Higher-order EW/QCD corrections for W/Z production in hadron and e^+e^- colliders

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25 September - 1 October 2016

Precision theory for precise measurements at LHC and future colliders

ICISE, Quy-Nhon, Vietnam

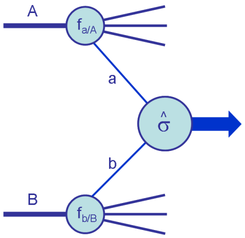
Two strategies of looking for New Physics

- bump hunting, if hypothetical new particles are in the investigated energy domain
  - analysis data driven
- if new BSM threshold is higher than the available energy
  - look for deviations from SM predictions in the tails of distributions
  - measure the SM couplings and parameters with the highest possible precision in order to discover internal inconsistencies
  - **both above cases require the most possible precision in theoretical predictions**

From SM Lagrangian to collider phenomenology

$$\sigma^{\text{exp}} \equiv \frac{1}{\int \mathcal{L} dt} \frac{N^{obs}}{A \, \epsilon} = \sigma^{\text{theory}}$$

$$\sigma^{\text{theory}} = \sum_{a,b} \int_0^1 dx_1 dx_2 f_{a,H_1}(x_1, \mu_F^2, \mu_R^2) f_{b,H_2}(x_2, \mu_F^2, \mu_R^2) \times$$
$$\times \int_\Phi d\hat{\sigma}_{a,b}(x_1, x_2, Q^2/\mu_F^2, Q^2/\mu_R^2) + \mathcal{O}\left(\frac{\Lambda^n_{QCD}}{Q^n}\right)$$



Campbell, Huston, Stirling, hep-ph/0611148

- PDF's fitted from data

  see PDF session

- $\hat{\sigma}$ calculated perturbatively

$$\sigma = \sigma_0(1 + \alpha_s \delta_1^{\text{QCD}} + \alpha_s^2 \delta_2^{\text{QCD}} + \alpha \delta_1^{\text{EWK}} + \ldots)$$

Higher order SM corrections

- **a powerful per cent level comparison between theoretical predictions and measurements requires the inclusion of perturbative higher order corrections**
- in particular, for observables inclusive on additional radiation, fixed order calculations are reliable
- for $2 \to 1$ and $2 \to 2$ scattering processes the QCD NNLO corrections have been recently calculated, fully exclusive on final state leptons

  see talk by G. Zanderighi

  - C.C. and N.C. Drell Yan

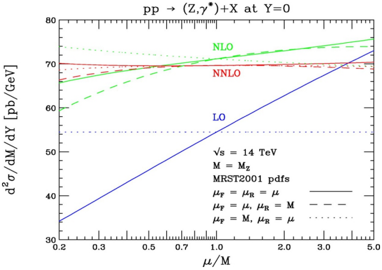
    Melnikov, Petriello, Phys. Rev. Lett. 96 (2006) 231803; Phys.Rev. D74 (2006) 114017; Li, Petriello, Phys.Rev. D86 (2012) 094034; Gavin, Li, Petriello, Quackenbush, Comput. Phys. Commun. 184 (2013) 208; Catani, Cieri, de Florian, Grazzini, Phys. Rev. Lett, 103 (2009) 082001;
  - $pp \to VV'$, $V, V' = Z, W, \gamma$

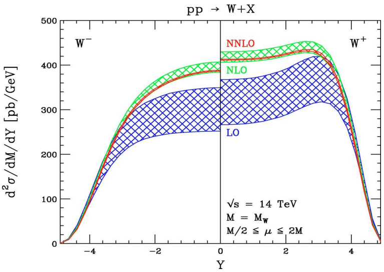
    Grazzini et al., Phys. Rev. Lett. 108 (2012) 072001; err. Phys. Rev. Lett. 117 (2016) 089901; Phys. Lett. B731 (2014) 204; JHEP 1507 (2015) 085; Phys. Lett. B735 (2014) 311; Phys. Rev. Lett. 113 (2014) 212001; Phys. Lett. B761 (2016) 179; JHEP 1608 (2016) 140
  - Wj, Zj production

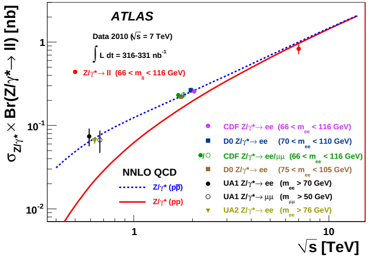
    Boughezal, Liu, Petriello, Phys. Rev. Lett. 115 (2015) 062002; Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello, Phys. Rev. Lett. 116 (2016) 152001

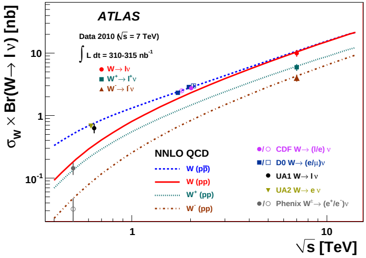
fully differential NNLO QCD corrections to DY

DYNNLO, FEWZ







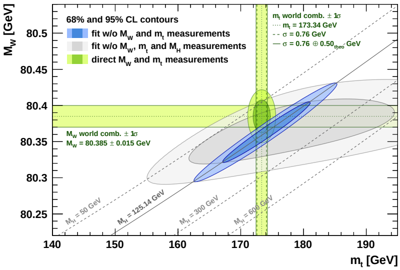


also electroweak corrections enter the game, in two ways

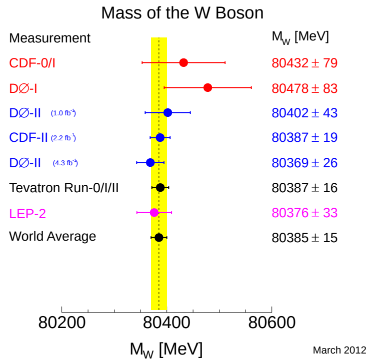
- ($\alpha_{e.m.} \sim \alpha_s^2 \Longrightarrow$ NLO EWK $\sim$ NNLO QCD)
  - usually largest effects from QED radiation from external legs $\sim \alpha \log\left(\frac{Q^2}{m^2}\right)$
  - EWK effects particularly relevant for observables (partially) insensitive to QCD corrections, e.g.
    - ★ transverse mass in the charged DY process
  - NLO EWK radiative corrections to $2 \to 2$, $2 \to 3$ and few $2 \to 4$ processes are already known
- LHC run2 is exploring (with enough statistics) regions of phase space with scales $Q^2 >> M_W^2$

see later

M_W direct measurement: crucial for a SM stress-test



Gfitter, EPJC 74 (2014) 3046

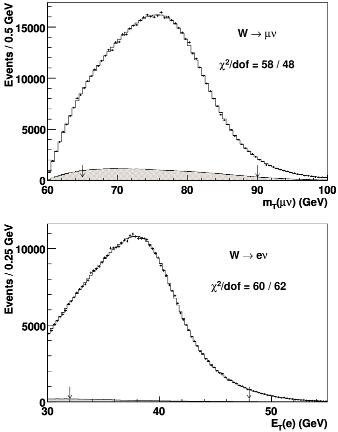


TeVatron EWWG, arXiv:1204.0042

- A precise ($\delta M_W < 10$ MeV) M_W measurement at LHC Run2 and beyond will be an important goal of the LHC precision physics pogramme

M_W measurement: relevant observables

- M_W from the $p_\perp^\ell$ distribution, showing a (Jacobian) peak at $M_W/2$
- more reliable is $M_T^W = \sqrt{2 p_\perp^\ell p_\perp^\nu (1 - \cos\phi_{\ell\nu})}$ (**mildly sensitive to QCD RC**)



2.2/fb, CDF, PRL 108 (2012) 151803

- M_W is extracted with a template fit technique to M_T and/or $p_\perp^\ell$ distributions

⋆ EW corrections (mainly QED FSR) can distort the shape → the extracted M_W is affected

⋆ with high lumi M_T can be experimentally challenging (uncertainties in E_{miss}^T from pile up)

Uncertainty sources breakdown from Tevatron

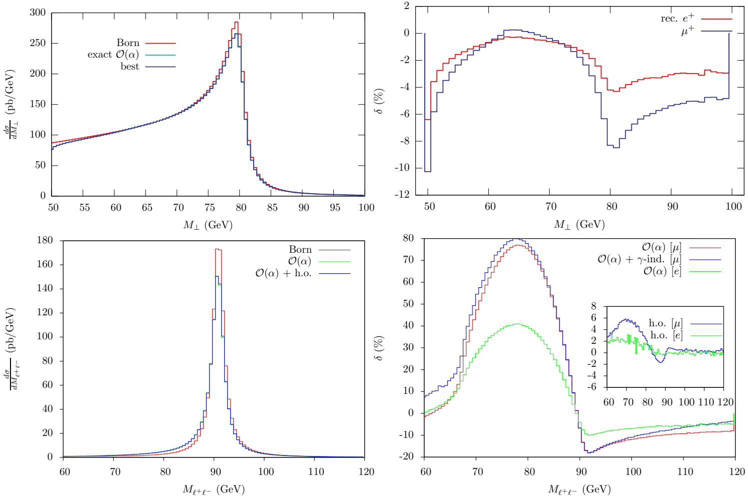
p_T^ℓ fit uncertainties			
Source	$W \to \mu\nu$	$W \to e\nu$	Common
Lepton energy scale	7	10	5
Lepton energy resolution	1	4	0
Lepton efficiency	1	2	0
Lepton tower removal	0	0	0
Recoil scale	6	6	6
Recoil resolution	5	5	5
Backgrounds	5	3	0
PDFs	9	9	9
W boson p_T	9	9	9
Photon radiation	4	4	4
Statistical	18	21	0
Total	25	28	16

CDF, arXiv:1311.0894

Source	Section	m_T	p_T^e	$\not\!\!E_T$
Experimental				
Electron Energy Scale	VII C 4	16	17	16
Electron Energy Resolution	VII C 5	2	2	3
Electron Shower Model	V C	4	6	7
Electron Energy Loss	V D	4	4	4
Recoil Model	VII D 3	5	6	14
Electron Efficiencies	VII B 10	1	3	5
Backgrounds	VIII	2	2	2
$\sum$(Experimental)		18	20	24
W Production and Decay Model				
PDF	VI C	11	11	14
QED	VI B	7	7	9
Boson p_T	VI A	2	5	2
$\sum$(Model)		13	14	17
Systematic Uncertainty (Experimental and Model)		22	24	29
W Boson Statistics	IX	13	14	15
Total Uncertainty		26	28	33

D0, arXiv:1310.8628

Effects of EW corrections on W and Z production



- EW $\mathcal{O}(\alpha)$ change the shape $\rightarrow$ $\delta M_W \simeq 100$ MeV
- effects dominated by QED FSR (strongly dependent on event selection)

Carloni Calame et al., PRD 69 (2004) 037301, JHEP 0710 (2007)

Higher-order corrections

$$
\begin{aligned}
d\sigma &= d\sigma_0 \\
&+ d\sigma_{\alpha_s} + d\sigma_{\alpha} \\
&+ d\sigma_{\alpha_s^2} + d\sigma_{\alpha\alpha_s} + d\sigma_{\alpha^2} + \ldots
\end{aligned}
$$

- multi-photon emission from the final state $\rightarrow \delta M_W \simeq 10$ MeV for $\mu\nu_\mu$ final state

Carloni Calame et al., PRD 69 (2004) 037301, JHEP 0710 (2007)

- mixed QCD-EWK corrections
- NNLO EWK effects
  - EWK input scheme
  - lepton pair emission

QCD-EWK interference

- the $\mathcal{O}(\alpha\alpha_s)$ calculation involves as building blocks
  - NNLO virtual corrections at $\mathcal{O}(\alpha\alpha_s)$ (not yet available)
    - ⋆ necessary two-loop master integrals (with $m = 0$ external particles and $M_W = M_Z$) just appeared

      R. Bonciani et al., arXiv:1604.08581, see talk by S. Di Vita
  - NLO EW corrections to $l\bar{l}^{(\prime)}+$ jet
  - NLO QCD corrections to $l\bar{l}^{(\prime)} + \gamma$
  - double real contributions $l\bar{l}^{(\prime)} + \gamma +$ jet
  - PDF's with NNLO accuracy at $\mathcal{O}(\alpha\alpha_s)$ (not yet available)
    - ⋆ very recent calculation of NLO mixed QCD-QED corrections to the Altarelli-Parisi evolution kernels

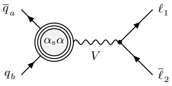
      De Florian, Sborlini and Rodrigo, arXiv:1512.00612
- what is available:
  - fixed order dominant $\mathcal{O}(\alpha_s\alpha)$ corrections to DY in pole approximation

    Dittmaier, Huss, Schwinn, NPB 885 (2014) 318, NPB 904 (2016) 216
  - Monte Carlo estimates through NLO QCD $\otimes$ NLO EW (with higher orders)

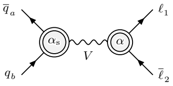
    L. Barzè et al., JHEP 1204 (2012) 037, Eur. Phys. J. C73 (2013) 2474

fixed order $\mathcal{O}(\alpha_s\alpha)$ in pole approximation

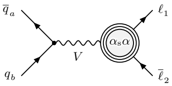
- two main classes of contributions:
  - factorizable
  - non-factorizable



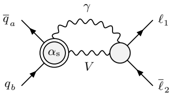
(a) Factorizable initial–initial corrections



(b) Factorizable initial–final corrections



(c) Factorizable final–final corrections



(d) Non-factorizable corrections

S. Dittmaier, A. Huss and C. Schwinn, arXiv:1601.02027

a) not known but expected to be very small

($\mathcal{O}(\alpha)$ corrections in PA $\Longrightarrow$ $M_\perp$ and $M(l^+l^-)$ insensitive to QED ISR
in addition $M_\perp$ and $M(l^+l^-)$ mildly affected by NLO QCD corrections)

b) this gives the bulk of the contribution

c) no real contributions $\Longrightarrow$ no impact on shape of $M_\perp$ and $M(l^+l^-)$

d) numerical impact below 0.1%

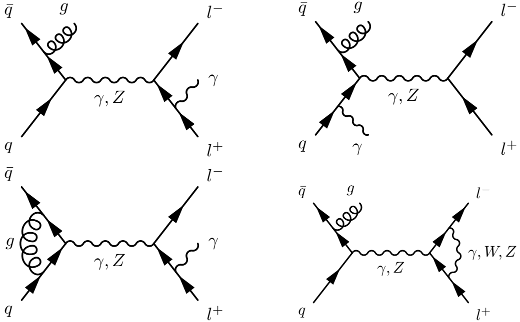
effects on M_W

	bare muons		dressed leptons	
	$M_{\mathrm{W}}^{\mathrm{fit}}$ [GeV]	ΔM_{W}	$M_{\mathrm{W}}^{\mathrm{fit}}$ [GeV]	ΔM_{W}
LO	80.385	} − 90 MeV	80.385	} − 40 MeV
$\mathrm{NLO_{ew}}$	80.295		80.345	
$\mathrm{NLO_{s\oplus ew}}$	80.374	} − 14 MeV	80.417	} − 4 MeV
NNLO	80.360		80.413	

Dittmaier, Huss, Schwinn, NPB 904 (2016) 216

$\mathcal{O}(\alpha_s\alpha)$ corrections through Monte Carlo

- The POWHEG-BOX includes NLO QCD & EW corrections interfaced to QCD/QED shower, i.e. NLOPS EW $\oplus$ QCD accuracy
  1. POWHEG_W_ew_BMNNP, CC DY
     Barzè et al, JHEP 1204 (2012) 037
  2. POWHEG_W_ew_BW, CC DY
     Bernaciak and Wackeroth, PRD 85 (2012) 093003
  3. POWHEG_Z_ew_BMNNPV, NC DY
     Barzè et al, EPJC 73 (2013) 6, 2474
- correctly taken into account the NLO contribution with one additional radiation in the soft/collinear limit



with the MC we can

C.M. Carloni Calame et al., in progress

- compare the shifts induced by $\mathcal{O}(\alpha_s\alpha)$ terms of the pole approximation with the ones induced by the factorized prescription through Monte Carlo (POWHEG with PYTHIA/PHOTOS)
  - using PHOTOS with only one emissions on top of NLO QCD, the difference with the complete $\mathcal{O}(\alpha_s\alpha)$ in pole approximation is of 3 MeV out of 14 MeV
  - above result obtained without a cut on p_T^W, which is used in the experimental analysis

Dittmaier, Huss, Schwinn, NPB 904 (2016) 216

- quantify the uncertainty of the practical solution used in the experimental analysis of QCD NLOPS⊗QEDLL
  - at Tevatron: ResBos+PHOTOS
  - at LHC, in present preliminary investigations:
    - ⋆ ATLAS: POWHEG+PYTHIA&PHOTOS
    - ⋆ CMS: POWHEG+PYHTIA

Results

	Templates: NLO-QCD+$\mathrm{QCD_{PS}}$		M_W shifts (MeV)			
			$W^+ \to \mu^+\nu$		$W^+ \to e^+\nu$(dres)	
	Pseudodata accuracy	QED FSR	M_T	p_T^ℓ	M_T	p_T^ℓ
1	NLO-QCD+(QCD+QED)$_{\mathrm{PS}}$	PYTHIA	-95.2 ±0.6	-400 ±3	-38.0 ±0.6	-149 ±2
2	NLO-QCD+(QCD+QED)$_{\mathrm{PS}}$	PHOTOS	-88.0 ±0.6	-368 ±2	-38.4 ±0.6	-150 ±3
3	NLO-(QCD+EW)+(QCD+QED)$_{\mathrm{PS}}$	PYTHIA	-101.8 ±0.4	-423 ±2	-45.0 ±0.6	-179 ±2
4	NLO-(QCD+EW)+(QCD+QED)$_{\mathrm{PS}}$	PHOTOS	-94.2 ±0.6	-392 ±2	-45.2 ±0.6	-181 ±2
5	NLO-(QCD+EW)+(QCD+QED)$_{\mathrm{PS}}$(two-rad)	PYTHIA	-89.0 ±0.6	-371 ±3	-38.8 ±0.6	-157 ±3
6	NLO-(QCD+EW)+(QCD+QED)$_{\mathrm{PS}}$(two-rad)	PHOTOS	-88.6 ±0.6	-370 ±3	-39.2 ±0.6	-159 ±2

- 1 vs 2: Genuine difference between the predictions of Pythia and Photos QED models.
- 1 vs 5 and 2 vs 6: gives an estimation of the effect of the missing mixed EW-QCD correction in the pure shower approach. Notice that this effect depends on the QED shower model used. The PHOTOS model provides a closer model to the full precision one.
- 5 vs 6: The description with EW NLO accuracy of the photon radiation makes the prediction independent of the QED shower model used (the difference between the models becomes a higher order effect).

Results

- When using the pure shower approach i.e. (POWHEG(NLO QCD)+PYTHIA(QCD)+QED model), the uncertainty due to the missing mixed QCD-EW corrections, is estimated to be (in MeV)

	LHC				Tevatron			
	$W^+ \to \mu^+\nu$		$W^+ \to e^+\nu$		$W^+ \to \mu^+\nu$		$W^+ \to e^+\nu$	
	M_T	p_T^ℓ	M_T	p_T^ℓ	M_T	p_T^ℓ	M_T	p_T^ℓ
PYTHIA QED	$\approx 6 \pm 1$	$\approx 29 \pm 4$	$\approx 1 \pm 1$	$\approx 8 \pm 4$	$\approx 5 \pm 1$	$\approx 17 \pm 5$	$\approx 1 \pm 1$	$\approx 1 \pm 5$
PHOTOS	$\approx 1 \pm 1$	$\approx 2 \pm 4$	$\approx 1 \pm 1$	$\approx 9 \pm 4$	$\approx 2 \pm 1$	$\approx 8 \pm 6$	$\approx 1 \pm 2$	$\approx 1 \pm 4$

- If ones uses the version with full EW corrections (POWHEG(NLO QCD NLO EW)+PYTHIA(QCD)+QED model), the remaining mixed EW-QCD is reduced.

Fixed order calculations not always reliable

- in regions of phase space where large scale differences appear, e.g.
  - $p_T \ll M_V$ in DY
  - small x, $Q^2/s \ll 1$
  - in regions of phase space where the radiation is tightly constrained, e.g.
    - large x, $Q^2/s \to 1$

  large logs appear which spoil perturbation theory
- solution:
  - analytic resummation, $\alpha_s^n \log^{2n}$ (LL), $\alpha_s^n \log^{2n-1}$ (NLL), $\ldots$
  - resummation through a Parton Shower merged with NLO fixed order
    - for DY recent developments in reaching NNLOPS accuracy

Karlberg, Re, Zanderighi, arXiv:1407.2940

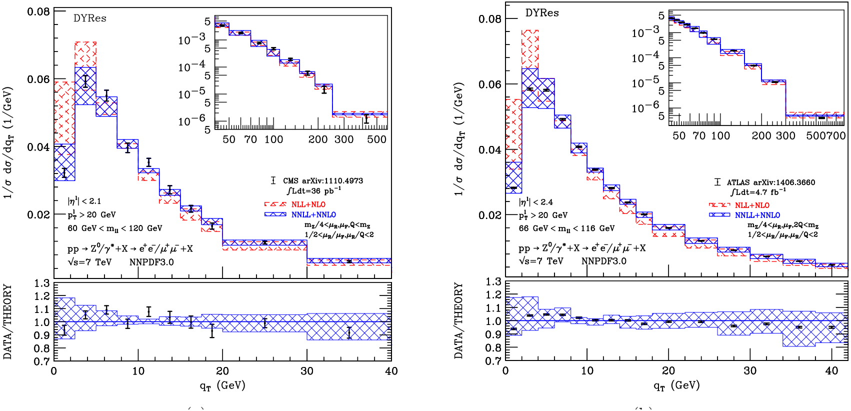
Höche, Li, Prestel, arXiv:1405.3607

see talks by G. Zanderighi and S. Prestel

Recent detailed comparisons among codes for DY in S. Alioli et al., arXiv:1606.02330

e.g.: q_T resumm. with DYRES, comparison with LHC data

- NNLL resummation with NNLO normalization



Catani, De Florian, Ferrera, Grazzini, arXiv:1507.06937

From LHC to future machines: FCC-hh and FCC-ee

- "Physics at a 100 TeV pp collider: Standard Model processes",

  M.L. Mangano, G. Zanderighi et al., arXiv:1607.01831

- "First Look at the Physics Case of TLEP",

  M. Bicer et al., arXiv:1308.6176

FCC-hh @100 TeV: the rise of EWK corrections

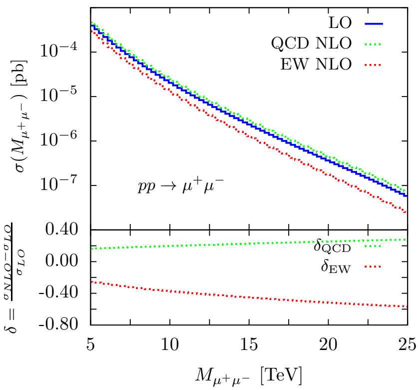
- At $\sqrt{Q^2} \lesssim$ EW scale higher order EW corrections to scattering processes are dominated by QED corrections
- genuine EW corrections (exchange/emission of massive gauge bosons) usually $\lesssim \mathcal{O}(\%)$
- for extremely high scales ($\sqrt{Q^2} \gg$ EW scale) EW corrections enter a new regime where they can become very large
- actually virtual EW corrections contain terms of the form

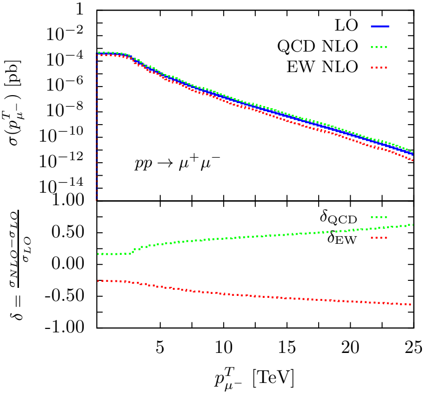
$$\frac{\alpha}{4\pi} \ln^2 \left(\frac{M_V^2}{s} \right) \quad (V = W, Z) \quad \text{Sudakov logs}$$

$$\sim 0.3\% @ \sqrt{s} = 1 \text{ TeV} \quad \sim 5\% @ \sqrt{s} = 10 \text{ TeV} \quad \sim 10\% @ \sqrt{s} = 100 \text{ TeV}$$

- W/Z masses act as IR cutoff: meaningful to consider only virtual corrections
- Experimentally weak boson emission usually can be separated (see later)
- contrary to IR divergences of QED and QCD, which cancel in inclusive quantities, Sudakov logs do not: final states are not $SU(2)$ singlets
- moreover PDF's give different weights to the initial states

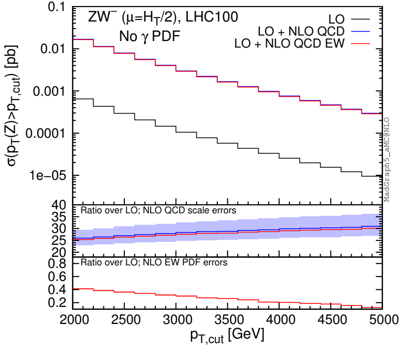
NC Drell-Yan $M(\mu^+\mu^-) \geq 5$ TeV

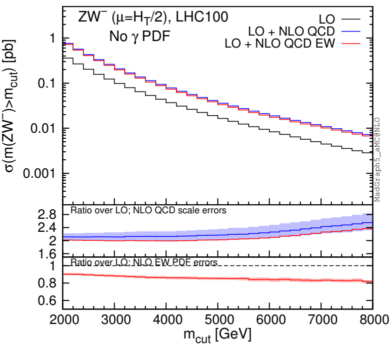




- for invariant masses in the multi-TeV region EW corrections $\sim -50\%$

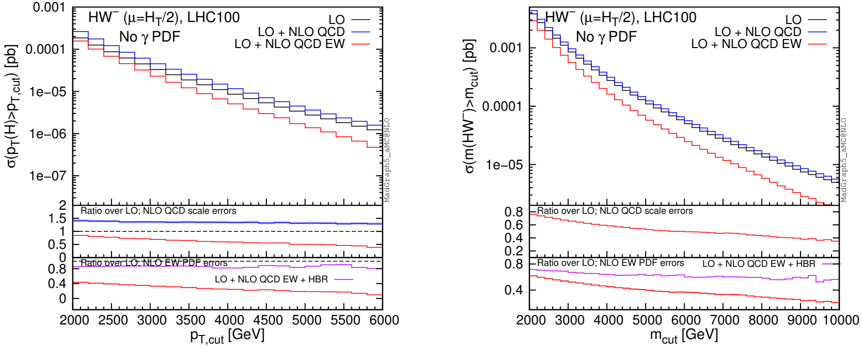
Diboson production: ZW^-





- huge QCD corrections for large p_T^Z: at NLO QCD the dominant configuration is the Z recoiling against a hard parton which emits a soft/collinear $W \Longrightarrow \alpha_s \ln^2 \frac{p_T^Z}{M_W}$ enhancement
- EW corrections large for p_T^Z and moderate for diboson invariant mass, which is dominated by $t-$channel configurations with small p_T^Z
- **similar effects for $\mathbf{W^+W^-}$ and ZZ final states**

Higgs associate production: HW^-

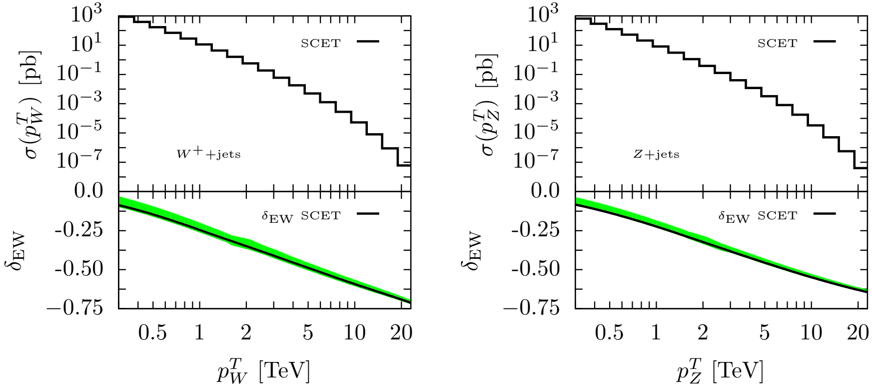


- no strong enhancements on NLO QCD corr's to p_T^H due to absence of tree-level $pp \to Hj$
- large EW corrections to p_T^H and $m(HW^-)$: no LO $t-$channel diagrams $\Longrightarrow$ large $m(HW^-) \sim$ large p_T^H
- similar consideration apply to $pp \to ZH$

Resumming Sudakov logs

- in several cases EW corrections become very large $\Longrightarrow$ the fixed order calculation becomes not reliable
- resummation is needed $\Longrightarrow$ SCET offers a general framework for resumming Sudakov logs

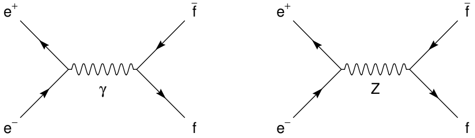
Chiu, Golf, Kelley, Manohar, Phys. Rev. D78 (2008) 073006

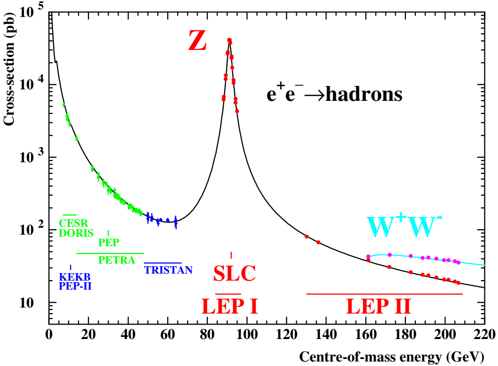


T. Becher, X. Garcia i Tormo, Phys. Rev. D88 (2013) 013009

FCC-ee: e^+e^- at high luminosity

basic processes studied at the Z peak





LEP EWWG, SLD WG, ALEPH, DELPHI, L3, OPAL, hep-ph/0509008

Electroweak Fit – Experimental inputs

- Latest experimental inputs:
  - Z-pole observables: from LEP / SLC
    [ADLO+SLD, Phys. Rept. 427, 257 (2006)]
  - M_W and Γ_W from LEP/Tevatron
    [arXiv:1204.0042, arXiv:1302.3415]
  - m_{top} latest avg from Tevatron
    [arXiv:1305.3929]
  - m_c, m_b world averages (PDG)
    [PDG, J. Phys. G33,1 (2006)]
  - $\Delta\alpha_{had}^{(5)}(M_Z^2)$ including α_S dependency
    [Davier et al., EPJC 71, 1515 (2011)]
  - M_H from LHC
    [arXiv:1207.7214, arXiv:1207.7235]
- 7 (+2) free fit parameters:
  - M_H, M_Z, $\alpha_S(M_Z^2)$, $\Delta\alpha_{had}^{(5)}(M_Z^2)$, m_t, m_c, m_b
  - 2 theory nuisance parameters
    - δM_W (4 MeV), $\delta\sin^2\theta^l_{eff}$ (4.7x10^{-5})

M_H [GeV]°	125.7 ± 0.4	LHC
M_W [GeV]	80.385 ± 0.015	Tevatron
Γ_W [GeV]	2.085 ± 0.042	
M_Z [GeV]	91.1875 ± 0.0021	LEP
Γ_Z [GeV]	2.4952 ± 0.0023	
σ^0_{had} [nb]	41.540 ± 0.037	
R^0_ℓ	20.767 ± 0.025	
$A^{0,\ell}_{FB}$	0.0171 ± 0.0010	
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	SLC
$\sin^2\theta^\ell_{eff}(Q_{FB})$	0.2324 ± 0.0012	
A_c	0.670 ± 0.027	SLC
A_b	0.923 ± 0.020	
$A^{0,c}_{FB}$	0.0707 ± 0.0035	LEP
$A^{0,b}_{FB}$	0.0992 ± 0.0016	
R^0_c	0.1721 ± 0.0030	
R^0_b	0.21629 ± 0.00066	
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	
m_t [GeV]	173.20 ± 0.87	Tevatron
$\Delta\alpha^{(5)}_{had}(M_Z^2)$ $^{(\dagger\triangle)}$	2756 ± 10	

Max Baak (CERN) The ElectroWeak fit of Standard Model 5

- electroweak fit based on *derived (pseudo-)observables* (allow easy combination among experiments and easy comparison data/theory within and beyond the SM)
- primary measured observables: cross section and asymmetries

From measured observables to pseudo-observables

$$\sigma_{\mathrm{T}}(s) = \int_{z_0}^{1} dz H(z;s) \hat{\sigma}_{\mathrm{T}}(zs)$$

$$A_{FB}(s) = \frac{\pi \alpha^2 Q_e^2 Q_f^2}{\sigma_{\mathrm{tot}}} \int_{z_0}^{1} dz \frac{1}{(1+z)^2} H_{\mathrm{FB}}(z;s)\, \hat{\sigma}_{\mathrm{FB}}(zs)$$

- Radiator function known up to $\mathcal{O}(\alpha^3)$
  1. additive form
  2. factorized form

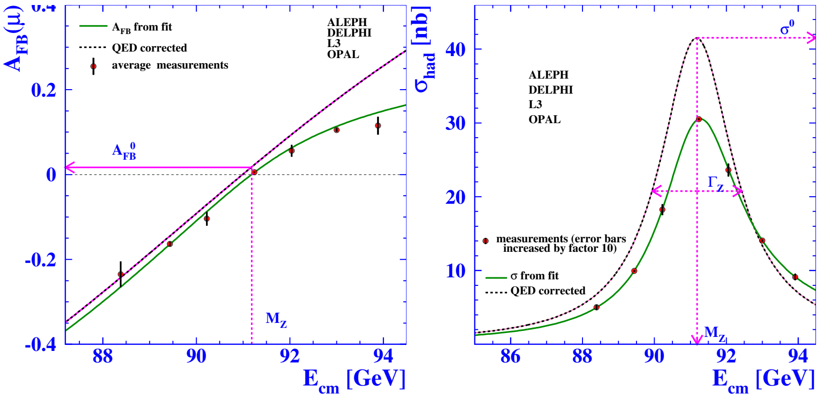
G. Montagna, O. Nicrosini, F.P., PLB 406, (1997) 243

M. Skrzypek, Acta Phys. Pol. B23 (1992) 135

- H_{FB} known up to $\mathcal{O}(\alpha^2)$

Theoretical control on the derived observables with a certain precision requires the process of "deconvolution" of ISR and FSR with the same level of precision

Effect of QED deconvolution



Deconvolution performed at LEP by means of

- TOPAZ0

  G. Montagna, O. Nicrosini, G. Passarino, F.P., R. Pittau, 1993, 1996, 1999

- ZFITTER

  D. Bardin et al., 1989, 1991, 1992, 1994

model-independent parameterization

$$A_{\rm SM} = A_\gamma + A_Z + \text{non-factorizable}$$

- aim: write the Z-line shape in a model independent way

Borrelli, Consoli, Maiani, Sisto, NPB333 (1990) 357

$$\sigma^Z_{f\bar f} = \sigma^{\rm peak}_{f\bar f} \frac{s\Gamma_Z^2}{(s-M_Z)^2 + s^2\Gamma_Z^2/M_Z^2}$$

$$\sigma^{\rm peak}_{f\bar f} = \frac{\sigma^0_{f\bar f}}{R_{\rm QED}}; \qquad \sigma^0_{f\bar f} = \frac{12\pi}{M_Z^2}\frac{\Gamma_{ee}\Gamma_{f\bar f}}{\Gamma_Z^2}$$

- what is not factorizable on the Z-exchange tree-level has to be taken for fixed SM parameters
- $\longrightarrow$ model independence is lost. At LEP the remainders taken from the SM show dependence on the SM Lagrangian parameters below the 0.1% level

Towards higher precision

- during the last decay great technological advances for the calculation of higher order (beyond one-loop) radiative corrections
- however, a bottleneck will be represented by the hadronic contributions to the vacuum polarization
  - breakdown of theoretical uncertainties on luminosity (small angle Bhabha scattering) at LEP

Type of correction/error	(%)	(%)	updated (%)
missing photonic $O(\alpha^2 L)$	0.100	0.027	0.027
missing photonic $O(\alpha^3 L^3)$	0.015	0.015	0.015
vacuum polarization	0.040	0.040	0.040
light pairs	0.030	0.030	0.010
Z-exchange	0.015	0.015	0.015
total	0.110	0.061	0.054

I column: S. Jadach, O. Nicrosini et al. Physics at LEP2 YR 96-01, Vol. 2
A. Arbuzov et al., Phys. Lett. B389 (1996) 129
II column: B.F.L. Ward, S. Jadach, M. Melles, S.A. Yost, hep-ph/9811245
III column: G. Montagna et al., Nucl. Phys. B547 (1999) 39

Vacuum Polarization: bottleneck for future precision

- $\alpha \to \alpha(q^2) \equiv \frac{\alpha}{1-\Delta\alpha(q^2)}$ $\quad \Delta\alpha(q^2) = \Delta\alpha_{e,\mu,\tau,\mathrm{top}}(q^2) + \Delta\alpha^{(5)}_{\mathrm{had}}(q^2)$
- $\Delta\alpha^{(5)}_{\mathrm{had}}$ is an intrinsically non-perturbative contribution. It can be calculated from $e^+e^- \to hadrons$ data using dispersion relations

$$\Delta\alpha^{(5)}_{\mathrm{had}}(q^2) = -\frac{q^2\alpha}{3\pi}\left[P\!\!\!\!\int_{4m_\pi^2}^{E^2_{cut}} \frac{R^{data}_{had}(s)}{s(s-q^2)}ds + P\!\!\!\!\int_{E^2_{cut}}^{\infty} \frac{R^{pQCD}_{had}(s)}{s(s-q^2)}ds \right]$$

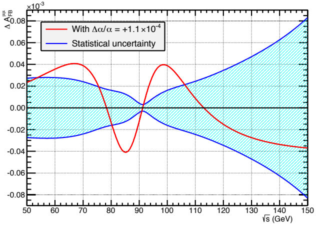
- it is affected by an uncertainty, due to low energy data on $\sigma_{had}(s)$ $\Longrightarrow$ it reflects on Bhabha predictions $\Longrightarrow$ Z-observables
- an historical perspective on the evolution of the error
  - $\Delta\alpha(M_Z^2) = 0.0280 \pm 0.0007 \Longrightarrow \alpha^{-1}(M_Z^2) = 128.89 \pm 0.09$ H. Burkhardt and B. Pietrzyk, Phys. Lett. B356 (1995) 398
  - $\Delta\alpha(M_Z^2) = 0.02750 \pm 0.00033$ H. Burkhardt and B. Pietrzyk, Phys. Rev. D84 (2011) 037502
  - $\Delta\alpha(M_Z^2) = 0.027498 \pm 0.000135[0.027510 \pm 0.000218]$ F. Jegerlehner, arXiv:1107.4683
  - $\Delta\alpha(M_Z^2) = 0.02757 \pm 0.0001 \Longrightarrow \alpha^{-1}(M_Z^2) = 128.952 \pm 0.014$ Davier, Hoecker, Malaescu, Zhang, arXiv:1010.4180
  - $\Delta\alpha(M_Z^2) = 0.027626 \pm 0.000138$ T. Teubner et al., Nucl. Phys. Proc. Suppl. 225 (2012) 282

recent new proposal

P. Janot, JHEP 1602 (2016) 053

- idea: measuring $\alpha(M_Z^2)$ from a measurement of A_{FB} below and above peak

$$\frac{\Delta\alpha}{\alpha} = \frac{\Delta A_{\rm FB}^{\mu\mu}}{A_{\rm FB}^{\mu\mu} - A_{\rm FB,0}^{\mu\mu}} \times \frac{\mathcal{Z}+\mathcal{G}}{\mathcal{Z}-\mathcal{G}} \simeq \frac{\Delta A_{\rm FB}^{\mu\mu}}{A_{\rm FB}^{\mu\mu}} \times \frac{\mathcal{Z}+\mathcal{G}}{\mathcal{Z}-\mathcal{G}},$$



- preliminary investigations show that an accuracy of the order of 10^{-5} can be reached

Summary

- LHC: the precision of experimental measurements in electroweak gauge boson production requires already now the best available theoretical accuracy in QCD and EWK sector of the SM
- future colliders
  - FCC-hh: at proton collision energy of 100 TeV, for several observables EWK radiative corrections become huge, due to the presence of Sudakov logs; resummation needed also in the EWK sector (the genuinely EWK part, not only QED)
  - FCC-ee:
    - ⋆ the exceptional recent progress in the calculation of higher order corrections for LHC makes it plausible thinking that future progress in higher order electroweak corrections can meet the projected experimental accuracy
    - ⋆ an issue is given by the uncertainty in the hadronic contribution to the vacuum polarization
    - ⋆ recent promising proposal to determine $\alpha(M_Z^2)$ from A_{FB} below and above the Z peak