Introduction to Effective Field Theory

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Georgi, EFT, ARNPP 43(93) 209
(one of my all-time favourite papers)

1. Effective Field Theory
   - *what* is it? (perturbation theory in scale ratios)
   - *how* to implement in QFT (?loops with $p_{loop} \to \infty$)

     to organise the SM/NP calculation, need: $\left\{ \begin{array}{l} \text{basis of } d > 4 \text{ operators,} \\ \text{recipe for changing scale} \end{array} \right.$
   - *why*: two perspectives: $\left\{ \begin{array}{l} \text{top} - \text{down} \\ \text{bottom} - \text{up} \end{array} \right.$
2. precision in EFT
   - (SM interactions...controllable)
   - NP : not know the couplings — what to do?

     $\Leftrightarrow$ (when) are dim 6 operators a good approximation to NP?
3. (and what about the Higgs?)

NP $\equiv$ New Physics , $\hat{s}$ = partonic centre-of-mass energy , **dim** = dimension

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  1. choose *appropriate* variables to describe *relevant* dynamics (*eg* use $\vec{E}, \vec{B}$ and currents for radio waves, electrons and photons at LEP)
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Example : leptogenesis in the early Universe of age τ_U ($\tau_U \sim 10^{-24}$ sec)

⋆ processes with $\tau_{int} \gg \tau_U$ ...neglect!

⋆ processes with $\tau_{int} \ll \tau_U$ ...assume in thermal equilibrium!

⋆ processes with $\tau_{int} \sim \tau_U$ ...calculate this dynamics

⋆ can then do pert. theory in slow interactions and departures from thermal equil.

Pre-implementation of EFT in the SM , and for NP

- take scale to be energy E : GeV $\to \Lambda_{NP}(\gtrsim$ few TeV) (then do pert. theory in $E/M, m/E$ for $m \ll E \ll M$)

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  ... *but resulting EFT depends on details of how (eg* put, or not, $M \gg E$ particles in loops?)

  $\star$ I use dimensional regularisation ; restricts/defines the EFT I construct.

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$\Rightarrow$ like in SM, EFT coupling constants (= operator coefficients) live in $\mathcal{L}$ rather than real world, are *not* observables...

Can parametrise NP@LHC in S-matrix-based approach = "pseudo-observables"/(form factors), more general, less QFT-detail-dependent, more difficult?

EFT for the SM and heavy NP ($\Lambda_{NP} \gg m_W$)

1. choose energy scale E of interest

$\Lambda_{NP} \gtrsim$ few TeV

$m_W \sim m_h \sim m_t$

GeV $\sim m_c, m_b, m_\tau$

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4. perturb in E/M (and m/E): allow $d > 4$ local operators $\Leftrightarrow$ exchange of $M \gg E$ particles

d counts field dims in interaction: $(\overline{\psi}\psi)(\overline{\psi}\psi) \leftrightarrow$ dim 6

$\Lambda_{NP} \gtrsim$ few TeV

$f', \gamma, g, Z, W, h, t$ $\qquad$ $\mathcal{L}_{SM}$ $\qquad$ $+\mathcal{L}(\text{SM invar. operators})$

$m_W \sim m_h \sim m_t$

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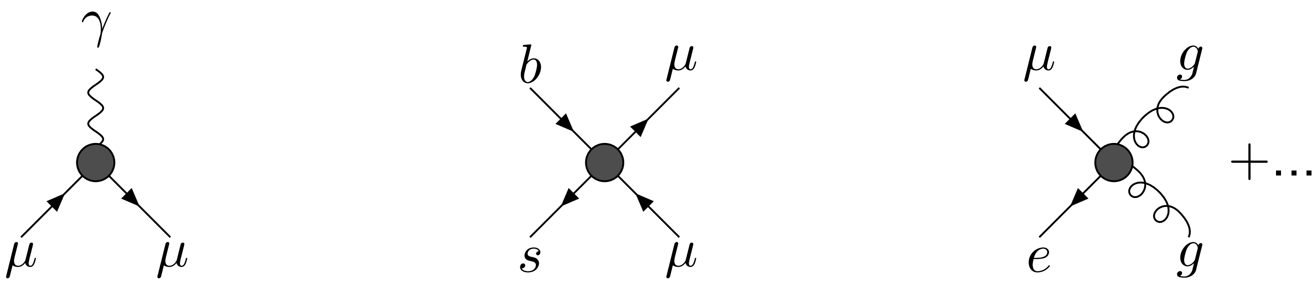
1. $E < m_W$: 3- and 4-point interactions of $f', \gamma, g \Leftrightarrow$ dimension 5,6,7 QCD*QED-invariant operators:



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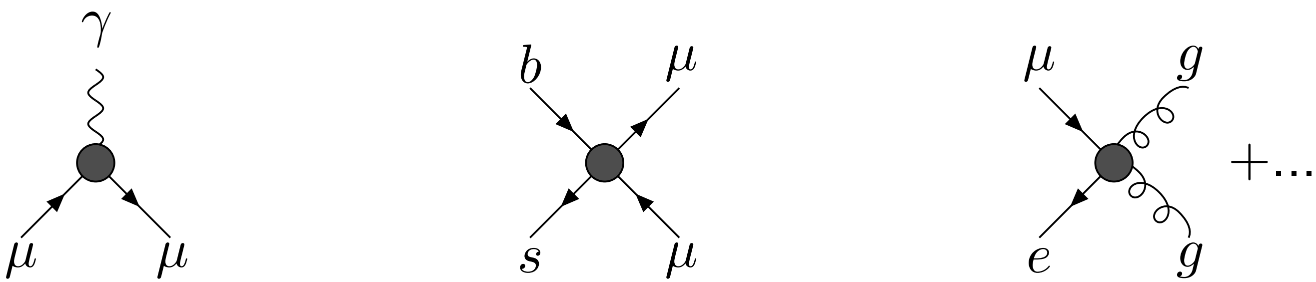
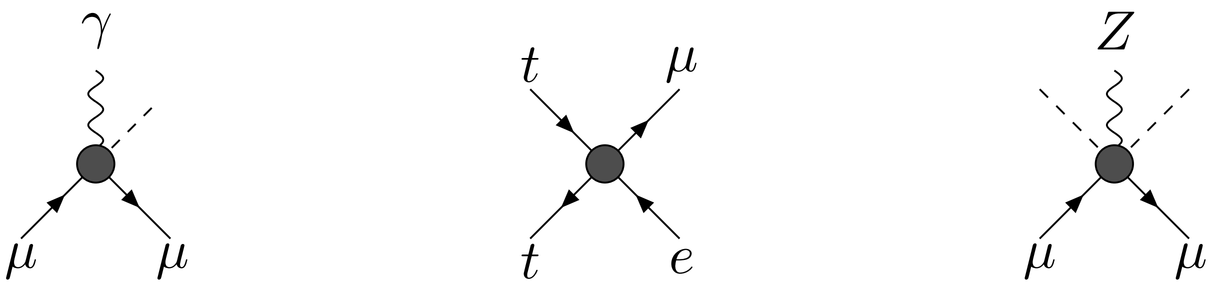


2. $E > m_W$: dim 6 $SU(3) \times SU(2) \times U(1)$-invariar operators

BuchmullerWyler
Grzakowski etal

(neglect Majorana ν mass operators)

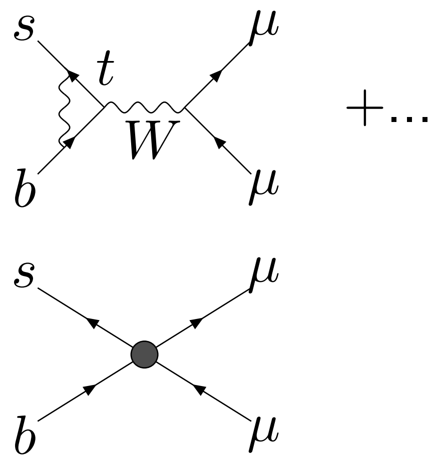


To implement in practise, need operator basis + **recipe to change scale**

need a recipe to relate EFTs at different scales

1. when change EFTs (eg at m_W):
   $match$ (= set equal) Greens functions
   in both EFTs at the matching scale

$$\Rightarrow C_V \sim \frac{V_{ts}}{16\pi^2}$$



$G_F C_V (\bar{s}\gamma b)(\bar{\mu}\gamma\mu)$

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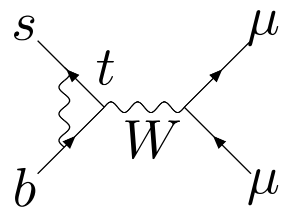

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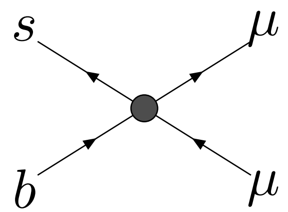

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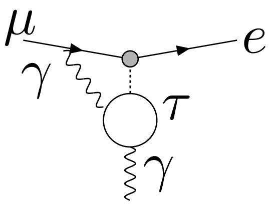
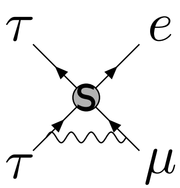
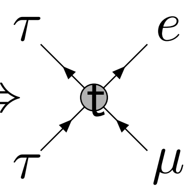
2. Within an EFT: couplings (= operator coefficients) *run* and *mix* with scale. Can mix to other operators, (better?) constrained at other scales

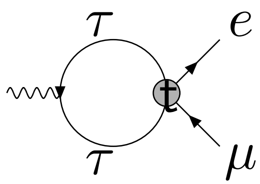




+... $\Rightarrow$



$(\bar{\tau}\sigma\tau)(\bar{e}\sigma\mu)$



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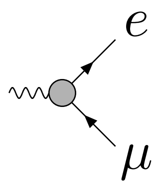

sensitivity of $\mu \to e\gamma$ to scalar $\bar{\tau}\tau\bar{e}\mu$ operator !

(replace $\tau \to t$ if you like)

Why do EFT: top-down vs bottom-up

Two perspectives in EFT:

top-down: EFT as the simple way to get the right answer

know the high-scale theory = can calculate the coefficients of dim > 4 operators (because know cplings ⇔ other perturbative expansions)

recall: EFT is perturbative expansion in scale ratios (eg m_B/m_W)

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bottom-up: EFT as a parametrisation of ignorance
not know NP masses, or couplings = other perturbative expansions

$\Rightarrow$ use lowest order EFT expansion (in scale ratio m_{SM}/Λ_{NP}) to parametrise ... (?we hope??) many models

$\Rightarrow$ how well does bottom-up EFT work?

How precise is bottom-up EFT?

(top-down: just do perturbative expansion to sufficient order...)

1. How precisely are the SM dynamics included?
   (non-trivial problem: perturb in loops$+$ Yukawa$+$ gauge cplings $y_t^2/16\pi^2 \sim y_c^2$.
   In addition, matching at m_W delicate due to appearance of Higgs vev which changes operator dimensions)

2. How good a parametrisation of New Physics, is lowest order EFT (dim 6 operators)?

First: what parameter space of dimension six operators can be probed?

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($\epsilon = 10^{-2}$ in the plot)

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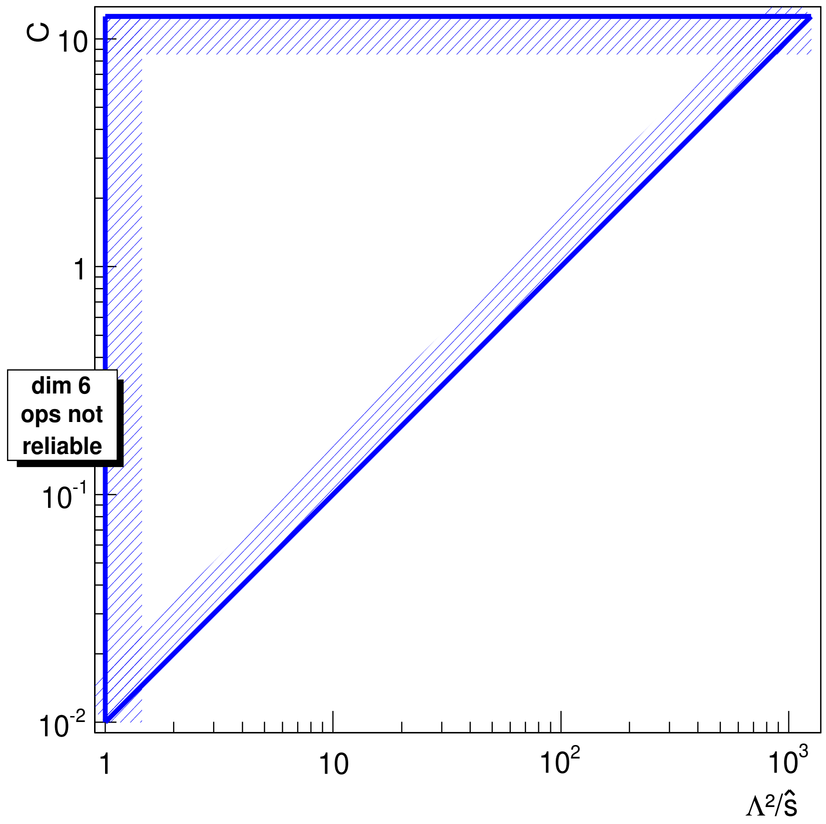

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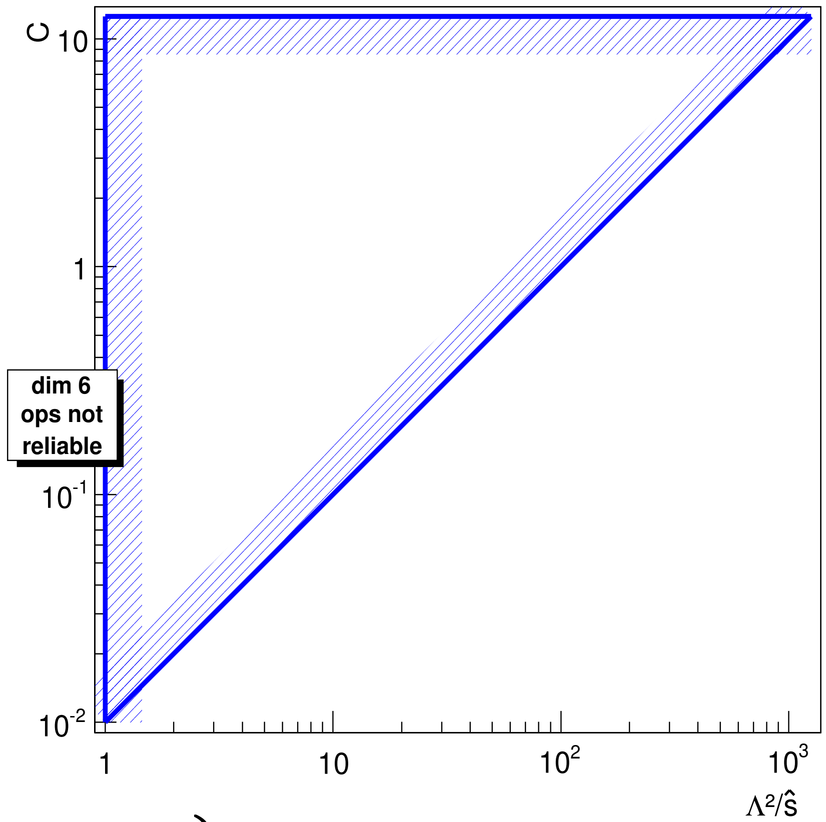
Ex: $BR(h \to \tau^+\mu^-) \sim .01$, induced by $\frac{C}{\Lambda_{NP}^2} H^\dagger H \overline{\ell_\mu} H \tau_R$:

$$\sqrt{BR} y_b < C\frac{m_h^2}{\Lambda_{NP}^2} < 4\pi$$



...can probe $\left\{ \begin{array}{c} C \gtrsim 1 \\ C \gtrsim 0.1 \end{array} \right\}$ for $\left\{ \begin{array}{c} \Lambda_{NP} \gtrsim 10 m_h \\ \Lambda_{NP} \gtrsim 3 m_h \end{array} \right\}$.

If a model induces dim-6 ops in that triangle,
are they a good approx to the model?

? maybe ? I think no answer in EFT — depends on model

EFT is a perturbative expansion in scale ratios ($eg\ \hat{s}/\Lambda^2_{NP}$)
...but measure $C_6\frac{\hat{s}}{\Lambda^2_{NP}}$, C_6 unknown (model-dep)
$\Rightarrow$ size of $C_8\frac{\hat{s}^2}{\Lambda^4_{NP}}$model-dependent too ??

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to get an idea if dim 6 ops are a good approximation:

1. Consider the formula for your favourite observable in your favourite model
2. expand in $\frac{1}{\Lambda^2_{NP}}$
3. check if the $\mathcal{O}(\frac{1}{\Lambda^2_{NP}})$ terms are a good approximation?

Repeat many times.

Are lowest order operators a good approximation? (examples)

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   m_h^2/m_t^2 is not small...
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2. $h \to \tau^+\mu^-$ and $\tau \to \mu\gamma$ in the 2HDM with LFV, decoupling limit. arXiv:1601.01949

   Matching 2HDM to dim-6 operators works fine for Higgs decay. But for $\tau \to \mu\gamma$:

$$\frac{\text{dim } 8}{\text{dim } 6} \sim \quad \tan\beta \frac{v^2}{\Lambda_{NP}^2} \quad , \quad \frac{v^2}{\Lambda_{NP}^2} \ln^2\left(\frac{v^2}{\Lambda_{NP}^2}\right)$$

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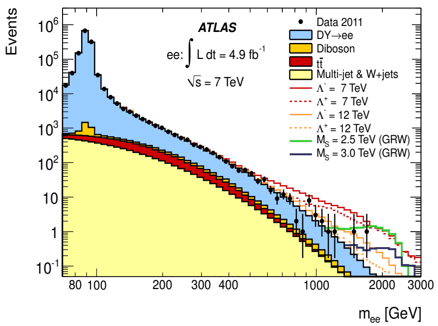
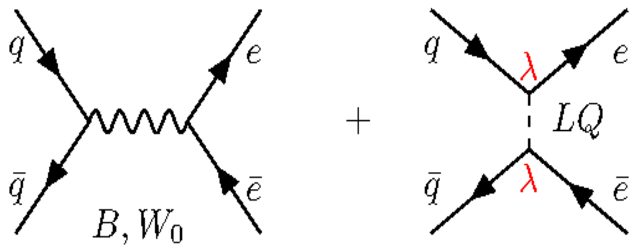
3. high-$\hat{s}$ tail $pp \to \ell^+\ell^-$, mediated by a t-channel leptoquark with $m^2 \gtrsim \hat{s}_{max}$





   dim-6 contact interaction=poor approximation (expected, $\hat{s} \approx \Lambda^2$ side of triangle)
   $\Rightarrow$ more info in distribution tails than in published contact interaction bounds :(

On the interest of many searches for New Physics

- observables often depend on linear combinations of operators coefficents
- coefficients run and mix with scale

$\Rightarrow$ need diverse observations to independently $\left\{ \begin{array}{c} \text{constrain all} \\ \text{determine non}-\text{zero} \end{array} \right\}$ coefficients

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At Λ_{NP}: $\mathcal{L}_{SM} \quad + \quad \frac{C_h}{\Lambda^2_{NP}} H^\dagger H \overline{\ell_\mu} H e \qquad\qquad + \quad \frac{C_{meg}}{\Lambda^2_{NP}} \overline{\ell_\mu} H \sigma \cdot F e$

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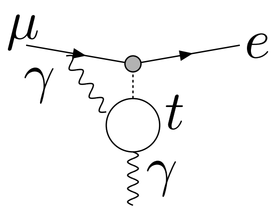
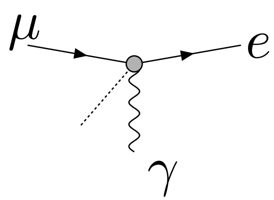
At m_h: h decays to $\mu^\pm e^\mp$; current LHC sensitivity $\sim \frac{C_h v^2}{\Lambda_{NP}^2} \gtrsim 10^{-3}$.

At m_μ:



$+$



$$BR(\mu \to e\gamma) \Rightarrow \left| \frac{e\alpha}{8\pi^3 Y_\mu} C_h + C_{meg} \right| \lesssim 10^{-8} \frac{\Lambda^2}{v^2} \quad , \quad \frac{e\alpha}{8\pi^3 Y_\mu} \sim 10^{-2}$$

$\mu \to e\gamma$ sensitive to $C_h v^2/\Lambda^2 \gtrsim 10^{-6}$...but if you admit cancellation up to one part per mil($\sim \alpha/(4\pi)$log) between C_h and C_{meg}, LHC can see $h \to \mu^\pm e^\mp$ soon.
$h \to \mu e$ at LHC *independent* constraint from $\mu \to e\gamma$:)

Summary

EFT is the way we do physics:

1. chose a scale E and relevant variables
2. perturb in scale ratios, eg E/M for $M \gg E$

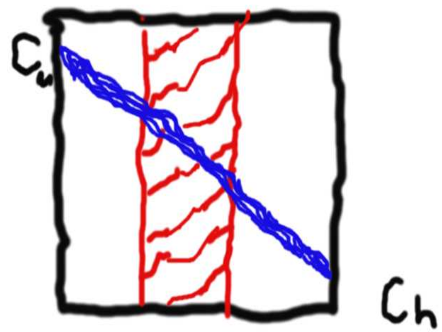
   works for β-decay, quark flavour physics, etc

At the LHC, use lowest order EFT (dimension 6 SM-gauge-invariant operators) from a "bottom-up" perspective, as a parametrisation of New Physics

If you know the high-scale theory (top-down perpective), the EFT expansion in scale ratios is a simple way to get the answer to the desired accuracy = precision can be estimated
(just work to required order in all expansions)

precision harder to quantify "bottom-up": to determine whether dim 6 operators (within the blue triangle) are a good approx to NP at the LHC: does EFT reproduce your favourite model? (if not, explore your favourite model differently—simplified models, form factors, pseudo-observables etc)

there are many operator coefficients,
want to know them all⇔ (almost)
every independent measurement is interesting :)



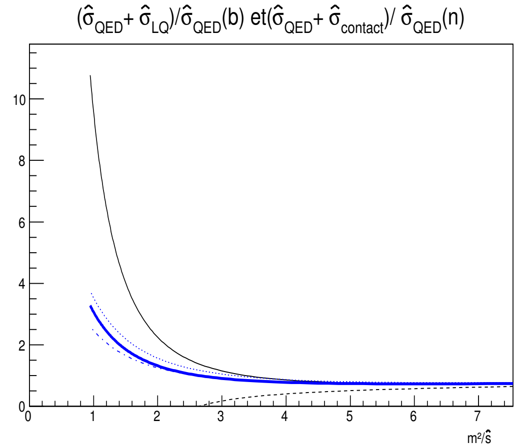
Backup

Looking for NP in the tails of distributions: leptoquarks and $pp \to \ell^+\ell^-$

1307.5068 = etal+ Santiago
1409.2772, 1410.4798

At 8 TeV LHC:
1. $m_{LQ} \gtrsim 800$ GeV bound on 1st generation LQ ($eg\ \overline{u_c}S_o e$), from absence of strong pair production.
2. Contact int. search in $pp \to e^+e^-$, with $\sqrt{\hat{s}_{max}} \lesssim 2$ TeV: $\Lambda_{CI} \gtrsim 10-20$ TeV.
Can this bound be applied to t-channel leptoquark exchange?

Not reliably for 800 GeV $< m_{LQ} < 4$ TeV !
Contact Int. poor approx to LQ exchange:
$\star$ $\mathcal{A}_{SM} \sim \mathcal{A}_{CI} \Rightarrow$ include $SM * CI + CI^2$
$\star$ t-channel: $\frac{1}{p^2-m^2_{LQ}} \sim -\frac{1}{\hat{s}+m^2_{LQ}}$



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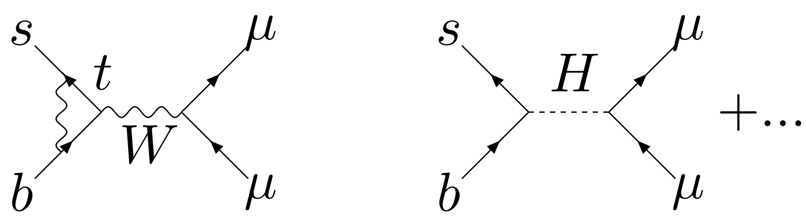
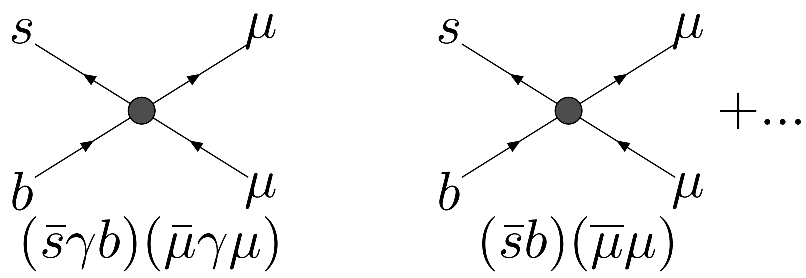
distribution tails among the places where fitting to form factors/simplfied models/pseudo-observables = a better summary of the data ?

Example 2: "top-down" matching in $B_s \to \bar{\mu}\mu$

At $E > m_W$, in SM, (or beyond) :



At $E \sim m_{B_s}$, replace
$M \gg m_{B_s}$ particles by contact int:



$E \simeq m_B \sim 5$ GeV, 0^{th} order theory $= QCD \times QED$ for $\{q^i, \ell^i, \nu^i, \gamma, g\}$
include effects of W, Z, h, t as four-fermion operators:

$$\mathcal{L}(m_B) \supset \bar{b}\, \not{D}\, b + ... - 2\sqrt{2}G_F\Big\{C_V(\bar{s}\gamma b)(\bar{\mu}\gamma\mu) + C_S(\bar{s}b)(\bar{\mu}\mu) + ... + h.c.\Big\}$$

Determine $C_V \sim \frac{V_{ts}}{16\pi^2}$, $C_S \sim \frac{\lambda_{bs}\lambda^*_{\mu\mu}v^2}{m_H^2}$ by equating Greens functions in both theories
(at scale m_W, same QCD/QED loop order in both theories)