LHC Higgs Boson Production at N3LO

XIIth Recontres du Vietnam

Precise Theory for Precise Experiments

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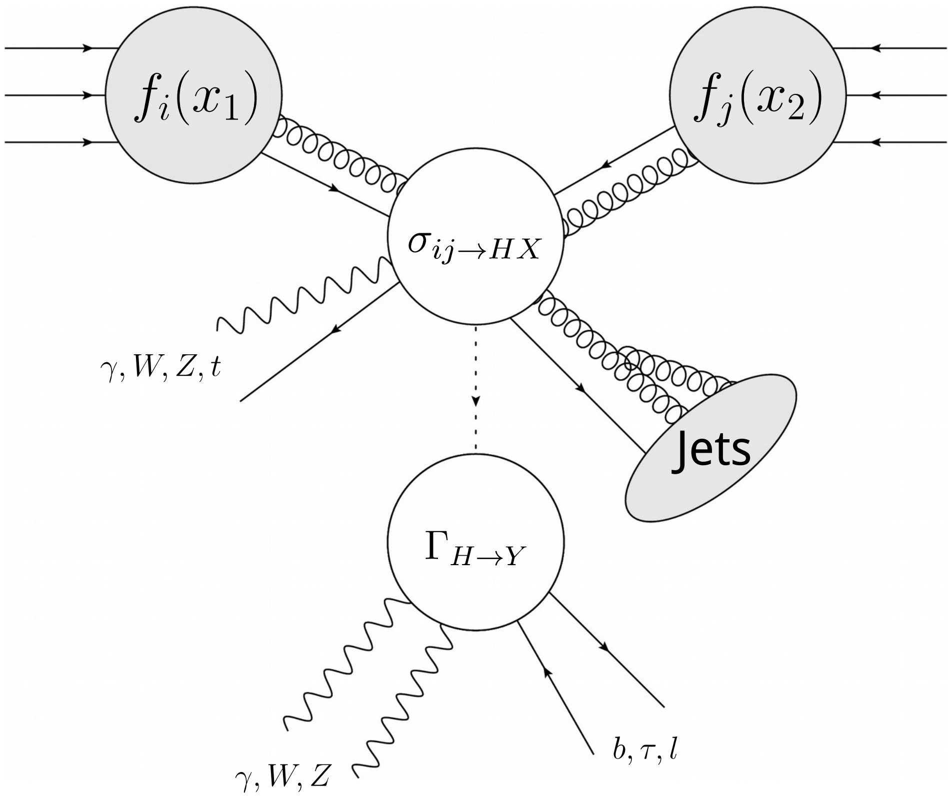
Higgs Production in Theory

Protons collide:

Higgs is produced:

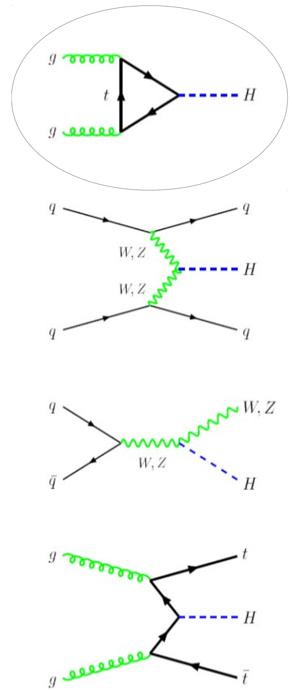
Higgs decays:

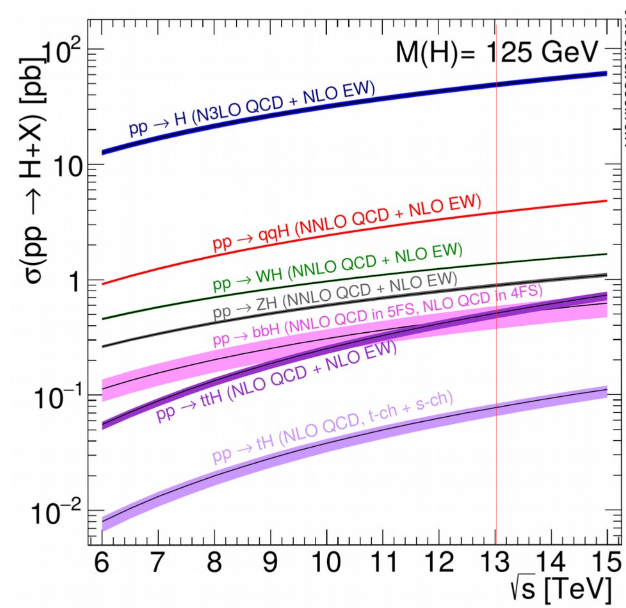


Master Formula:

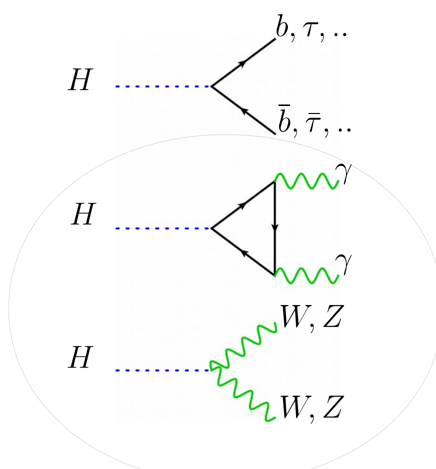
$$\sigma_{PP\to HX\to Y}[J] = \sum_{ij} \int_J f_i(x_1)\, f_j(x_2)\, d\sigma_{ij\to HX}(Sx_1x_2, m_H, ..)\, d\Gamma_{H\to Y}(m_H, ..)$$

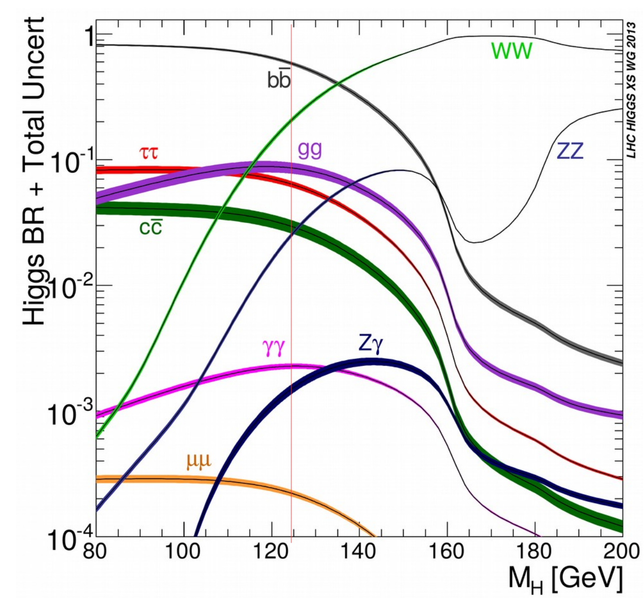
Production



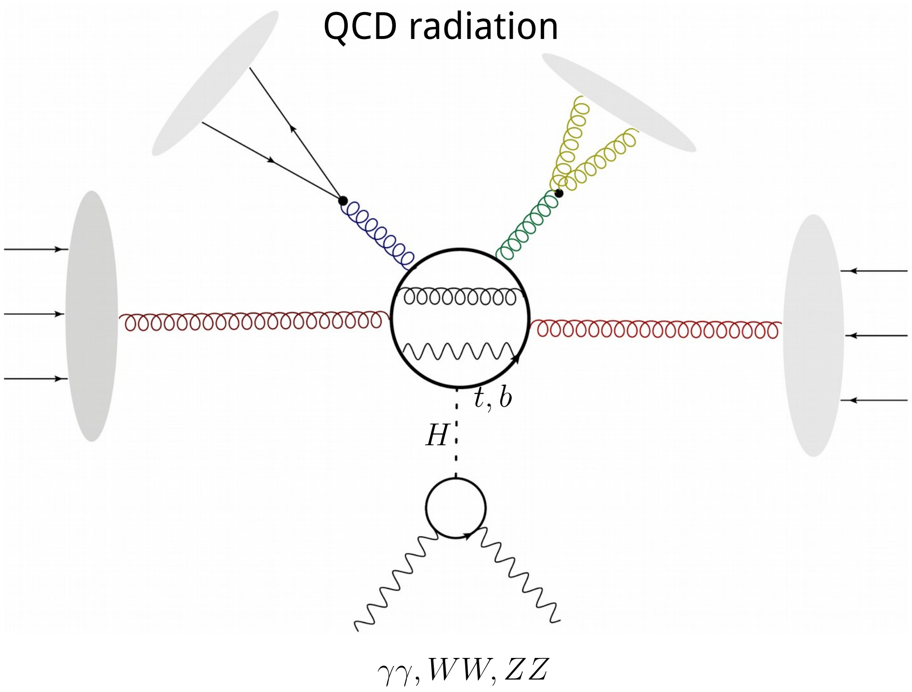


Decays



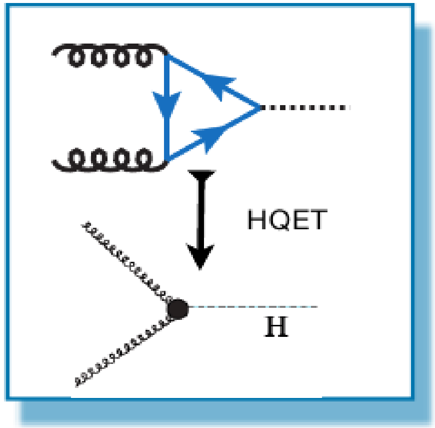


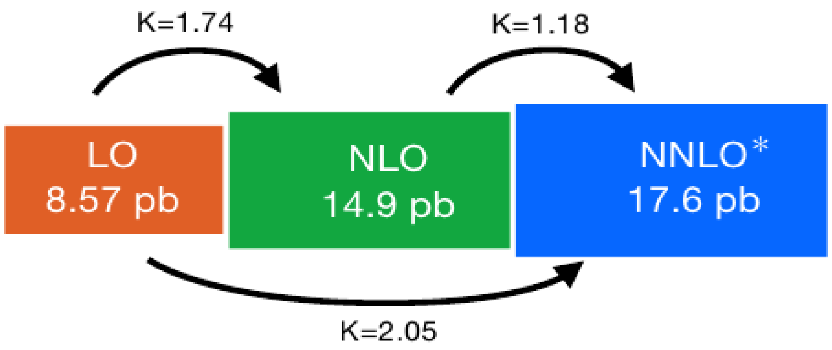
Gluon Fusion



- Cross Section is large enough to afford photonic and leptonic decay channels
- Total partonic Cross Section and decay are theoretically clean in fixed order perturbation theory
- Transverse momentum, energy, distributions and jet tagging require resummation at low scales (normalized by total cross section)

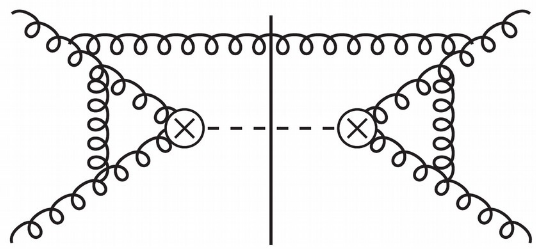
Gluon Fusion: Perturbative Growth





QCD corrections Convergence is rather slow!
N3LO is needed.
EW corrections only of order 5%

Status at N3LO

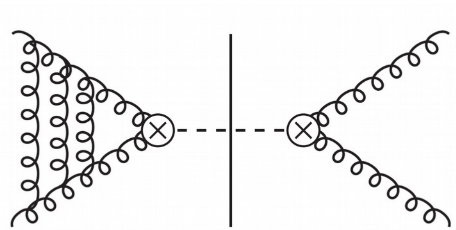


Real-Virtual Squared

Known [Anastasiou, Duhr, Dulat, FH, Mistlberger; Kilgore]

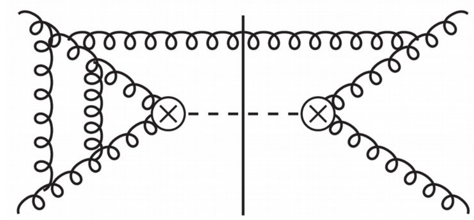
+UV and IR counter terms

Known[Pak, Rogal, Steinhauser; Anastasiou, Buehler, Duhr, FH; Höschele, Hoff, Pak, Steinhauser, Ueda; Buehler, Lazopoulos]



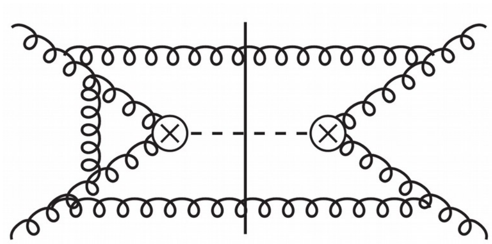
Triple Virtual

Known from QCD Form Factor
[Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]



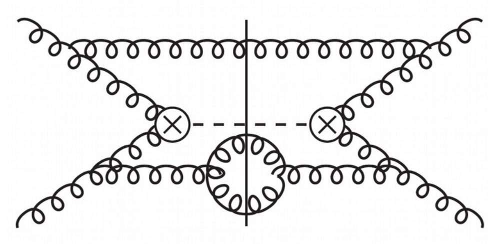
Double Virtual- Real

Known [Dulat, Mistlberger; Duhr, Gehrmann]



Double Real - Virtual

2 terms in soft expansion [Anastasiou, Duhr, Dulat, FH, Mistlberger,Furlan; Li, Mantueffel, Schabinger, Zhu]
37 terms [Anastasiou, Duhr, Dulat, FH, Mistlberger]
Known [to be published]



Triple Real

2 terms in soft expansion [Anastasiou, Duhr, Dulat, Mistlberger; Zhu]
37 terms [Anastasiou, Duhr, Dulat, FH, Mistlberger]

qq` channel known [Chihaya Anzai, Alexander Hasselhuhn, Maik Hoschele, Jens Hoff, William Kilgore, Matthias Steinhauser, Takahiro Ueda]

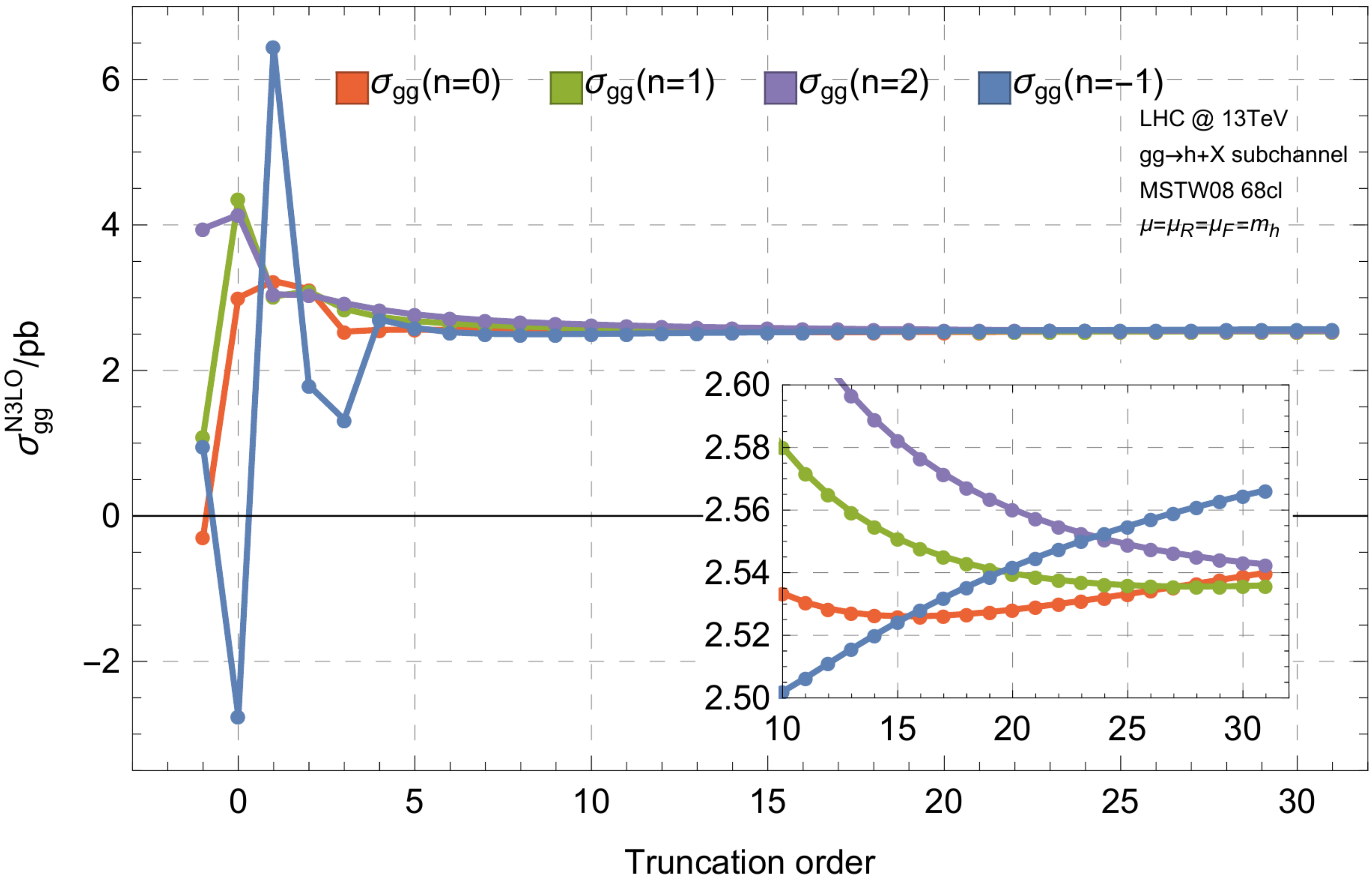
Why was it only possible last year?

- Have used all the tricks in the box and invented new ones:
  - Reverse Unitarity
  - Differential equations
  - Mellin Barnes Representations
  - Hopf Algebra of Generalized Polylogs
  - Number Theory
  - Soft Expansion by Region
  - Optimised Algorithm for IBP reduction and powerful computing resources

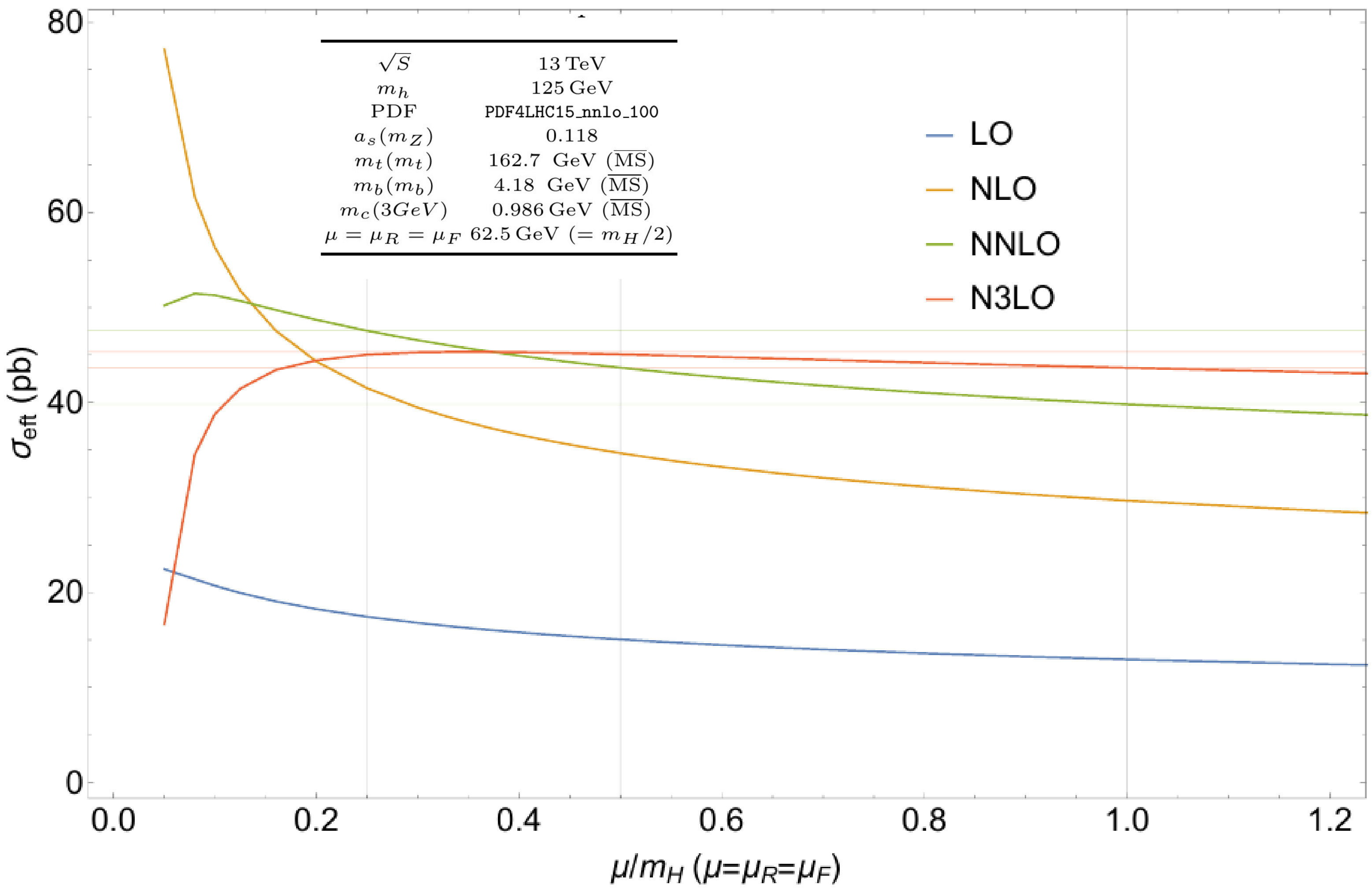
Integral Statistics

	NNLO	N3LO
#diagrams	~1.000	~100.000
#integrals	~50.000	517.531.178
#masters	27	1.028
#soft masters	5	78

Convergence of Soft Expansion



Scale Variation at N3LO



Sources of Uncertainty

Factorisation Master Formula:

$$\sigma_{PP\to X}[J] = \sum_{ij} \int dx_1 dx_2\, f_i(x_1, \mu_F) f_j(x_2, \mu_F) \times d\sigma_{ij\to X}(x_1, x_2, \mu_R, \mu_F, \{\alpha\}, \{p\}) \times J(\{p\}) \left(1 + \mathcal{O}(\Lambda_{QCD}/Q)\right)$$

Gives rise to several sources of uncertainties:

- **PDF Uncertainties:**
  Due to our limited knowledge of the Parton Distributions
- **Couplings and Masses:**
  Due to our limited knowledge of couplings and masses
- **Perturbative Uncertainties:**
  Due to our limited ability to calculate EW and QCD perturbative Corrections
- **Non-perturbative Uncertainties:**
  Breakdown of Series in special kinematic regions → Resummation
  Factorisation breaking?

Our Determination of the Uncertainty for SM Higgs at 13TeV LHC

$\delta(\text{scale})$	$\delta(\text{trunc})$	$\delta(\text{PDF-TH})$	$\delta(\text{EW})$	$\delta(t,b,c)$	$\delta(1/m_t)$
$^{+0.10\text{ pb}}_{-1.15\text{ pb}}$	± 0.18 pb	± 0.56 pb	± 0.49 pb	± 0.40 pb	± 0.49 pb
$^{+0.21\%}_{-2.37\%}$	$\pm 0.37\%$	$\pm 1.16\%$	$\pm 1\%$	$\pm 0.83\%$	$\pm 1\%$

[arxiv: 1602.00695]

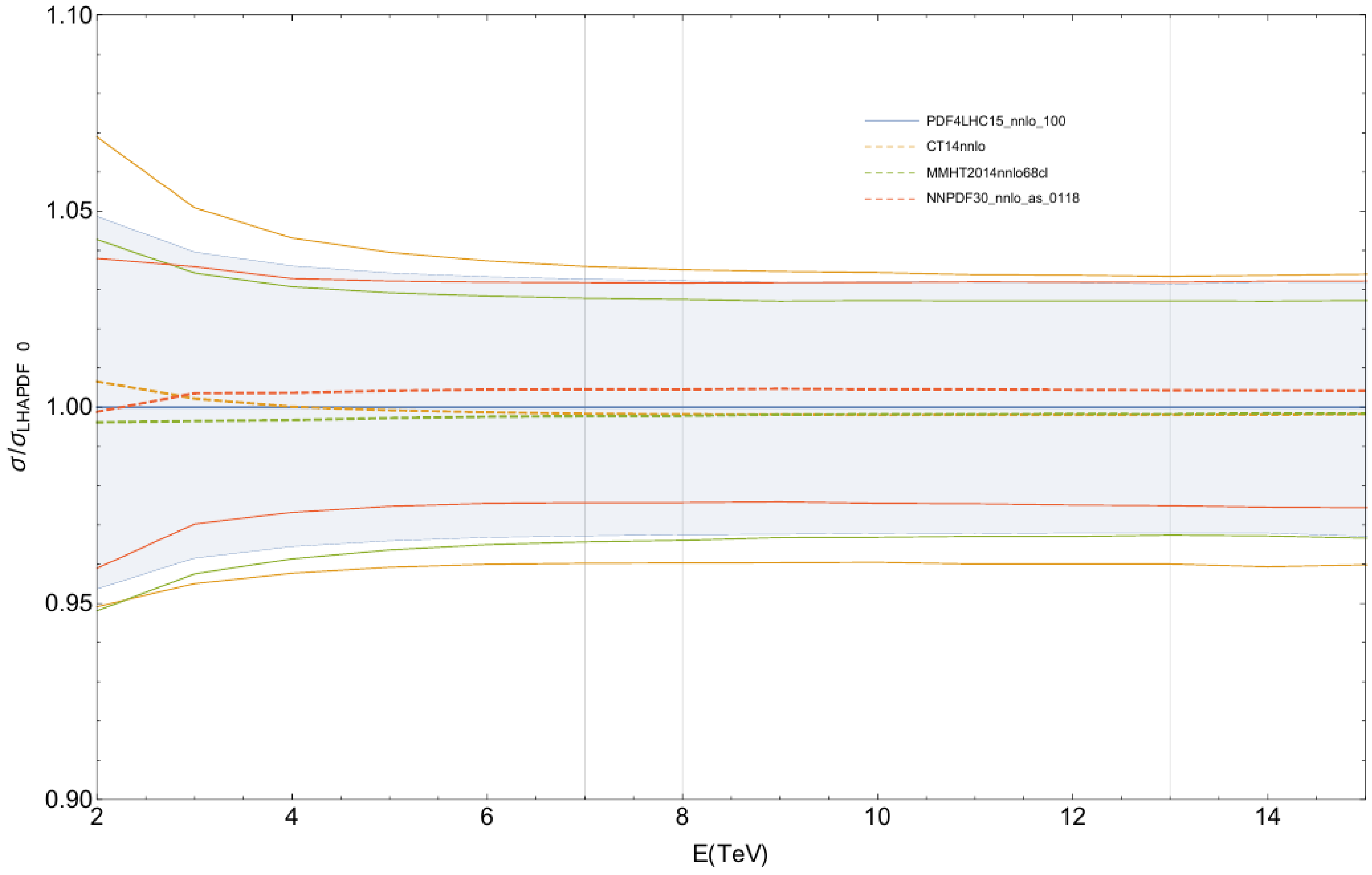
Individual Uncertainties are small but highly correlated!
We combine most of them **linearly**.

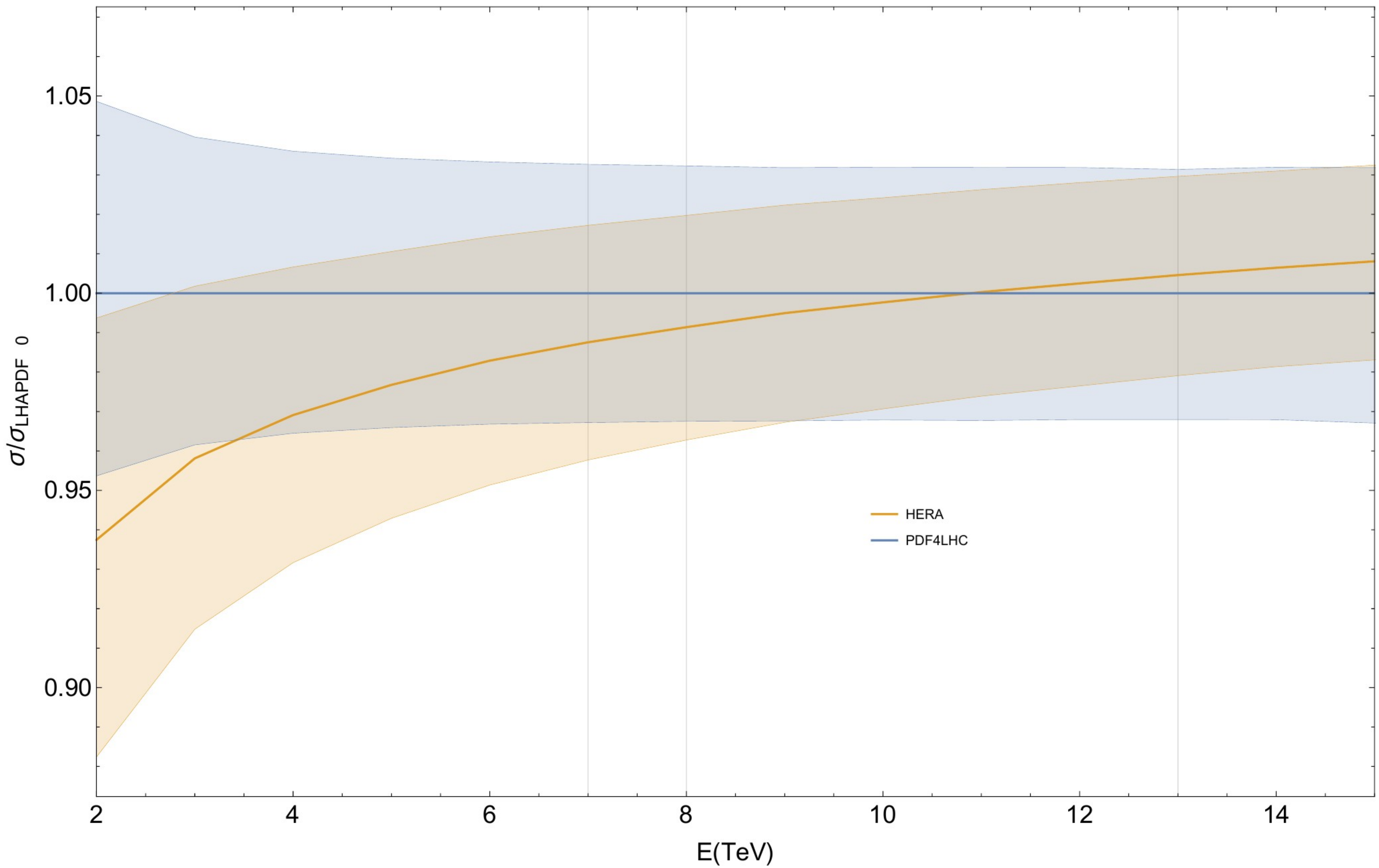
The Final Number

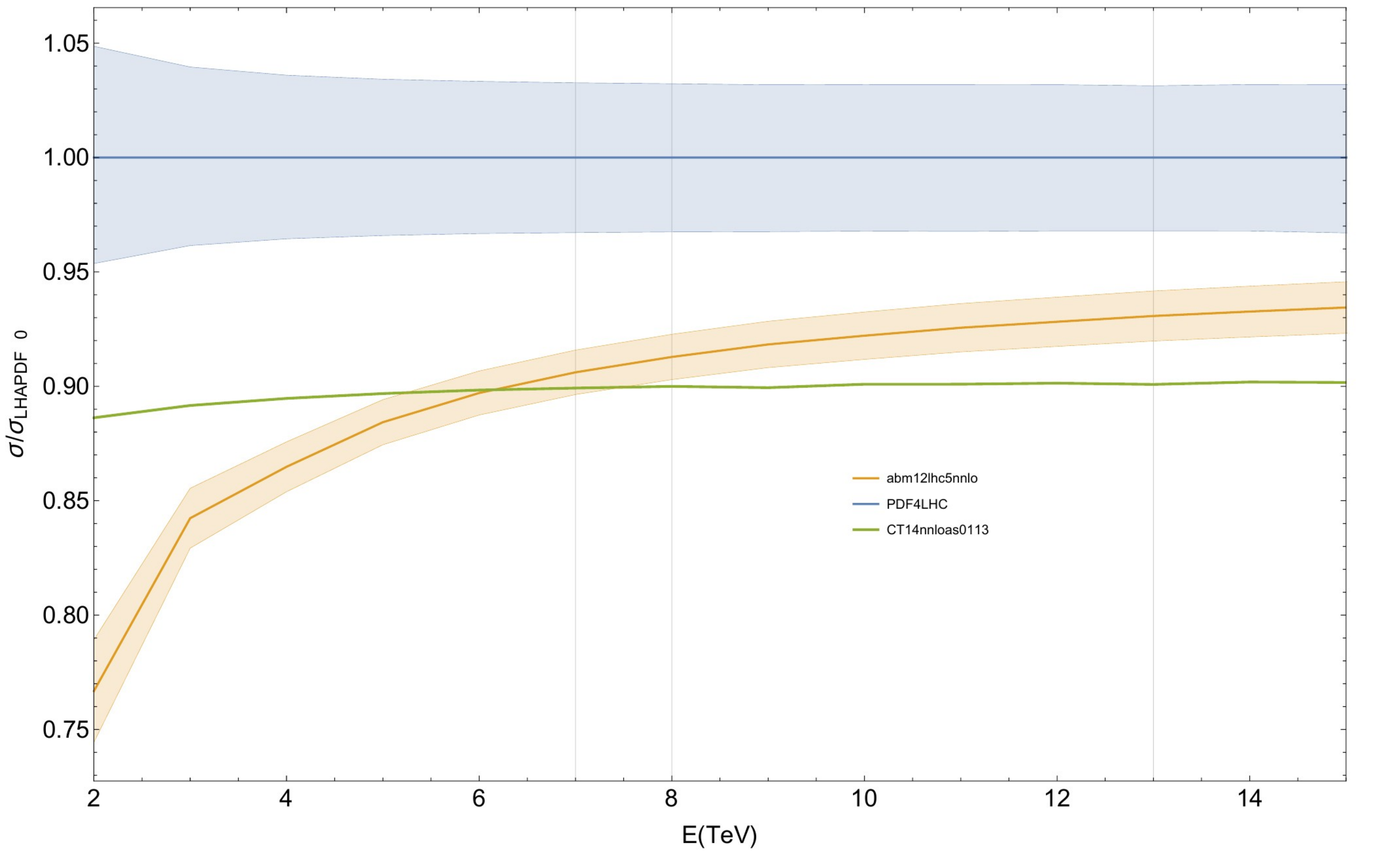
$$\sigma = 48.58\,\text{pb}^{+2.22\,\text{pb}\,(+4.56\%)}_{-3.27\,\text{pb}\,(-6.72\%)}\ (\text{theory}) \pm 1.56\,\text{pb}\,(3.20\%)\ (\text{PDF}+\alpha_s)\,.$$

$$\begin{aligned}
48.58\,\text{pb} = \quad & 16.00\,\text{pb} & (+32.9\%) \quad & (\text{LO, rEFT}) \\
& +20.84\,\text{pb} & (+42.9\%) \quad & (\text{NLO, rEFT}) \\
& -\ \ 2.05\,\text{pb} & (-4.2\%) \quad & ((t,b,c),\ \text{exact NLO}) \\
& +\ \ 9.56\,\text{pb} & (+19.7\%) \quad & (\text{NNLO, rEFT}) \\
& +\ \ 0.34\,\text{pb} & (+0.2\%) \quad & (\text{NNLO},\ 1/m_t) \\
& +\ \ 2.40\,\text{pb} & (+4.9\%) \quad & (\text{EW, QCD-EW}) \\
& +\ \ 1.49\,\text{pb} & (+3.1\%) \quad & (\text{N}^3\text{LO, rEFT})
\end{aligned}$$

PDF Uncertainties



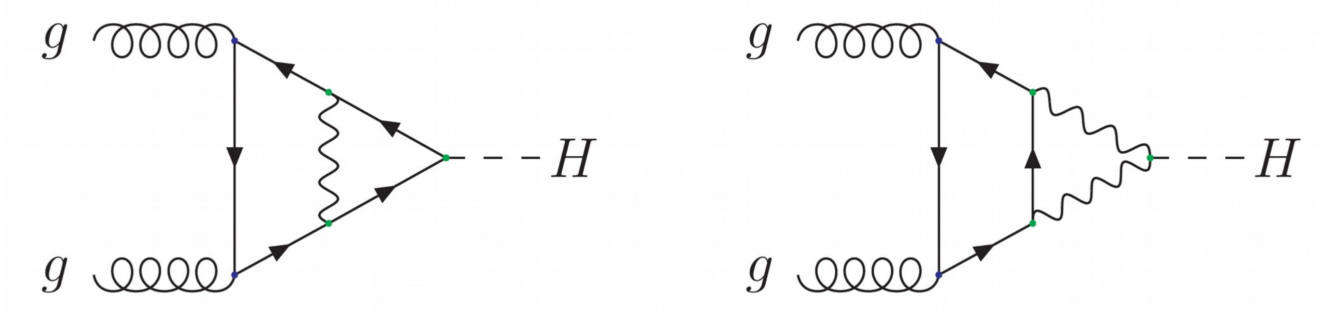

1.05
1.00
0.95
0.90
σ/σLHAPDF 0
2
4
6
8
10
12
14
E(TeV)
HERA
PDF4LHC



1.05
1.00
0.95
0.90
0.85
0.80
0.75
$\sigma/\sigma_{LHAPDF\ 0}$
2
4
6
8
10
12
14
E(TeV)
abm12lhc5nnlo
PDF4LHC
CT14nnloas0113

EW Corrections

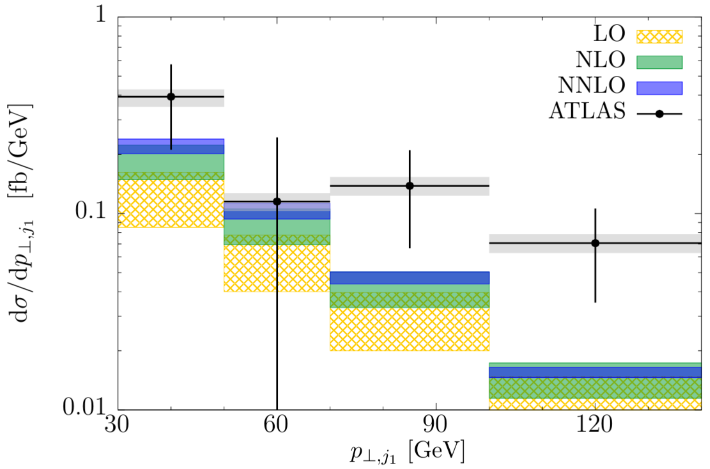
- LO is already 2 loop!
  [Passarino, Actis, Sturm, Uccirati; Degrassi, Maltoni; Aglietti, Bonciani, Degrassi, Vicini]
  calculation tough; still not available analytically!



- **BUT:** must expect QCD corrections to this channel similarly big as for LO QCD!
- Our prediction is precisely based on this assumption (factorisation)!
  But evidence from from invalid EFT for W,Z is shaky <=> to account for this we combine uncertainties linearly.
- Certainty will come only from an exact calculation!

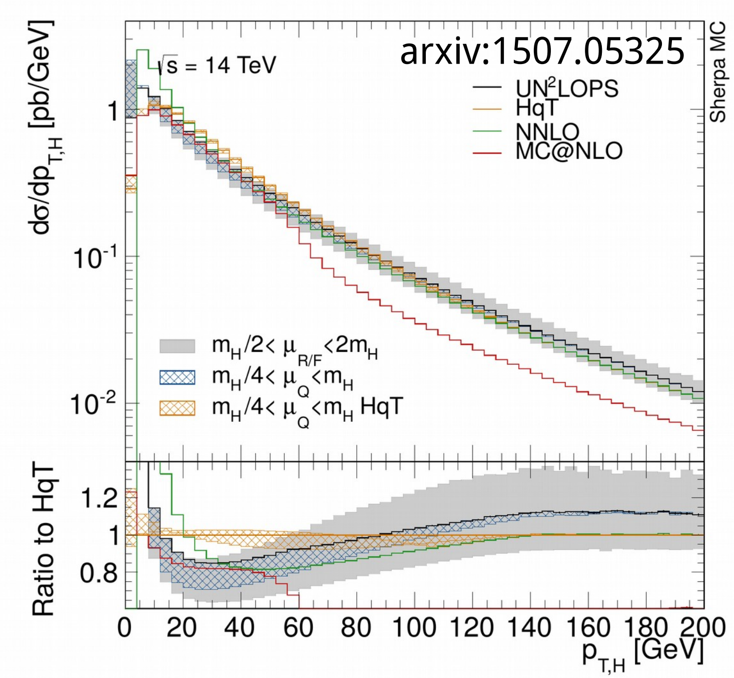
Differential Observables

- H+J @ NNLO
  [Boughezal, Caola, Melnikov,Petriello,Schulze;Boughezal,Focke,Giele,Liu,Petriello; Chen, Gehrmann, Glover, Jaquier]
  - 3 different methods, completed last year
  - pt distribution and single jet rate

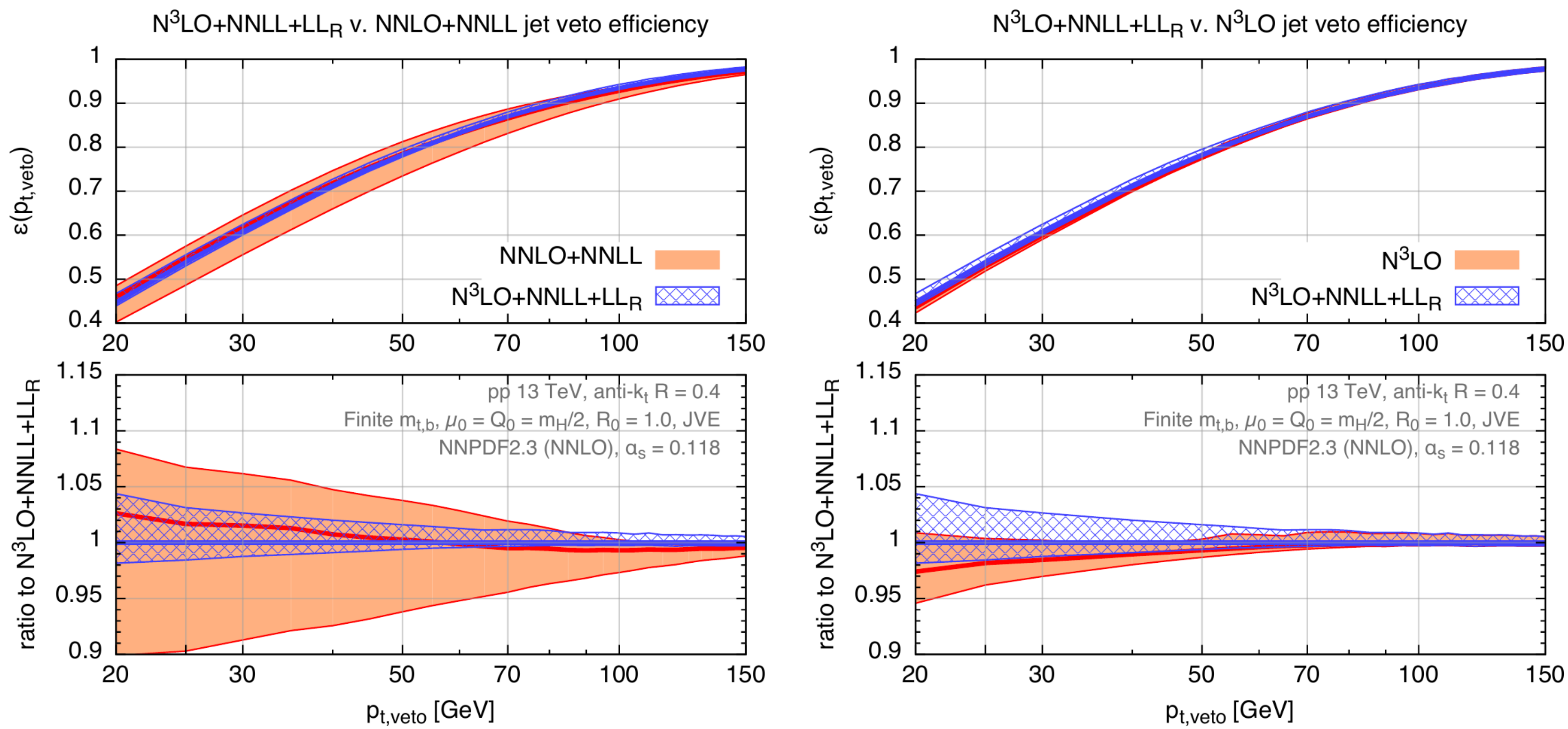


- Pt resummation @ NNLL
  - Public Codes: Hres, CuTe, ..
  - With traditional methods and SCET

- Parton Shower matched to fully differential NNLO
  - PowHegNNLOPS with MinLo
  - UN2LOPS



First N3LO differential observable: Jet veto at N3LO+NNLL+LL(R)



Arxiv: 1511.02886

[Banfi, Caola, Dreyer, Monni, Salam, Zanderighi, Dulat]

How to interpret theoretical uncertainties?

- We advocate a flat uncertainty band(not a flat prior)
  - Is this **feasible** to use for experiments?

- One alternative is a Gaussian[Yellow Report 4]
  - What are the differences when comparing to flat uncertainty band?

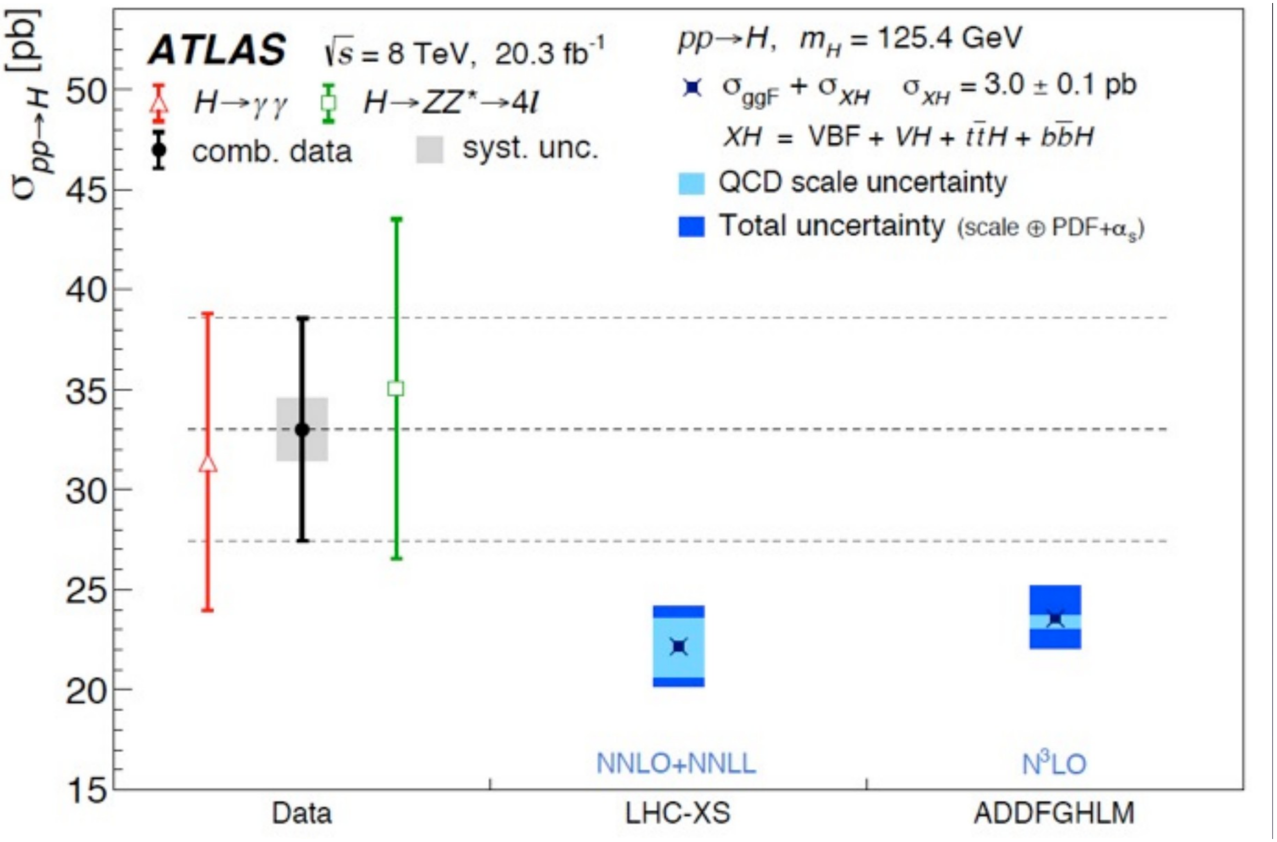
- Other Alternatives?

Conclusions & Outlook

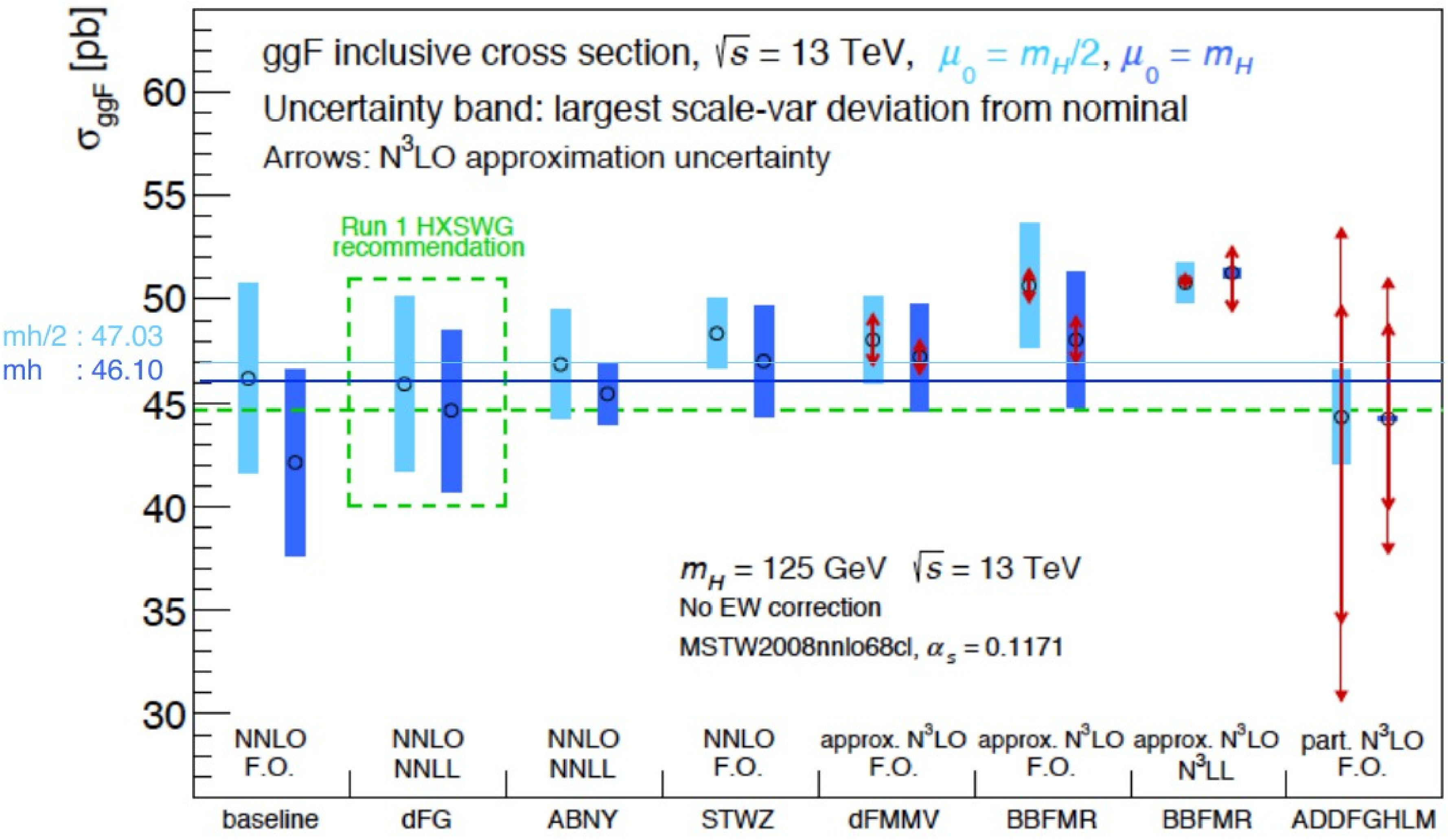
- Reviewed Progress for total Cross Section Calculation:
  - Differential and inclusive Obervables are in very good shape
  - First N3LO calculation has been completed for the LHC
- Analysed theoretical uncertainties:
  - Theoretical Uncertainities of individual components are small but highly correlated
  - Our prescriptions have largely been endorsed by the LHCHXWG. but alternative prescription has been suggested.
  - It remains an open problem how to interprete and estimate theoretical uncertainties
- Many improvements will be needed in the future:
  - exact t,b-mass NNLO(also differential!)
  - mixed QCD-EW corrections (also differential!)
  - fully differential N3LO
  - 4-loop splitting functions for N3LO evolution
  - N3LO PDFs, Drell-Yan@ N3LO, DiJet ..a whole field to be explored..
    ..Resummations at NNNLL?

Backup

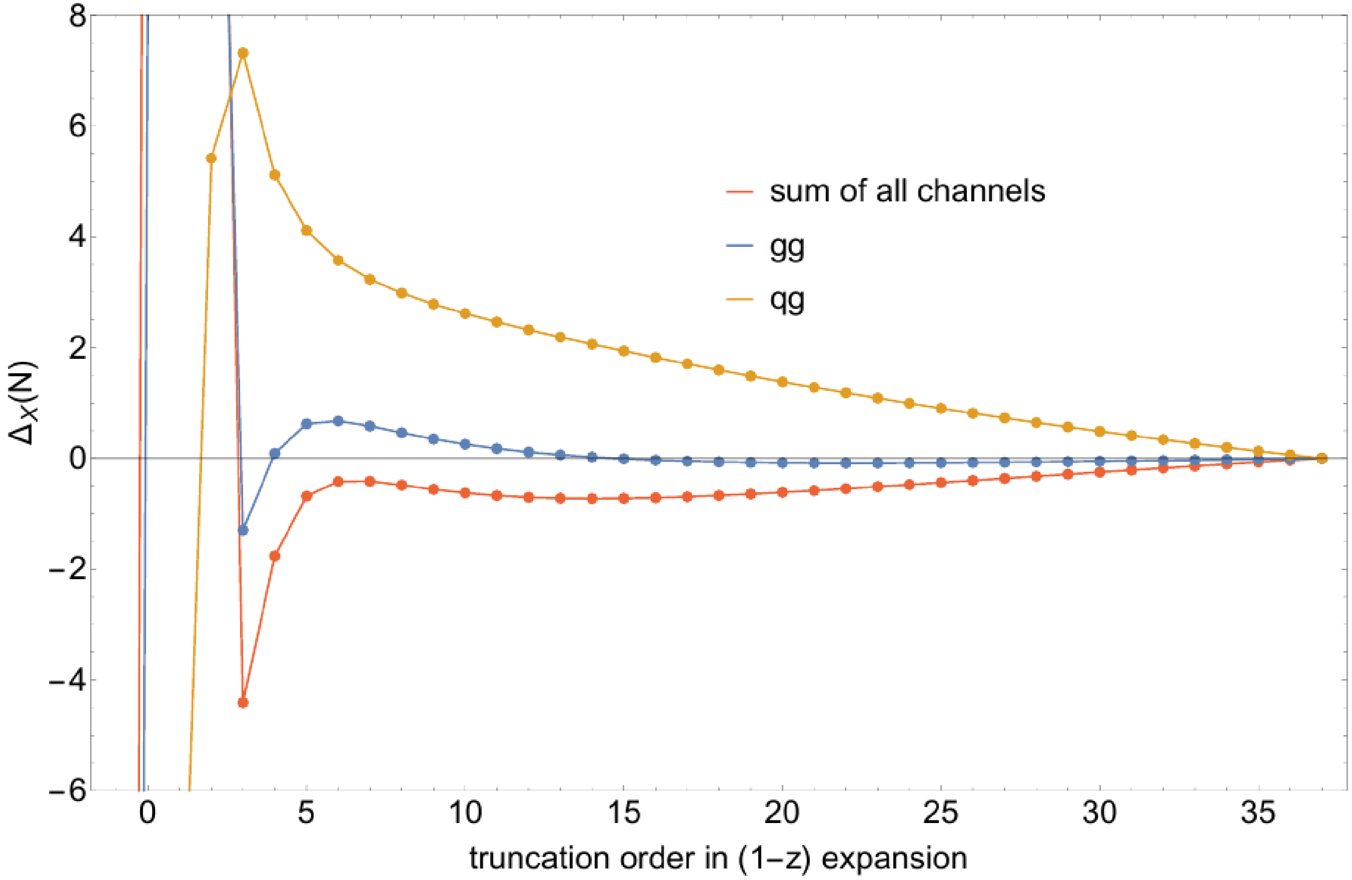
Comparison with 8 TeV Data



Comparison with Approximate Results



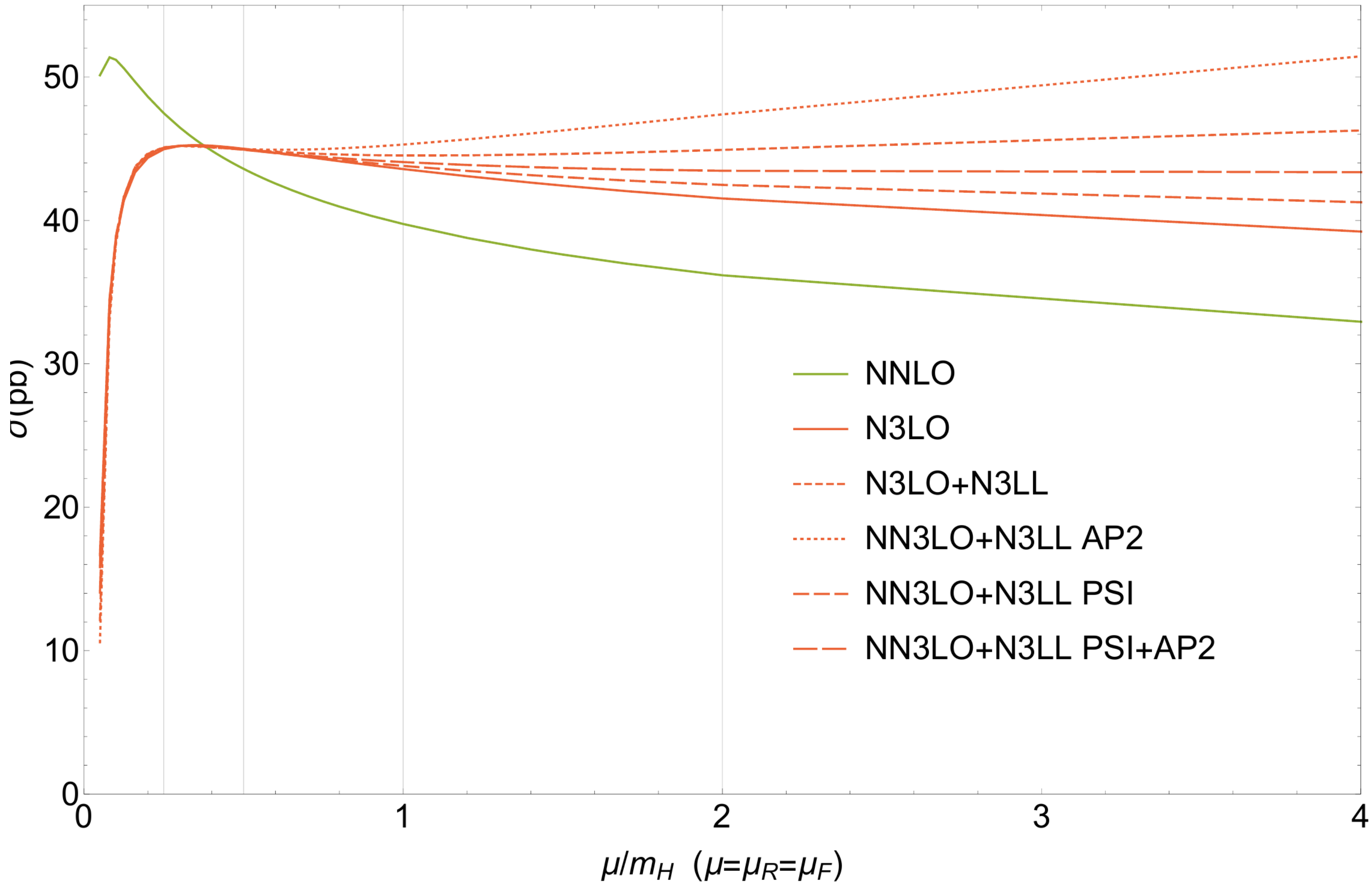
Convergence in Different Channels



$$\Delta_X(N) \equiv \frac{\sigma^{(3)}_{X,EFT}(N) - \sigma^{(3)}_{X,EFT}(N_{\text{last}})}{\sigma^{(3)}_{X,EFT}(N_{\text{last}})}\,100\%\,.$$

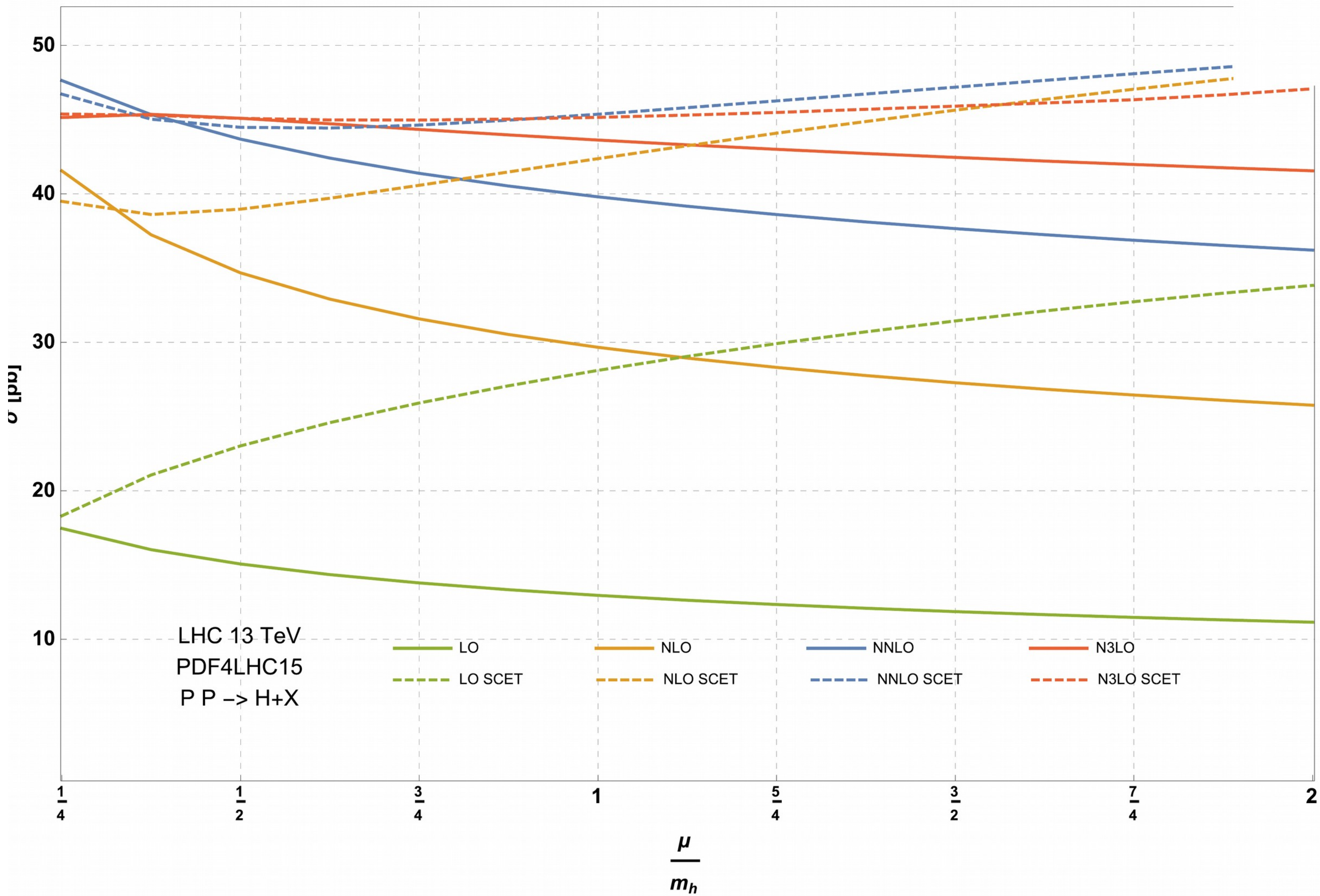
Threshold Resummation

Scheme Dependence



Threshold Resummation

a la SCET (Becher/Neubert)



t,b,c mass effects

$$\delta(tbc)^{\overline{\rm MS}} = \pm \left| \frac{\delta\sigma^{NLO}_{ex;t} - \delta\sigma^{NLO}_{ex;t+b+c}}{\delta\sigma^{NLO}_{ex;t}} \right| \left(R_{LO}\delta\sigma^{NNLO}_{EFT} + \delta_t\hat{\sigma}^{NNLO}_{gg+qg,EFT}\right) \simeq \pm 0.31\,\mathrm{pb}$$

$$\delta(t,b,c) = 1.3\,\delta(t,b,c)^{\overline{\rm MS}}$$

1.3 motivated from 30% scheme dependence at NLO

Negligibilty of Parameteric Mass Uncertainties

Top quark			Bottom quark			Charm quark		
$\delta m_t = 1$ GeV	$\sigma^{NLO}_{ex;t+b+c}$	34.77	$\delta m_b = 0.03$ GeV	$\sigma^{NLO}_{ex;t+b+c}$	34.77	$\delta m_c = 0.026$ GeV	$\sigma^{NLO}_{ex;t+b+c}$	34.77
$m_t + \delta m_t$	$\sigma^{NLO}_{ex;t+b+c}$	34.74	$m_b + \delta m_b$	$\sigma^{NLO}_{ex;t+b+c}$	34.76	$m_c + \delta m_c$	$\sigma^{NLO}_{ex;t+b+c}$	34.76
$m_t - \delta m_t$	$\sigma^{NLO}_{ex;t+b+c}$	34.80	$m_b - \delta m_b$	$\sigma^{NLO}_{ex;t+b+c}$	34.79	$m_c - \delta m_c$	$\sigma^{NLO}_{ex;t+b+c}$	34.78

Truncation zoomed in