Impact of LHC on the Parton Distribution Functions

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Precision theory for precise measurements at LHC and future colliders

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Parton Distribution Functions

Parton Distribution Functions (PDFs) are of crucial for precision physics at hadron colliders because:

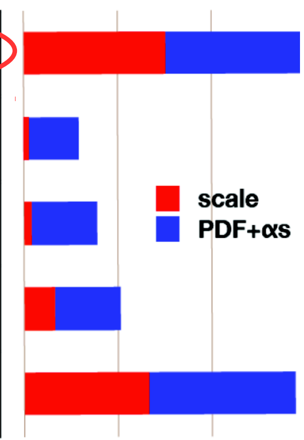
→ **reach of new physics** searches depends on PDF knowledge at high Bjorken-x

→ PDFs limit **the accuracy of the SM predictions** (including Higgs, W mass)

PDF uncertainties in Higgs productions at LHC are significant

→ similarly to top quarks, Higgs cross section is strongly gluon and α_s dependent

		σ (8 TeV)	uncertainty
NNLL QCD +NLO EW	gg→H	19.5 pb	14.7%
	VBF	1.56 pb	2.9%
NNLO QCD +NLO EW	WH	0.70 pb	3.9%
	ZH	0.39 pb	5.1%
NLO QCD	ttH	0.13 pb	14.4%



→ agreement with Standard Model depends on how well we know PDFs and α_s

Parton Distribution Functions

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QCD factorisation:

measured cross section =

PDF | hard-scattering ME

$$\sigma(\alpha_s,\mu_R^2,\mu_F^2)=\sum_{a,b}\int_0^1 f_a(x_1,\mu_F^2)f_b(x_2,\mu_F^2)\,\hat{\sigma}(x_1,x_2;\alpha_s,\mu_R^2,\mu_F^2)+\ldots$$

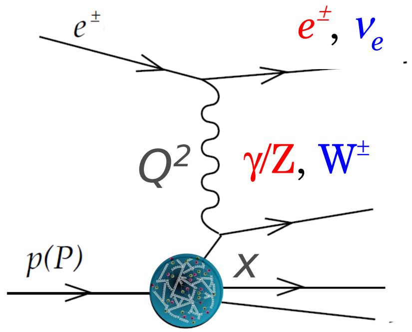
precision measurements of hadron collider data

PDF determination, heavy quark treatment, QCD analysis tools

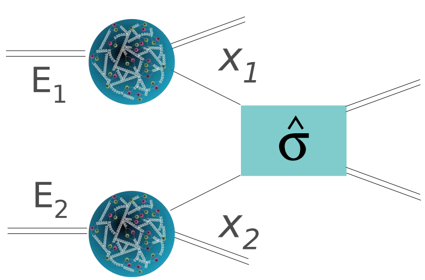
parton cross section (calculable in pQCD)

Experimental data: from HERA to LHC

Deep Inelastic Scattering (**DIS**): unique opportunity to study PDFs

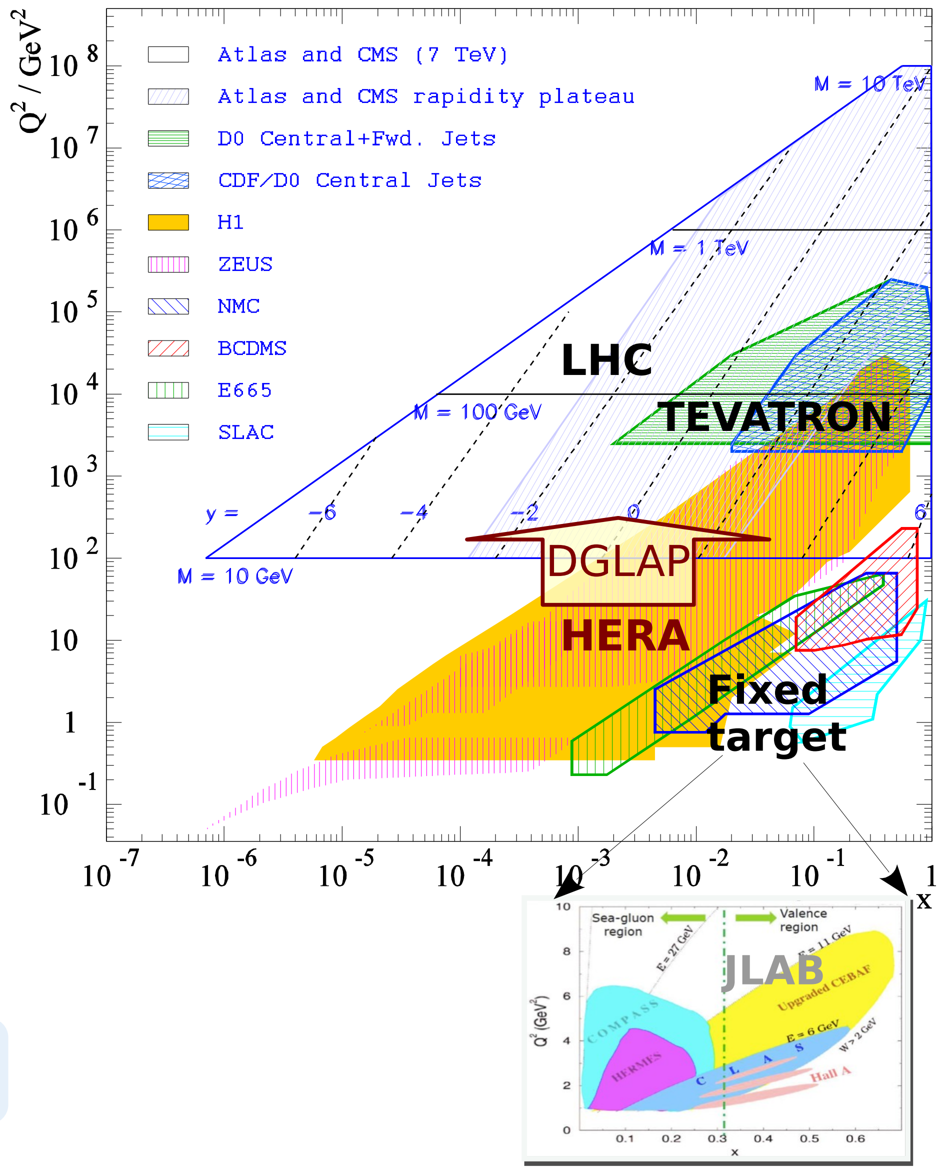


same PDFs can be used to predict *pp* collisions



$\hat{\sigma}$ – perturbative QCD cross section

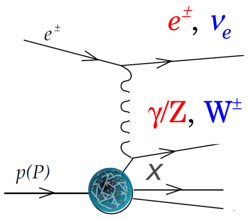
Kinematic plane of the experimental data (in x,Q^2)



LHC data can improve PDFs further

Data in PDF fits

Deep Inelastic Scattering:



ep data: quarks and gluon at small x (F_L), flavour separation (CC)

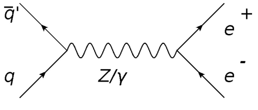
jets → gluons (moderate x) and α_s

heavy quarks → gluons, tests of heavy quark schemes, mass determination

fixed target data: higher x

neutrino DIS: flavour decomposition, x>0.01

Drell-Yan production:

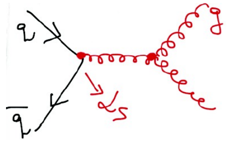


different PDF combinations (low/mid/high x), deuterium target – $\bar{u}/\bar{d}$ asymmetry

W/Z ratio, asymmetries → quark flavour separation

V+ heavy flavour → sensitivity to s quark

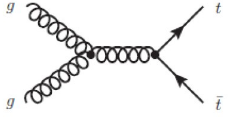
Inclusive jets, multi-jets and ratios:



high x gluon, α_s

Isolated photon → gluon at medium and high x

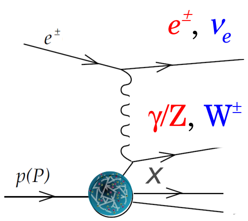
ttbar, single top:



gluon at high x, u and d quarks, α_s

Data in PDF fits

Deep Inelastic Scattering:



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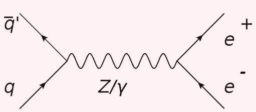

different PDF combinations (low/mid/high x), deuterium target – $\bar{u}/\bar{d}$ asymmetry

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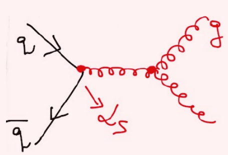
Inclusive jets, multi-jets and photons:

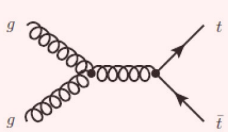


high x gluon, α_s

Isolated photon → gluon at medium and high x

LHC

ttbar, single top:



gluon at high x, u and d quarks, α_s

→ only selected examples will be shown

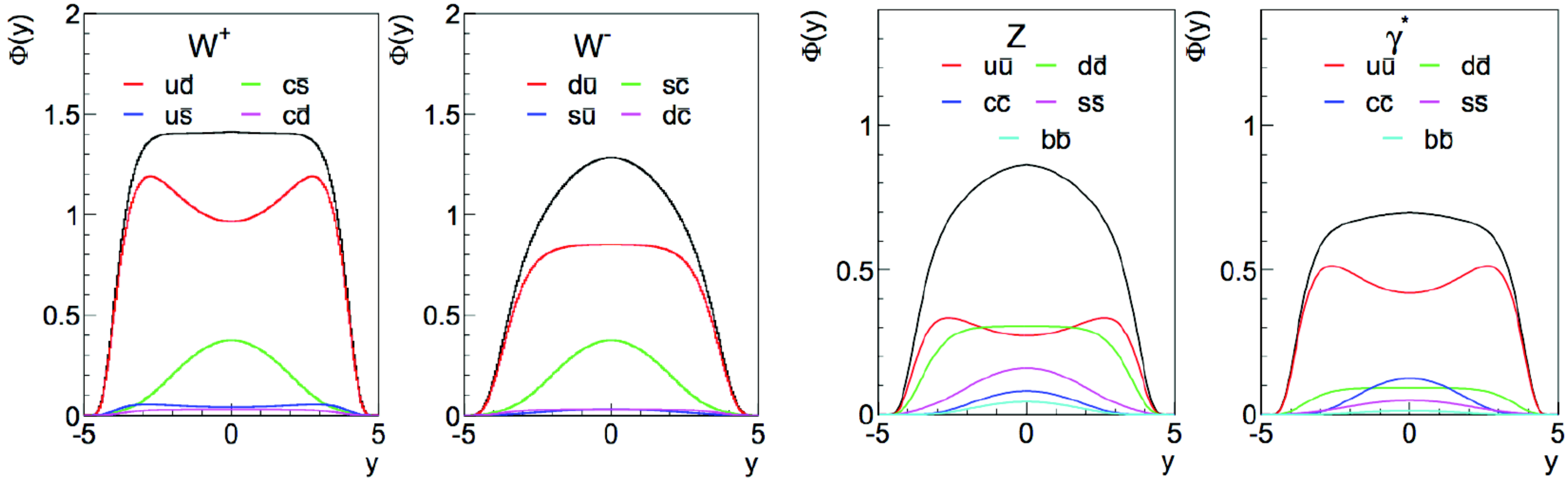
W and Z production at LHC

Z and W production at LHC

→ probe different flavour combinations
→ potential to improve quark PDFs

$$W^+ \approx 0.95(u\bar{d} + c\bar{s}) + 0.05(u\bar{s} + c\bar{d})$$
$$W^- \approx 0.95(d\bar{u} + s\bar{c}) + 0.05(d\bar{c} + s\bar{u})$$

$$Z \approx 0.29(u\bar{u} + c\bar{c}) + 0.37(d\bar{d} + s\bar{s} + b\bar{b})$$
$$\gamma^* \approx 0.44(u\bar{u} + c\bar{c}) + 0.11(d\bar{d} + s\bar{s} + b\bar{b})$$



(A.Glazov/V.Radescu)

→ u and d quarks dominate for W, all flavours contribute to Z

W charge asymmetry

W lepton charge asymmetry at LHC

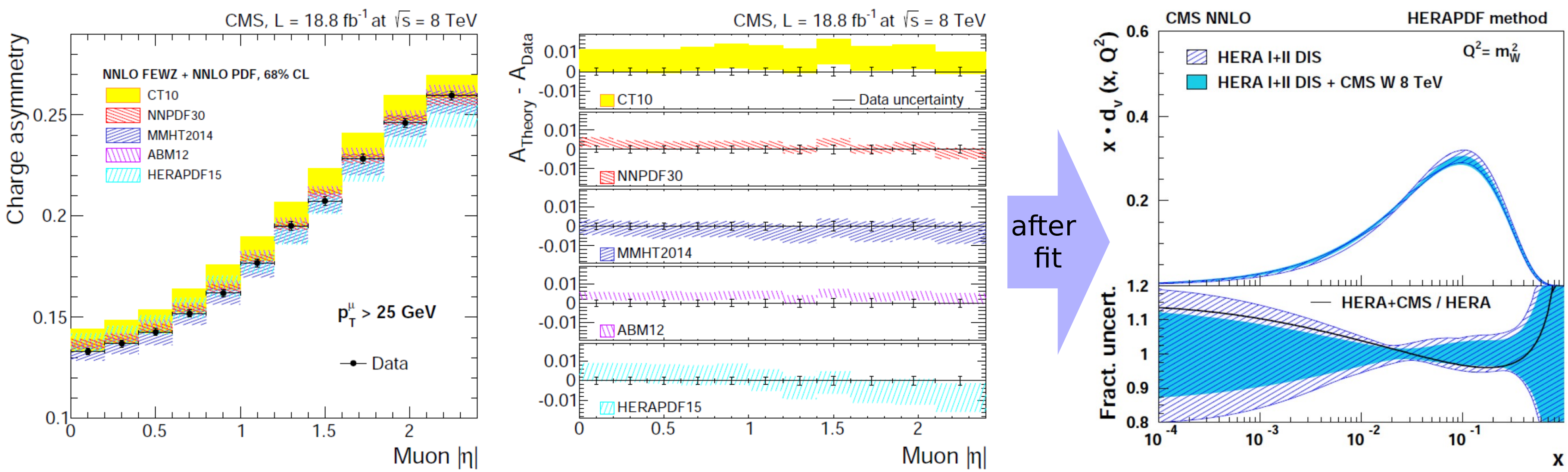
→ overall excess of W^+ over W^- due to presents of two valence *u* quarks in the proton

→ probe valence quarks and PDFs rations (u_v, d_v, d/u, d_v/u_v, dbar/ubar):

$$A_W = \frac{W^+ - W^-}{W^+ + W^-} \approx \frac{u_v - d_v}{u_v + d_v + 2u_{sea}}$$

CMS W muon asymmetry data (8 TeV)

arXiv:1603.01803, accepted by EPJC



note: comparison is done with HERA data alone

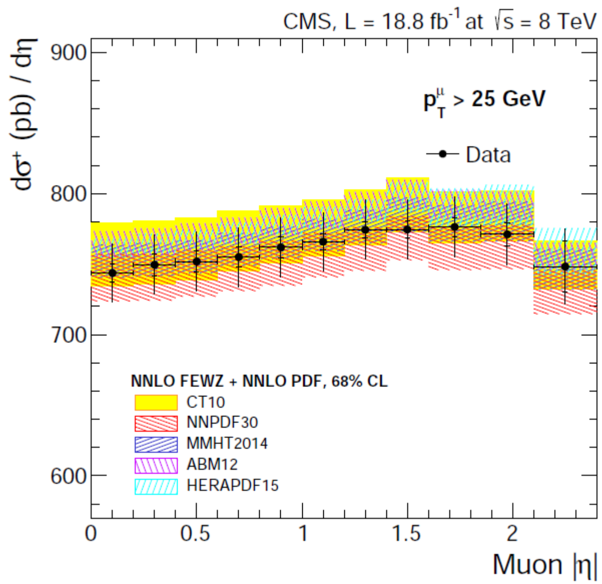
CMS W^+ and W^- distributions

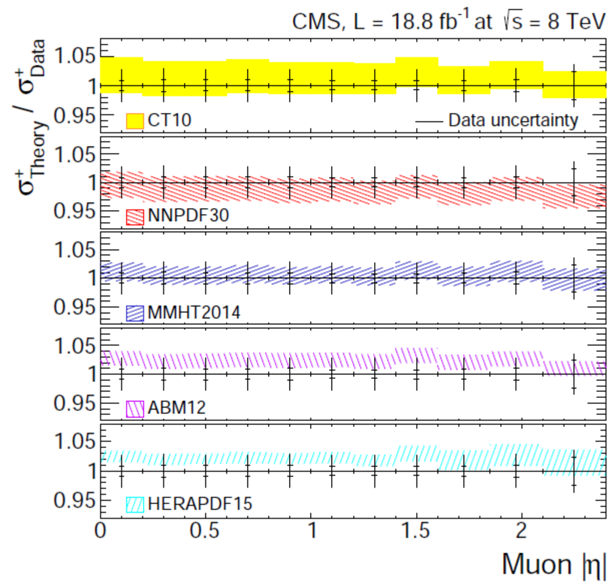
arXiv:1603.01803

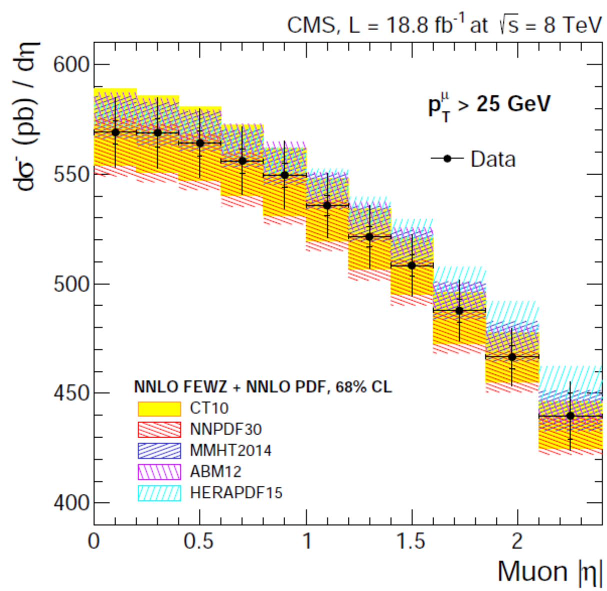
→ compared with theory predictions (FEWZ) at NNLO using different PDFs

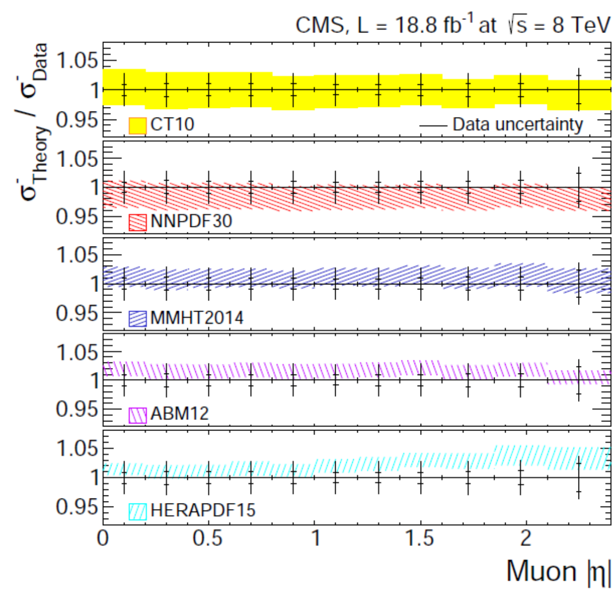
→ good agreement predictions obtained with all PDFs

(systematic correlations provided in the paper)





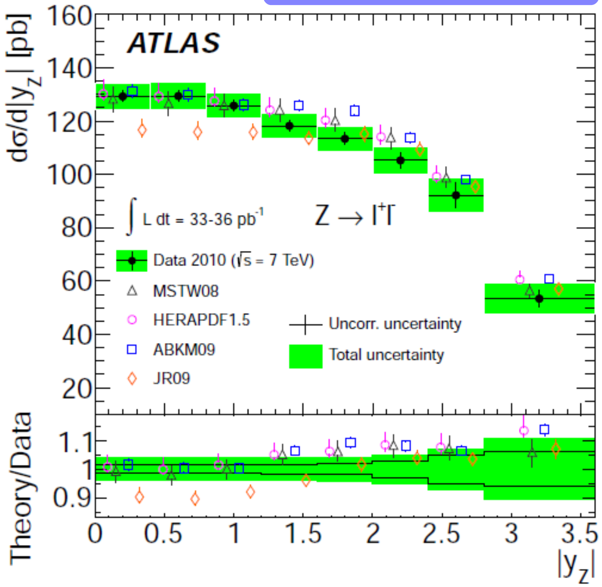




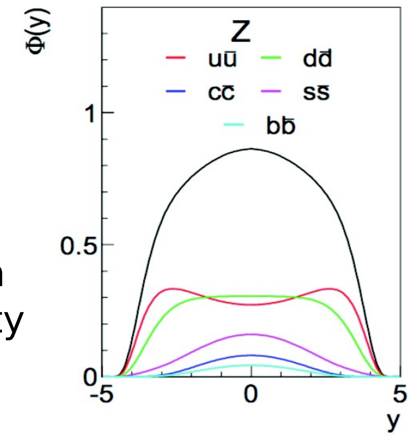
W and Z production: sensitivity to s-quark

ATLAS $W^{\pm}$ and Z inclusive differential cross sections (35 pb^{-1})

PRD 85 (2012) 072004



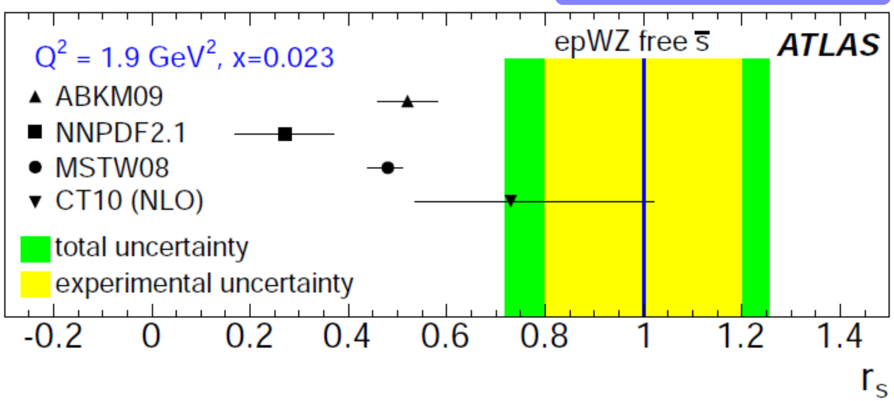
s-quark is poorly known, main constraints come from fixed-target experiments (e.g. NOMAD, Nucl.Phys. B876 (2013) 339) which suggest suppressed s-quark, i.e. less s than d at low x



s-quark contribution largest in Z production at central rapidity

$$r_s = 0.5(s + \bar{s})/\bar{d}$$

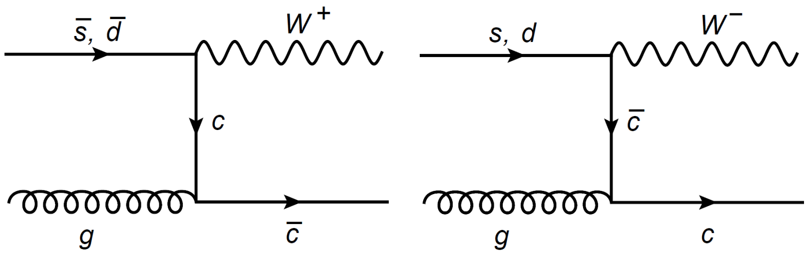
PRL 109 (2012) 012001



QCD analysis at NNLO of ATLAS W,Z data
→ results support symmetric light-sea

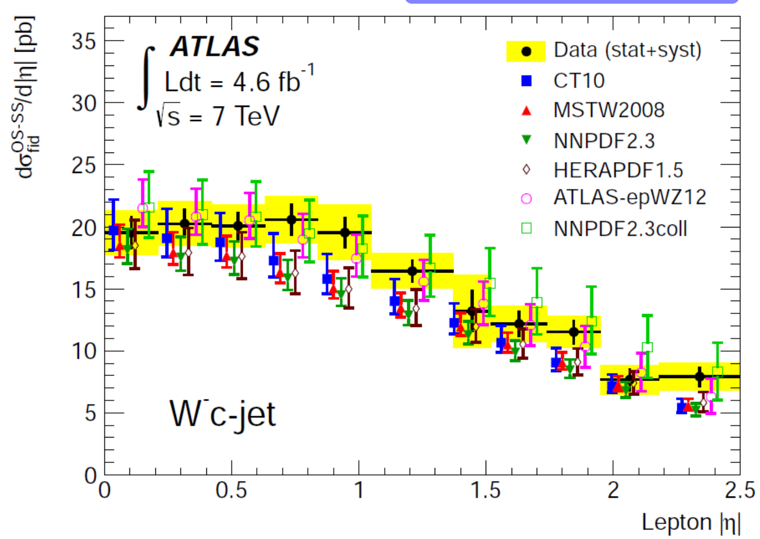
W + charm Production at LHC

Measurement of W+c at LHC provide additional constrains to the s quark

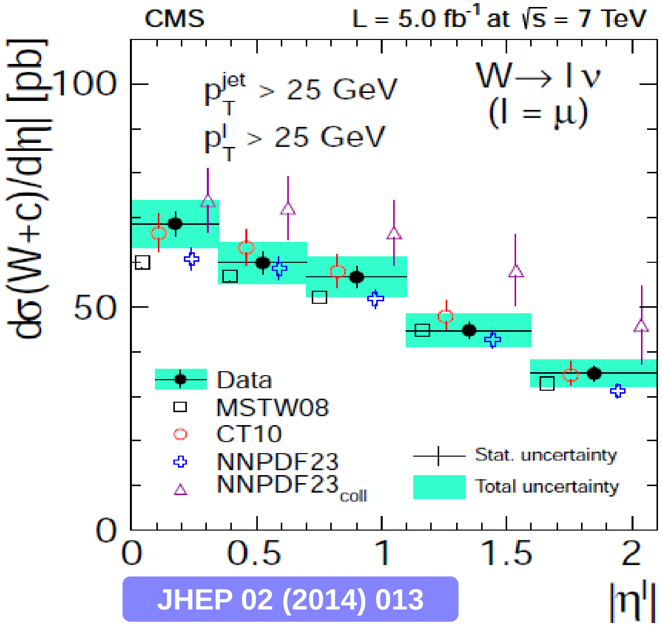


ATLAS W+ charm data (4.6 fb^{-1})

JHEP 05 (2014) 068



CMS W+ charm data (5 fb^{-1})



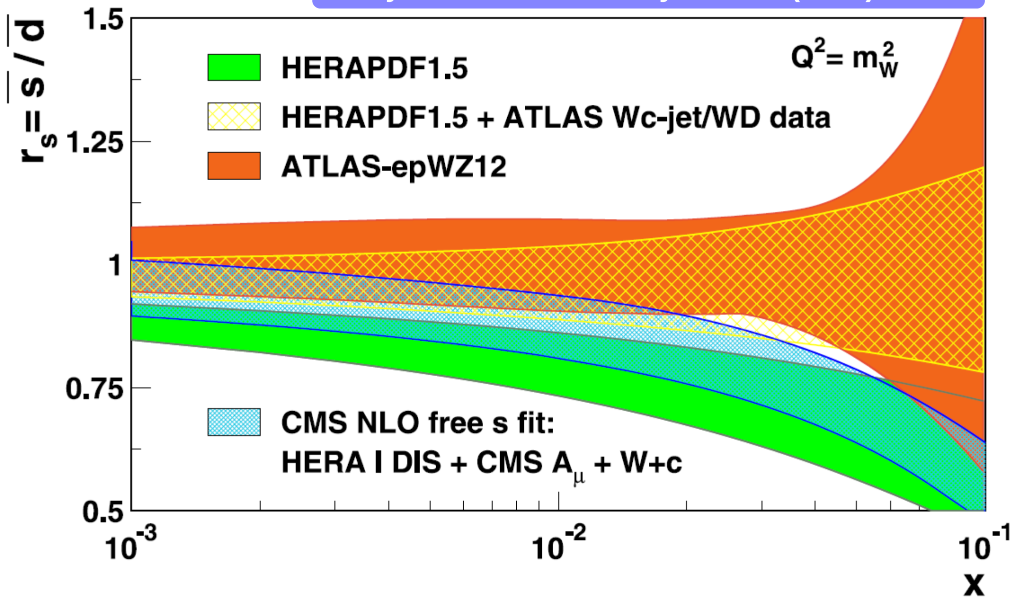
JHEP 02 (2014) 013

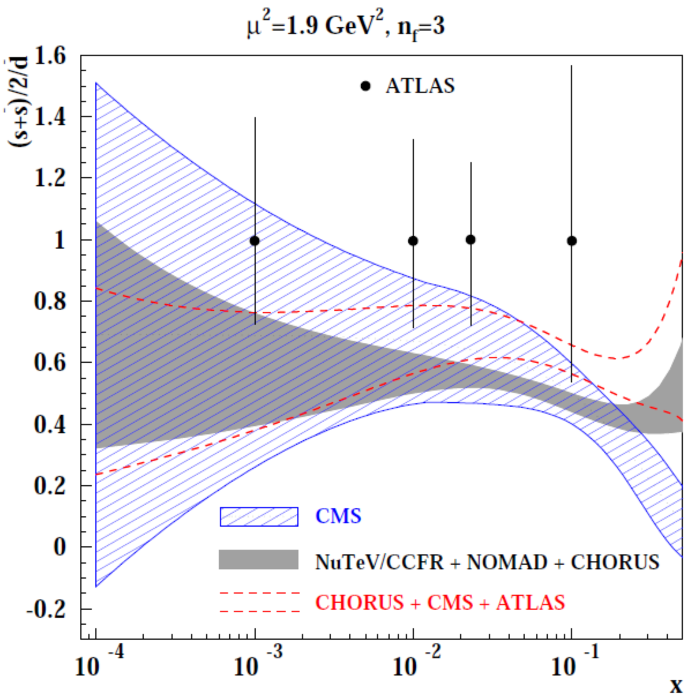
CMS in good agreement with CT10 while ATLAS data is above - indication of enhanced s fraction

Strange quark PDF

Comparison of the s-quark fraction determined by ATLAS and CMS (no fixed-target data, no additional assumptions) and with determination using fixed-target data

J. Phys.G: Nucl. Part. Phys. 42 05 (2015) 013103





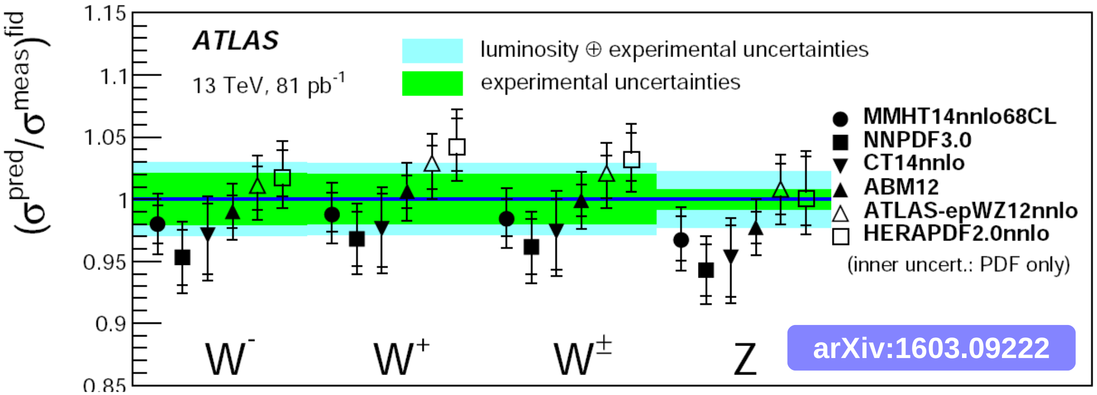
PRD 91 (2015) 094002

New data from LHC will bring more information about the s-quark

W and Z production (13 TeV)

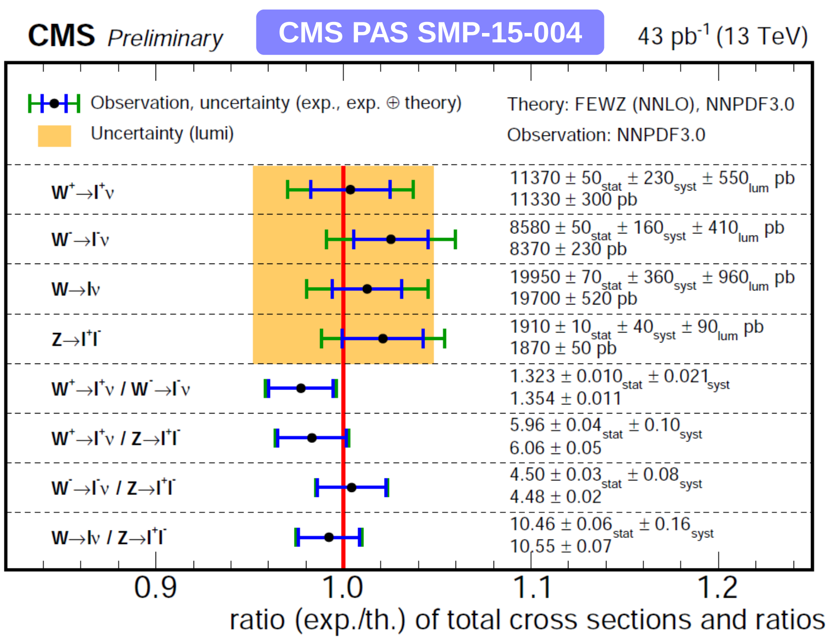
New 13 TeV measurement of W and Z boson cross sections with $\int L = 43 \pm 2\ pb^{-1}$ (CMS) and $\int L = 81\ pb^{-1}$ (ATLAS)

Measurements are compared to theory predictions at NNLO QCD+NLO QED using different PDFs



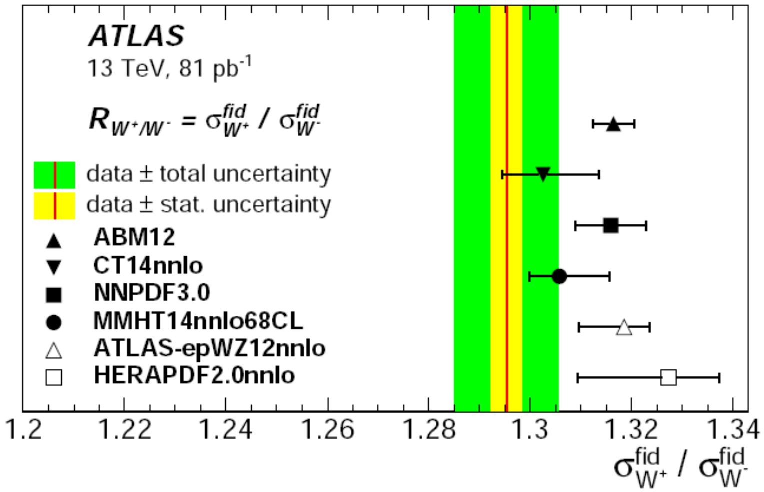
Experimental uncertainties in cross-section ratios largely cancel

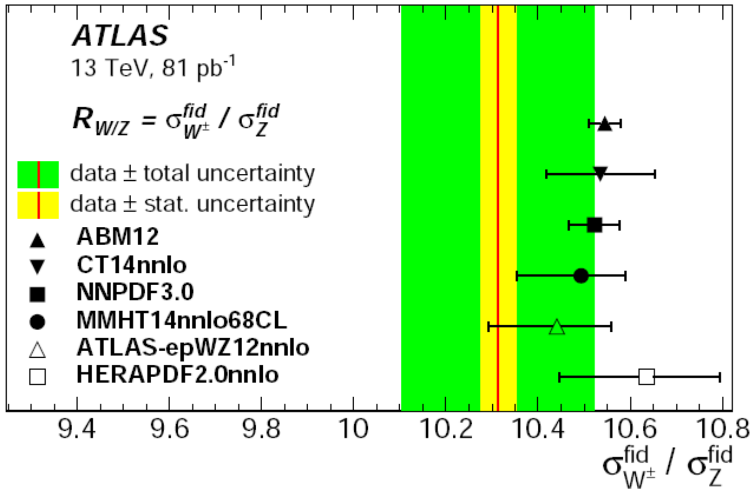
→ (alternative) constraints to PDFs



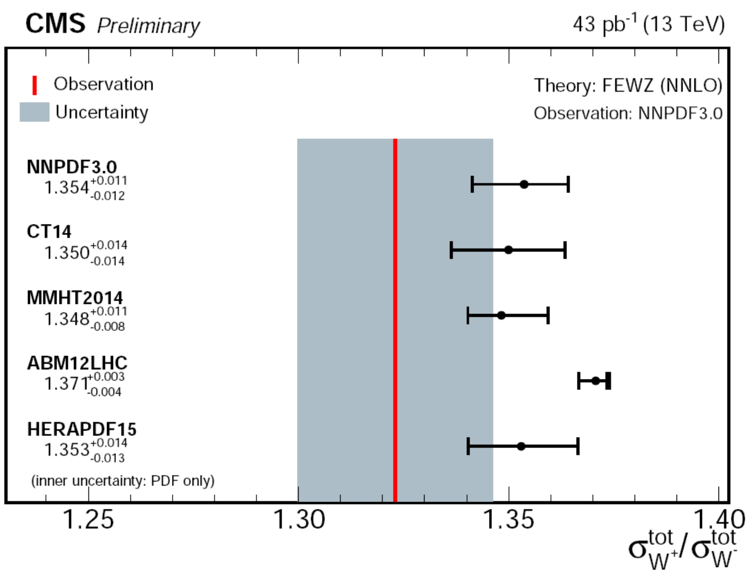
W and Z production (13 TeV)

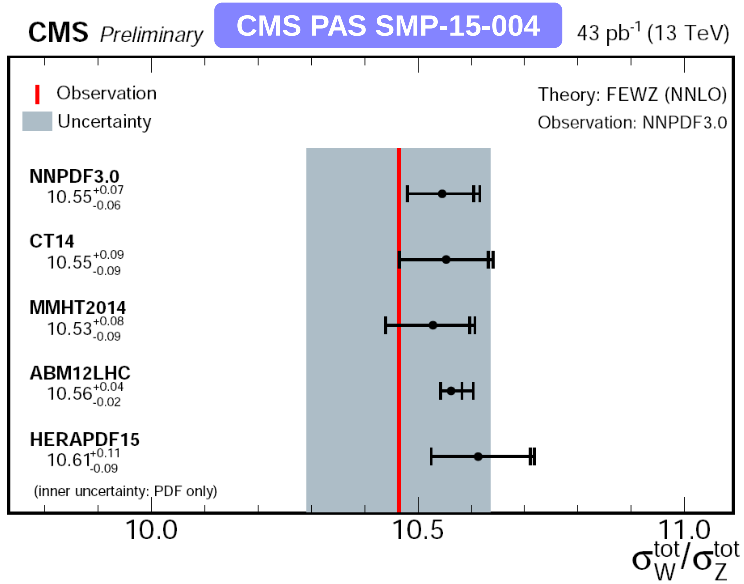
arXiv:1603.09222





CMS PAS SMP-15-004



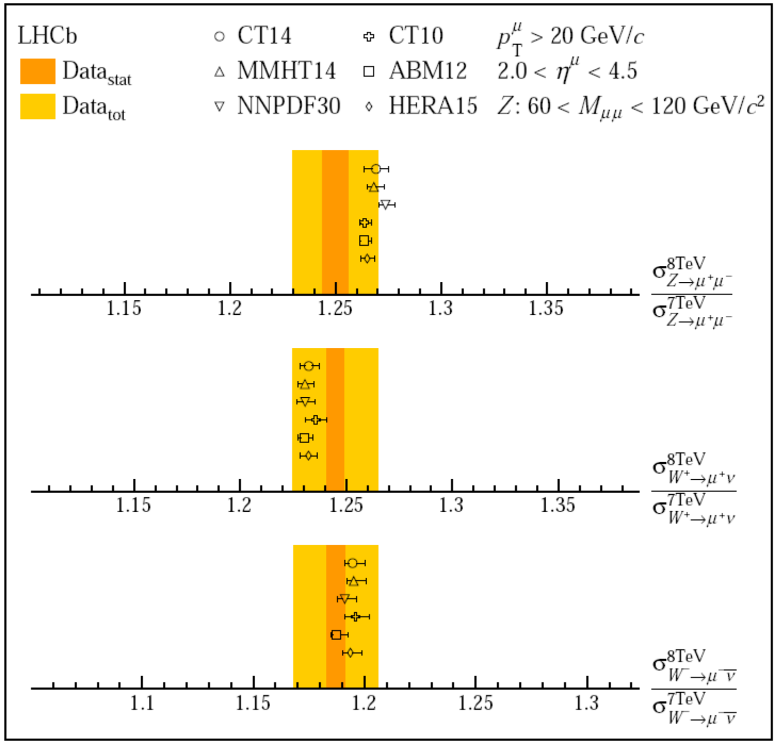


W^+ cross section tends to be slightly below the NNLO predictions
(similar in ATLAS and CMS)

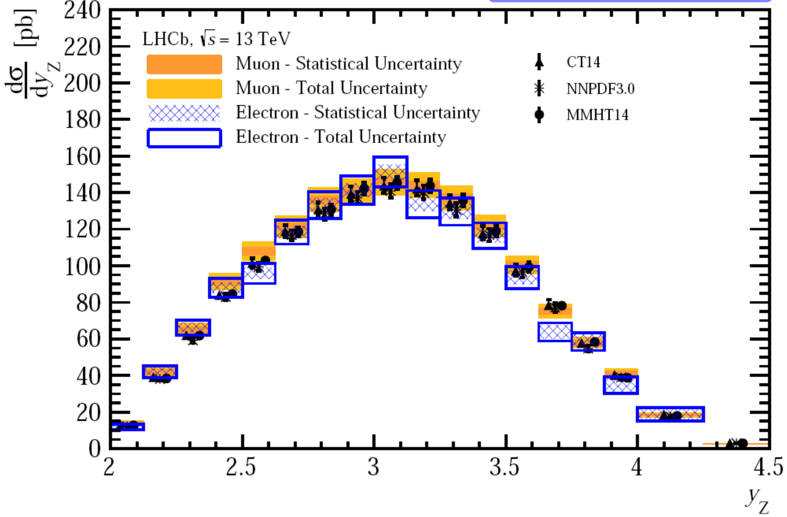
W and Z production in the forward region

The forward acceptance of the LHCb detector allows the PDFs to be constrained at low Bjorken-x

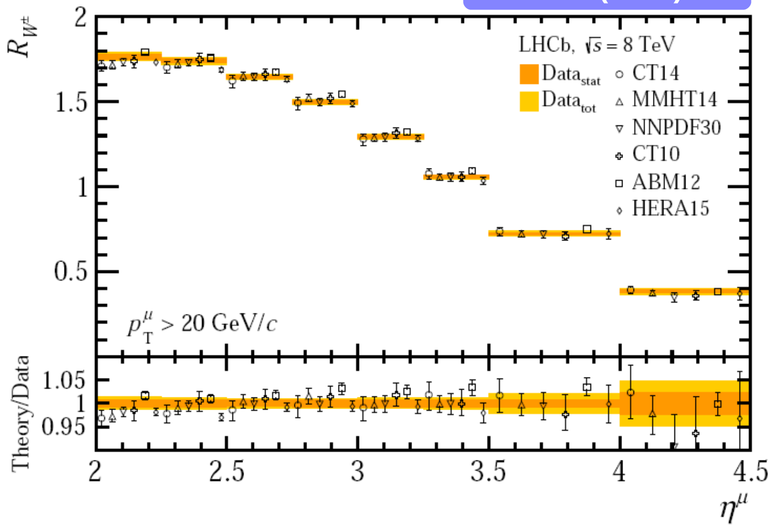
→ *available W, Z, ratios and asymmetry measurements*



arXiv:1607.06495



JHEP 01 (2016) 155



→ large correlation between the uncertainties allows for sub-percent determination of the cross-section ratios

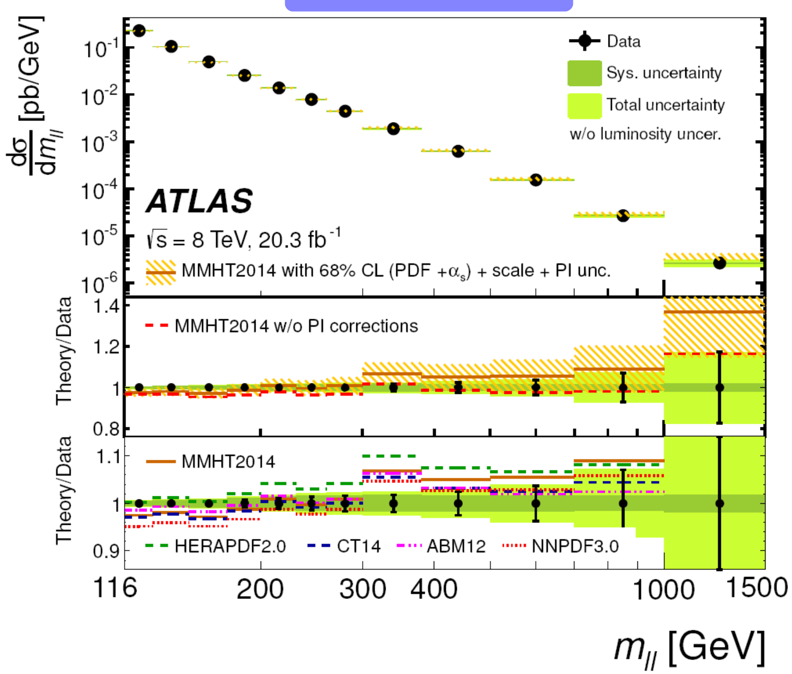
W and Z production (low-mass, high-mass)

Drell Yan data mass spectra:

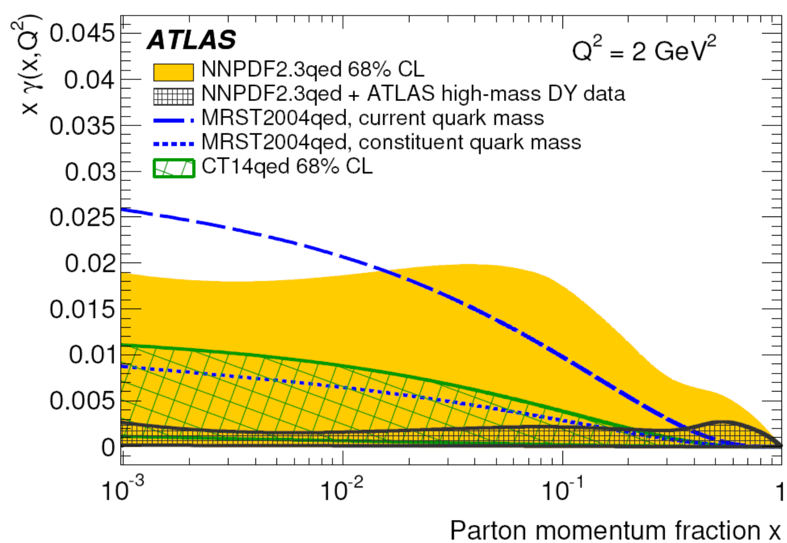
→ high-mass: sensitive to sea quarks at high-x (and thus to new physics at high-scale)
→ low-mass: similar at the low-x region
(EW corrections are important)

ATLAS DY high-mass measurement at √s =8 TeV

arXiv:1606.01736



→ sensitivity to the photon PDF through the photon-induced (PI) process $\gamma\gamma \rightarrow l^+l^-$



→ at large m_{ll} the measurements offer constraints on the large-x antiquark PDFs

→ sensitivity study with the Bayesian reweighting (without fitting the data)

Z+charm production

New CMS measurement at $\sqrt{s}$ = 8 TeV ($\int L$ =19.7 0.5 fb^{-1}) of the production cross section of a Z boson and at least one jet originating from a *c*-quark

→ Z-boson candidates are identified through their decay into a pair of electrons or muons

→ heavy flavour jets in the kinematic region p^{jet}_T > 25 GeV, $|\eta^{jet}|$ < 2.5

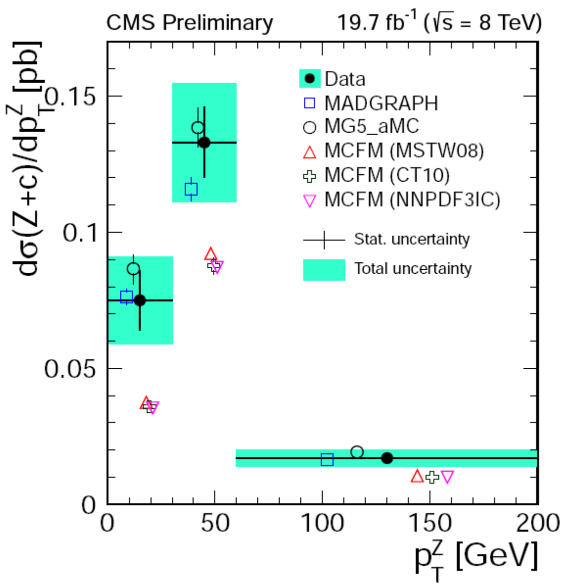
→ can address question of the intrinsic charm quark component in the proton

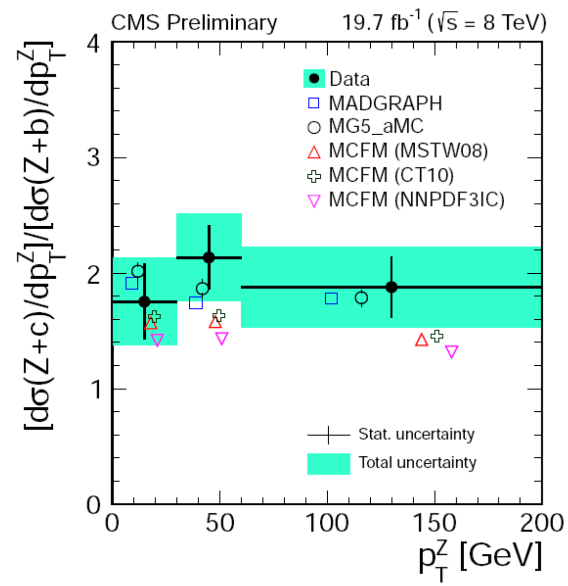
CMS PAS SMP-15-009

Theory predictions (NLO) obtained with the MCFM using the NLO PDF sets MSTW08, CT10, NNPDF3IC and NNPDF3nIC

→ all predictions are smaller than measured cross sections

→ no significant differences in the predictions using either NNPDF3IC or NNPDF3nIC PDFs





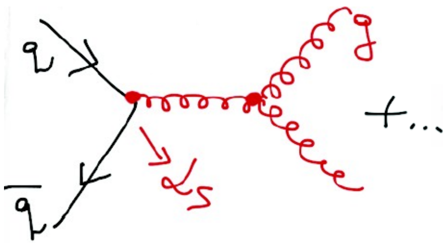
Jet Production at LHC

Jet production at LHC

→ provides information about hard QCD, PDFs, strong coupling constant α_s

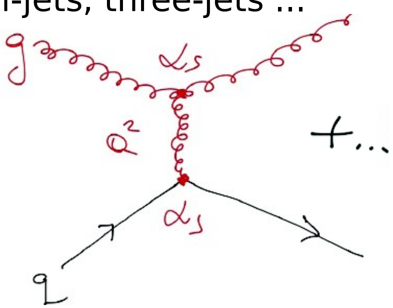
→ PDFs and α_s depend on scale of the process → P_T of the jet

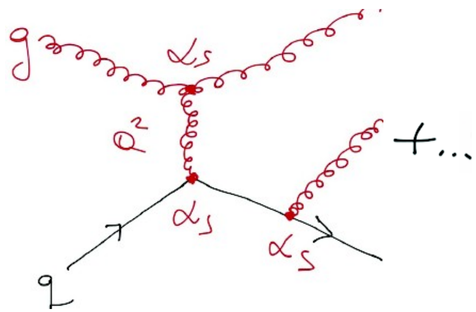
Inclusive jet production at LO



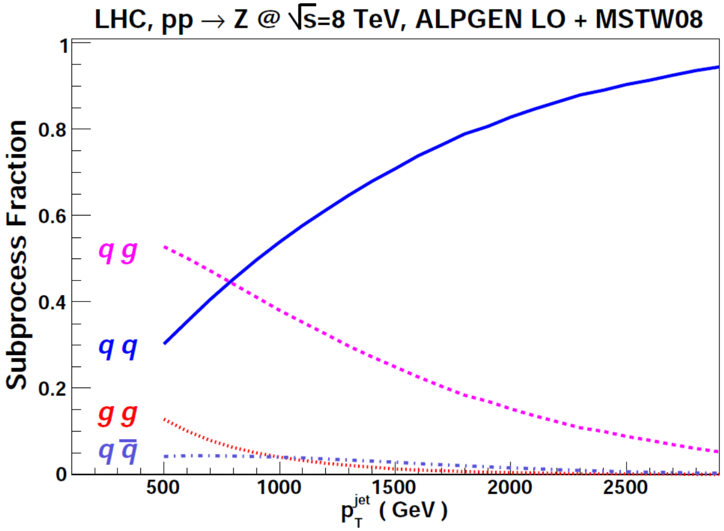
(G. Dissertori)

di-jets, three-jets ...





JHEP 08 (2012) 101



... and ratios (smart way of canceling large part of e.g. jet scale uncertainty)

→ LHC jet data provide constrains in high-x region

→ at high scales may reveal new physics (depend how well gluon at high x is known)

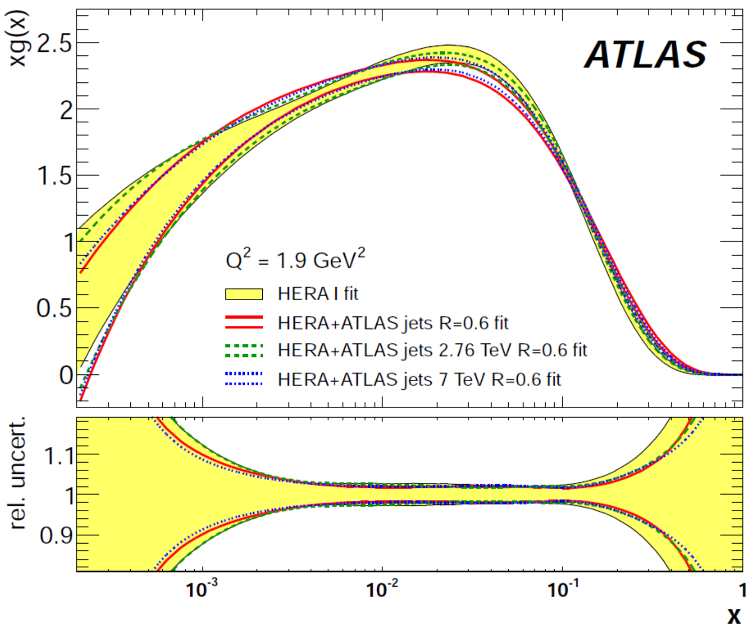
Inclusive Jet Production

Inclusive jet measurements and QCD analysis of LHC data

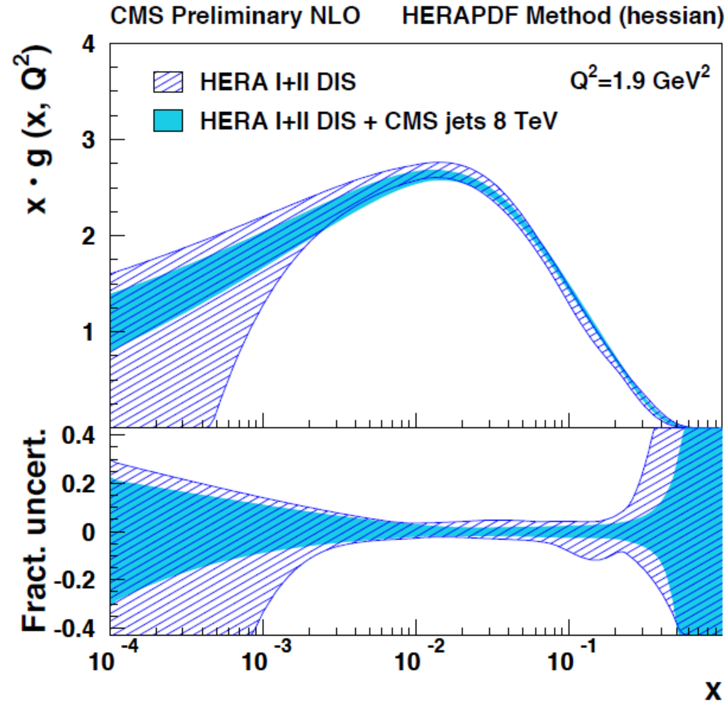
QCD analyses at NLO with (HERA and) inclusive jet data performed by

ATLAS (2.7 TeV and 7 TeV data) and CMS (8 TeV data, 19.7 fb^{-1})

EPJC (2013) 73 2509



arXiv:1609.05331



→ jet data can help to improve gluon distribution function in high-x region and provides possibility to extract strong coupling constant α_s

Inclusive Jet Production (13 TeV)

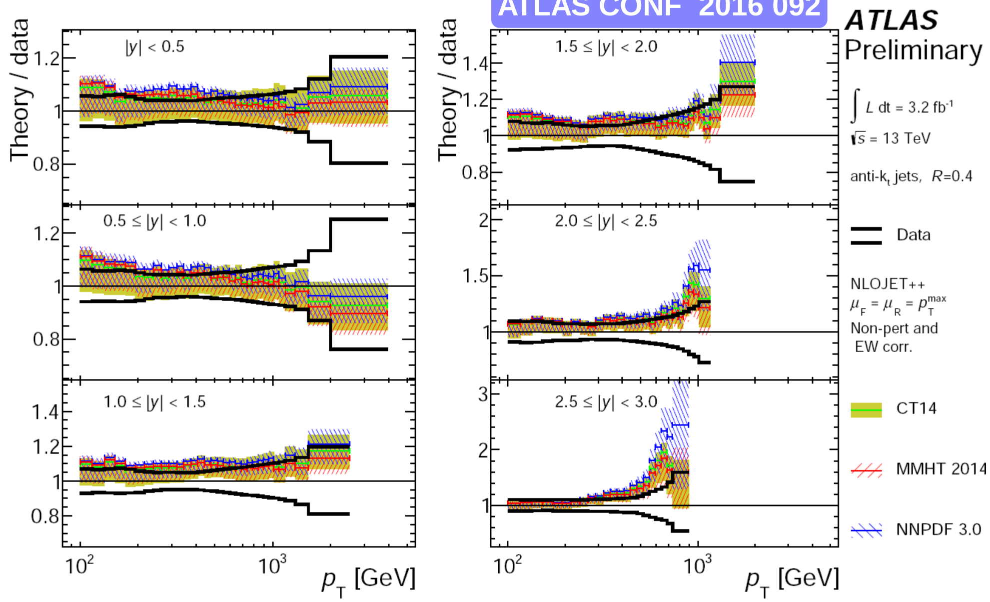
Latest inclusive jet measurements at 13 TeV from ATLAS and CMS

→ compared to predictions at NLO QCD (corrected for non-perturbative and EW effects) using different PDFs

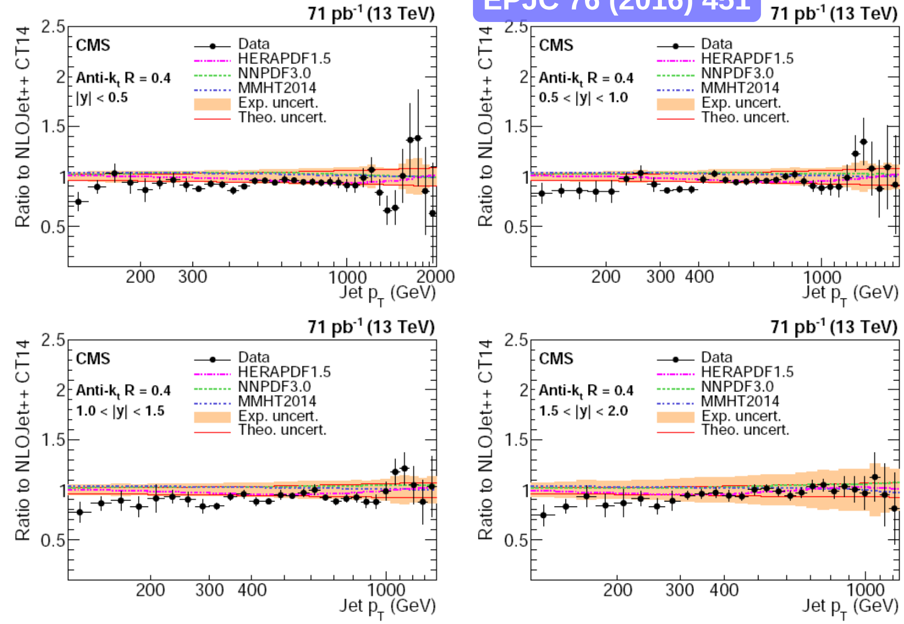
→ dominant systematic uncertainty: jet energy calibration, luminosity

→ NNLO accuracy for theory predictions is needed

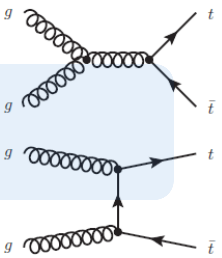
ATLAS CONF 2016 092



EPJC 76 (2016) 451



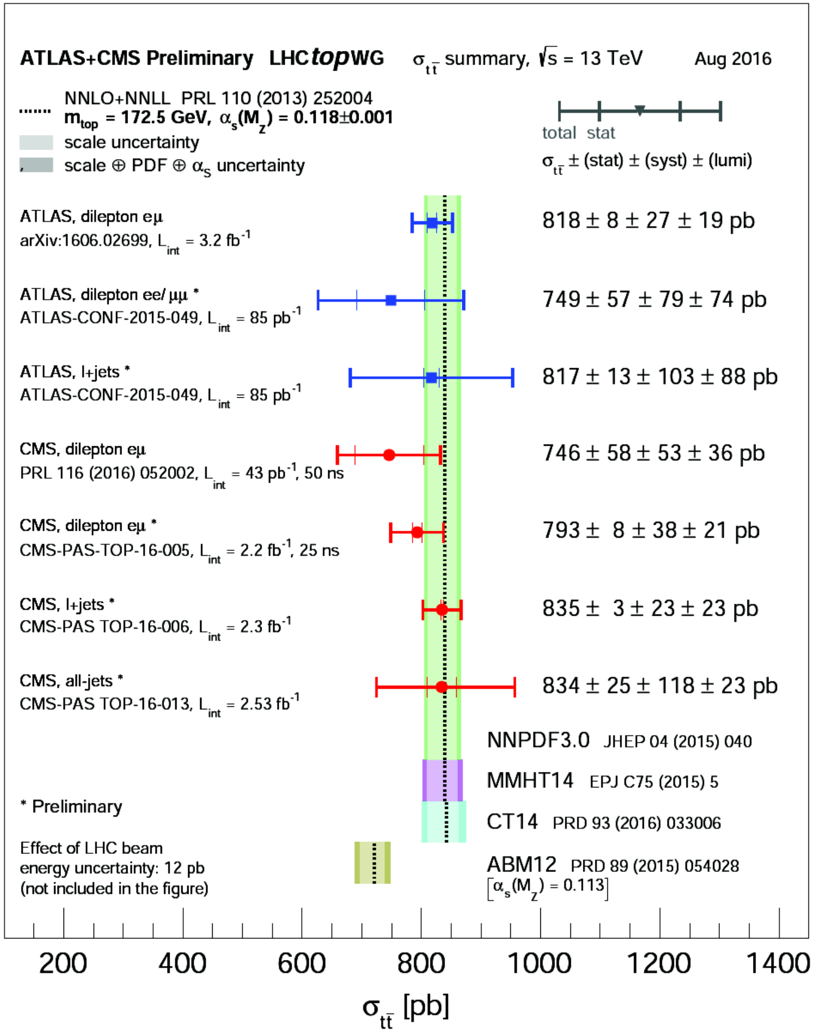
Top Quark at LHC



LHC provides possibility for high statistics of top quark pairs ($t\bar{t}$) to be measured (gluon-gluon fusion is a dominant sub-process)

→ tests of pertubative QCD, sensitive to new physics effects

→ probe of high-x gluon (high correlation between gluon, α_s and top quark mass)

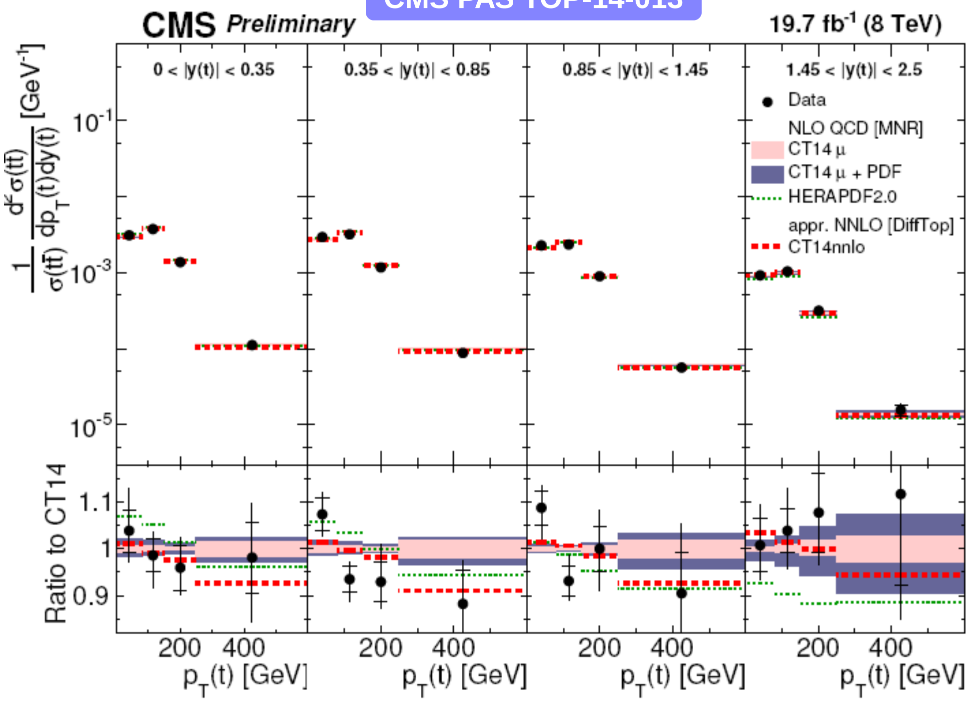


Many new single-top and ttbar measurements at $\sqrt{s}$ = 5, 7, 8 and 13 TeV from ATLAS and CMS

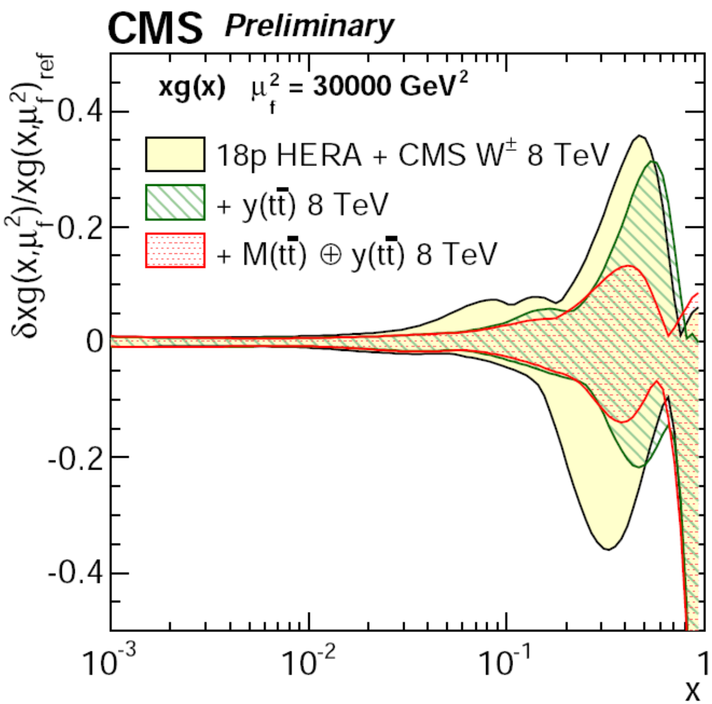
Top Quark at LHC: impact on PDFs

Differential $t\bar{t}$ production cross sections (dilepton $e^{\pm}\mu^{\mp}$ decay mode) from CMS at $\sqrt{s}$ = 8 TeV, 19.7 fb^{-1}

CMS PAS TOP-14-013



PDF constraints (with approx NNLO theory)



→ measured in the full phase space as a function of six different combinations of variables *(p_T(t),y(t); y(t),M(tt); y(tt),M(tt); Δη(t,t),M(tt); p_T(tt),M(tt); Δϕ(t,t),M(tt))*

→ significant impact on the gluon distributions at large values of x is observed, in particular when the distribution of *y(tt)* in the *M(tt)* ranges

Impact of LHCb Heavy Flavour Data to PDFs

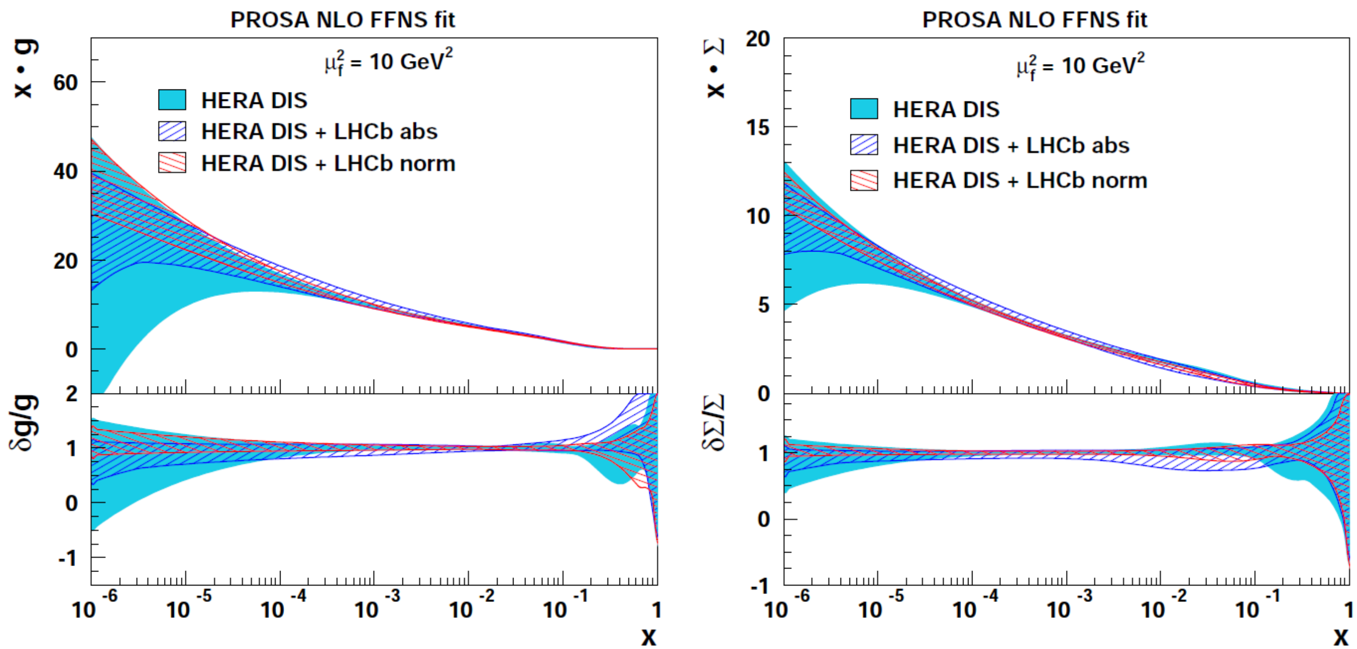
LHCb heavy-flavour data impose additional constraints on the gluon and the sea-quark distributions at low x

→ first time used to constrain PDFs   Nucl.Phys.B871 (2013)   JHEP 08 (2013) 117

→ NLO QCD analysis (together with HERA data) with the fixed-flavour number scheme

→ absolute and normalised cross sections

EPJC 75 (2015) 8, 396



→ significant reduction of the gluon uncertainty at very low x ($x \sim 5 \times 10^{-6}$)

Summary and Outlook

LHC data provide important constraints to PDFs

→ a wealth of precise LHC measurements (2.76, 5, 7, 8 and first 13 TeV) available, many of which are already used in different PDF fits

	CT14	MMHT14	NNPDF3.0	HERAPDF2.0	ABM12(ABMP)	CJ12(15)	JR14
HERA data	HERA I+ charm	HERA I charm jets	HERA I+ H1 and ZEUS II charm	HERA I+II	HERA I charm	HERA I	HERA I charm jets
Fix. Target DIS	✓	✓	✓	✗	✓	JLAB, high x ✓	JLAB, high x ✓
Tevatron W,Z	✓	✓	✓	✗	✗/✓	✓	✗
Tevatron Jets	✓	✓	✓	✗	✗	✗	✓
Fix. Target DY	✓	✓	✓	✗	✓	✓	✓
LHC WZ	✓	✓	✓	✗	✓	✗	✗
LHC jets	✓	✓	✓	✗	✗	✗	✗
LHC top	✗	✓	✓	✗	✓	✗	✗
LHC charm	✗	✗	✓	✗	✗/✓	✗	✗
References	arXiv:1506.07443	arXiv:1412.3989	arXiv:1410.8849	arXiv:1506.06042	arXiv:1310.3059	arXiv:1212.1702	arXiv:1403.1852

V. Radescu, QCD@LHC16

→ LHC data sensitivity studies within the global PDF fit see P. Nadolsky, L. Harland-Lang, L. Del Debbio's talks

Summary and Outlook

LHC data provide important constraints to PDFs

→ a wealth of precise LHC measurements (2.76, 5, 7, 8 and first 13 TeV) available, many of which are already used in different PDF fits

→ also can probe other QCD and EW aspects (e.g. boson polarisation, A_{FB} asymmetry, angular coefficients, MC tuning, ..)

→ only a subset of available data sensitive to PDFs were shown

(not included e.g. V+jets, multiple jet data, inclusive photon, proton-lead data, ..)

→ additional details in F. Giuli, D. Hai Nguyen's and other parallel session talks

→ most of PDF sensitivity studies at LHC were obtained in comparison with HERA data alone (expected lesser effect in a global PDF fit)

→ most studies performed by experiments were done with xFitter (*A. Glazov's talk*)

Many new LHC measurements still to come, more precision, further improvements on PDFs

Back-up slides

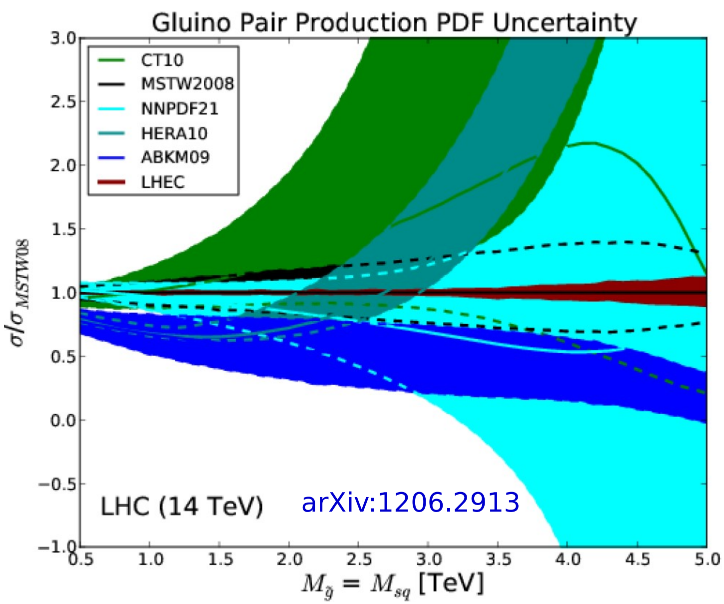
Parton Distribution Functions

Parton Distribution Functions (PDFs) are of crucial for precision physics at hadron colliders because:

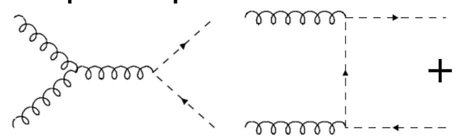
→ PDFs limit **the accuracy of the SM predictions** (including Higgs, W mass)

→ **reach of new physics** searches depends on PDF knowledge at high Bjorken-x

For example, the production of SUSY colored particles (squarks and gluinos) are sensitive to gluon at high $x=2m_X/\sqrt{s} \sim 0.2$ -0.7

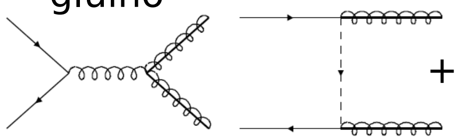


squark production



\+ ...

gluino



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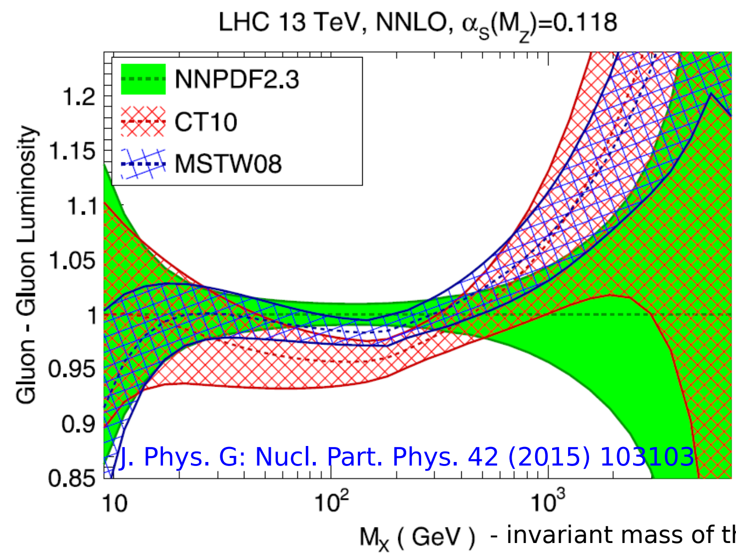


- invariant mass of the final state

Recent CMS DY Measurements (8 and 13 TeV)

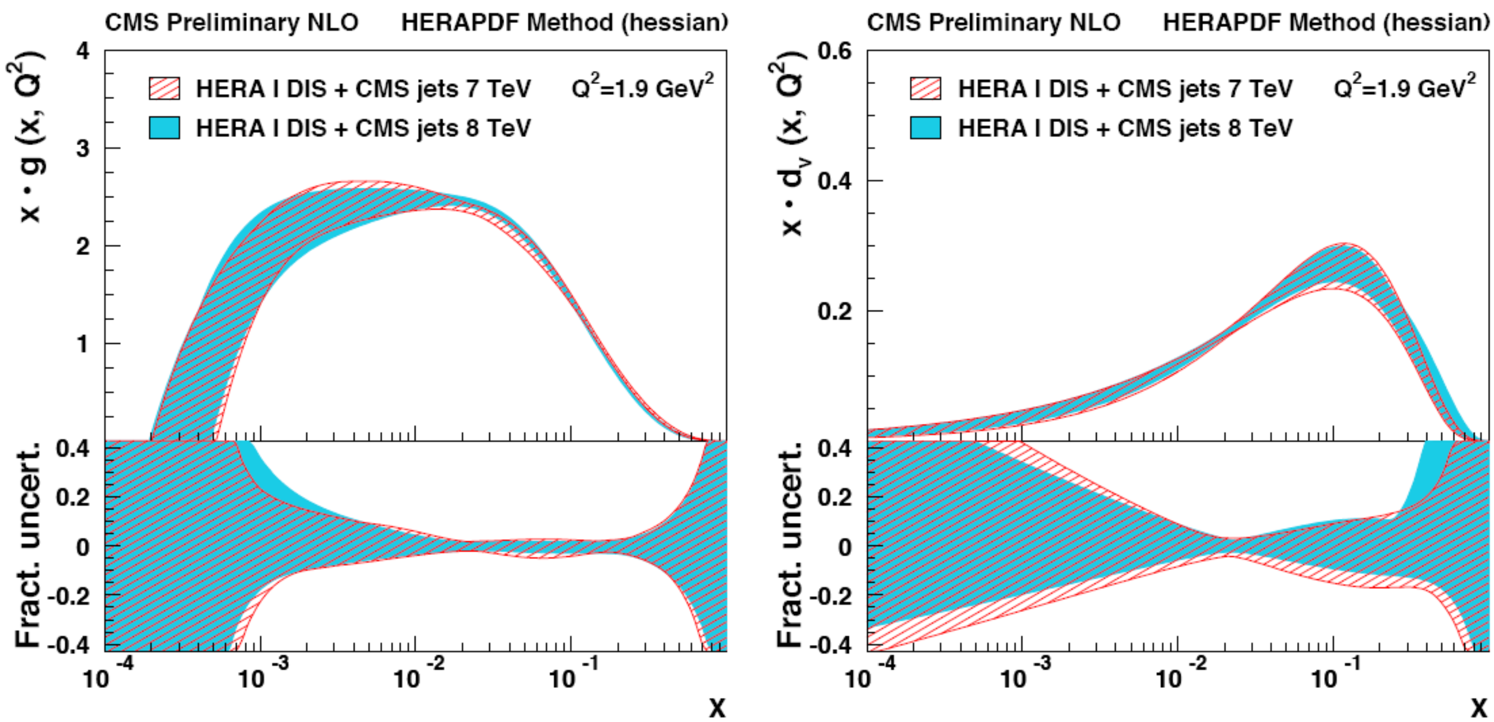
- **Differential cross section and charge asymmetry for pp → WX production at $\sqrt{s}$ = 8 TeV**
  arXiv:1603.01803
- **Z + charm production in pp collisions at $\sqrt{s}$ = 8 TeV**
  CMS PAS SMP-15-009
- **W boson production cross section in association with two b jets at $\sqrt{s}$ = 8 TeV**
  CMS PAS SMP-14-020
- **Inclusive W and Z boson production cross sections at $\sqrt{s}$ = 13 TeV**
  CMS PAS SMP-15-004
- **Inclusive and differential Z boson production cross sections ($d\sigma/dy$, $d\sigma/dp_T$) at $\sqrt{s}$ = 13 TeV**
  CMS PAS SMP-15-011
- Differential Drell-Yan cross section ($d\sigma/dm$) in pp collisions $\sqrt{s}$=13 TeV
  CMS PAS SMP-16-009
- Transverse momentum spectra of weak vector bosons ($d\sigma/dp_T$) at $\sqrt{s}$ = 8 TeV
  arXiv:1606.05864
- Differential cross section for the production of a W ($\to\mu\nu$) boson in association with jets at $\sqrt{s}$ = 13 TeV
  CMS PAS SMP-16-005
- Differential cross section of W ($\to\mu\nu$) boson in association with jets at $\sqrt{s}$ = 8 TeV
  CMS PAS SMP-14-023

Inclusive Jet Production at CMS

Inclusive jet measurements and QCD analysis of LHC data

QCD analyses at NLO with (HERA and) inclusive jet data performed by

arXiv:1609.05331 CMS (8 TeV data, 19.7 fb^{-1})



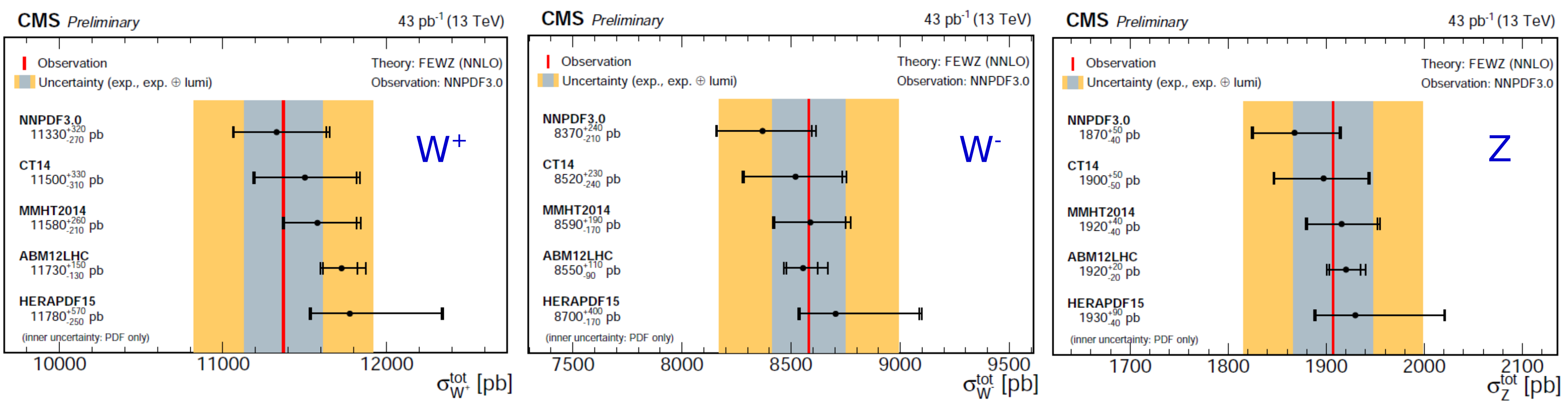
→ alternative fit reproducing 7 TeV QCD analysis settings (replacing 7 TeV by 8 TeV data), similar constraints on gluon distribution are observed

CMS W and Z cross sections at 13 TeV

New 13 TeV measurement of W and Z boson cross sections with $\int L = 43 \pm 2\ \mathrm{pb}^{-1}$

→ dominant systematic uncertainty: luminosity

CMS PAS SMP-15-004



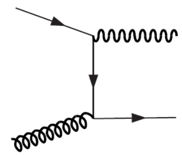
Compared with FEWZ (NNLO QCD and NLO order EW calculations) with various PDFs:

	NNPDF3.0	CT14	MMHT2014	ABM12LHC	HERAPDF15
σ^{tot}_{W+} [pb]	11330^{+320}_{-270}	11500^{+330}_{-310}	11580^{+260}_{-210}	11730^{+150}_{-130}	11780^{+570}_{-250}
σ^{tot}_{W-} [pb]	8370^{+240}_{-210}	8520^{+230}_{-240}	8590^{+190}_{-170}	8550^{+110}_{-90}	8700^{+400}_{-170}
σ^{tot}_{W} [pb]	19700^{+560}_{-470}	20020^{+560}_{-550}	20170^{+430}_{-390}	20280^{+260}_{-220}	20480^{+960}_{-410}
σ^{tot}_{Z} [pb]	1870^{+50}_{-40}	1900^{+50}_{-50}	1920^{+40}_{-40}	1920^{+20}_{-20}	1930^{+90}_{-40}
$\sigma^{tot}_{W+} / \sigma^{tot}_{W-}$	$1.354^{+0.011}_{-0.012}$	$1.350^{+0.014}_{-0.014}$	$1.348^{+0.011}_{-0.008}$	$1.371^{+0.003}_{-0.004}$	$1.353^{+0.014}_{-0.013}$
$\sigma^{tot}_{W+} / \sigma^{tot}_{Z}$	$6.06^{+0.04}_{-0.05}$	$6.06^{+0.06}_{-0.06}$	$6.04^{+0.05}_{-0.05}$	$6.11^{+0.02}_{-0.01}$	$6.10^{+0.06}_{-0.06}$
$\sigma^{tot}_{W-} / \sigma^{tot}_{Z}$	$4.48^{+0.03}_{-0.02}$	$4.49^{+0.03}_{-0.03}$	$4.48^{+0.03}_{-0.04}$	$4.46^{+0.02}_{-0.01}$	$4.51^{+0.04}_{-0.03}$
$\sigma^{tot}_{W} / \sigma^{tot}_{Z}$	$10.55^{+0.07}_{-0.06}$	$10.55^{+0.09}_{-0.09}$	$10.53^{+0.08}_{-0.09}$	$10.56^{+0.04}_{-0.02}$	$10.61^{+0.11}_{-0.09}$

Prompt Photon Production at LHC

Prompt photon data at LHC is sensitive to gluon content at high x

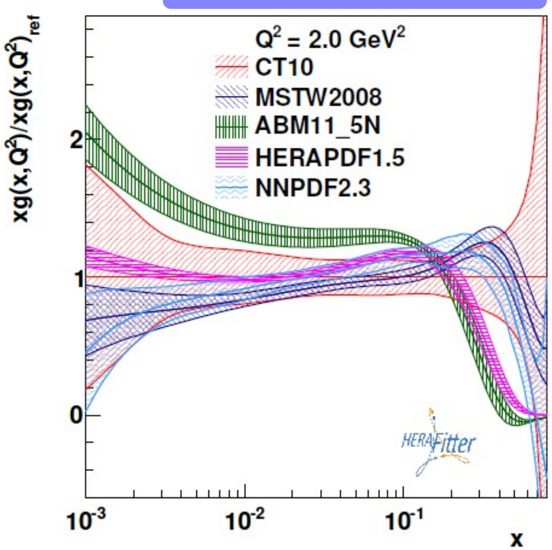
→ dominantly via Compton-like process $qg \rightarrow q\gamma$



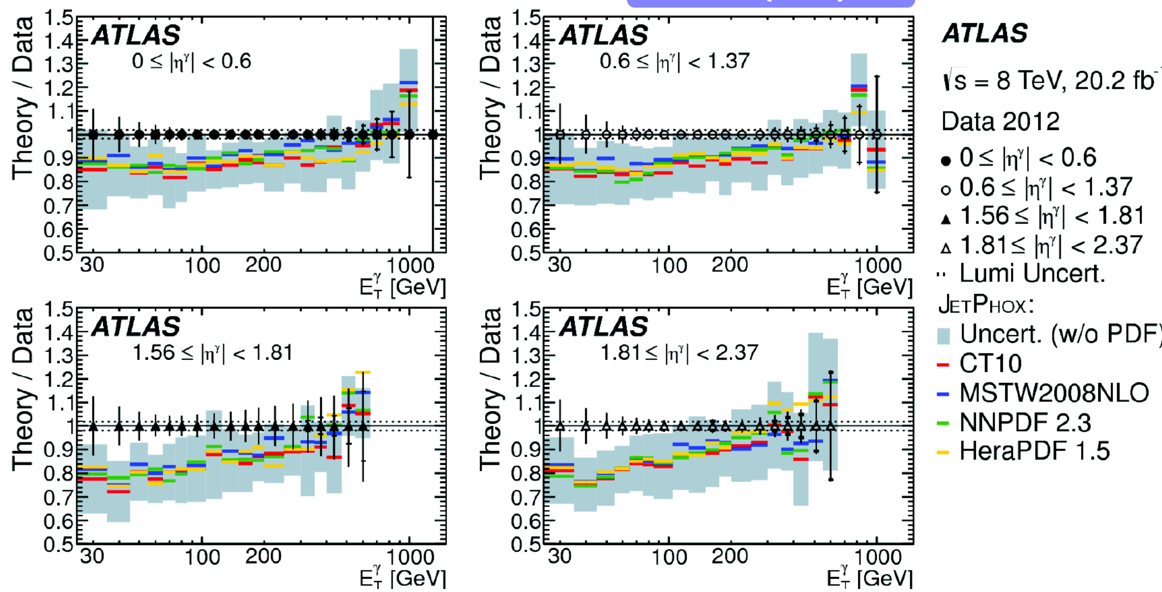
ATLAS study of the inclusive photon data sensitivity to parton distributions

→ quantitative data to theory assessment (χ^2)

ATL-PHYS-PUB-2013-018



JHEP 06 (2016) 005



→ about half smaller uncertainties in new 8 TeV measurement

→ data show potential to improve gluon distribution

CMS W+$b\bar{b}$ measurement at 8 TeV

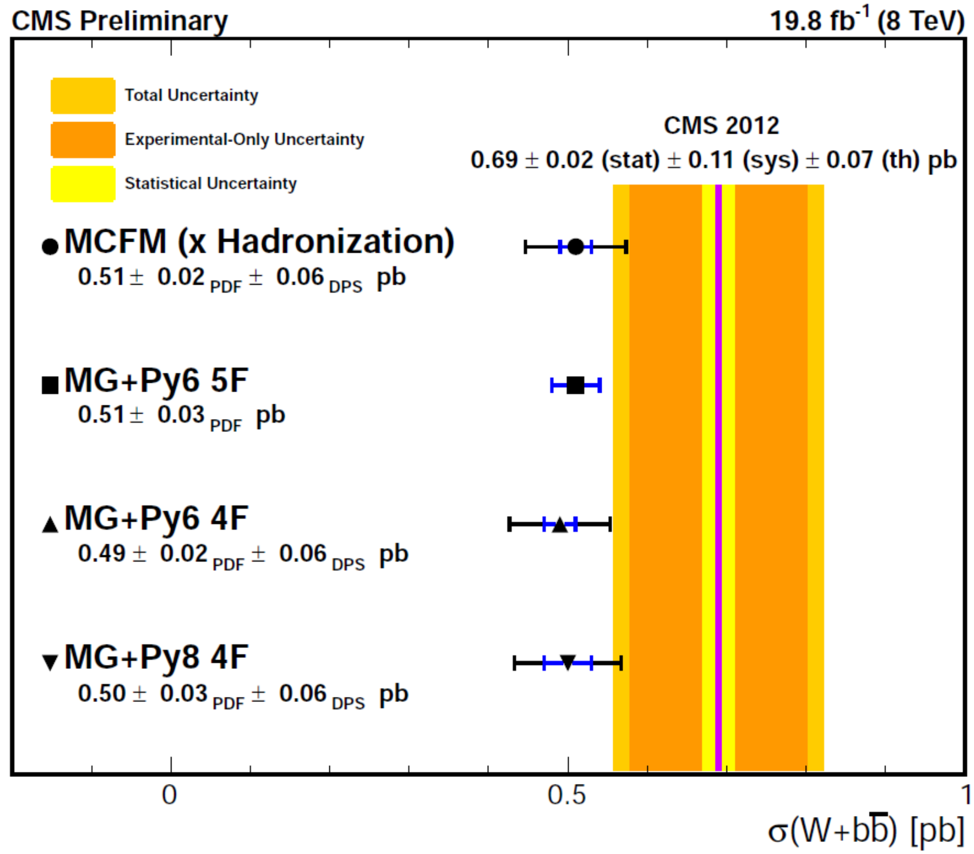
CMS PAS SMP-14-020

Extension of earlier W+$b\bar{b}$ measurement (with muons) with both, muon and electron, W decay channels and $\int L = 19.8\ fb^{-1}$

→ analysis with $p^l_T > 30$ GeV, $|\eta^l| < 2.1$ and b-tagged jets $p_T > 25$ GeV and $|\eta| < 2.4$

Comparison with predictions (hadron level) including the estimated hadronization and double parton scattering (DPS) corrections

→ agree within 1 standard deviation



→ useful to test PDFs with different number of flavours

CMS differential Z measurement at 13 TeV

CMS PAS SMP-15-011

Inclusive and differential Z production at 13 TeV in the μ final state with ∫L =2.3 fb^{-1}

Inclusive cross section (in the dilepton mass range of 60 to 120 GeV) and comparison with NNLO predictions (FEWZ) using different PDFs

$$\sigma(\mathrm{pp} \to \mathrm{ZX}) \times \mathcal{B}(\mathrm{Z} \to \ell^+\ell^-) = 1870 \pm 2\ (\mathrm{stat}) \pm 35(\mathrm{syst}) \pm 51\ (\mathrm{lumi})\ \mathrm{pb}$$

	σ_Z^{tot} [pb]
NNPDF3.0	1870^{+50}_{-40}
CT14	1900^{+50}_{-50}
MMHT2014	1920^{+40}_{-40}
ABM12LHC	1920^{+20}_{-20}
HERAPDF15	1930^{+90}_{-40}

Inclusive and **differential** Z production at 13 TeV in the μ final state with ∫L =2.3 fb^{-1}

→ measurement as a function of p_T, angular variable φ*, $y^{\mu+\mu-}$ *and* $p_T^{\mu+\mu-}$

→ quark-gluon scattering dominates at high p_T (low p_T range is governed by ISR and the transverse momentum of the initial-state parton inside the proton)

→ angular variable φ* is expressed via pseudo-rapidity of muon pair

$$\phi^*_\eta = \tan\left(\frac{\pi - \Delta\phi}{2}\right) \cdot \sin(\theta^*_\eta) \qquad \cos(\theta^*_\eta) = \tanh\left(\frac{\eta^- - \eta^+}{2}\right)$$

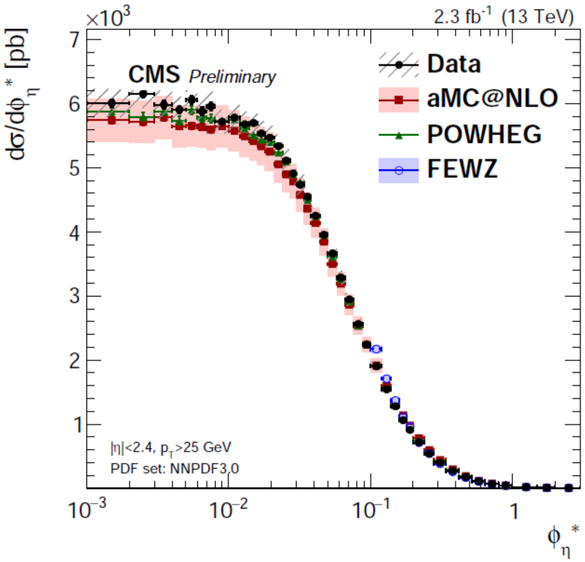
probes Z boson p_T but depends on direction of muon → smaller exp. uncertainty

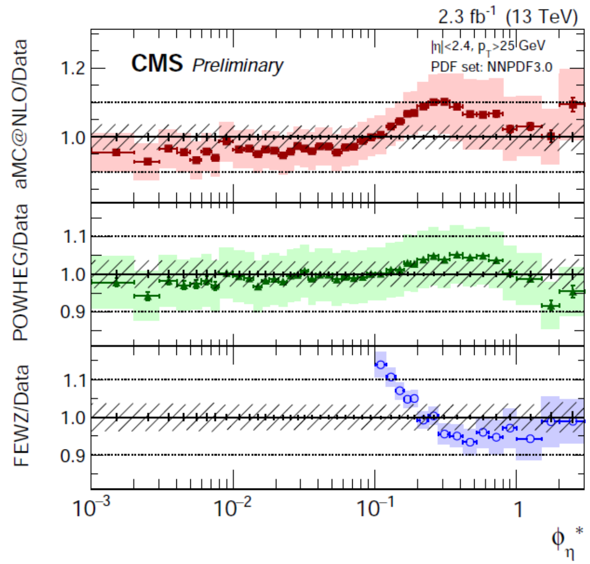
CMS differential Z measurement at 13 TeV

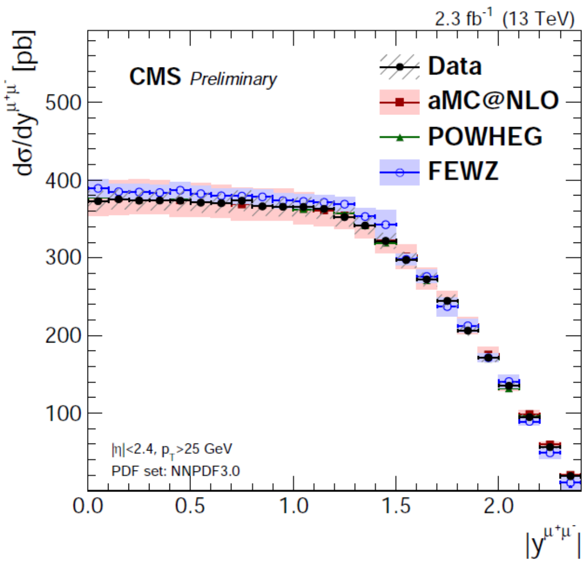
CMS PAS SMP-15-011

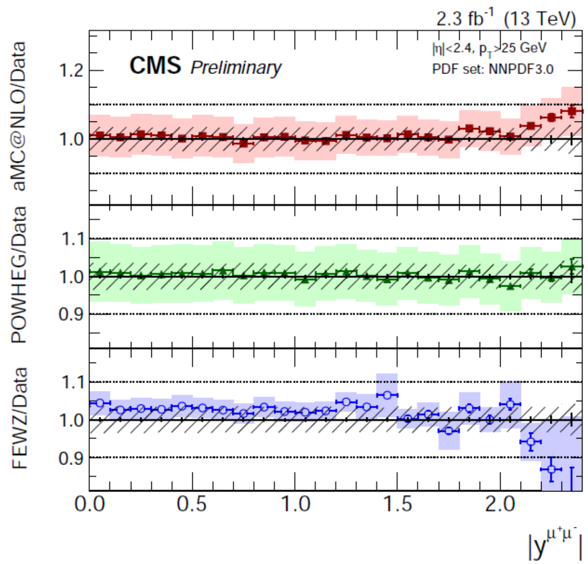
Data compared to MADGRAPH5_AMC@NLO, POWHEG and FEWZ

→ no generator is able to describe the data in all of the studied phase-space







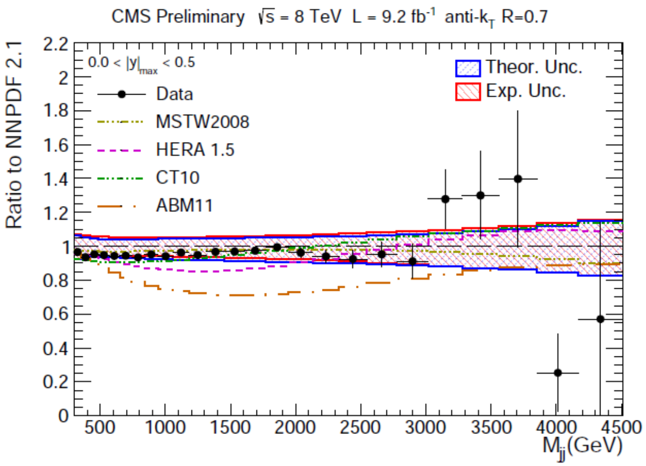


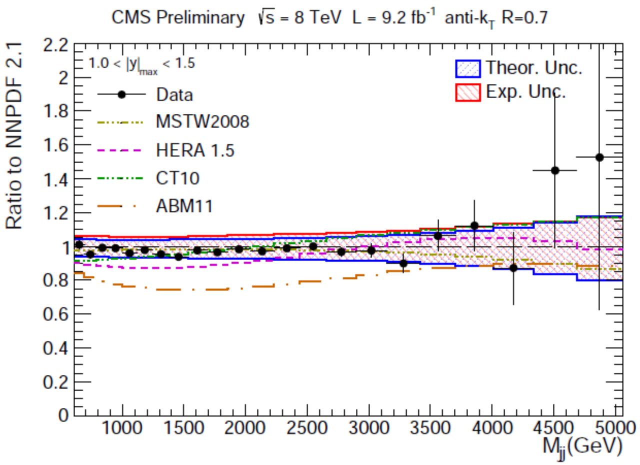
Di- and three-jet Measurements

Recent di-jet and three-jet measurements from ATLAS and CMS

→ comparison with different PDFs: some tension with e.g. ABM11 PDF observed

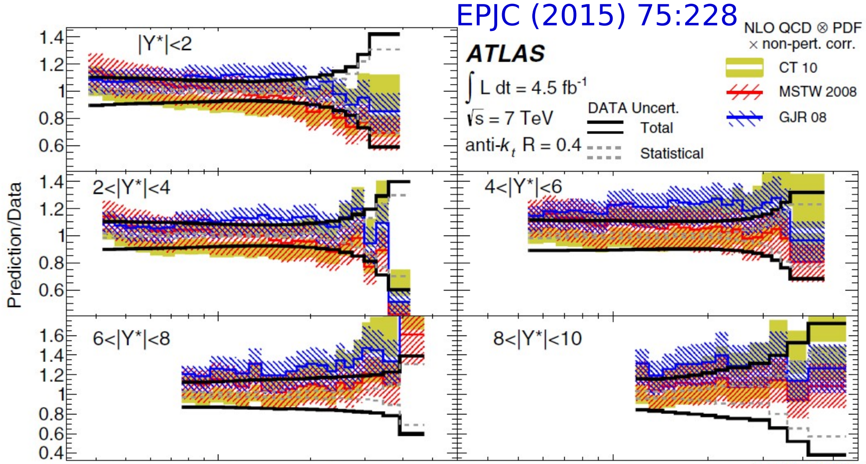
Di-jet CMS PAS SMP 14-002

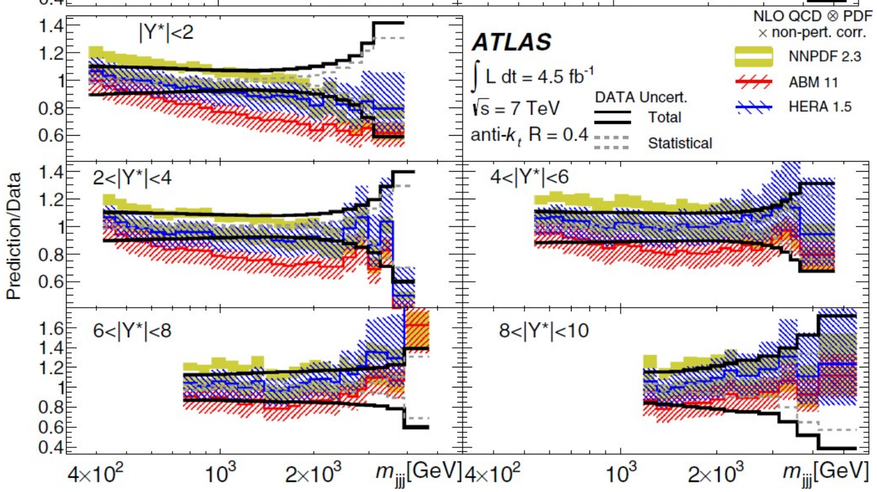




Three-jet (probe different phase space due to different combination of the initial-state partons)

EPJC (2015) 75:228





Heavy Quark Treatment in PDFs

QCD factorisation:

measured cross section =

PDF — hard-scattering ME

$$\sigma(\alpha_s,\mu_R^2,\mu_F^2)=\sum_{a,b}\int_0^1 f_a(x_1,\mu_F^2)f_b(x_2,\mu_F^2)\,\hat{\sigma}(x_1,x_2;\alpha_s,\mu_R^2,\mu_F^2)+\ldots$$

a, b - partons in the proton (g, q ,qbar) of different flavours

- if #flavours fixed: **Fixed Flavour Number Scheme (FFNS)**

  only light flavours in the proton: i = 3 (4)
  c- (b-) quarks massive, produced in boson-gluon fusion,
  $Q^2 \gg m_{HQ}^2$: can be less precise, NLO coefficients contain terms ~ $\ln(Q/m_{HQ})$

- if #flavours variable: **Variable Flavour Number Scheme (VFNS)**

  - Zero Mass VFNS: all flavours massless. Breaks down at $Q^2 \sim m_{HQ}^2$
  - Generalized Mass VFNS: different implementations provided by PDF groups, smooth matching with FFNS for $Q^2 \to m_{HQ}^2$ must be assured

→ m_c is a parameter (M_C)

treatment of heavy quarks is important in PDFs

PDF Tools

Often perturbative higher-order calculations are extremely time consuming
→ not possible to include into PDF fits

solution: fast grid techniques

→ based on assumption that PDF can be approximated by a set of the interpolation functions

→ after first time (full) calculation, technique with interpolation functions can be used for the fast theory prediction calculations (for any PDF)

Currently available tools: FastNLO Eur.Phys.J. C19 , 289 (2001), hepph/0609285

and APPLGRID hepph/0510324, arXiv:0911.2985

PDFs ($f_{a/h}$) approximated by linear combination of the eigenfunctions $E^{(i)}$:

$$f_{a/h}(x) \simeq \sum_i f_{a/h}(x_i)\ E^{(i)}(x)$$

M. Wobisch, DESY-THESIS-2000-049