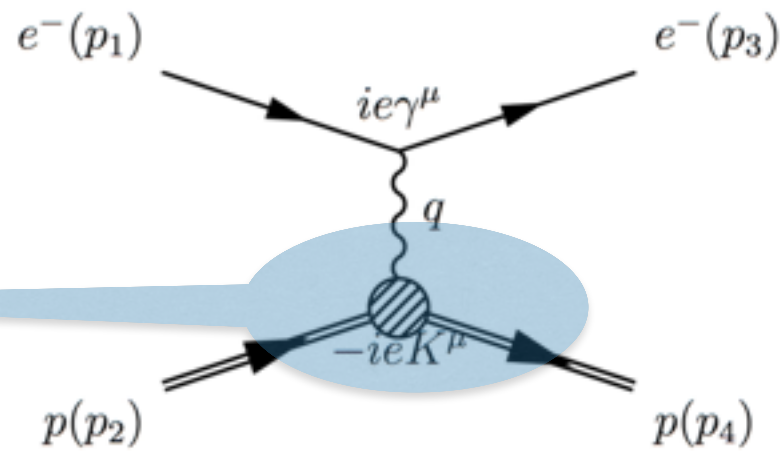
Parton Model

Journey Continues .....

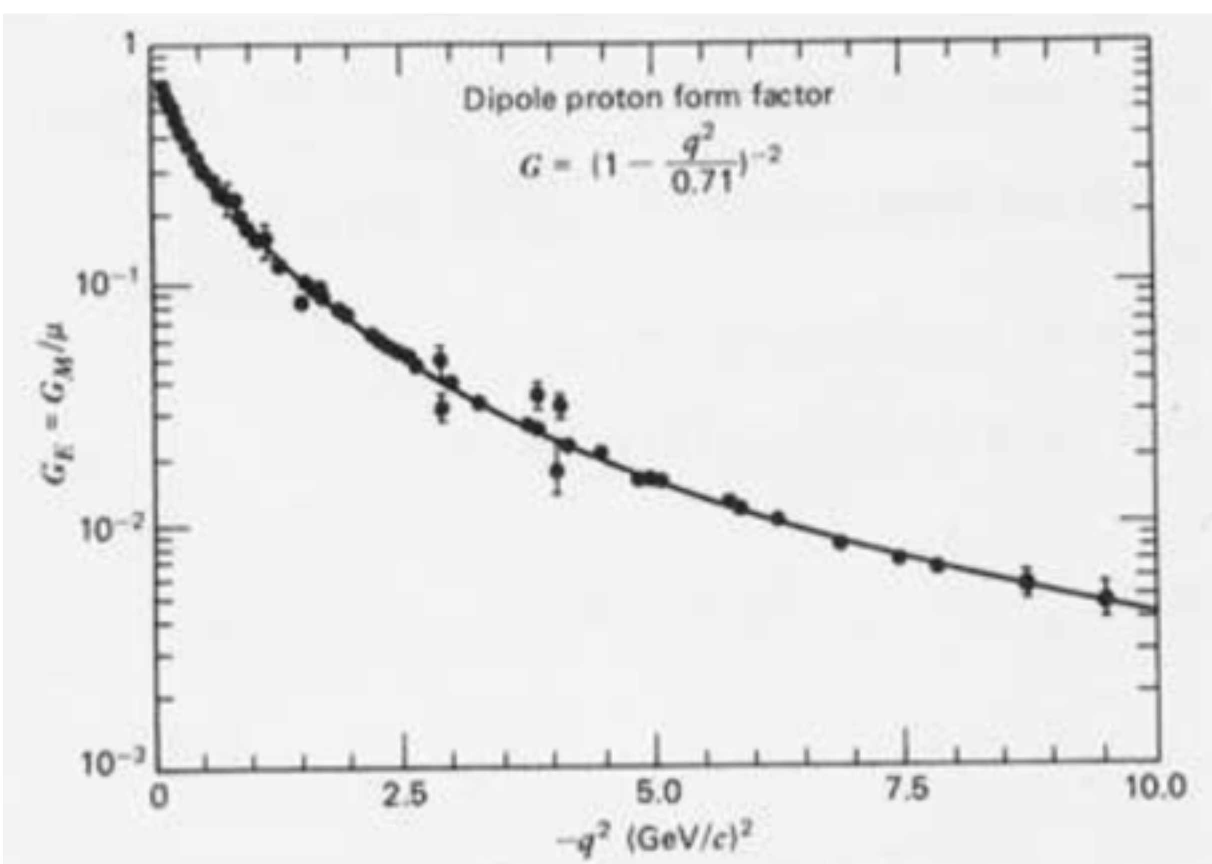
Form Factors of hadrons

Elastic Scattering

$$\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{point}} |F(q^2)|^2$$



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\frac{\theta}{2}} \frac{E_3}{E_1} \left\{ \left(F_1^2 - \frac{\kappa_p^2 q^2}{4m_p^2} F_2^2 \right) \cos^2\frac{\theta}{2} - \frac{q^2}{2m_p^2} (F_1 + \kappa_p F_2)^2 \sin^2\frac{\theta}{2} \right\}$$



Normalisation

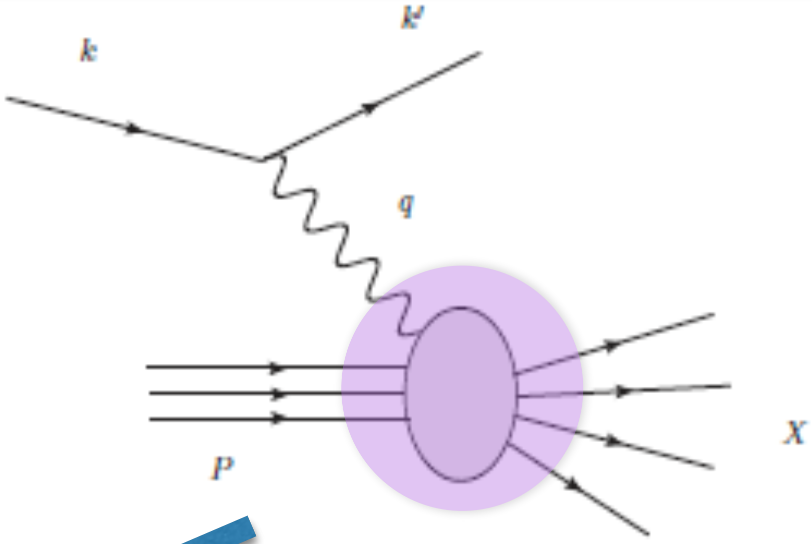
$$F_1^p(0) = 1, F_2^p(0) = 1$$

$$\mu_p = \frac{(1+\kappa_p)e}{2m_p} \qquad \kappa_p = 1.79$$

κ_P is the anomalous magnetic moment

Bjorken Scaling

Deep Inelastic Scattering



Hadronic Tensor

$$W^{\mu\nu}(P,q) = \int d^4\xi \; e^{iq\cdot\xi} < P|J^\mu(\xi)J^\nu(0)|P >$$

$$= \left(-g^{\nu\mu} + \frac{q^\nu q^\mu}{q^2}\right)W_1 + \left(P^\nu - \frac{P\cdot q}{q^2}q^\nu\right)\left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right)W_2$$

Bjorken Limit: $-q^2 \to \infty, P\cdot q \to \infty$ *with* $x = \dfrac{-q^2}{2P.q}$ *fixed*

$$W_1(P,q) = F_1(x),$$
$$P.q\; W_2(P,q) = F_2(x)$$

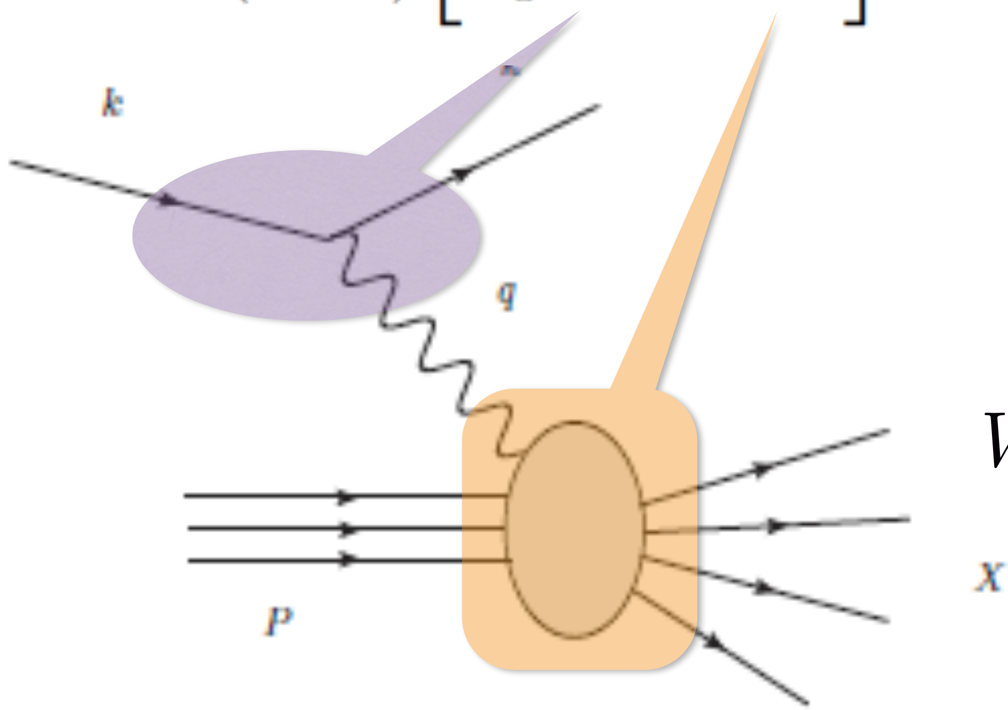
Bjorken Scaling

Birth of Parton Model

Structure Functions from DIS

Inelastic Scattering Factorises

$$d\sigma = \frac{1}{4(k\cdot P)}\left[\frac{4\pi e^4}{q^4}L_{\mu\nu}W^{\mu\nu}\right]\frac{d^3k'}{2E'(2\pi)^3}$$



Leptonic Tensor

$$L_{\mu\nu} = 2\left[k_\mu k'_\nu + k'_\mu k_\nu - \frac{Q^2}{2}g_{\mu\nu}\right]$$

Hadronic Tensor

$$W^{\mu\nu} = \left(-g^{\nu\mu} + \frac{q^\nu q^\mu}{q^2}\right)W_1 + \left(P^\nu - \frac{P\cdot q}{q^2}q^\nu\right)\left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right)W_2$$

$W_i(\nu, Q^2)$ $\quad i = 1, 2$ Structure Function

$$x = \frac{Q^2}{2p.q}; \quad y = \frac{p.q}{p.k}; \quad Q^2 = xys$$

Not calculable

Inclusive Cross section

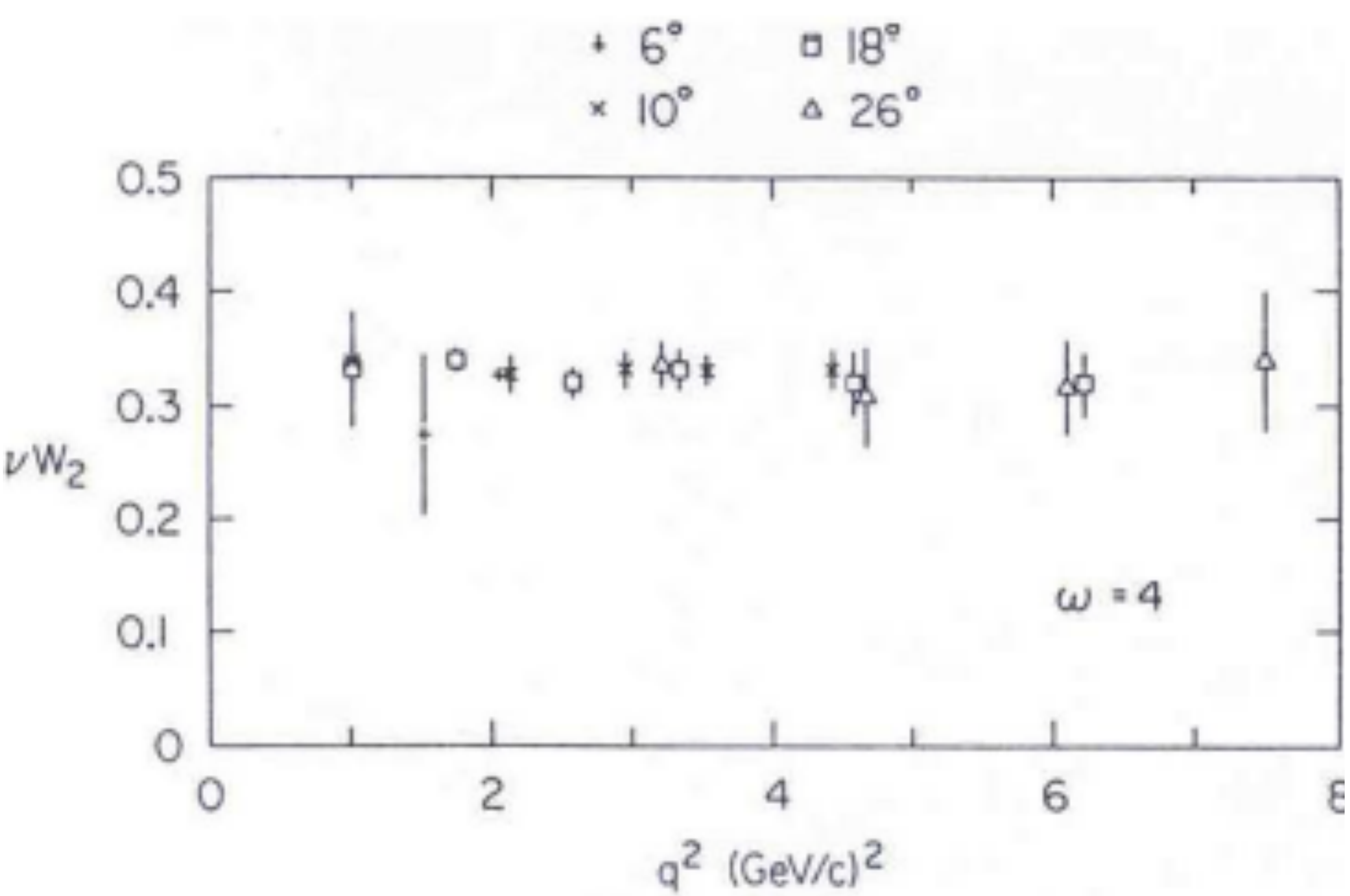
$$\frac{d\sigma}{dE_3 d\Omega} = \frac{\alpha^2}{4E_1^2\sin^4\frac{\theta}{2}}\left(W_2(\nu,Q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,Q^2)\sin^2\frac{\theta}{2}\right)$$

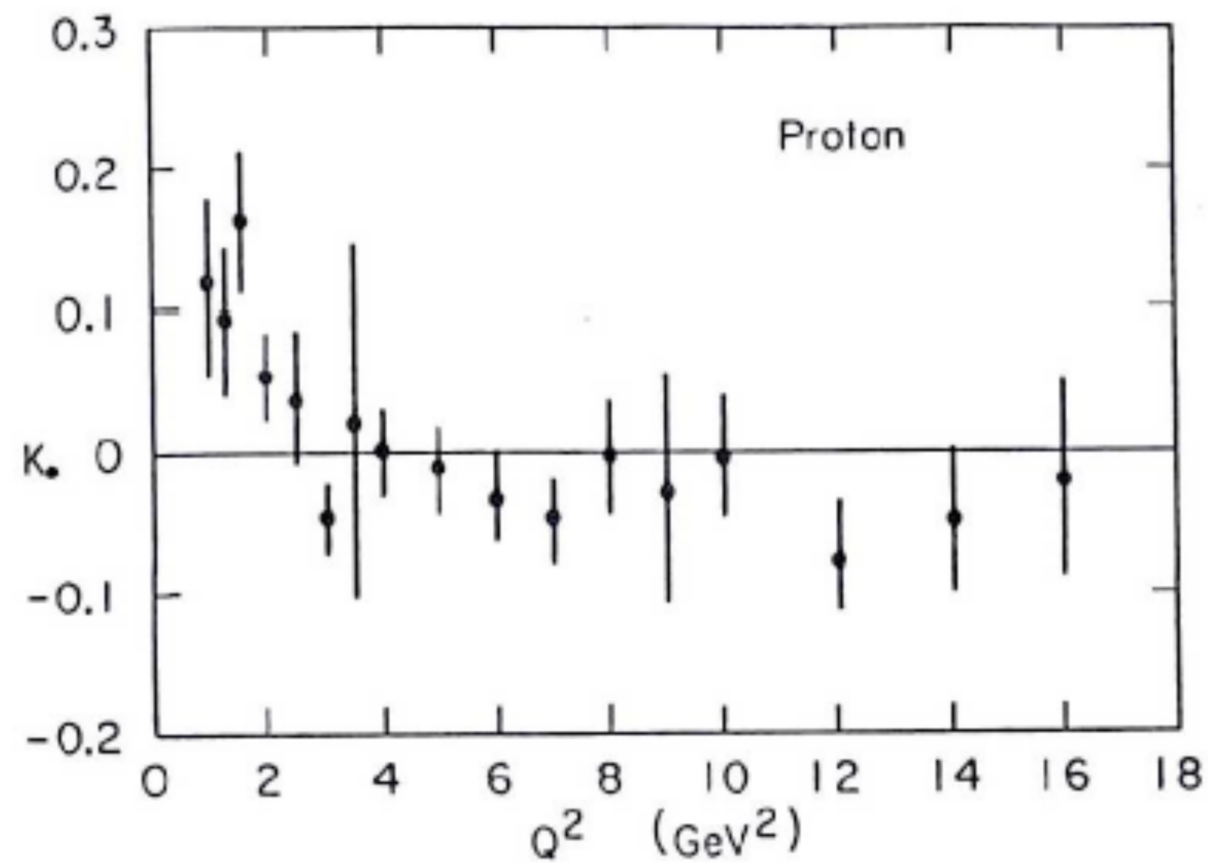
$$m_p W_1(\nu,Q^2) \rightarrow F_1(x) \qquad \nu W_2(\nu,Q^2) \rightarrow F_2(x)$$

Deep Inelastic Scattering



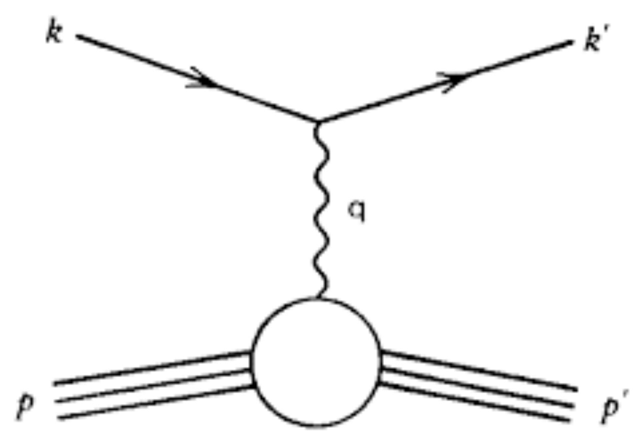
$Q^2 \approx 3M_N^2$ J. Friedman *1930 H. Kendall (1926-1999) R. Taylor *1929 (1968/69)

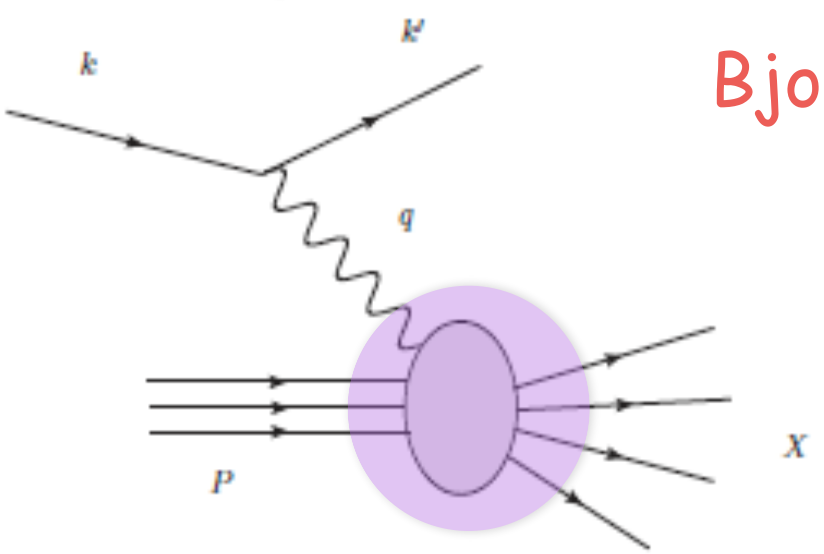




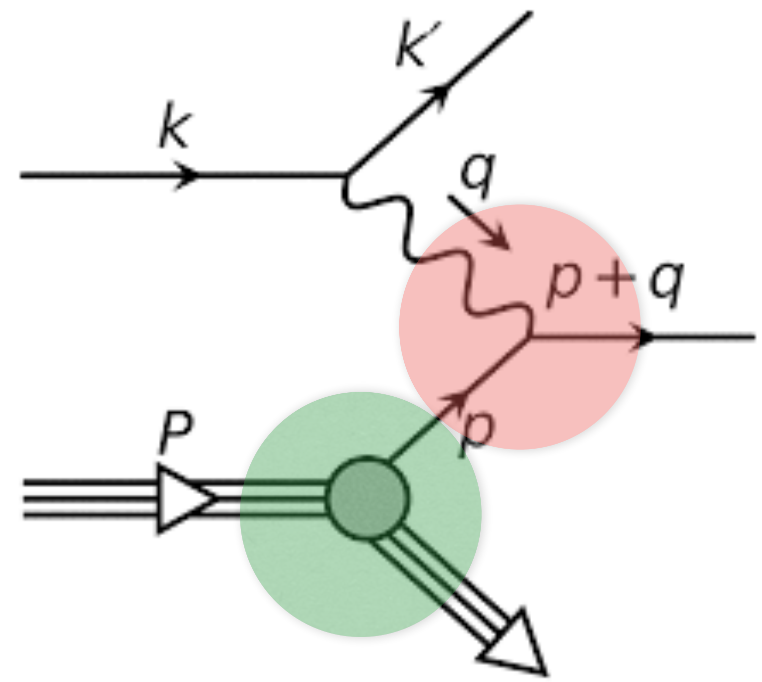
Parton Model

Elastic Scattering





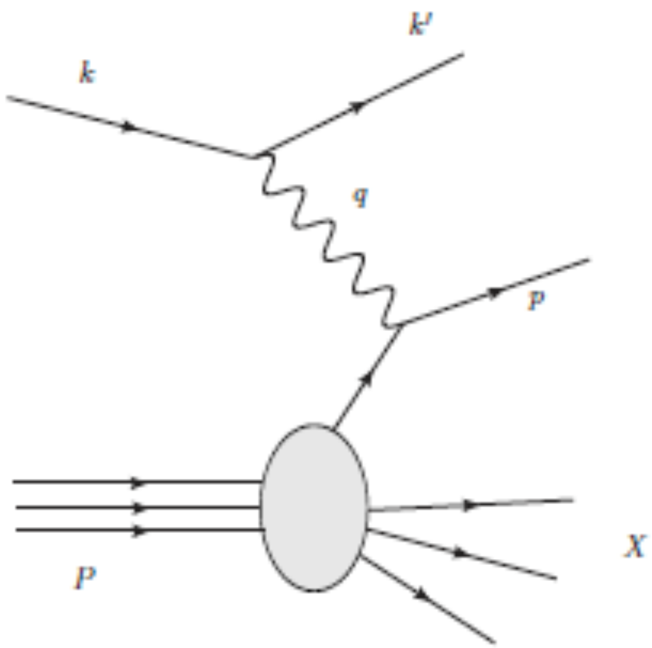
Bjorken Scaling:



Deep Inelastic Scattering

Parton Model Picture

Naive Parton Model



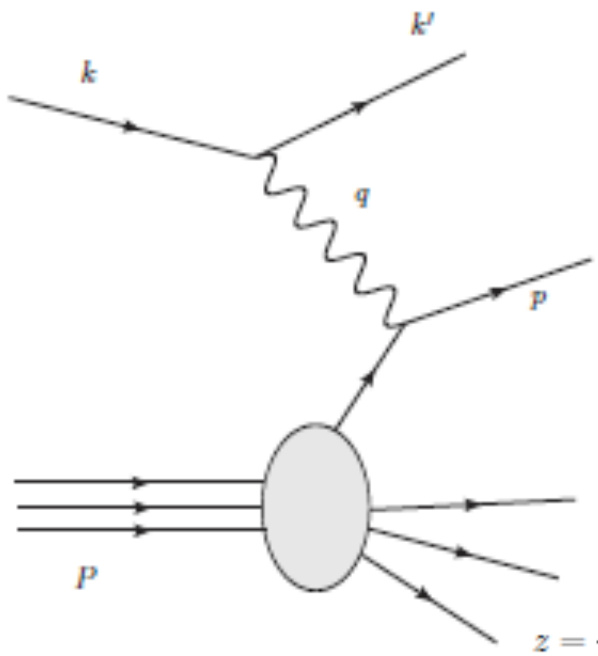
$$d\sigma^{DIS}(P,q) = \sum_i \int_x^1 dz\, f_i(z)\, d\hat{\sigma}_i(zP,q)$$

- Elastic scattering cross section with i-th parton
- Does not depend on the details of the target proton - Target Independent

$f_i(z)$ Parton Distribution Function (PDF)

- Probability of finding i-th parton with momentum fraction z of proton
- Does not depend on the future course of action of the i-th parton - Process Independent

Parton Model



$$z = \frac{Q^2}{2p_i \cdot q} = \frac{x}{y}$$

$$x = \frac{Q^2}{2p.q}; \quad y = \frac{p.q}{p.k}; \quad Q^2 = xys$$

Parton Model - Master Formula

$$W_{\mu\nu}(P,q) = \sum_i \int_0^1 \frac{dy}{y} f_i(y) \hat{W}^{(i)}_{\mu\nu}(yP,q)\,,$$

Dimension-less

$$m_P W_1(\nu, Q^2) = F_1(x, Q^2),$$
$$\nu W_2(\nu, Q^2) = F_2(x, Q^2)$$

$$F_2(x,Q^2) = \sum_i \int_x^1 \frac{dy}{y} f_i(y) \hat{F}_2(x/y, Q^2)\,,$$

Parton level Cross sections

$$\hat{F}_1(x) = \frac{1}{2} e_q^2 \delta(x - \xi)\,,$$
$$\hat{F}_2(x) - 2x\hat{F}_1(x) = 0\,.$$

Bjorken scaling

$$2xF_1(x) = F_2(x) = \sum_i e_i^2 \; x \; f_i(x)$$

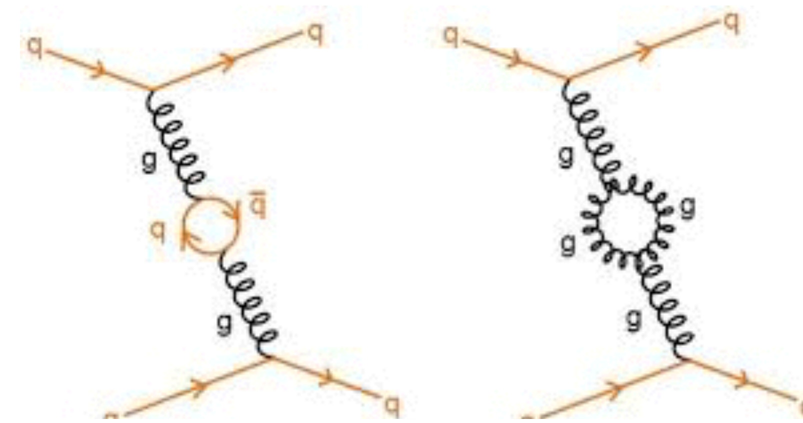
Quantum Chromodynamics

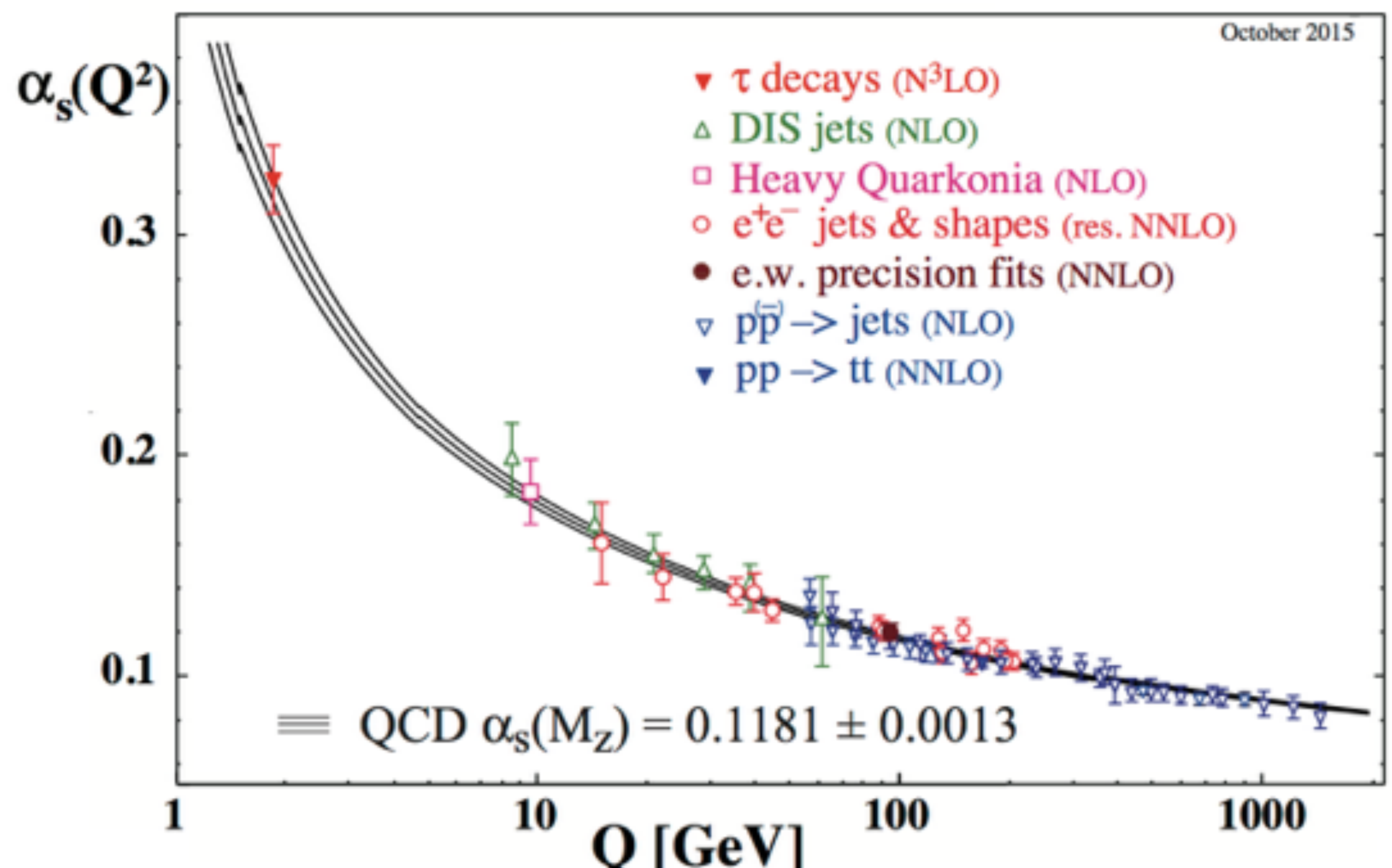
The Nobel Prize in Physics 2004

"for the discovery of asymptotic freedom in the theory of the strong interaction"

David J. Gross	H. David Politzer	Frank Wilczek

Non-Abelian Gauge theory - SU(3)





Asymptotic Freedom

$Q^2 \to \infty$ $\qquad$ $\alpha_s(Q^2) \to 0$

Accommodates Bjorken Scaling

Momentum sum rule

$$2xF_1(x) = F_2(x) = \sum_i Q_i^2 \; x \; f_i(x)$$

Measurements for proton and neutron

SU(2) symmetry

$$\int_0^1 F_2^p(x)dx = \frac{4}{9}f_u + \frac{1}{9}f_d = 0.18$$

$$\int_0^1 F_2^n(x)dx = \frac{4}{9}f_d + \frac{1}{9}f_u = 0.12$$

where $f_q = \int_0^1 dx \; x \; f_q(x)$

Contribution to hadron momentum

$f_u = 0.36$ $\qquad$ $f_d = 0.18$ $\qquad$ Only about 50% from quarks!

Gluons also contribute significantly to momentum

Charged current DIS

Neutrino-Nucleon DIS can bring in parity violating SF

$$\frac{d\sigma^{CC}}{dxdy}(\nu N) = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\nu}(x) + y^2 x F_1^{\nu}(x) \pm y\left(1-\frac{y}{2}\right) x F_3^{\nu}\right]$$

Parton Model gives

$$F_2^{\nu p}(x) = 2x\left[d(x) + \bar{u}(x)\right]$$

$$F_2^{\nu n}(x) = 2x\left[u(x) + \bar{d}(x)\right]$$

Number of Valence quarks inside the Nucleon

$$\int_0^1 F_3^{\nu N}(x)dx = \int_0^1 \left[u(x) - \bar{u}(x) + d(x) - \bar{d}(x)\right] dx = 3$$

Parton Model in QCD

Hadronic Cross section:

$$\sigma^A(\tau, m_A^2) = \sigma^{A,(0)}(\mu_R^2) \sum_{a,b=q,\bar{q},g} \int_\tau^1 dy\, \Phi_{ab}(y, \mu_F^2) \Delta_{ab}^A\left(\frac{\tau}{y}, m_A^2, \mu_R^2, \mu_F^2\right)$$

Partonic Flux:

$$\Phi_{ab}(y, \mu_F^2) = \int_y^1 \frac{dx}{x} f_a(x, \mu_F^2) f_b\left(\frac{y}{x}, \mu_F^2\right),$$

Partonic cross section:

Precision Measurements

Precise Results

PDFs

Parametrisation of PDFs

Standard form

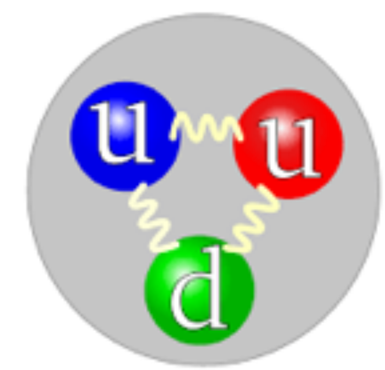
at initial scale μ_0

$$f(x,\mu_o^2) = \alpha_o x^{\alpha_1}(1-x)^{\alpha_2} P(x)$$

where $P(x) = (1 + \alpha_3 x + \alpha_4 x^2 + \cdots)e^{\beta_1 x}(1 + e^{\beta_4} x)^{\beta_5}$

Simple Constraints

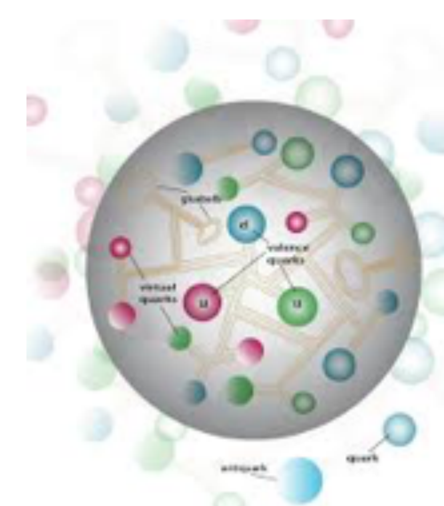
$$\int_0^1 dx\,(f_{u/P}(x,\mu^2) - f_{\overline{u}/P}(x,\mu^2)) = 2 \qquad \int_0^1 dx\left(f_{d/P}(x,\mu^2) - f_{\overline{d}/P}(x,\mu^2)\right) = 1$$



$$\int_0^1 dx\,(f_{s/P}(x,\mu^2) - f_{\overline{s}/P}(x,\mu^2)) = 0$$

Momentum sum rule

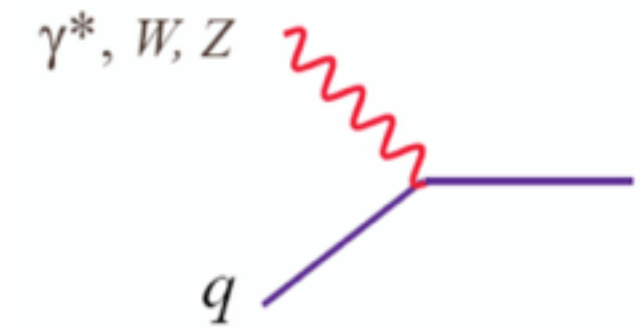
$$\int_0^1 dx\; x\left(\sum_i (f_{q_i/P}(x,\mu^2) - f_{\overline{q}_i/P}(x,\mu^2)) + f_{g/P}(x,\mu^2)\right) = 1$$



Observables for PDF exactraction

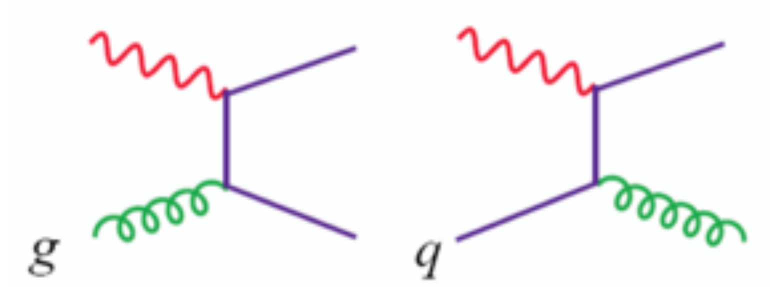
$DIS - eN, \mu N$

(CDHS,CHARM,CCFR,CHORUS,NuTeV)



$DIS - \nu N, \overline{\nu} N$

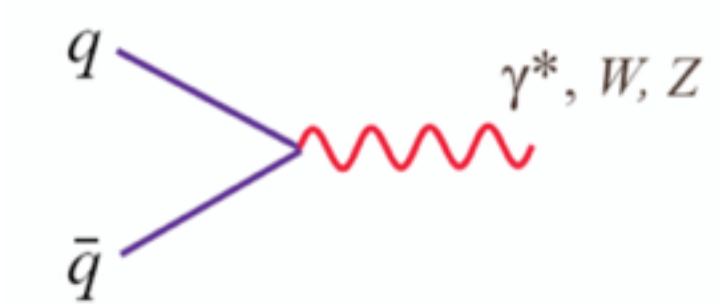
(SLAC,BCDMS,NMC,E665,H1,ZEUS)



$p\overline{p} \rightarrow jets$

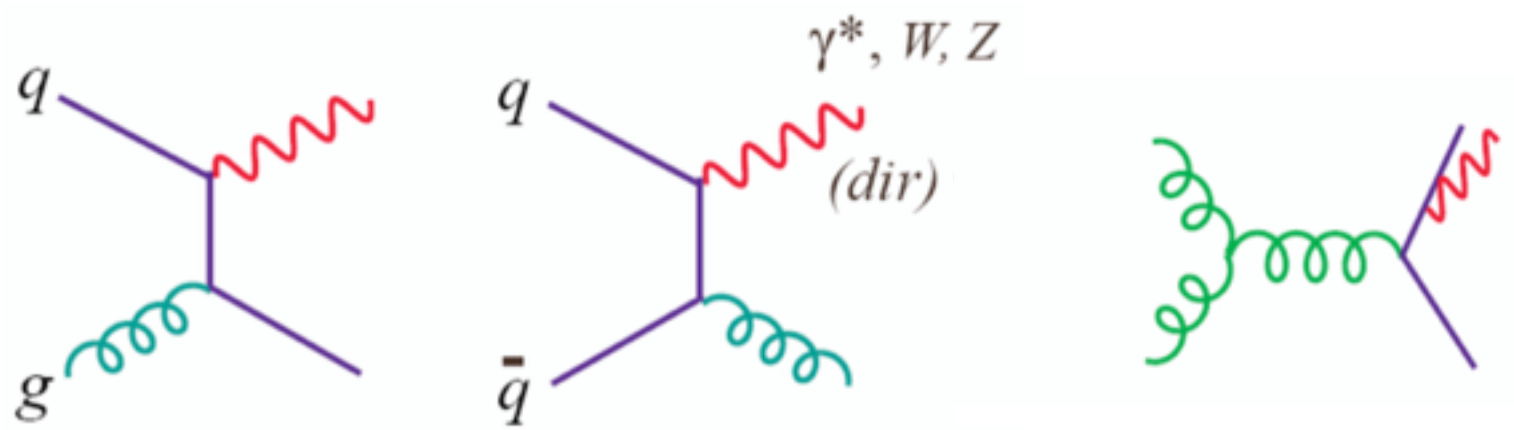
(CDF,D0)

Drell-Yan



Prompt photon

(WA70,UA6,E706)



PDF extraction

GRV, GJR ...

MRST, MSTW ...

CTEQ, CT# ...

NNPDF

ABM, ABKM

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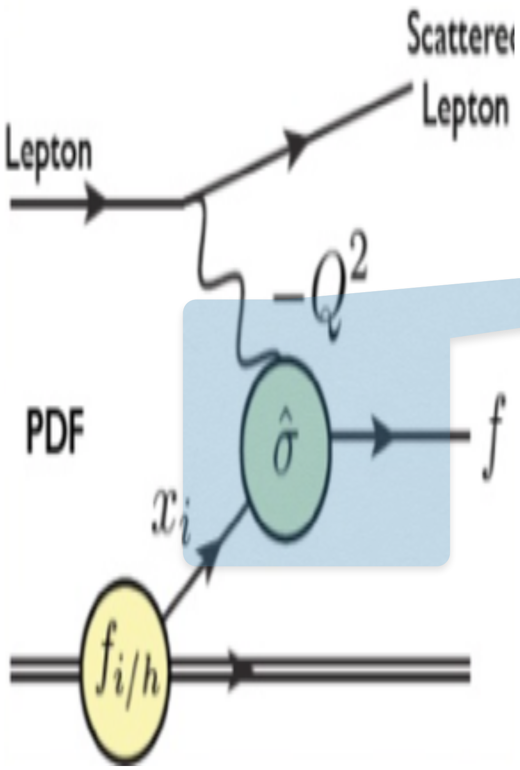
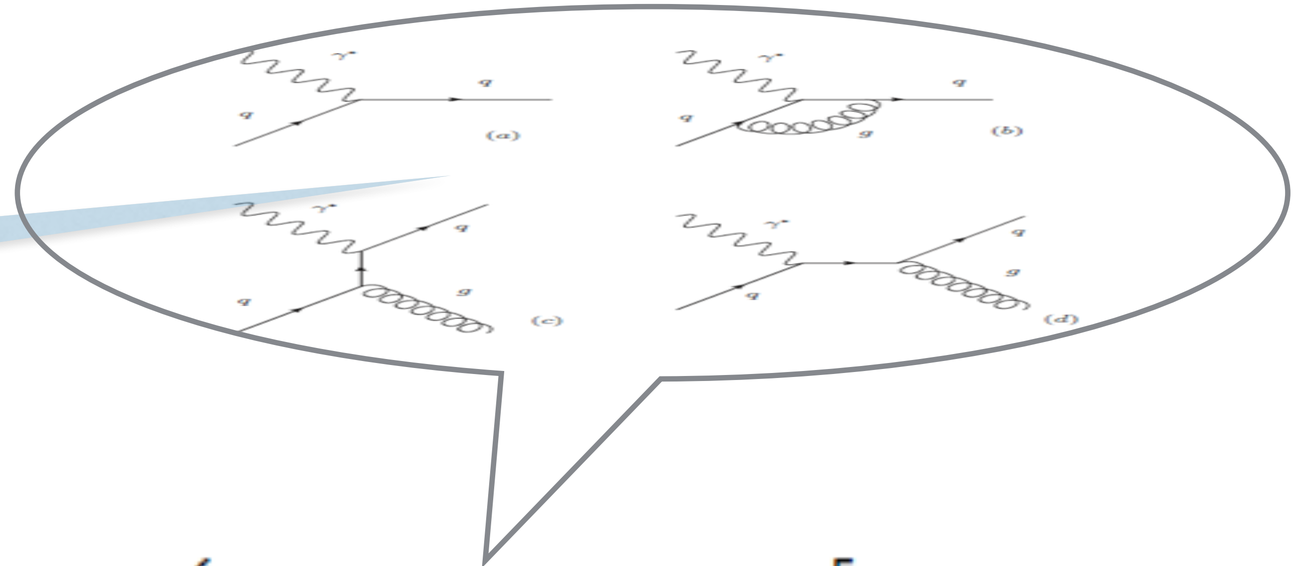
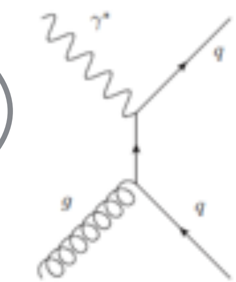
QCD improved Parton Model







$$F_2(x,Q^2) = x\sum_{q,\bar{q}} e_q^2 \left\{ q_0(x) + \frac{\alpha_s}{2\pi}\int_0^1 \frac{dz}{z} q_0\left(\frac{x}{z}\right)\left[P(z)\log\left(\frac{Q^2}{m^2}\right) + C(z)\right] + \dots \right\}$$

Collinear Renormalisation

Factorisation Scale

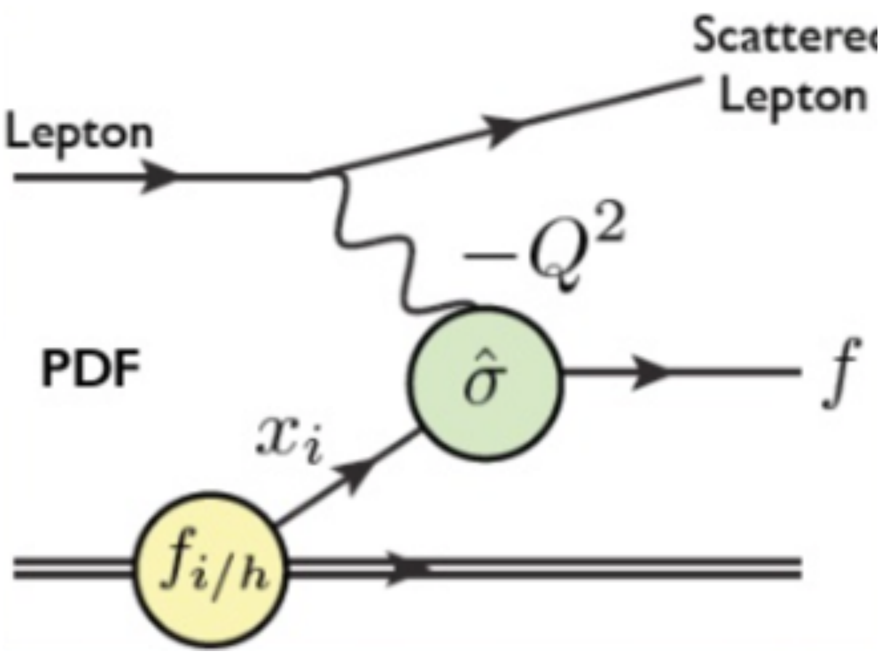
$$q(x,\mu) = q_0(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z} q_0\left(\frac{x}{z}\right)\left[P(z)\log\left(\frac{\mu^2}{m^2}\right) + C(z)\right] + \dots$$

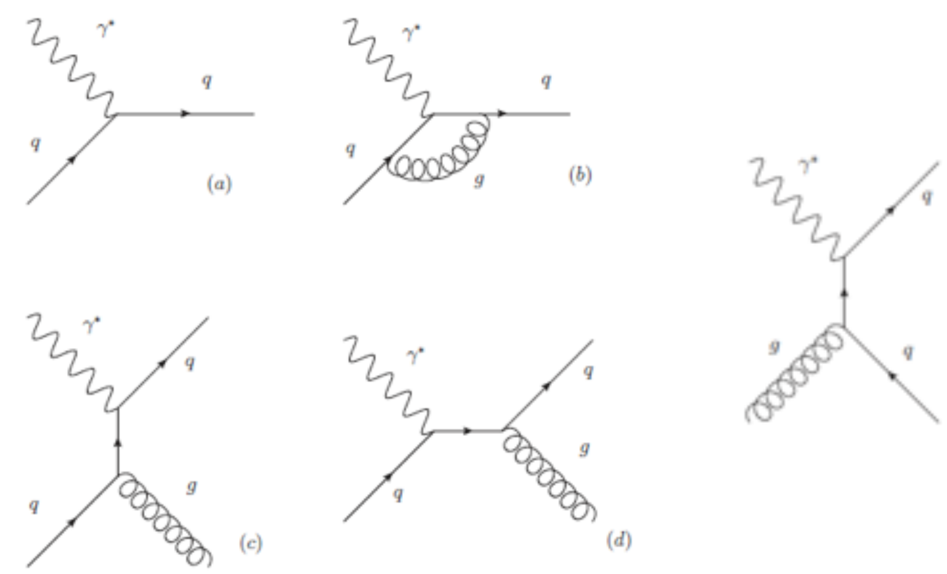
$$\log\frac{Q^2}{m^2} = \log\frac{Q^2}{\mu^2} + \log\frac{\mu^2}{m^2}.$$

$$\begin{aligned} F_2(x,Q^2) \;&=\; x\sum_{q\bar{q}} e_q^2 \int_x^1 \frac{dz}{z} q\left(\frac{x}{z},Q^2\right)\left[\delta(1-z) + \frac{\alpha_s}{2\pi}C_q^{\overline{MS}}(z) + \dots\right] \\ &\quad + x\sum_{q\bar{q}} e_q^2 \int_x^1 \frac{dz}{z} g\left(\frac{x}{z},Q^2\right)\left[\frac{\alpha_s}{2\pi}C_g^{\overline{MS}}(z) + \dots\right] \end{aligned}$$

Factorisation Theorem

$$F_2(x,Q^2) = x\sum_{q\bar{q}} e_q^2 \int_x^1 \frac{dz}{z} q\left(\frac{x}{z},Q^2\right)\left[\delta(1-z)+\frac{\alpha_s}{2\pi}C_q^{\overline{MS}}(z)+\ldots\right] + x\sum_{q\bar{q}} e_q^2 \int_x^1 \frac{dz}{z} g\left(\frac{x}{z},Q^2\right)\left[\frac{\alpha_s}{2\pi}C_g^{\overline{MS}}(z)+\ldots\right]$$





μ_F - Factorisation Scale

μ_R - Renormalisation Scale

$$\sigma^P(x,Q^2) = \sum_{i=q,\bar{q},g} \int_x^1 \frac{dz}{z} C_i\left(z,Q^2,\mu_R^2,\mu_F^2\right) f_{i/P}\left(\frac{x}{z},\mu_F^2\right)$$

Process Dependent Coefficient function
Perturbqtively Calculable to all orders

Only Parton and Target dependent
Non-Perturbative

DGLAP Evolution

Collinear Renormalisation

$$q(x,\mu) = q_0(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z} q_0\left(\frac{x}{z}\right)\left[P(z)\log\left(\frac{\mu^2}{m^2}\right) + C(z)\right] + \dots$$

Arbitrariness in the choice of $\mu = \mu_F$

$$\mu^2 \frac{d}{d\mu^2} q_0(z) = 0$$

Collinear Renormalisation Group Equation

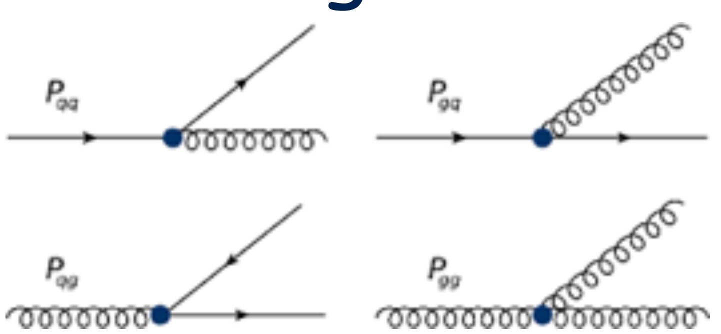
DGLAP Evolution Equation

$$\frac{\partial}{\partial \log \mu^2}\begin{pmatrix} q_i \\ g \end{pmatrix}(x,\mu^2) = \frac{\alpha_s}{2\pi}\int_x^1 \sum_{j=q,\bar{q}} \frac{d\xi}{\xi}\begin{pmatrix} P_{ij}\left(\frac{x}{\xi},\alpha_s\right) & P_{ig}\left(\frac{x}{\xi},\alpha_s\right) \\ P_{gj}\left(\frac{x}{\xi},\alpha_s\right) & P_{gg}\left(\frac{x}{\xi},\alpha_s\right) \end{pmatrix}\begin{pmatrix} q_j \\ g \end{pmatrix}(\xi,\mu^2),$$

In QCD perturbation

$$P_{ij}^{N^mLO}(x,\mu^2) = \sum_{k=0}^{m} a_s^{k+1}(\mu^2) P_{ij}^{(k)}(x).$$

Leading Order

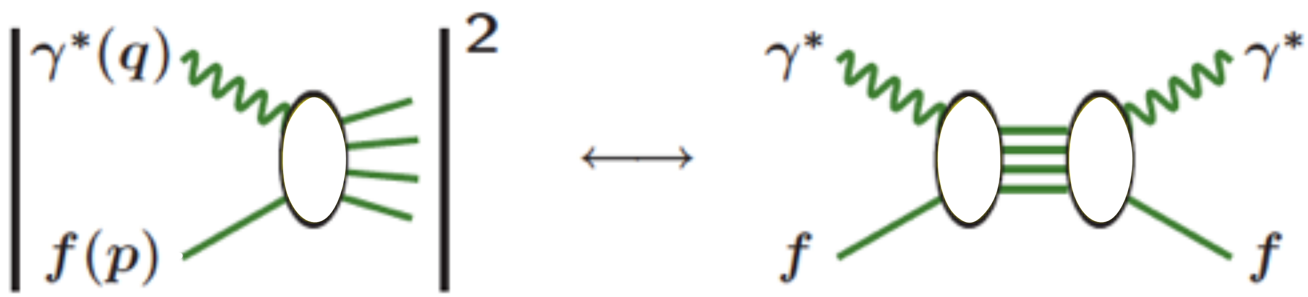


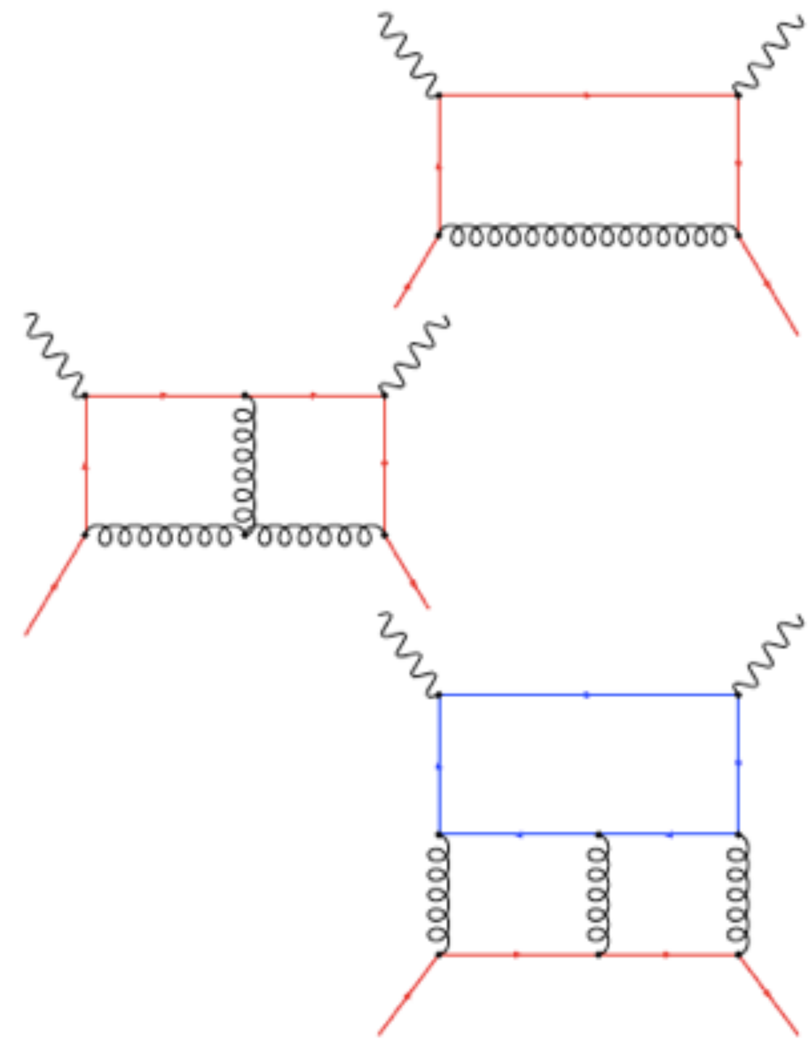
NNLO Results

[Moch,Vogt, Vermaseren]

Optical Theorem

UV + IR Poles in Dim. Regularisation





Poles in Dim. Regularisation

$$
\begin{aligned}
P_{ij}(x,\mu^2) &= a_s(\mu^2) P_{ij}^{(0)}(x) \\
&+ a_s^2(\mu^2) P_{ij}^{(1)}(x) \\
&+ a_s^3(\mu^2) P_{ij}^{(2)}(x)
\end{aligned}
$$

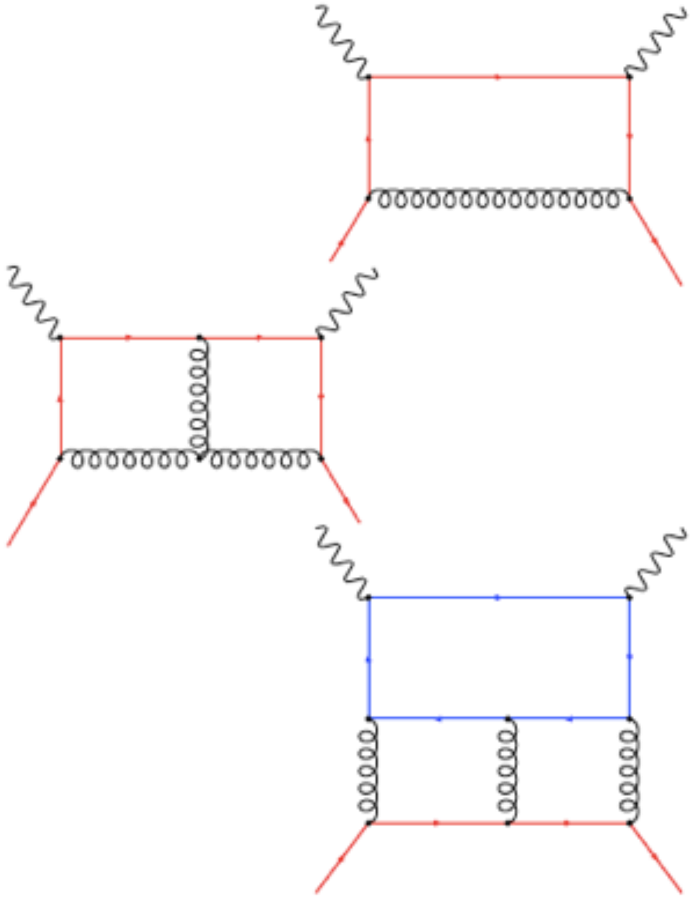
Finite part

$$C_j(x,\mu^2) = C_j^{(0)}(x,\mu^2) + a_s(\mu^2) C_j^{(1)}(x,\mu^2) + a_s^2(\mu^2) C_j^{(2)}(x,\mu^2)$$

NNLO splitting functions

[Moch,Vogt, Vermaseren]

	tree	1-loop	2-loop	3-loop
qγ	1	3	25	359
gγ		2	17	345
hγ			2	56
qW	1	3	32	589
qϕ		1	23	696
gϕ	1	8	218	6378
hϕ		1	33	1184
sum	3	18	350	9607



NNLO splitting functions

[Moch,Vogt, Vermaseren]

$$P_{gg}^{(2)}(x) =$$

$$16C_A C_F n_f\Big(x^2\Big[\frac{4}{9}H_2 + 3H_{1,0} - \frac{97}{12}H_1 + \frac{8}{3}H_{-2,0} - \frac{2}{3}H_0\zeta_2 + \frac{103}{27}H_0 - \frac{16}{3}\zeta_2 + 2H_3$$
$$-6H_{-1,0} + 2H_{2,0} + \frac{127}{18}H_{0,0} - \frac{511}{12}\Big] + p_{gg}(x)\Big[2\zeta_3 - \frac{55}{24}\Big] + \frac{4}{3}(\frac{1}{x} - x^2)\Big[\frac{17}{24}H_{1,0} - \frac{43}{18}H_0$$
$$-\frac{521}{144}H_1 - \frac{6923}{432} - \frac{1}{2}H_{2,1} + 2H_1\zeta_2 + H_0\zeta_2 - 2H_{1,0,0} + \frac{1}{12}H_{1,1} - H_{1,1,0} - H_{1,1,1}\Big] - \frac{175}{12}H_2$$
$$+6H_{-1,0} + 8H_0\zeta_3 - 6H_{-2,0} - \frac{53}{6}H_0\zeta_2 - \frac{49}{2}H_0 + \frac{185}{4}\zeta_2 + \frac{511}{12} - \frac{1}{2}H_{2,0} - 3H_{1,0} - 4H_{0,0,0,0}$$
$$-\frac{67}{12}H_{0,0} + \frac{43}{2}\zeta_3 - H_{2,1} + \frac{97}{12}H_1 - 4\zeta_2^2 - \frac{9}{2}H_3 - 8H_{-3,0} + \frac{33}{2}H_{0,0,0} + \frac{4}{3}(\frac{1}{x} + x^2)\Big[\frac{1}{2}H_2 - H_{2,0}$$
$$+\frac{11}{3}H_{-1,0} + H_{-2,0} + \frac{19}{6}\zeta_2 + 2\zeta_3 - H_{-1}\zeta_2 - 4H_{-1,-1,0} - \frac{1}{2}H_{-1,0,0} - H_{-1,2}\Big] + (1-x)\Big[9H_1\zeta_2$$
$$+12H_{0,0,0,0} - \frac{293}{108} + \frac{61}{6}H_0\zeta_2 - \frac{7}{3}H_{1,0} - \frac{857}{36}H_1 - 9H_0\zeta_3 + 16H_{-2,-1,0} - 4H_{-2,0,0} + 8H_{-2}\zeta_2$$
$$-\frac{13}{2}H_{1,0,0} + \frac{3}{4}H_{1,1} - H_{1,1,0} - H_{1,1,1}\Big] + (1+x)\Big[\frac{1}{6}H_{2,0} - \frac{95}{3}H_{-1,0} - \frac{149}{36}H_2 + \frac{3451}{108}H_0$$
$$-7H_{-2,0} + \frac{302}{9}H_{0,0} + \frac{19}{6}H_3 - \frac{991}{36}\zeta_2 - \frac{163}{6}\zeta_3 - \frac{35}{3}H_{0,0,0} + \frac{17}{6}H_{2,1} - \frac{43}{10}\zeta_2^2 + 13H_{-1}\zeta_2$$
$$+18H_{-1,-1,0} - H_{3,1} - 6H_4 - 4H_{-1,2} + 6H_{0,0}\zeta_2 + 8H_2\zeta_2 - 7H_{2,0,0} - 2H_{2,1,0} - 2H_{2,1,1} - 4H_{3,0}$$
$$-9H_{-1,0,0}\Big] - \frac{241}{288}\delta(1-x)\Big) + 16C_A n_f^2\Big(\frac{19}{54}H_0 - \frac{1}{24}xH_0 - \frac{1}{27}p_{gg}(x) + \frac{13}{54}(\frac{1}{x} - x^2)\Big[\frac{5}{3} - H_1\Big]$$
$$+(1-x)\Big[\frac{11}{72}H_1 - \frac{71}{216}\Big] + \frac{2}{9}(1+x)\Big[\zeta_2 + \frac{13}{12}xH_0 - \frac{1}{2}H_{0,0} - H_2\Big] + \frac{29}{288}\delta(1-x)\Big)$$
$$+16C_A^2 n_f\Big(x^2\Big[\zeta_3 + \frac{11}{9}\zeta_2 + \frac{11}{9}H_{0,0} - \frac{2}{3}H_3 + \frac{2}{3}H_0\zeta_2 + \frac{1639}{108}H_0 - 2H_{-2,0}\Big] + \frac{1}{3}p_{gg}(x)\Big[\frac{10}{3}\zeta_2$$
$$-\frac{209}{36} - 8\zeta_3 - 2H_{-2,0} - \frac{1}{2}H_0 - \frac{10}{3}H_{0,0} - \frac{20}{3}H_{1,0} - H_{1,0,0} - \frac{20}{3}H_2 - H_3\Big] + \frac{10}{9}p_{gg}(-x)\Big[\zeta_2$$
$$+2H_{-1,0} + \frac{3}{10}H_0\zeta_2 - H_{0,0}\Big] + \frac{1}{3}(\frac{1}{x} - x^2)\Big[H_3 - H_0\zeta_2 - \frac{13}{3}H_2 + \frac{5443}{108} - 3H_1\zeta_2 + \frac{205}{36}H_1$$
$$-\frac{13}{3}H_{1,0} + H_{1,0,0}\Big] + (\frac{1}{x} + x^2)\Big[\frac{151}{54}H_0 - \frac{8}{3}\zeta_2 + \frac{1}{3}H_{-1}\zeta_2 - \zeta_3 + 2H_{-1,-1,0} - \frac{2}{3}H_{-1,0,0}$$
$$-\frac{37}{9}H_{-1,0} + \frac{2}{3}H_{-1,2}\Big] + (1-x)\Big[\frac{5}{6}H_{-2,0} + H_{-3,0} + 2H_{0,0,0} - \frac{269}{36}\zeta_2 - \frac{4097}{216} - 3H_{-2}\zeta_2$$
$$-6H_{-2,-1,0} + 3H_{-2,0,0} - \frac{7}{2}H_1\zeta_2 + \frac{677}{72}H_1 + H_{1,0} + \frac{7}{4}H_{1,0,0}\Big] + (1+x)\Big[\frac{193}{36}H_2 - \frac{11}{2}H_{-1}\zeta_2$$
$$+\frac{39}{20}\zeta_2^2 - \frac{7}{12}H_3 - \frac{53}{9}H_{0,0} + \frac{7}{12}H_0\zeta_2 - \frac{5}{2}H_{0,0}\zeta_2 + 5\zeta_3 - 7H_{-1,-1,0} + \frac{77}{6}H_{-1,0} + \frac{9}{2}H_{-1,0,0}$$
$$+2H_{-1,2} - 3H_2\zeta_2 - \frac{2}{3}H_{2,0} + \frac{3}{2}H_{2,0,0} + \frac{3}{2}H_4\Big] + \frac{1}{9}\zeta_2 + 7H_{-2,0} + 2H_2 + \frac{458}{27}H_0 + H_{0,0}\zeta_2$$
$$+\frac{3}{2}\zeta_2^2 + 4H_{-3,0} - x\Big[\frac{131}{12}H_{0,0} - \frac{8}{3}H_0\zeta_2 + \frac{7}{2}H_3 - H_{0,0,0,0} + \frac{7}{6}H_{0,0,0} + \frac{1943}{216}H_0 + 6H_0\zeta_3\Big]$$
$$-\delta(1-x)\Big[\frac{233}{288} + \frac{1}{6}\zeta_2 + \frac{1}{12}\zeta_2^2 + \frac{5}{3}\zeta_3\Big]\Big) + 16C_A^3\Big(x^2\Big[33H_{-2,0} + 33H_0\zeta_2 - \frac{1249}{18}H_{0,0}$$
$$-44H_{0,0,0} - \frac{110}{3}H_3 - \frac{44}{3}H_{2,0} + \frac{85}{6}\zeta_2 + \frac{6409}{108}H_0\Big] + p_{gg}(x)\Big[\frac{245}{24} - \frac{67}{9}\zeta_2 - \frac{3}{10}\zeta_2^2 + \frac{11}{3}\zeta_3$$
$$-4H_{-3,0} + 6H_{-2}\zeta_2 + 4H_{-2,-1,0} + \frac{11}{3}H_{-2,0} - 4H_{-2,0,0} - 4H_{-2,2} + \frac{1}{6}H_0 - 7H_0\zeta_3 + \frac{67}{9}H_{0,0}$$
$$-8H_{0,0}\zeta_2 + 4H_{0,0,0,0} - 6H_1\zeta_3 - 4H_{1,-2,0} + 10H_{2,0,0} - 6H_{1,0}\zeta_2 + 8H_{1,0,0,0} + 8H_{1,1,0,0} + 8H_4$$
$$+\frac{134}{9}H_{1,0} + \frac{11}{6}H_{1,0,0} + 8H_{1,2,0} + 8H_{1,3} + \frac{134}{9}H_2 - 4H_2\zeta_2 + 8H_{3,1} + 8H_{2,2} + \frac{11}{6}H_3 + 10H_{3,0}$$
$$+8H_{2,1,0}\Big] + p_{gg}(-x)\Big[\frac{11}{2}\zeta_2^2 - \frac{11}{6}H_0\zeta_2 - 4H_{-3,0} + 16H_{-2}\zeta_2 - 12H_{-2,2} - \frac{134}{9}H_{-1,0} + 2H_2\zeta_2$$
$$+8H_{-2,-1,0} + 12H_{-1}\zeta_3 - 18H_{-2,0,0} + 8H_{-1,-2,0} - 16H_{-1,-1}\zeta_2 + 24H_{-1,-1,0,0} + 16H_{-1,-1,2}$$
$$+18H_{-1,0}\zeta_2 - 16H_{-1,0,0,0} - 4H_{-1,2,0} - 16H_{-1,3} - 5H_0\zeta_3 - 8H_{0,0}\zeta_2 + 4H_{0,0,0,0} + 2H_{3,0}$$
$$-\frac{67}{9}\zeta_2 + \frac{67}{9}H_{0,0} + 8H_4\Big] + \Big(\frac{1}{x} - x^2\Big)\Big[\frac{16619}{162} + \frac{22}{3}H_{2,0} - \frac{55}{2}\zeta_3 - \frac{11}{2}H_0\zeta_2 - \frac{67}{9}H_2 - \frac{67}{9}H_{1,0}$$
$$-\frac{413}{108}H_1 - \frac{11}{2}H_1\zeta_2 + \frac{33}{2}H_{1,0,0}\Big] + 11(\frac{1}{x} + x^2)\Big[\frac{71}{54}H_0 - \frac{1}{6}H_3 - \frac{389}{198}\zeta_2 - \frac{2}{3}H_{-2,0} - \frac{1}{2}H_{-1}\zeta_2$$
$$+H_{-1,-1,0} - \frac{523}{198}H_{-1,0} + \frac{8}{3}H_{-1,0,0} + H_{-1,2}\Big] + (1-x)\Big[\frac{31}{36}H_1 + \frac{27}{2}H_{1,0} - \frac{25}{2}H_{1,0,0} - 4H_{-3,0}$$
$$-\frac{263}{12}H_{0,0} - \frac{29}{3}H_{0,0,0} - \frac{19}{3}H_{-2,0} - \frac{11317}{108} - 4H_{-2}\zeta_2 - 8H_{-2,-1,0} - 12H_{-2,0,0} - \frac{3}{2}H_1\zeta_2\Big]$$
$$+(1+x)\Big[\frac{27}{2}H_0\zeta_2 - \frac{43}{6}H_3 + \frac{29}{3}H_{2,0} + \frac{4651}{216}H_0 - \frac{329}{18}\zeta_2 + \frac{11}{2}(1+x)\zeta_3 - \frac{43}{5}\zeta_2^2 - \frac{215}{6}H_{-1,0}$$
$$-22H_{0,0}\zeta_2 - 8H_0\zeta_3 - 3H_{-1,-1,0} + 38H_{-1,0,0} + 25H_{-1,2} + 10H_{2,0,0} - 4H_2\zeta_2 + 16H_{3,0} + 26H_4$$
$$-\frac{158}{9}H_2 - \frac{53}{2}H_{-1}\zeta_2\Big] - 29H_{0,0} - \frac{40}{3}H_{0,0,0} + 27H_{-2,0} + \frac{41}{3}H_0\zeta_2 - 20H_3 - 24H_{2,0} + \frac{53}{6}\zeta_2$$
$$+\frac{601}{12}H_0 + 24\zeta_3 + 2\zeta_2^2 + 27H_2 - 4H_{0,0}\zeta_2 - 16H_0\zeta_3 - 16H_{-3,0} + 28xH_{0,0,0,0} + \delta(1-x)\Big[\frac{79}{32}$$
$$-\zeta_2\zeta_3 + \frac{1}{6}\zeta_2 + \frac{11}{24}\zeta_2^2 + \frac{67}{6}\zeta_3 - 5\zeta_5\Big]\Big) + 16C_F n_f^2\Big(\frac{2}{9}x^2\Big[\frac{11}{6}H_0 + H_2 - \zeta_2 + 2H_{0,0} - 9\Big] + \frac{1}{3}H_2$$
$$-\frac{1}{3}\zeta_2 - \frac{10}{3}H_0 - \frac{1}{3}H_{0,0} + 2 + \frac{2}{9}(\frac{1}{x} - x^2)\Big[\frac{8}{3}H_1 - 2H_{1,0} - H_{1,1} - \frac{77}{18}\Big] - (1-x)\Big[\frac{1}{3}H_{1,0} + \frac{1}{6}H_{1,1}$$
$$+\frac{4}{9} + \frac{13}{6}H_1 + xH_1\Big] + \frac{1}{3}(1+x)\Big[\frac{68}{9}H_0 - \frac{4}{3}H_2 + \frac{4}{3}\zeta_2 + \frac{29}{6}H_{0,0} - \zeta_3 + 2H_0\zeta_2 - H_{0,0,0} - 2H_3$$
$$-H_{2,1} - 2H_{2,0}\Big] + \frac{11}{144}\delta(1-x)\Big) + 16C_F^2 n_f\Big(\frac{4}{3}x^2\Big[\frac{163}{16} + \frac{27}{8}H_0 + \frac{7}{2}H_{0,0} - H_{2,0} - \zeta_2 + \frac{9}{4}H_{1,0}$$
$$-H_{2,1} + \frac{1}{2}H_{0,0,0} + \frac{85}{16}H_1 + H_2 - 2H_{-2,0} - \frac{3}{2}\zeta_3\Big] + \frac{4}{3}(\frac{1}{x} - x^2)\Big[\frac{31}{16}H_1 - \frac{11}{16} - \frac{5}{4}H_{1,0} + \frac{1}{2}H_{1,0,0}$$
$$-H_1\zeta_2 - H_{1,1} + H_{1,1,0} + H_{1,1,1} + \zeta_3\Big] + \frac{4}{3}(\frac{1}{x} + x^2)\Big[H_{-1}\zeta_2 + 2H_{-1,-1,0} - H_{-1,0,0}\Big] + \frac{215}{12}H_{0,0}$$
$$+\frac{20}{3}H_0 - \frac{131}{6} + 3H_{2,0} + \frac{205}{12}x\zeta_2 - 3H_{1,0} + H_{2,1} - \frac{85}{12}H_1 + \frac{11}{4}H_2 + 8H_{-2,0} + 2\zeta_2^2 - H_0\zeta_2$$
$$+H_3 + 6H_0\zeta_3 + 8H_{-3,0} - 4xH_{0,0,0} + (1-x)\Big[\frac{107}{12}H_1 - \frac{5}{6}H_{1,0} - 4\zeta_2 + H_0\zeta_3 - 8H_{-2,-1,0}$$
$$-4H_{-2}\zeta_2 + 4H_{-2,0,0} - 4H_1\zeta_2 + \frac{7}{2}H_{1,0,0} - \frac{7}{12}H_{1,1} + H_{1,1,0} + H_{1,1,1}\Big] + (1+x)\Big[\frac{5}{4}H_2 + \frac{33}{8}$$
$$-\frac{99}{4}H_{0,0} - 8H_{2,0} - \frac{541}{24}H_0 - 4H_{2,1} - \frac{3}{2}H_{0,0,0} - 2x\zeta_3 + \frac{9}{2}\zeta_2^2 + 5H_0\zeta_2 - 5H_3 - 4H_{-1}\zeta_2$$
$$-8H_{-1,-1,0} + \frac{67}{3}H_{-1,0} + 4H_{-1,0,0} + 2H_{0,0}\zeta_2 - 2H_{0,0,0,0} - 4H_2\zeta_2 + 3H_{2,0,0} + 2H_{2,1,0}$$
$$+2H_{2,1,1} + H_{3,1} - 2H_4\Big] + \frac{1}{16}\delta(1-x)\Big)$$

MVV (2004)

Going beyond NNLO

[Moch et al]

Large x double log behaviour of Splitting and Coefficient functions

Trick: Use Physical Evolution Equations (PEE):

PEE: Differential equations w.r.t Q of Structure functions

$$\frac{d}{d\ln Q^2} F = \mathbf{K} F \equiv \sum_{\ell=0}^{\infty} a_s^{\ell+1} \begin{pmatrix} K_{22}^{(\ell)} & K_{2\phi}^{(\ell)} \\ K_{\phi 2}^{(\ell)} & K_{\phi\phi}^{(\ell)} \end{pmatrix} \cdot \begin{pmatrix} F_2 \\ F_\phi \end{pmatrix}$$

$$\mathbf{F} = \begin{pmatrix} F_2 \\ F_\phi \end{pmatrix}$$

Kernels are enhanced by single logs

Going beyond NNLO

[Vogt et al]

Large x Behaviour of 4-loop Splitting and Coefficient functions

From PEE of (F_2, F_ϕ)

4-loop results!

$$
\begin{aligned}
P^{(3)}_{\rm qg}(x) = & \ln^6(1-x) \cdot 0 \\
& + \ln^5(1-x)\Big[\frac{22}{27}\, C_{AF}^3 n_f - \frac{14}{27}\, C_{AF}^2 C_F n_f - \frac{4}{27}\, C_{AF}^2 n_f^2\Big] \\
& + \ln^4(1-x)\Big[\Big(\frac{293}{27} - \frac{80}{9}\zeta_2\Big) C_{AF}^3 n_f + \Big(\frac{4477}{162} - 8\zeta_2\Big) C_{AF}^2 C_F n_f \\
& - \frac{13}{81}\, C_{AF} C_F^2 n_f - \frac{116}{81}\, C_{AF}^2 n_f^2 + \frac{17}{81}\, C_{AF} C_F n_f^2 - \frac{4}{81}\, C_{AF} n_f^3\Big] \\
& + \mathcal{O}\left(\ln^3(1-x)\right)
\end{aligned}
$$

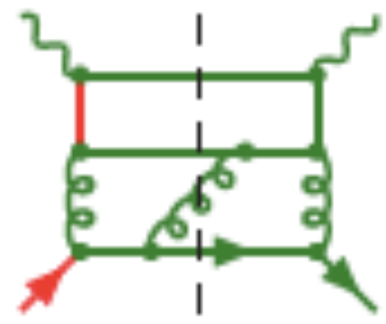
From PEE of (F_2, F_ϕ) $\quad and \quad$ (F_2, F_L)

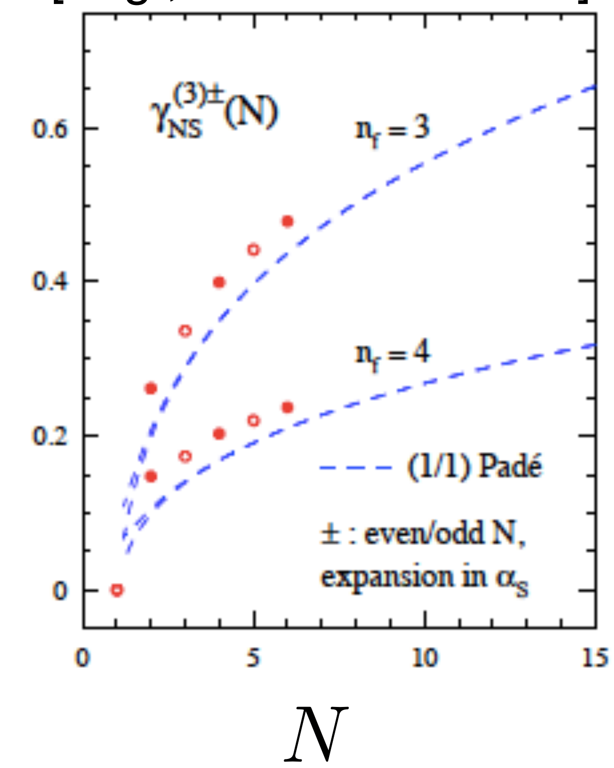
Predictions of $\log^{6,5,4}(1-x)$ $\quad of \quad$ $C_L^{(3)}$

Going beyond NNLO

[Vogt, Vermaseren et al]

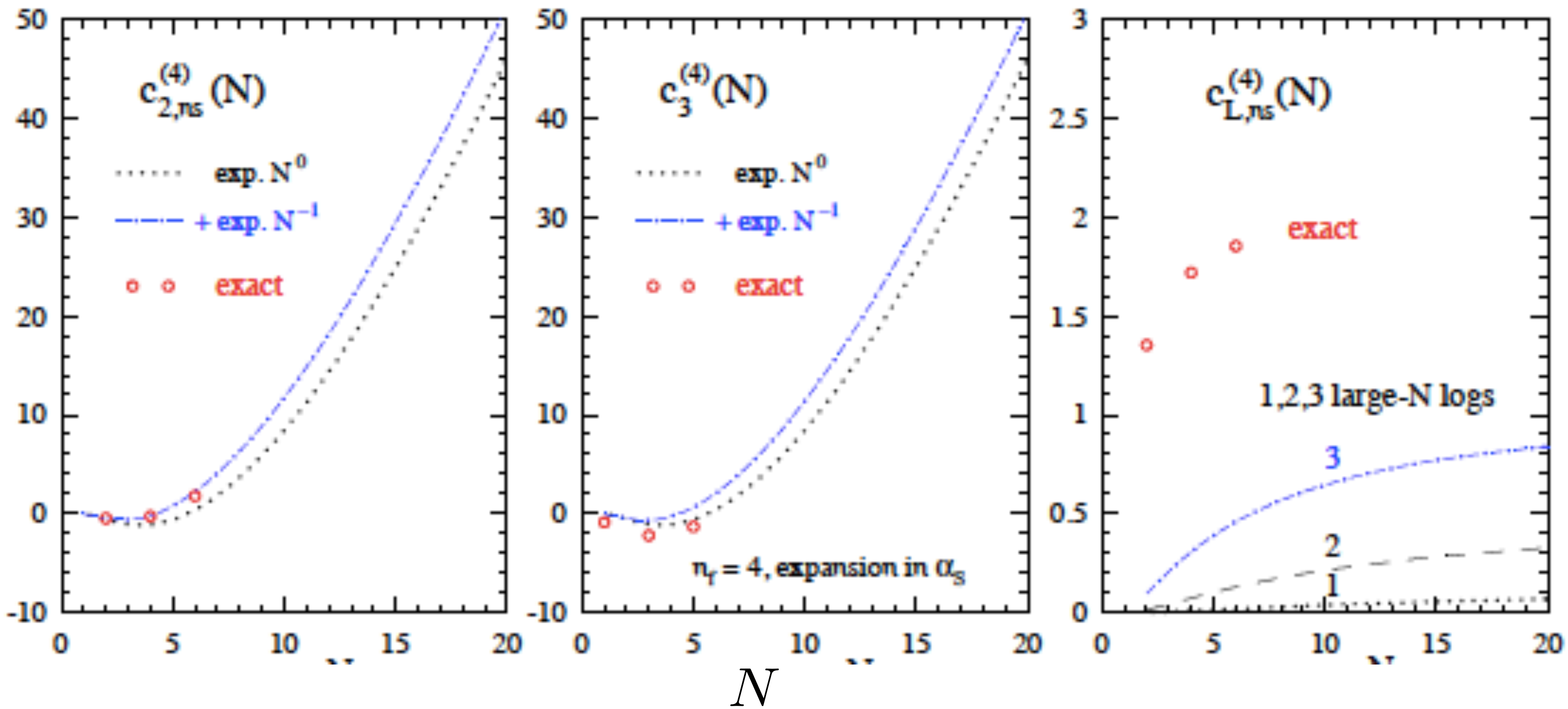
MINCER to FORCER
for 4 loop results





Third order contributions to Coefficient and splitting functions

$$C^{(3)}_{2,3,L} \qquad P^{(3)}_{ij}(x), \gamma^{(3)}_{ij}(N)$$



Going beyond NNLO

[Baikov et al, Velizhanin]

Non-Singlet Splitting function at 4-loops

$$\gamma_n^{approx:3} = \frac{{\gamma_n^{(2)}}^2}{\gamma_n^{(1)}}$$

$$\gamma_n = -\int dx x^{n-1} P(x)$$

$$\mathcal{O}_{\rm NS}^{a,\mu\nu\rho} = \bar{\psi}\lambda^a\gamma^\mu \mathcal{D}^\nu \mathcal{D}^\rho \psi\,,$$
$$\mathcal{O}_{\rm NS}^{a,\mu\nu\rho\sigma} = \bar{\psi}\lambda^a\gamma^\mu \mathcal{D}^\nu \mathcal{D}^\rho \mathcal{D}^\sigma \psi$$

n=2 moment

$$\gamma_2^{3;NS} = \frac{32}{9} a_s + \frac{9440}{243} a_s^2 + \left[\frac{3936832}{6561} - \frac{10240}{81}\zeta_3\right] a_s^3 + \left[\frac{1680283336}{1777147} - \frac{24873952}{6561}\zeta_3 + \frac{5120}{3}\zeta_4 - \frac{56969}{243}\zeta_5\right] a_s^4$$

n= 3, 4 moment

Pade` 3480

$$\gamma_{\rm NS}^{4-loop}(3, n_f = 4) = 5.55556\, a_s + 50.39095\, {a_s}^2 + 418.17201\, {a_s}^3 + 4322.89048\, {a_s}^4\,,$$
$$\gamma_{\rm NS}^{4-loop}(4, n_f = 4) = 6.97778\, a_s + 60.07233\, {a_s}^2 + 502.91174\, {a_s}^3 + 5066.33924\, {a_s}^4\,.$$

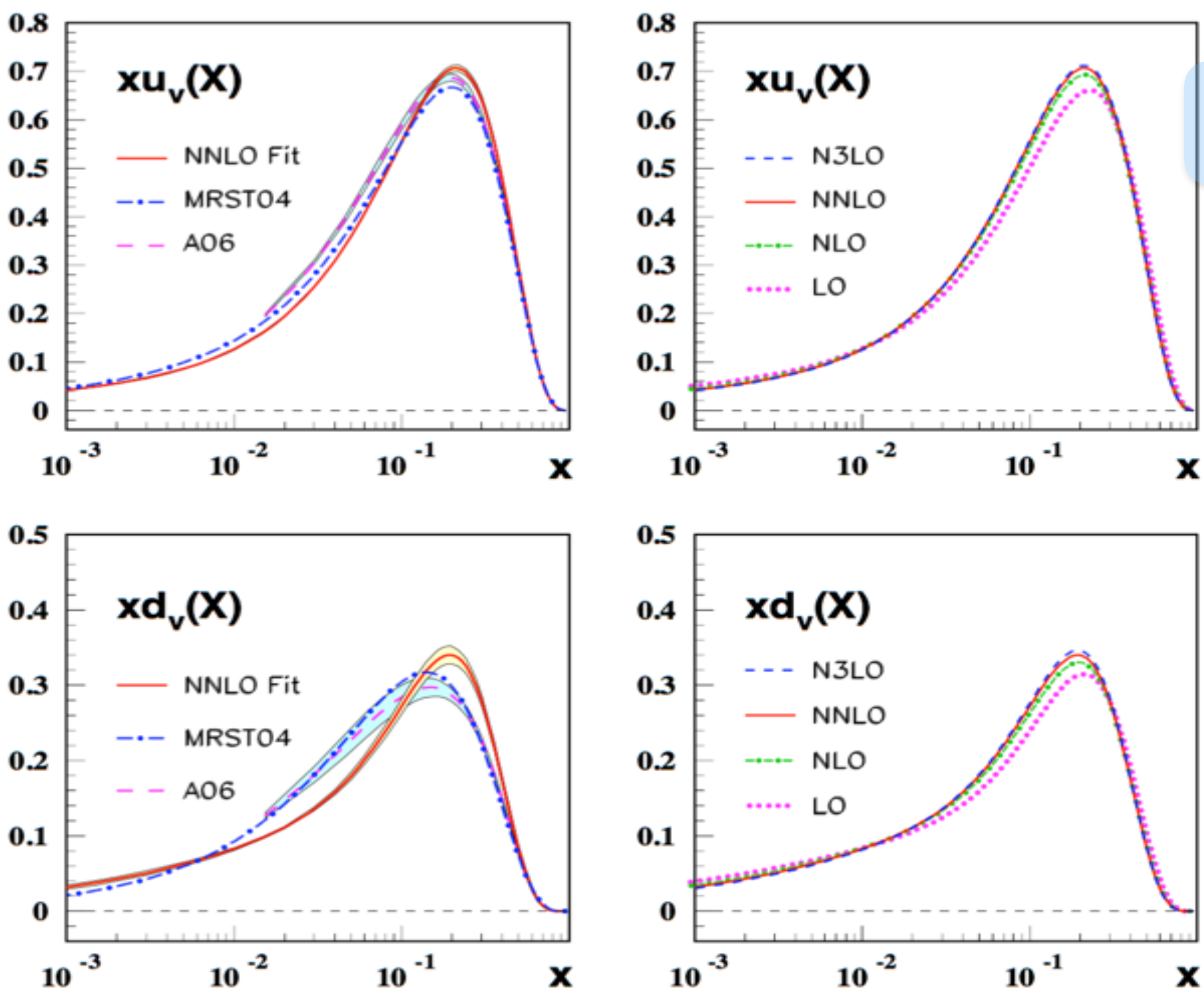
Pade` 4200

PDFs at approx. N3LO

[Bluemlein et al]

World Data: NS-analysis

W^2> 12.5 GeV^2 , Q^2 > 4 GeV^2



$$\alpha_s(M_Z) = 0.1145 \pm 0.0009 \text{ (exp.)}$$

Heavy Flavours to DIS

[Bluemlein et al]

Coefficient functions depend on m

$$F_{2,L}(x,Q^2) = \sum_j C_{j,2,L}\left(x, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right) \otimes f_j(x,\mu^2)$$

mass of the heavy flavour

Mellin space result

$$C_{j,2,L}\left(N, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right) = C_{j,2,L}\left(N, \frac{Q^2}{\mu^2}\right) + H_{j,2,L}\left(N, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right)$$

Light flavour

Heavy flavour

Factorisation

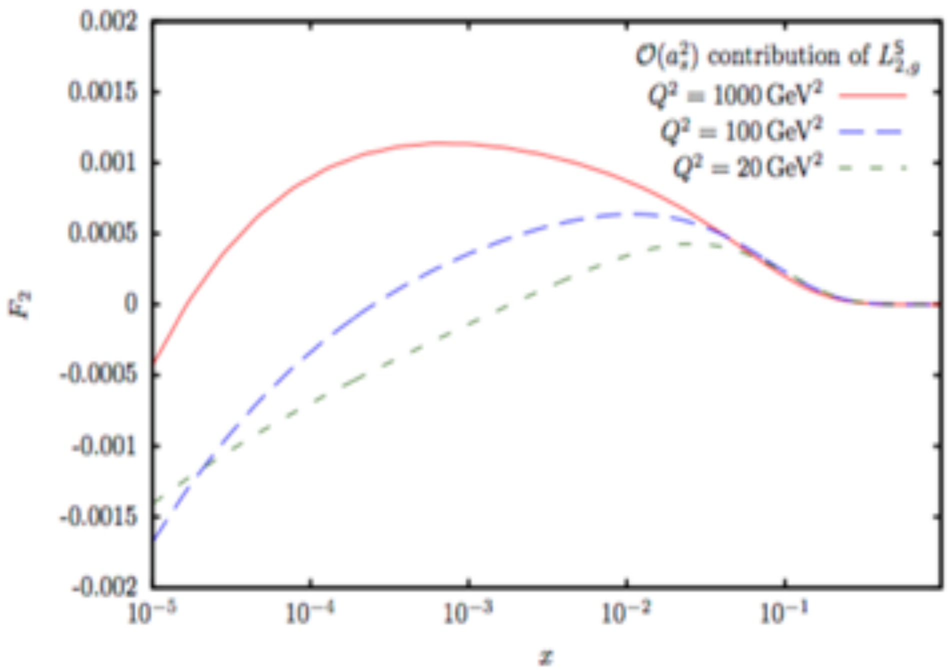
$$H_{j,2,L}\left(N, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right) = \sum_i C_{i,2,L}\left(N, \frac{Q^2}{\mu^2}\right) A_{ij}\left(\frac{m^2}{\mu^2}, N\right)$$

Operator

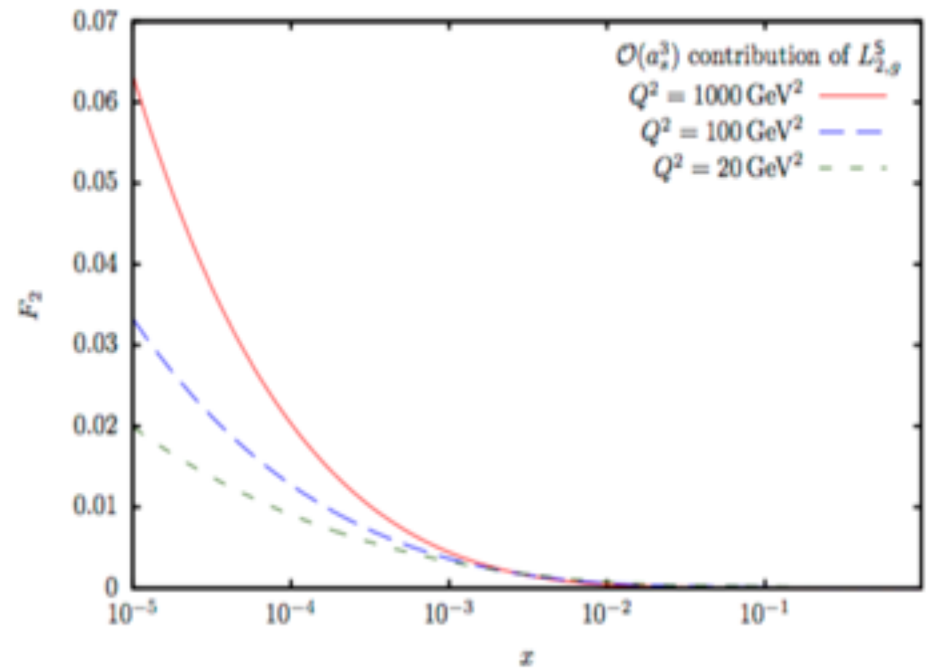
Perturbatively Calculable: $A_{ij}\left(\frac{m^2}{\mu^2}, N\right) = \langle j \mid O_i \mid j \rangle$.

Heavy Flavours to DIS

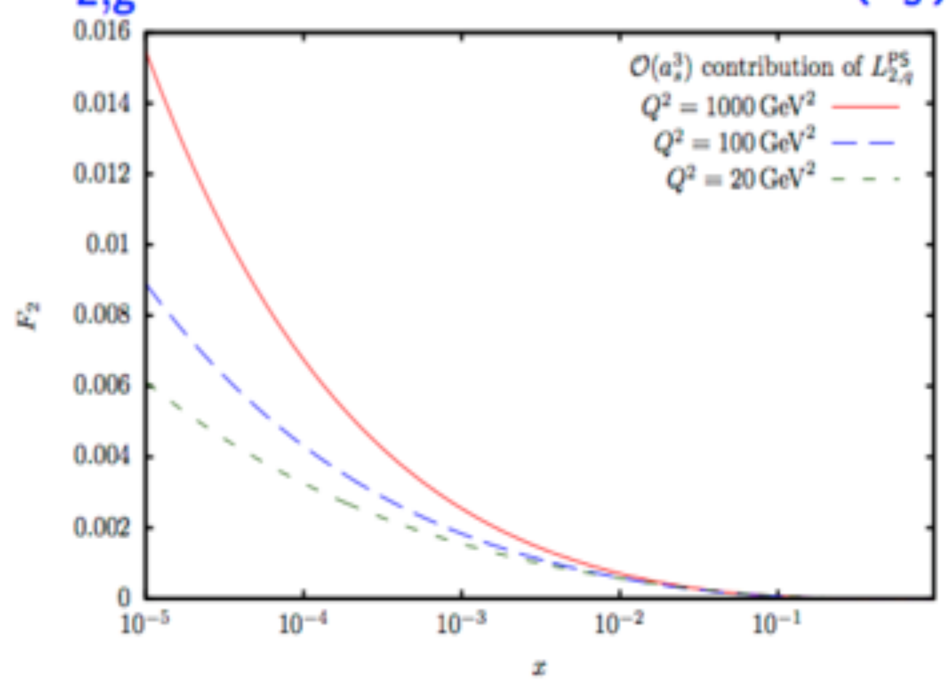
[Bluemlein et al]



$O(a_s^2)$ $L_{2,g}^{S}$



$O(a_s^3)$ $L_{2,g}^{S}$

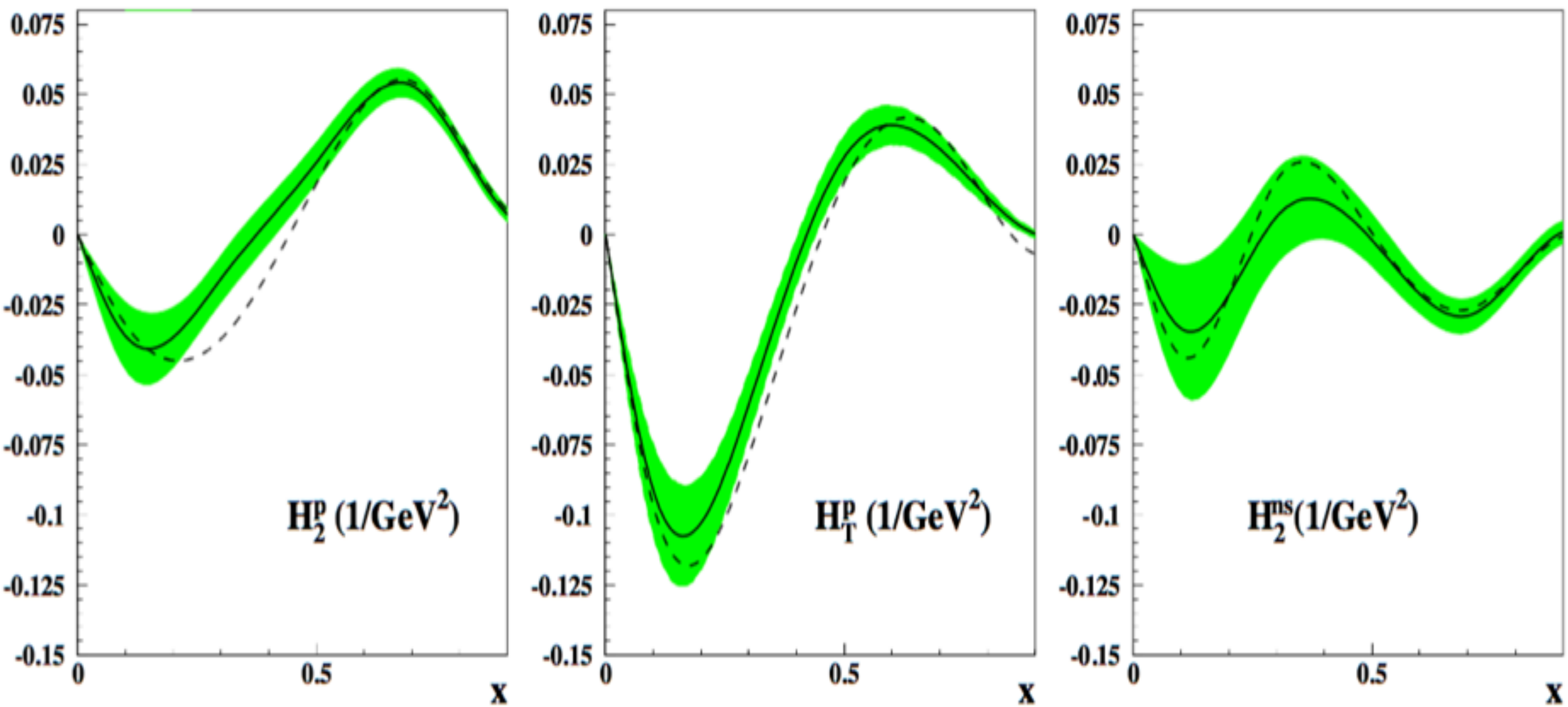


$L_{q,2}^{PS}$

Higher Twist to DIS

[Bluemlein et al]

$$F_i(x,Q^2) = F_i^{TMC,\tau=2}(x,Q^2) + \frac{H_i^4(x)}{Q^2} + \frac{H_i^6(x)}{Q^4} + \ldots$$



Resummationed PDFs

[Bonvini et al]

Coefficient functions in

$$F_2(x,Q^2) = x\sum_{n=0}^{\infty}\frac{\alpha_s^n(\mu_R^2)}{(2\pi)^n}\sum_{i=q,g}\int_x^1\frac{dz}{z}C_{2,i}^{(n)}(z,Q^2,\mu_R^2,\mu_F^2)\,f_{i/p}\left(\frac{x}{z},\mu_F^2\right)$$

$$\delta(1-z),\quad \left(\frac{\log^j(1-z)}{1-z}\right)$$

$$j = 0, \ldots, \infty$$

$$\frac{1}{z}\log^j z$$

Soft Gluons $z \to 1$

High Energy Gluons $z << 1$

$$\alpha_s^m(\mu_R^2)\ a(x)\log^n b(x) \approx 1$$

when for certain n = g(m)

RESUMMATION to all orders Reliable perturbations predictions

Resummationed PDFs

[Bonvini et al]

Soft Gluon Resummation: $z \to 1$ Or Mellin $N \to \infty$

$$C_i(z,Q^2,\mu^2) = C_i^{(0)}(z) \quad g_0(\alpha_s) \exp\left(\frac{1}{\alpha_s} g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots\right)$$

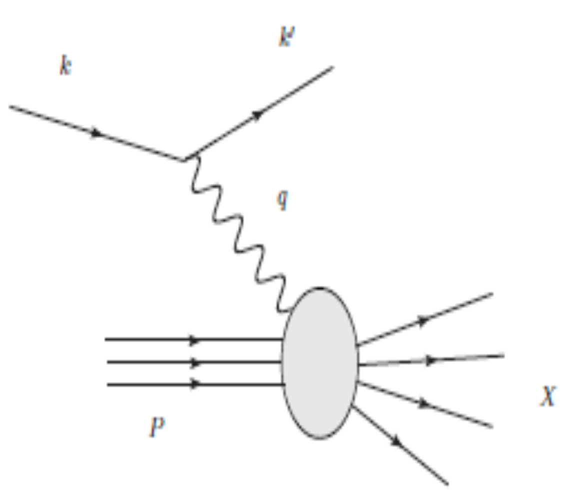
$$L = \frac{\beta_0}{4\pi} \log N$$

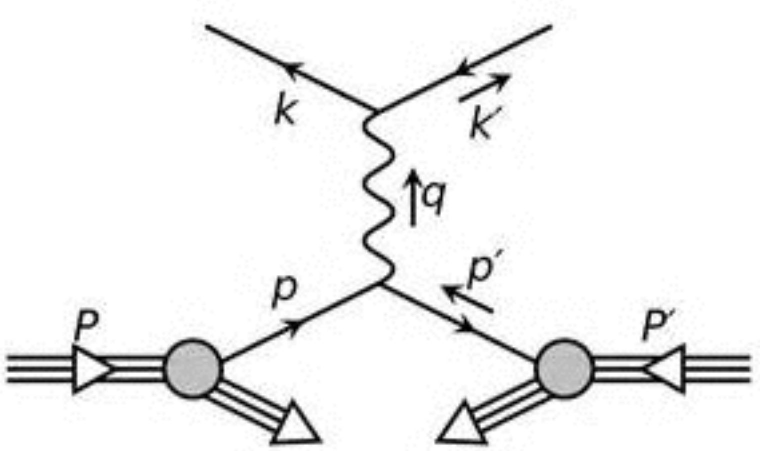
N independent

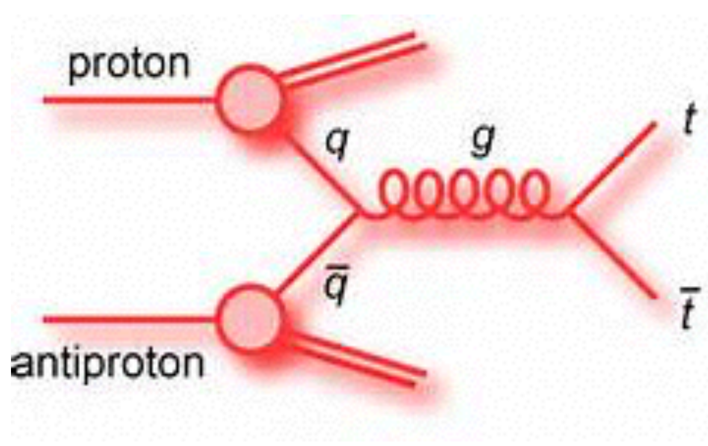
DIS prod.

Drell-Yan prod.

Top pair prod.

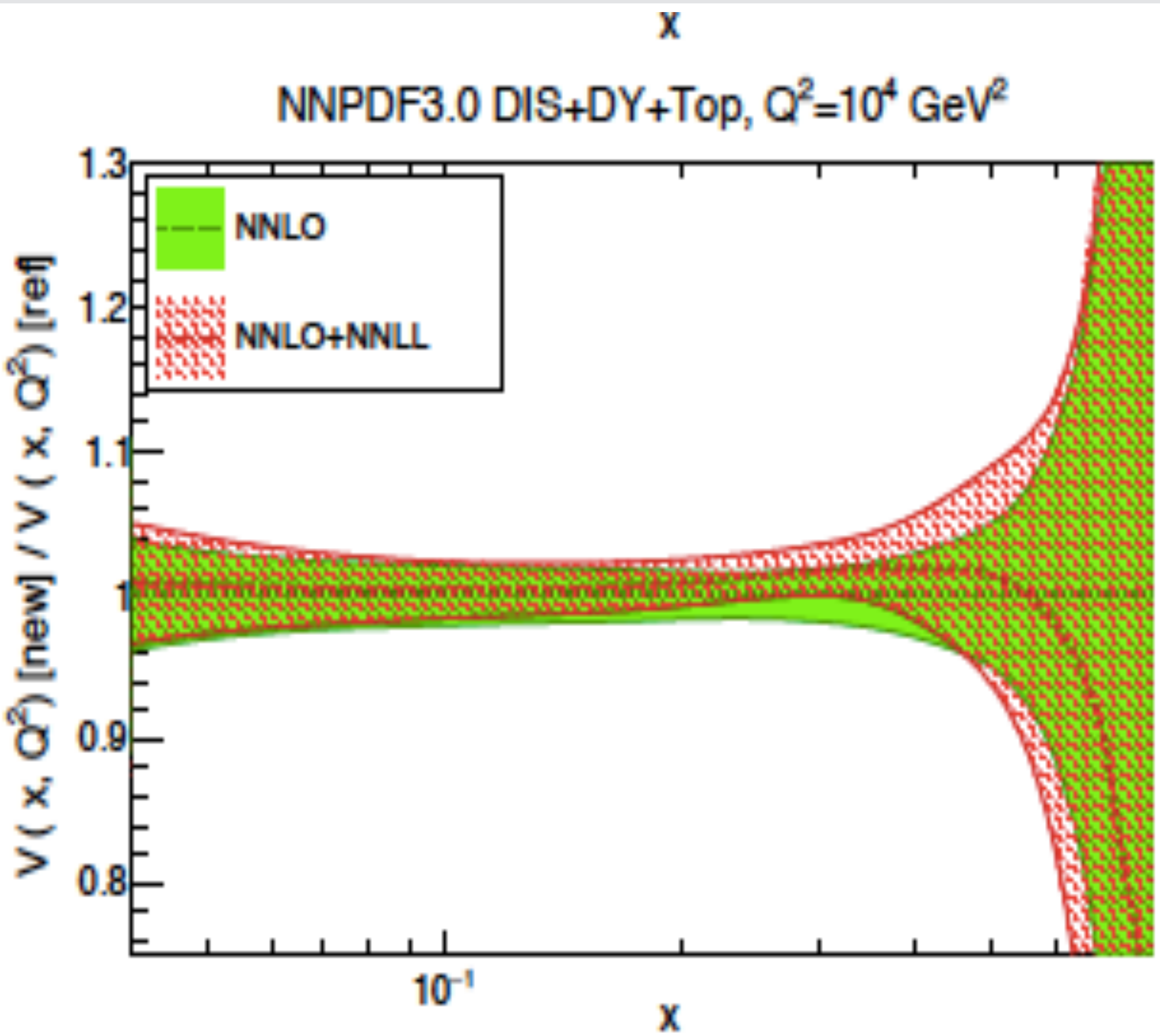


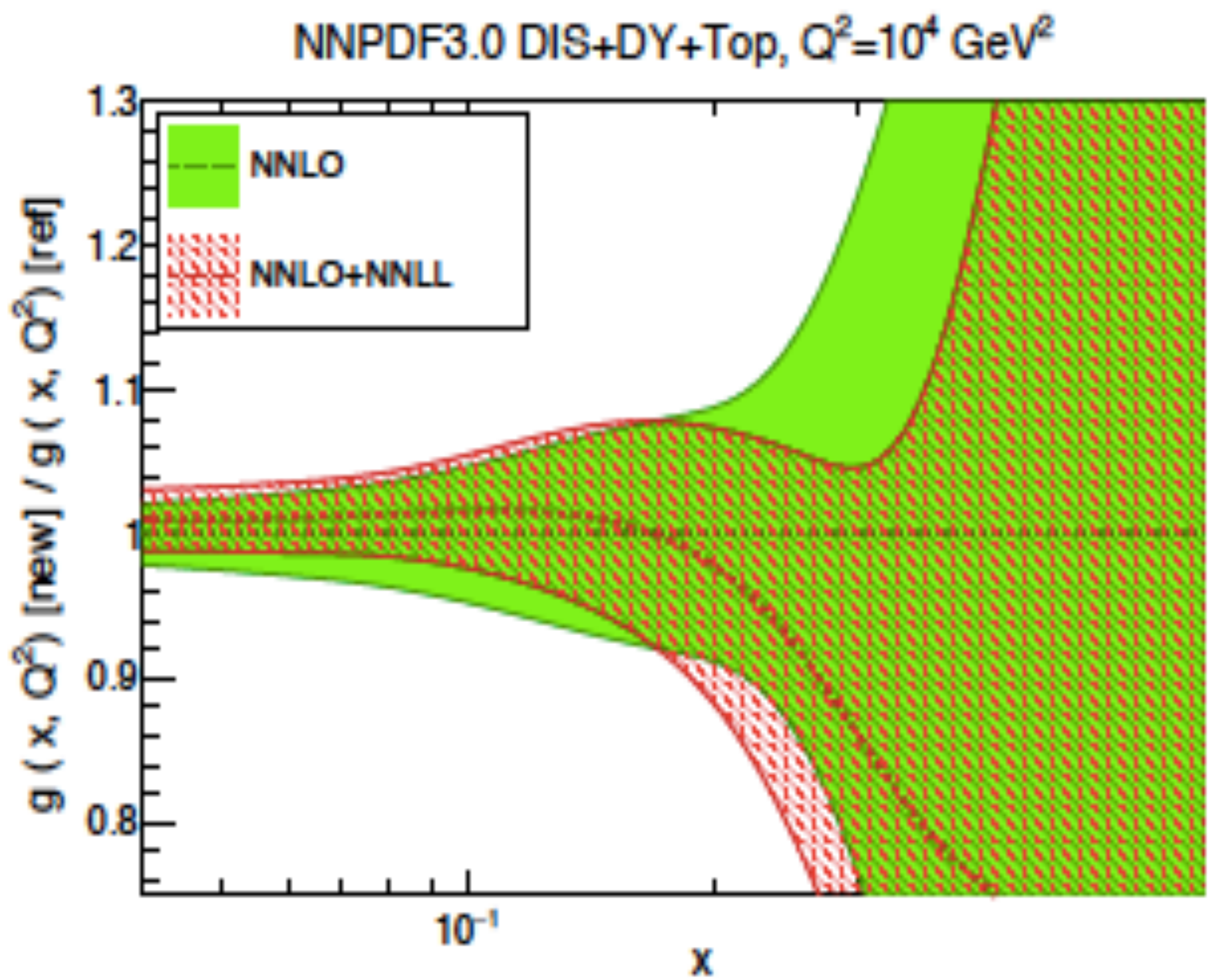




Resummationed PDFs

[Bonvini et al]





- Valence quark are less sensitive to Resumed Coefficient functions
- Large x behaviour of gluons gets modified

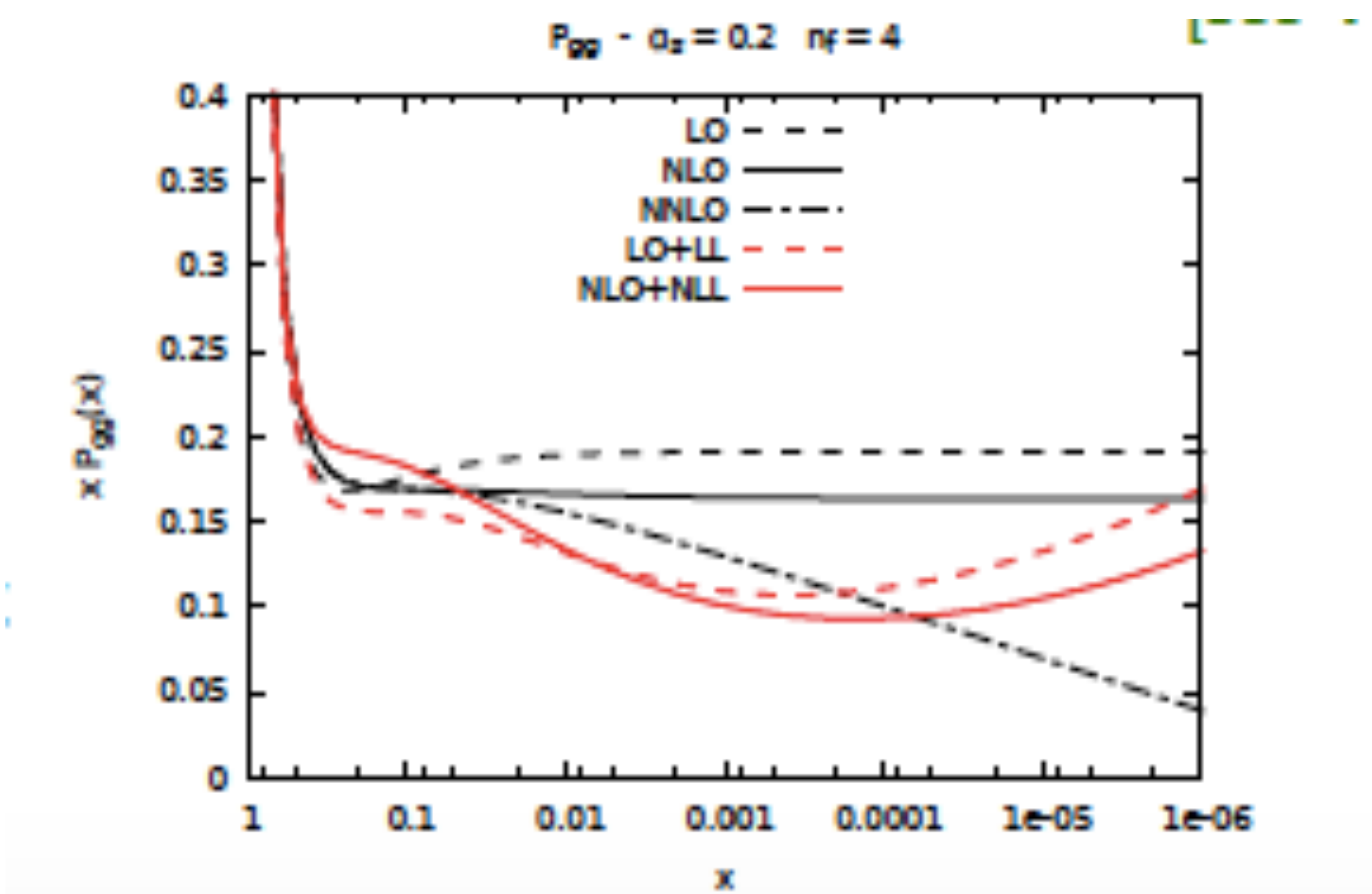
Resummationed PDFs

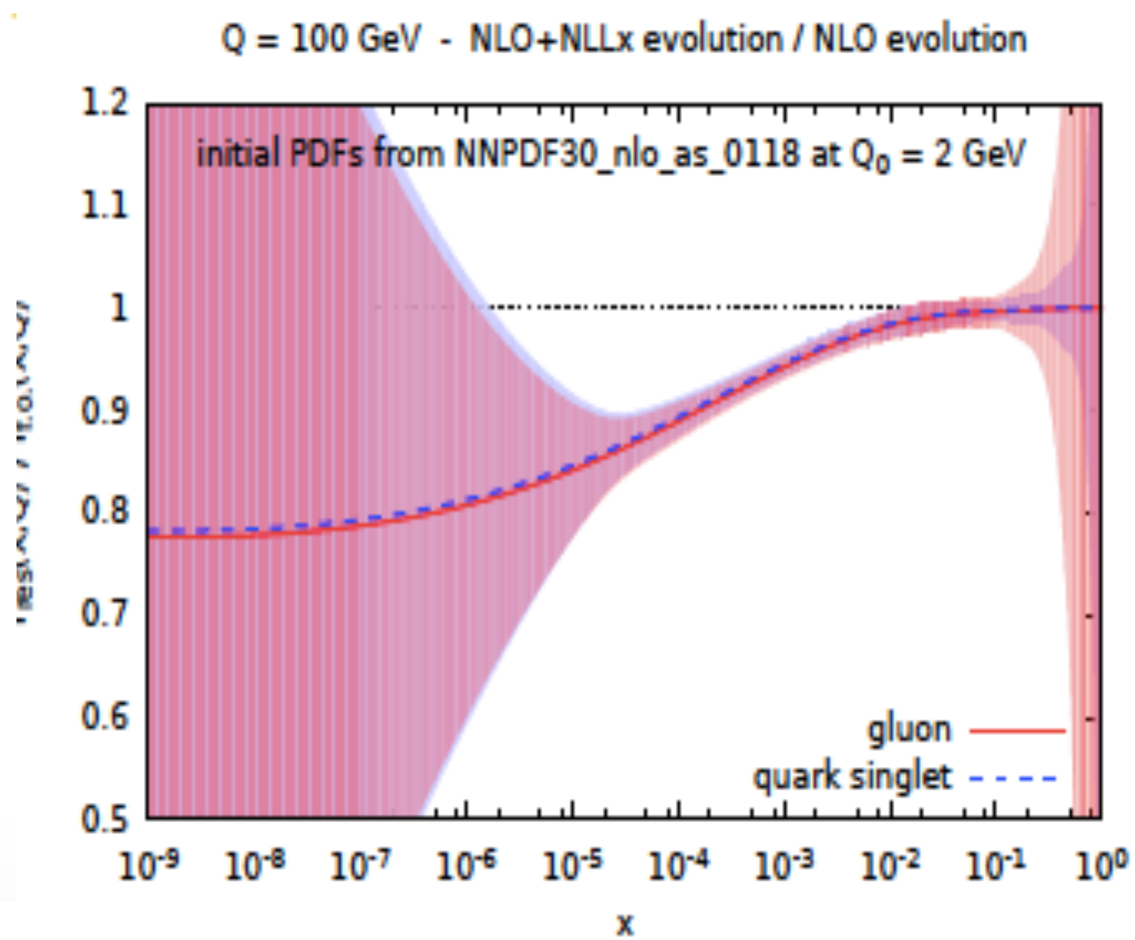
[Bonvini et al]

- Kt or BFKL approach to Small x Resummation
- Altarelli-Ball-Forte procedure to resum small x for both Coefficient and splitting functions

BFKL: $$x\frac{d}{dx}f(x,\mu^2) = \int \frac{d\nu^2}{\nu^2} K\left(x, \frac{\mu^2}{\nu^2}, \alpha_s(\cdot)\right) f(x,\nu^2)$$

HELL (High Energy Large Logarithm) interfaced with APFELL



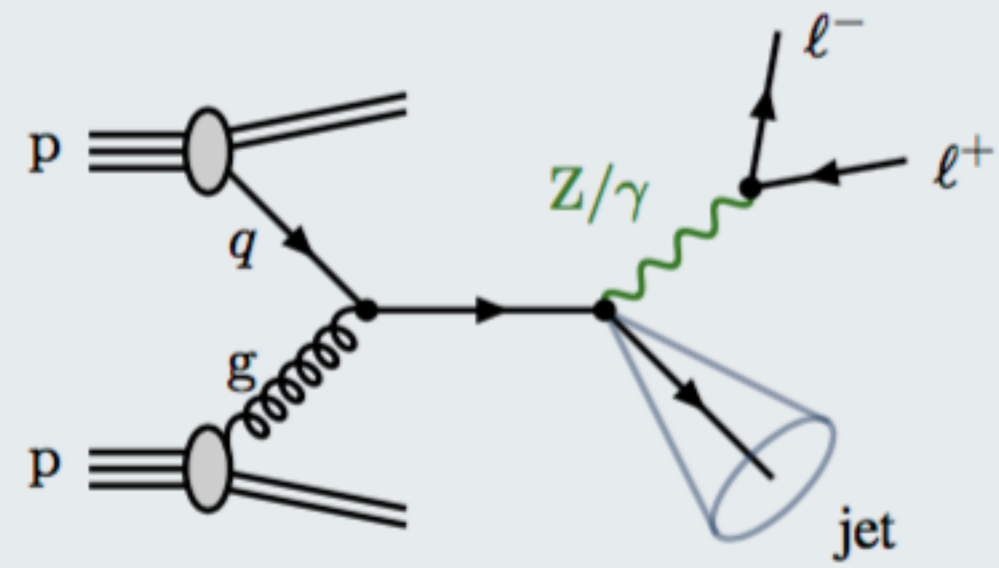


Jets at NNLO for PDF

$$p\,p \rightarrow Z/\gamma^* + \text{jet} \rightarrow \ell^-\ell^+ + \text{jet} + X$$

- large cross section
- clean leptonic signature

$+$jet $\rightsquigarrow$ sensitivity to α_s, gluon PDF,...

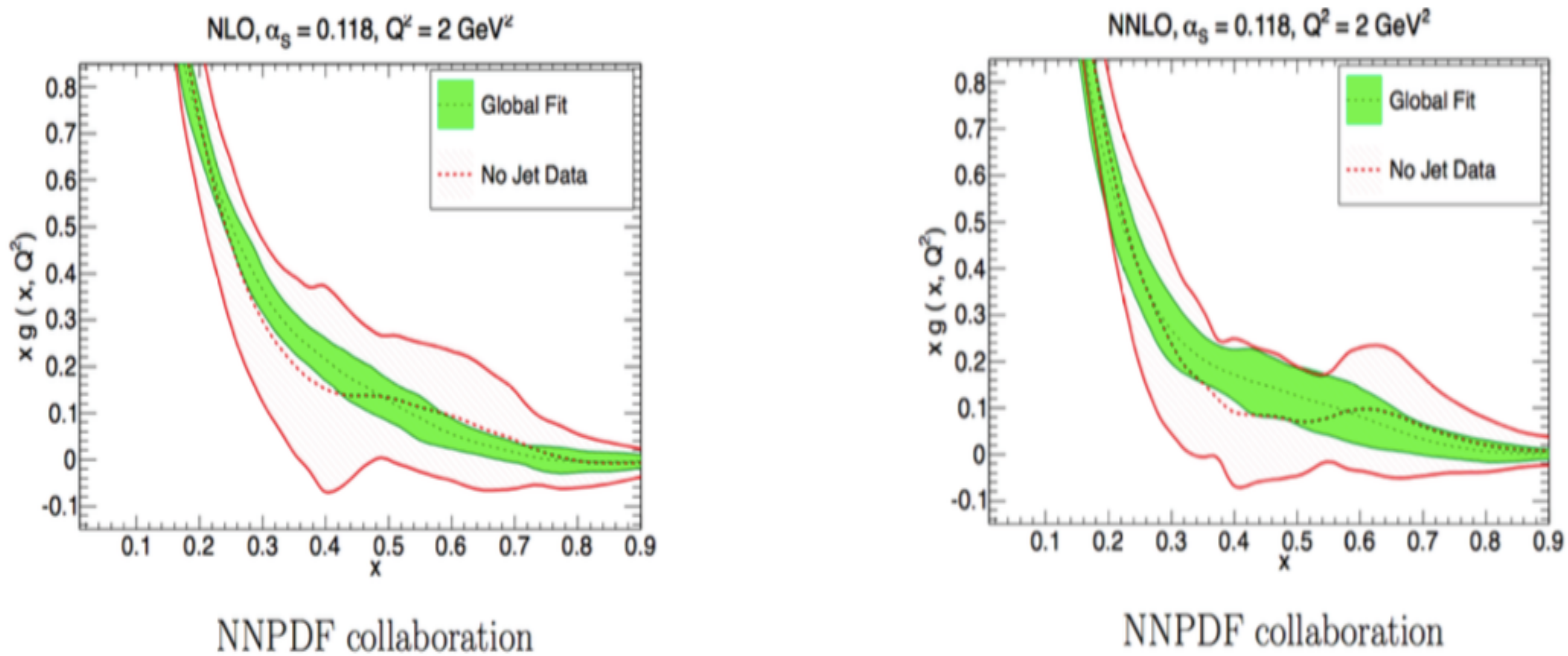


X. Chen, J. Cruz-Martinez, J. Currie, A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, AH, M. Jaquier, T. Morgan, J. Niehues, J. Pires

NLO QCD, Giele, Glover, Kosher 93
NLO EW Denner, Dittmaier, Kasprzik, Muck 11

NNLO QCD, Antenna substraction, ..Gehrmann-De Ridder,Gehrmann, Glover, Huss, Morgan
N-jettiness, Boughezal, campbell, Ellis, Focke, Giele, Liu, petriello,15
Boughezal, Liu, Petriello,16

Jet studies for PDFs



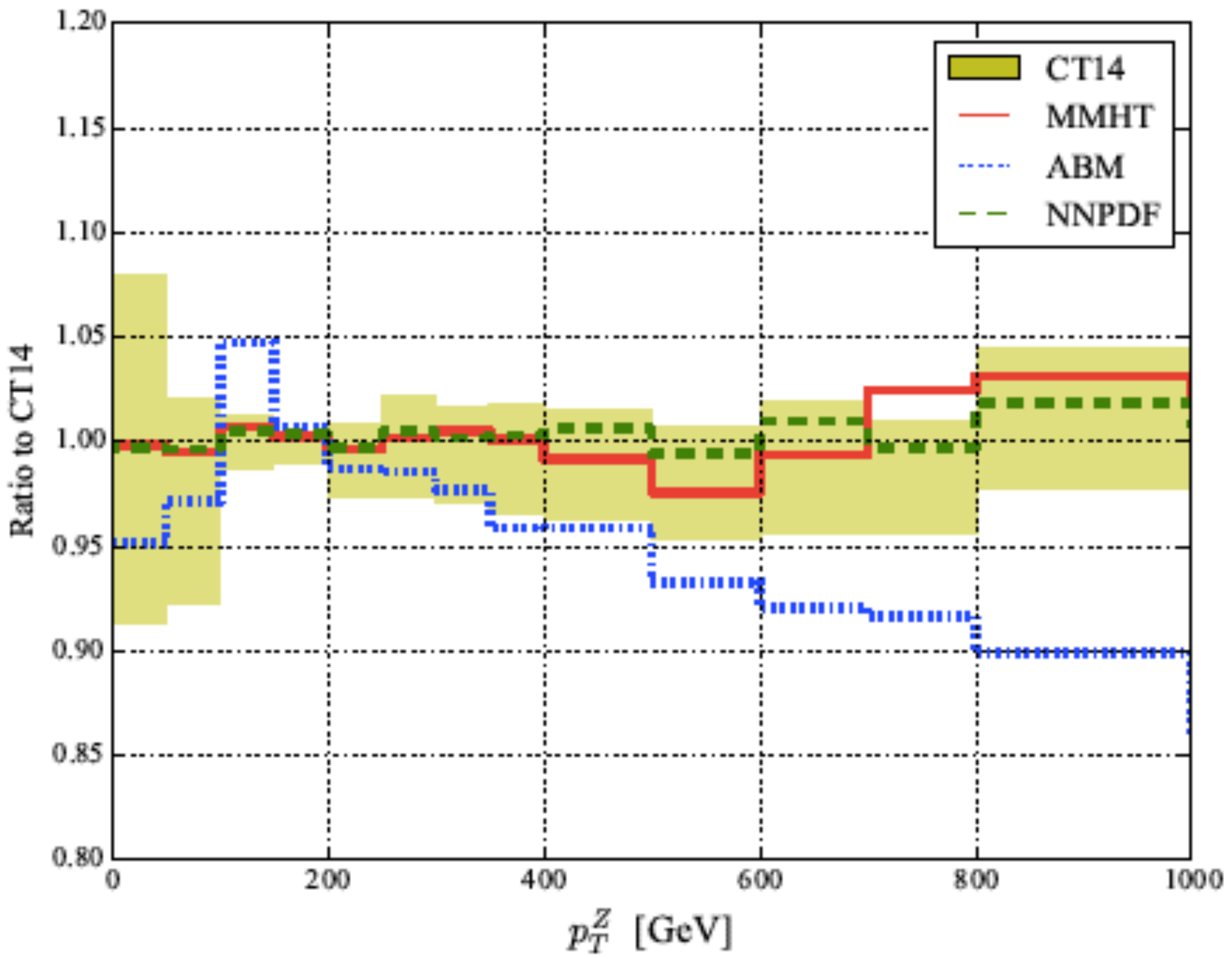
Jet data has a big impact on the medium to large-x gluon PDF

Need exact NNLO all-channel prediction to include full jet dataset

Pt of Z boson in DY for PDFs

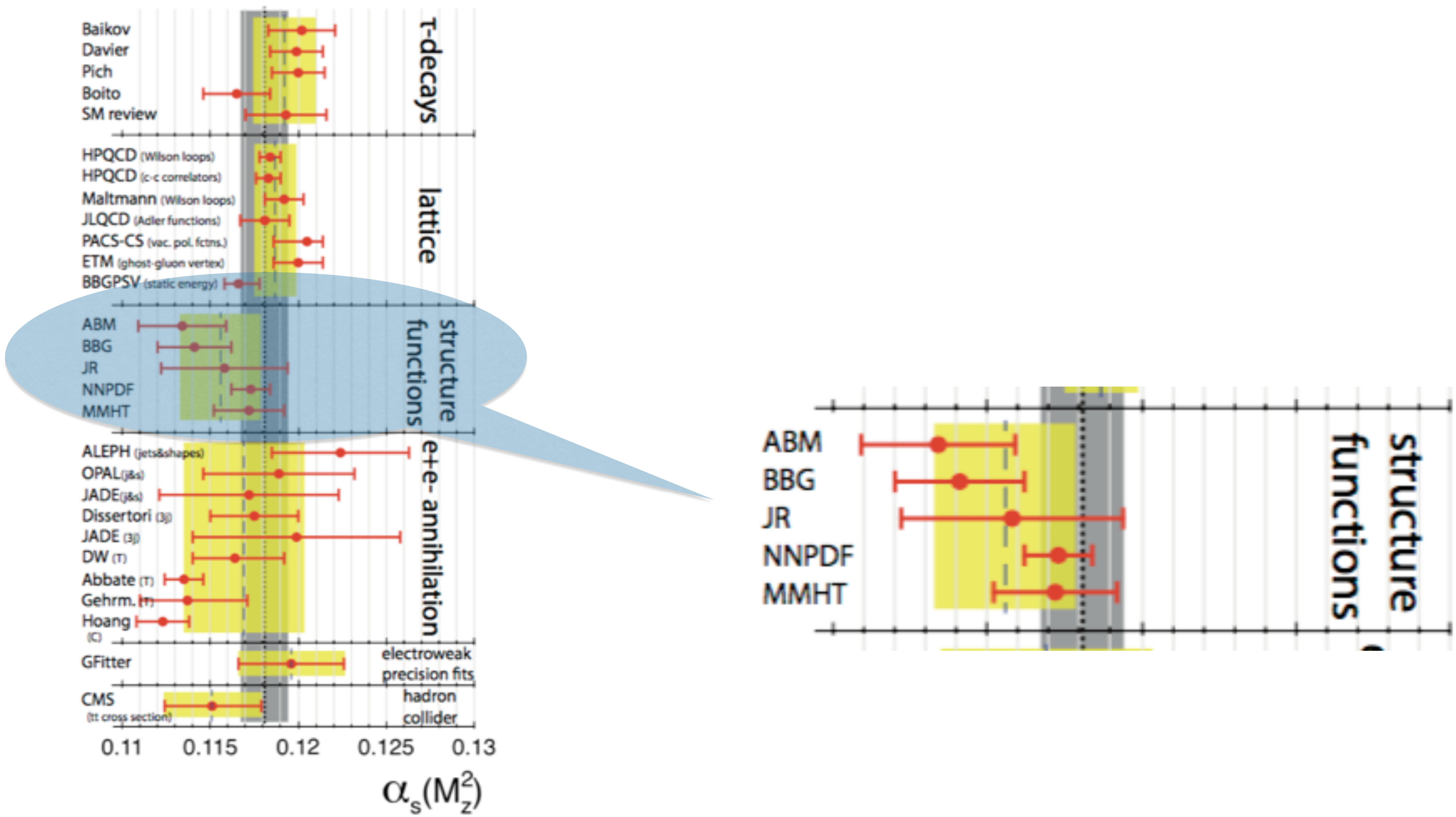
Boughezal, Liu, Petriello 2016

- Z boson transverse momentum depends on high x of the gluon
- Fiducial cross section is sensitive to NNLO
- Cross section is dominated at x 10^-2 which is closer to H production region



CT14	ABM12	MMHT2014	NNPDF3.0
15.54 pb	14.98 pb	15.66 pb	15.44 pb

Strong Coupling from PDFs



Conclusions

- Form Factor
- Deep Inelastic Scattering
  - Bjorken Scaling
  - Naive Parton Model
- QCD improved Parton Model
  - NLO Coefficient
  - DGLAP evolution
- NNLO and Beyond
- Higher twist, Heavy flavours