Phenomenology of precise Wilson coefficient determination

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TU Munich – IAS

“Precision theory for precise measurements at LHC and future colliders”
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Phenomenology of precise determination of Wilson coefficients

experimental data ⇔ **fit** ⇔ **parameters (pmr) in models**

Statistical procedures

pmr's of interest: $\vec{\theta}$ nuisance pmr's: $\vec{\nu}$

likelihood: $\mathcal{L}\left(\text{data}, \vec{\theta}, \vec{\nu}\right)$

Bayesian | **Frequentist**

posterior ← probabilities → prior | profiling the likelihood

$$P(\vec{\theta}\,|\,M, D) = \frac{\int d\vec{\nu}\;\; P(D\,|\,M, \vec{\theta}, \vec{\nu})\; P(\vec{\theta}, \vec{\nu}\,|\,M)}{\int d\vec{\nu}\, d\vec{\theta}\; P(D\,|\,M, \vec{\theta}, \vec{\nu})\; P(\vec{\theta}, \vec{\nu}\,|\,M)}$$

$$\mathcal{L}(\vec{\theta}) = \max_{\vec{\nu}}\; \mathcal{L}(\vec{\theta}, \vec{\nu})$$

⇒ posterior probability regions $\neq$ ⇒ confidence regions

Precision on Wilson coefficients ≈ **size of** probability or confidence regions, respectively

BUT complex numerical problem ⇒ no straight answer

Wilson coefficients in flavor physics

Origin of flavor in the standard model (SM)

$$\mathcal{L}_{\rm SM} = \underbrace{\mathcal{L}_{\rm gauge}}_{\text{flavor sym } G_{\rm flavor}} + \underbrace{\overline{Q}_L Y_U \tilde{\Phi} U_R + \overline{Q}_L Y_D \Phi D_R}_{\text{Yukawa's break } G_{\rm flavor}}$$

$$\Uparrow$$

- $Y_{U,D}$ origin of flavor in the SM $= 6 + 4$ parameters in quark sector
- $6 \times$ quark masses $\propto vev \times \text{diag}(Y_{U,D}) \Rightarrow$ very hierarchical
- $4 \times V_{\rm CKM} \Rightarrow$ off-diagonal entries strongly suppressed

$$G_{\rm flavor} = SU(3)_{Q_L} \otimes SU(3)_{U_R} \otimes SU(3)_{D_R} \otimes SU(3)_{L_L} \otimes SU(3)_{E_R} \otimes U(1)_{\rm PQ} \otimes U(1)_Y \otimes G_{\rm SM}$$

SM still invariant under $G_{\rm SM} \equiv U(1)_Y \otimes U(1)_B \otimes U(1)_L$

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SM still invariant under $G_{\rm SM} \equiv U(1)_Y \otimes U(1)_B \otimes U(1)_L$

$U_i = \{u, c, t\}$:

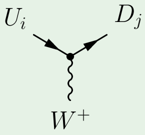
$Q_U = +2/3$

$D_j = \{d, s, b\}$:

$Q_D = -1/3$

$$\mathcal{L}_{\rm CC} = \frac{g_2}{\sqrt{2}} (\bar{u}\ \bar{c}\ \bar{t}) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \gamma^\mu P_L \begin{pmatrix} d \\ s \\ b \end{pmatrix} W^+_\mu$$

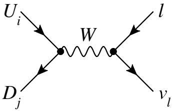
$\sim$ Cabibbo-Kobayashi-Maskawa (CKM) matrix

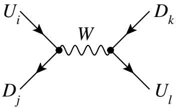


Specific pattern of CC- and FCNC-mediated decays

⇒ charged current (CC): $Q_i \neq Q_j$

Tree: only $U_i \to D_j$ & $D_i \to U_j$





$M_1 \to \ell\bar{\nu}_\ell$

$M_1 \to M_2 + \ell\bar{\nu}_\ell$

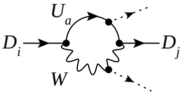
$M_1 \to M_2 M_3$

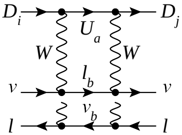
$Amp \sim G_F V_{ij}$

$\sim G_F V_{ij} V_{lk}^*$

⇒ neutral current (FCNC): $Q_i = Q_j$

Loop: $D_i \to D_j$ (& $U_i \to U_j$)





$M_1 \to M_2 + \{\gamma, Z, g\}$

$\{\gamma, Z, g\} \to \{\ell\bar{\ell}, \nu\bar{\nu}, M_3\}$

$M_1 \to \ell\bar{\ell}$

$M_1 \to M_2 + \{\ell\bar{\ell}, \nu\bar{\nu}, M_3\}$

$M^0 \leftrightarrow \overline{M}^0$ (= mixing)

$\sim G_F g \sum_a V_{ai} V_{aj}^* f(m_a)$

$\sim G_F g^2 \sum_{a,b} V_{ai} V_{aj}^* f(m_{a,b})$

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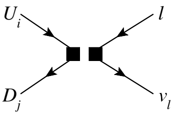

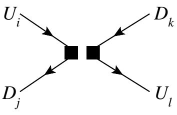

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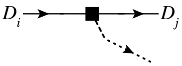

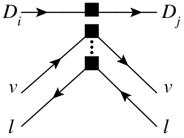

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$Amp \sim G_F C(V_{ij})$ $\qquad \sim G_F C(V_{ij})$ $\qquad \sim G_F\, C(V_{ij}, m_a)$ $\qquad \sim G_F\, C(V_{ij}, m_a, m_b)$

- **decoupling for $m_M \ll m_W$ ⇒ effective theory à la Fermi** [Fermi 1934]

  works for all quarks except top quark ($m_W < m_t$)

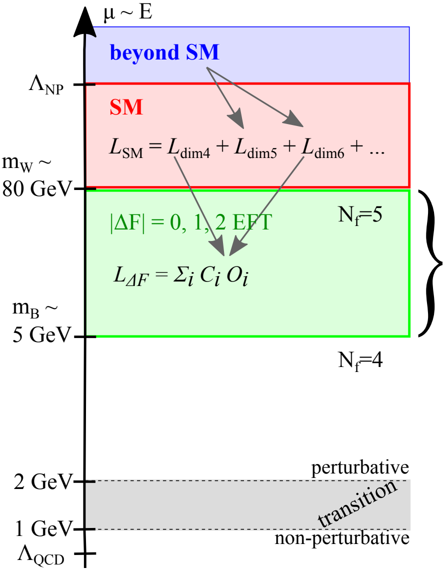
- short-distance (SD) couplings: ***C* = Wilson coefficients**

  depend on SD-parameters ⇒ in SM: CKM and heavy masses: m_W, m_Z, m_t

  ⇒ extract in measurement and calculate in specific UV completions

- overall rescaling factor **Fermi's constant $G_F \sim$ GeV^{-2}**, measured in $\mu \to e\bar{\nu}_e\nu_\mu$

Central theme: factorization via stack of effective theories (EFT)



for $\mu \leq m_W$ only QCD⊗QED gauge symmetry

perturbative ⇒ under control

- matching @ NNLO QCD + NLO EW
- RGE @ NNLO QCD + NLO QED

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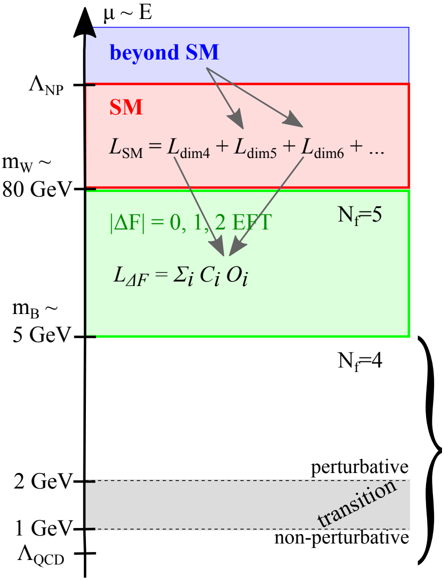

for $\mu \le m_W$ only QCD⊗QED gauge symmetry

***B*-physics:** "$1/m_b$ expansions"

exploit hierarchies $\Lambda_1 \ll \Lambda_2 \ll \ldots \ll m_B$

- Λ_i =process-specific scales, sometimes non-perturbative $\sim \Lambda_{\rm QCD} \approx 0.5$ GeV

⇒ factorization of matrix elements (ME) → hard coefficients ⊗ hadronic objects (decay constants, form factors, distribution amplitudes)

!!! @ leading order in $1/m_b$ usually

a few universal hadronic objects

⇓

from data or non-pert. methods

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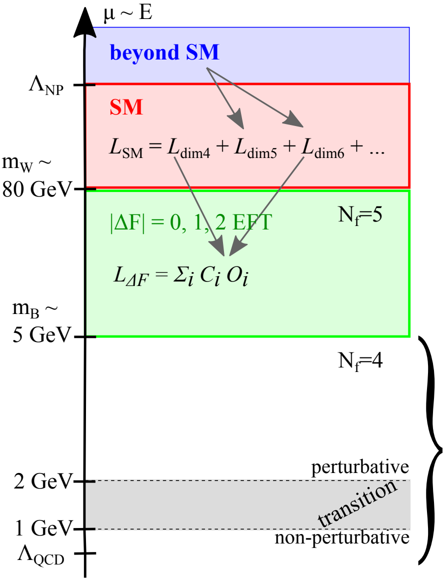

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***K*-physics:**

- direct calculation of ME's on LATTICE
  currently $N_f = 3$, but $N_f = 4$ under investigation
- @ very low mom. χ-PT ⇒ low energy constants (LEC)
  ⇓
  from data or non-pert. methods

$|\Delta F| = 0, 1, 2$ effective theories (EFT)

$$\mathcal{L}_{\rm eff}(\mu_{\rm low} \ll m_W) = \mathcal{L}_{\rm QED\times QCD}(u, d, s, c, b, e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau, ???)$$

$$+ G_F \sum_{\rm SM} (C_i + \Delta C_i) \mathcal{O}_i + G_F \sum_{\rm NP} C_j \mathcal{O}_j(???)$$

C_i, ΔC_i	=	SM & new physics (NP) Wilson coefficients
$\mathcal{O}_i$	=	$\|\Delta F\| = 0, 1, 2$ operators in SM & NP
???	=	additional light degrees of freedom ($\Leftarrow$ usually not pursued)
$\mu_{\rm low}$	=	low energy scale relevant for process ($\mu_{\rm low} \sim m_b$ in B decays, etc.)

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a possible **Procedure**

1) calculate observables within EFT for certain set of operators
2) fit $C_i(\mu_{\text{low}})$ from data of these observables

3) calculate $C_i(m_W)$ for specific UV-completion (dependence on SD parameters)
4) RG-evolution of $C_i(m_W) \to C_i(\mu_{\text{low}})$ within EFT

5) if know SD parameters, can compare fitted with calculated C_i

Precision on Wilson coefficients . . .

. . . depends strongly on our capability

to calculate **hadr. matrix elements (ME) within EFT**

- **QCD** requires nonperturbative methods (lattice, sum rules)
- **QED ISR / FSR** (initial + final state radiation)
  $\Rightarrow$ usually included in experimental analysis via PHOTOS [Golonka/Was hep-ph/056026]
- **QED structure-dependent** parts usually neglected (in principle nonperturbative)
  $\Rightarrow$ in Kaon physics via χ-PT
  $\Rightarrow$ first attempts on lattice for $\pi \to \ell \nu_\ell$ [Rome/Southampton 1502.00257]

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Of course Wilson coefficients can be calculated in the SM $\Rightarrow$

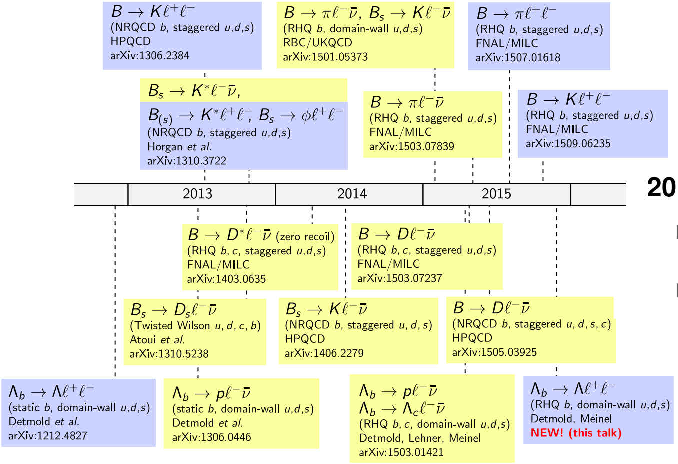
in **SM predictions** additional sources of uncertainties from SM SD parameters:

CKM, $m_{\text{top}}^{\text{MS}}$, $\sin\theta_W$, higher order's ...

Hadronic predictions from lattice

Tremendous amount of lattice results on . . .

- meson decay constants $\propto \langle 0|\bar{q}\Gamma b|B\rangle$
- $b \to c, u, d$ meson/baryon transition form factors (only at low recoil) $\propto \langle H|\bar{q}\Gamma b|B\rangle$
- $|\Delta F| = 2$ local matrix elements $\propto \langle B|[\bar{q}\Gamma b][\bar{q}\Gamma b]|\bar{B}\rangle$



2016:

- $\Delta B = 2$ FNAL/MILC 1602.03560
- . . .

[Stefan Meinel, talk, LHCb Implications 2015]

Leptonic Decays

- ... are helicity-suppressed in the SM $\propto m_\ell^2$
  - $\Rightarrow$ largest Br's for $\ell = \tau$, but experimentally challenging
  - $\Rightarrow$ enhanced sensitivity to scalar couplings
- ... depend only on B-meson decay constant (@ LO in QED) $\quad ip_\mu f_B \equiv \langle 0|\bar{q}\gamma_\mu\gamma_5 b|B(p)\rangle$
  - $\Rightarrow$ lattice calculations with $\mathcal{O}(2\%)$ uncertainty, in future $\lesssim 1\%$ [FLAG'16 1607.00299]

$$f_B = 190.0(4.3)\,\text{MeV} \qquad f_{B_s} = 228.4(3.7)\,\text{MeV} \qquad N_f = 2+1$$

$$f_B = 186.0(4.0)\,\text{MeV} \qquad f_{B_s} = 224.0(5.0)\,\text{MeV} \qquad N_f = 2+1+1$$

not yet in FLAG average ETM results [ETMC 1603.04306]

@ NLO QED $\Rightarrow$ first conceptual studies for lattice calculations [Carrasco et al. arXiv:1502.00257]

$$\boldsymbol{Br \propto f_B^2 \text{ "}\sum_i |C_i|^2\text{"}} \quad \Rightarrow \quad 4\% \text{ uncertainty on "}\sum_i |C_i|^2\text{"}$$

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$d_i \to u_j \ell \bar{\nu}_{\ell'}$

In SM: “$\sum_i |C_i|^2$”$\propto |V_{ub}|^2$

$$Br(B^- \to \tau\bar{\nu}_\tau)_{\rm exp} = (1.14 \pm 0.27) \times 10^{-4}$$

Belle/Babar PDG avg.

prospects @ Belle II few%

$d_i \to d_j \ell \bar{\ell}'$

In SM: “$\sum_i |C_i|^2$”$\propto |V_{tb} V_{tq}^* C_{10}(m_t)|^2$

$$Br(B_d \to \mu\bar{\mu})_{\rm exp} = (3.9^{+1.6}_{-1.4}) \times 10^{-10}$$

$$Br(B_s \to \mu\bar{\mu})_{\rm exp} = (2.8^{+0.7}_{-0.6}) \times 10^{-9}$$

CMS & LHCb 1411.4413

@ LHCb 50/fb 35% and 5%
@ CMS 300/fb 50% and 12%

Semileptonic Decays: $\ell\bar{\nu}_\ell$ & $\nu\bar{\nu}$ final states

- $B \to P, V$ form factors from lattice precise at low recoil = high q^2 (dilepton inv. mass)

... are relevant for exclusive decays:

$d_i \to u_j \ell \bar{\nu}_{\ell'}$ in SM: CC-tree decays $\propto V_{ji}$ $\Leftarrow$ **CKM determination**

- $B \to (D, D^*)\ell\bar{\nu}_\ell, \quad \Lambda_b \to \Lambda_c \ell\bar{\nu}_\ell$ [talk monday Soumitra Nandi]
- $B \to (\pi, \rho, \omega)\ell\bar{\nu}_\ell, \quad B_s \to K\ell\bar{\nu}_\ell, \quad \Lambda_b \to p\ell\bar{\nu}_\ell$

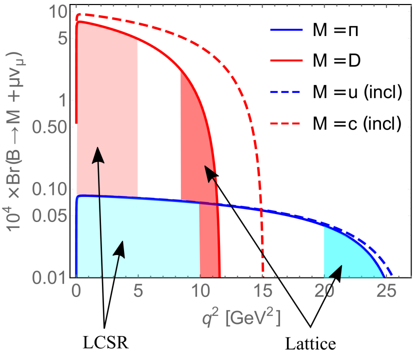
$d_i \to d_j \nu_\ell \bar{\nu}_{\ell'}$ in SM: FCNC-loop decays $\propto \left| V_{tb} V_{td}^* C_L(m_t) \right|^2$

- $Br(B \to K^{(*)}\nu\bar{\nu})$ prospects @ Belle II 20% [talk monday Chunhua Li]
- also "free" of hadronic uncertainties [Mescia/Smith 0705.2025]
  $K^+ \to \pi^+ \nu\bar{\nu}$
  $K_L \to \pi^0 \nu\bar{\nu}$ in SM: $\mathrm{Im}[V_{tb} V_{td}^* C_L(m_t)]$

$d_i \to d_j \ell \bar{\ell}'$ with $\ell \neq \ell'$ LFV decays in SM: absent

- $B \to (K, K^*) + (e\bar{\mu}, e\bar{\tau}, \mu\bar{\tau}), \ldots$

$B \to (P, V)$ form factors



$B \to (P, V)$ FF's calculated at

- low q^2 = large recoil:

  light-cone sum rules (LCSR)

  $\Rightarrow$ precision not expected much below 10%

- high q^2 = low recoil:

  lattice QCD (LQCD)

  $\Rightarrow$ high precision for QCD, but isospin & QED effects kick in $\approx 2\,\%$

$B \to P$ simpler than $B \to V$,
due to unstable $V \to P_1 P_2$ can arise

V_{cb} from $B \to D\ell\bar{\nu}_\ell$

$V_{cb}|_{B\to D} = 40.49(97) \times 10^{-3}$ (2.4%) [Bigi/Gambino 1606.08030]

- simultaneous fit of V_{cb} and FF-nuisance parameters from Babar/Belle data + Lattice results + unitarity constrains
- from incl. $B \to X_c\ell\bar{\nu}_\ell$ $\quad V_{cb}|_{\mathrm{incl}} = 42.00(64) \times 10^{-3}$ (1.5%) [Gambino/Healey/Turczyk 1606.06174]
- from excl. $B \to D^*\ell\bar{\nu}_\ell$ $\quad V_{cb}|_{B\to D^*} = 39.04(75) \times 10^{-3}$ (1.9%) [FNAL/MILC 1403.0635]
- tensions: $1.3\,\sigma$ for D vs. X_c, $\quad 3.0\,\sigma$ for D^* vs. X_c, $\quad 1.2\,\sigma$ for D vs. D^*

$B \to (P, V)$ form factors

FF	method	q^2/GeV^2	rel. unc. [%]	Ref.
		$B \to \pi$		
f_+	LCSR	$q^2 < 10$	≈ 7	Imsong et al. 1409.7816
$f_{+,0}$	LQCD_{2+1}	$16 < q^2$	≈ 28	HPQCD hep-lat/0601021
$f_{+,0}$	LQCD_{2+1}	$19 < q^2$	$8 - 14$	RBC/UKQCD 1501.05373
$f_{+,0,T}$	LQCD_{2+1}	$20 < q^2$	≈ 4	FNAL/MILC 1503.07839/1507.01618
f_0	LQCD_{2+1+1}	$q^2 = q^2_{\max}$	3	HPQCD 1510.07446
		$B \to \rho, \omega$		
V, A_i, T_j	LCSR	$q^2 < 14$	≈ 10 & 14	Bharucha et al. 1503.05534
		$B \to D$		
$f_{+,0}$	LCSR	$q^2 < 6$	≈ 27	Faller et al. 0809.0222
$f_{+,0}$	LQCD	$8.5 < q^2$	≈ 1.5	FNAL/MILC 1503.07237
$f_{+,0}$	LQCD	$9.5 < q^2$	≈ 5	HPQCD 1505.03925
		$B \to D^*$		
V, A_i	LCSR	$q^2 < 6$	≈ 27	Faller et al. 0809.0222
$\mathcal{F}(1)$	LQCD	$q^2 = q^2_{\max}$	1.4	FNAL/MILC 1403.0635

Also $B \to K$, $B \to K^*$, $B_s \to K$, $\Lambda_b \to \Lambda_c$, $\Lambda_b \to p$, . . .

Example: $|\Delta B| = 2$

EFT: $\mathcal{L} = -\sum_i C_i \mathcal{O}_i + \text{h.c.}$

$$\mathcal{O}_1^{\rm VLL} = [\bar{b}\gamma_\mu P_L d][\bar{b}\gamma^\mu P_L d]$$

$$\mathcal{O}_1^{\rm LR} = [\bar{b}\gamma_\mu P_L d][\bar{b}\gamma^\mu P_R d]$$
$$\mathcal{O}_2^{\rm LR} = [\bar{b}P_L d][\bar{b}P_R d]$$

$$\mathcal{O}_1^{\rm SLL} = [\bar{b}P_L d][\bar{b}P_L d]$$
$$\mathcal{O}_2^{\rm SLL} = -[\bar{b}\sigma_{\mu\nu} P_L d][\bar{b}\sigma^{\mu\nu} P_L d]$$

$+$ VLL $\to$ VRR and SLL $\to$ SRR

In SM: only non-zero

$$C_1^{\rm VLL}\Big|_{\rm SM} = \frac{G_F^2}{4\pi^2} m_W^2 \left(V_{td} V_{tb}^*\right)^2 S_0(m_t)$$

Neutral *B*-meson mass diff. for $B_{d,s}$

$$\Delta M_B \propto \left|\langle B|\mathcal{L}|\overline{B}\rangle\right| \propto \left|\sum_i C_i \langle B|\mathcal{O}_i|\overline{B}\rangle\right| \quad \text{with} \quad \langle B|\mathcal{O}_i|\overline{B}\rangle \propto f_B^2 B_i(\overline{m}_b)$$

Latest lattice ($N_f = 2+1$) uncertainties on $f_B^2 B_i(\overline{m}_b) \approx 9\,\%$ [FNAL/MILC 1602.03560]

Example: $|\Delta B| = 2$

EFT: $\mathcal{L} = -\sum_i C_i \mathcal{O}_i + \text{h.c.}$

$$\mathcal{O}_1^{\text{VLL}} = [\bar{b}\gamma_\mu P_L d][\bar{b}\gamma^\mu P_L d]$$

$$\mathcal{O}_1^{\text{LR}} = [\bar{b}\gamma_\mu P_L d][\bar{b}\gamma^\mu P_R d]$$
$$\mathcal{O}_2^{\text{LR}} = [\bar{b}P_L d][\bar{b}P_R d]$$

$$\mathcal{O}_1^{\text{SLL}} = [\bar{b}P_L d][\bar{b}P_L d]$$
$$\mathcal{O}_2^{\text{SLL}} = -[\bar{b}\sigma_{\mu\nu} P_L d][\bar{b}\sigma^{\mu\nu} P_L d]$$

+ VLL → VRR and SLL → SRR

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In SM precision on CKM V_{td} and V_{ts} (unitarity $|V_{tb}| \approx 1$) "quasi single-observable-single-parameter"

$$|V_{td}| = 8.00(33)(2)(3)(8) \times 10^{-3} \qquad (4.2\,\%)$$
$$|V_{ts}| = 39.0(1.2)(0.0)(0.2)(0.4) \times 10^{-3} \qquad (3.3\,\%)$$
$$|V_{td}/V_{ts}| = 0.2052(31)(4)(0)(10) \qquad (1.6\,\%)$$

with errors from: lattice, exp, SD-parameters (m_t, α_s, etc.), omission of c-sea quark

$b \to s\gamma$ and $b \to s\ell\bar{\ell}$

Hadronic matrix elements for $B \to K^{(*)}\bar{\ell}\ell$ – Part 1

Radiative & Semileptonic op's

$$\mathcal{O}_{7\gamma(7\gamma')} = m_b[\bar{s}\,\sigma^{\mu\nu} P_{R(L)}\, b] F_{\mu\nu}$$

$$\mathcal{O}_{9(9')} = [\bar{s}\,\gamma^{\mu} P_{L(R)}\, b][\bar{\ell}\,\gamma_{\mu}\,\ell]$$

$$\mathcal{O}_{10(10')} = [\bar{s}\,\gamma^{\mu} P_{L(R)}\, b][\bar{\ell}\,\gamma_{\mu}\gamma_5\,\ell]$$

Factorisation into well-defined hadronic objects (@ LO QED)
$\Rightarrow$ No conceptual problems !!!

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Hadronic amplitude $B \to K^{(*)}\bar{\ell}\ell$ (@ LO in QED)

$$\mathcal{A}_7 \propto C_7 L_\mu \frac{q_\nu}{q^2}\langle K_\lambda^{(*)}|[\bar{s}\,\sigma^{\mu\nu} P_R\, b]|B(p)\rangle \quad \propto C_7\, T_\lambda(q^2)$$

$$\mathcal{A}_9 \propto C_9 L_\mu \langle K_\lambda^{(*)}|[\bar{s}\,\gamma^\mu P_L\, b]|B(p)\rangle \quad \propto C_9\, V_\lambda(q^2)$$

- $q = p_B - p_K$ dilepton invariant mass
- $\lambda = K^{(*)}$ polarization
- V_λ and T_λ: $B \to K^{(*)}$ vector and tensor form factors (FF)

@ low q^2: FF's from LCSR (10 – 15)% accuracy

$B \to K$ [Ball/Zwicky hep-ph/0406232, Khodjamirian et al. 1006.4945]

$B \to K^*$ [Khodjamirian et al. 1006.4945, Bharucha/Straub/Zwicky 1503.05534]

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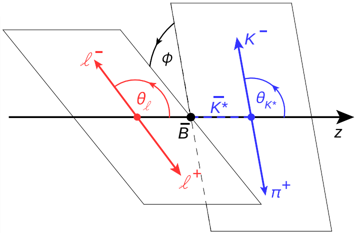
FF relations at low & high q^2

- allow to relate FF's $\Rightarrow$ reduce their number
- valid up to corrections of $\Lambda_{\rm QCD}/m_b \simeq 0.5/4 \approx 13\%$

$\Rightarrow$ "optimized observables" in $B \to K^*\bar{\ell}\ell$

Angular observables $J_i(q^2)$ in $B \to K^* [\to K\pi] + \bar{\ell}\ell$

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} \simeq J_{1s} \sin^2\theta_K + J_{1c} \cos^2\theta_K$$
$$+(J_{2s} \sin^2\theta_K + J_{2c} \cos^2\theta_K) \cos 2\theta_\ell + J_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi$$
$$+J_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi + J_5 \sin 2\theta_K \sin\theta_\ell \cos\phi$$
$$+(J_{6s} \sin^2\theta_K + J_{6c} \cos^2\theta_K) \cos\theta_\ell + J_7 \sin 2\theta_K \sin\theta_\ell \sin\phi$$
$$+J_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi + J_9 \sin^2\theta_K \sin^2\theta_\ell \sin 2\phi$$



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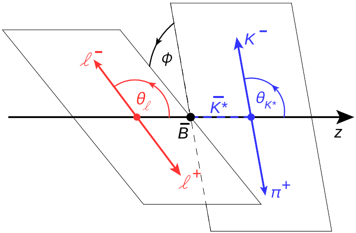

"Optimized observables" $\Rightarrow$ reduced FF sensitivity

- guided by large energy limit @ low-q^2 and Isgur-Wise @ high-q^2 FF-relations
- FF's cancel up to corrections $\sim \Lambda_{\rm QCD}/m_b$

@ low q^2

[Krüger/Matias hep-ph/0502060, Egede/Hurth/Matias/Ramon/Reece arXiv:0807.2589 + 1005.0571]
[Becirevic/Schneider arXiv:1106.3283]
[Matias/Mescia/Ramon/Virto arXiv:1202.4266]
[Descotes-Genon/Matias/Ramon/Virto arXiv:1207.2753]

$$A_T^{(2)} \equiv P_1 \equiv \frac{J_3}{2\, J_{2s}} \qquad A_T^{(\rm re)} \equiv 2\, P_2 \equiv \frac{J_{6s}}{4\, J_{2s}} \qquad A_T^{(\rm im)} \equiv -2\, P_3 \equiv \frac{J_9}{2\, J_{2s}}$$

$$P_4' \equiv \frac{J_4}{\sqrt{-J_{2c} J_{2s}}} \qquad P_5' \equiv \frac{J_5/2}{\sqrt{-J_{2c} J_{2s}}} \qquad P_6' \equiv \frac{-J_7/2}{\sqrt{-J_{2c} J_{2s}}} \qquad P_8' \equiv \frac{-J_8}{\sqrt{-J_{2c} J_{2s}}}$$

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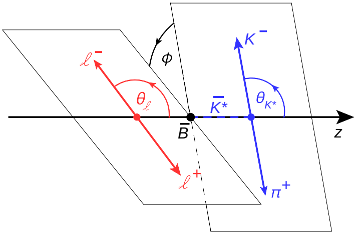

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@ high q^2

[CB/Hiller/van Dyk arXiv:1006.5013]
[Matias/Mescia/Ramon/Virto arXiv:1202.4266]
[CB/Hiller/van Dyk arXiv:1212.2321]

$$H_T^{(1)} \equiv P_4 \equiv \frac{\sqrt{2}J_4}{\sqrt{-J_{2c}(2J_{2s} - J_3)}}$$

$$H_T^{(2)} \equiv P_5 \equiv \frac{J_5/\sqrt{2}}{\sqrt{-J_{2c}(2J_{2s} + J_3)}} \qquad H_T^{(3)} \equiv \frac{J_{6s}/2}{\sqrt{(2J_{2s})^2 - (J_3)^2}}$$

$$H_T^{(4)} \equiv Q \equiv \frac{\sqrt{2}J_8}{\sqrt{-J_{2c}(2J_{2s} + J_3)}} \qquad H_T^{(5)} \equiv \frac{-J_9}{\sqrt{(2J_{2s})^2 - (J_3)^2}}$$

Hadronic matrix elements for $B \to K^{(*)}\bar{\ell}\ell$ – Part 2

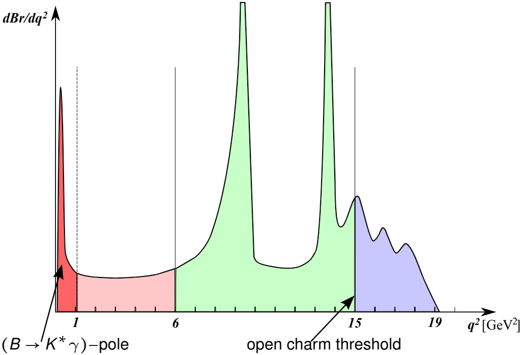
Nonleptonic = "the bad guys"

$$\mathcal{O}_{(1)2} = [\bar{s}\gamma^{\mu} P_L (T^a)\, c][\bar{c}\gamma_{\mu} P_L (T^a)\, b]$$

$$\mathcal{O}_{3,4,5,6} = [\bar{s}\,\Gamma_{sb} P_L (T^a)\, b] \sum_q [\bar{q}\,\Gamma_{qq} (T^a)\, q]$$

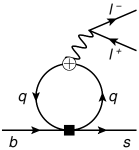
$$\mathcal{O}_{8g(8g')} = m_b [\bar{s}\,\sigma^{\mu\nu} P_{R(L)} T^a\, b] G^a_{\mu\nu}$$

- at LO in QED $\Rightarrow$ solved with different approaches depending on q^2

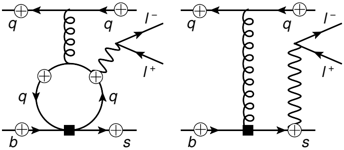


$$\mathcal{A}_{\lambda,\text{hadr}} = \frac{\alpha_e}{4\pi}\frac{L^{\mu}}{q^2} \int d^4x\, e^{i\,q\cdot x} \left\langle K_{\lambda}^{(*)} \middle| \mathrm{T}\left\{ j_{\mu}^{\text{em}}(x), \sum_i C_i \mathcal{O}_i(0) \right\} \middle| B(p) \right\rangle$$

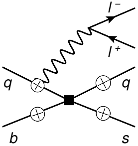
"resonant contributions"



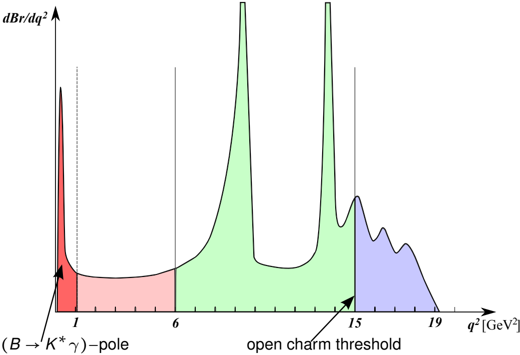
"spectator scattering"



"weak annihilation"



Hadronic matrix elements for $B \to K^{(*)}\bar{\ell}\ell$ – Part 2



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- at LO in QED ⇒ solved with different approaches depending on q^2

$$\mathcal{A}_{\lambda,\mathrm{hadr}} = \frac{\alpha_e}{4\pi}\frac{L^\mu}{q^2}\int \mathrm{d}^4x\, e^{i\,q\cdot x} \left\langle K^{(*)}_\lambda \middle| \mathrm{T}\left\{ j^{\mathrm{em}}_\mu(x), \sum_i C_i \mathcal{O}_i(0) \right\} \middle| B(p) \right\rangle$$

Large Recoil (low-q^2)

- very low-q^2 ($\lesssim 1$ GeV^2) dominated by $\mathcal{O}_7$
  low-q^2 ($[1,6]$ GeV^2) dominated by $\mathcal{O}_{9,10}$
- 1) QCD factorization or SCET
  2) LCSR
  3) non-local OPE of $\bar{c}c$-tails

[Beneke/Feldmann/Seidel hep-ph/0106067 + 0412400;
Lyon/Zwicky et al. 1212.2242 +1305.4797; Khodjamirian et al. 1006.4945 + 1211.0234 + 1506.07760]

Low Recoil (high-q^2)

- dominated by $\mathcal{O}_{9,10}$
- local OPE (+ HQET) ⇒ theory only for sufficiently large q^2-integrated obs's
  [Grinstein/Pirjol hep-ph/0404250, Beylich/Buchalla/Feldmann 1101.5118]

Uncertainties @ high-q^2

Hard momentum transfer $(q^2 \sim m_B^2) \Leftrightarrow x \to 0$ allows for local OPE (at each value of q^2)

$$\int d^4x \, \frac{e^{i\,q\cdot x}}{q^2} \, \mathrm{T}\Big\{ j^\mu_{\rm em}(x), \sum_i C_i \mathcal{O}_i(0) \Big\} \overset{x\to 0}{=} \sum_a \mathcal{C}_{3a} \mathcal{Q}^\mu_{3a} + \text{no dim-4} + \sum_b \mathcal{C}_{5b} \mathcal{Q}^\mu_{5b} + \mathcal{O}(\text{dim} > 5)$$

[Buchalla/Isidori hep-ph/9801456, Grinstein/Pirjol hep-ph/0404250, Beylich/Buchalla/Feldmann 1101.5118]

dim = 3 $\propto B \to K^{(*)}$ FF's $\Rightarrow$ from lattice & also NLO-α_s corrections known

dim = 5 suppressed by $(\Lambda_{\rm QCD}/m_b)^2 \sim 2\%$, explicite estimate @ $q^2 = 15$ GeV^2: $< 1\%$

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Duality violating (DV) effects $\Rightarrow$ go beyond those neglected in $\Lambda_{\rm QCD}/m_b$ and α_s in the OPE

$$\mathcal{B} \simeq \underbrace{|A_{10}|^2 + |A_{9,\rm OPE}|^2}_{\text{similar in size}} + \underbrace{2\,\mathrm{Re}(A_{9,\rm OPE}\Delta^*_{9,\rm DV})}_{0 \gtrless} + \underbrace{|\Delta_{9,\rm DV}|^2}_{0\le} + \ldots$$

- !!! no first principle methods to calculate $\Delta_{9,\rm DV}$
- $\Delta_{9,\rm DV}$ = oscillatory in q^2 $\Rightarrow$ hope to minimize DV effects by q^2 integration
- with exponential suppression for $q^2 \to q^2_{\rm max}$ $\Rightarrow$ or stay close to endpoint (not much data)
- using Shifman model for c-quark corr. $\Rightarrow \Delta_{9,\rm DV}$ affects integrated rate ($q^2 > 15$ GeV^2) by $\pm 2\%$ [Beylich/Buchalla/Feldmann 1101.5118]
- OPE predicts relations: $H_T^{(1)} \simeq 1$ and $H_T^{(2)} \simeq H_T^{(3)}$
  large breaking from DV can be checked for experimentally [CB/Hiller/van Dyk 1006.5013, 1212.2321]
- allowing for large DV does NOT improve goodness of global fits [Altmannshofer/Straub 1411.3161]

Uncertainties @ low-q^2: $1/m_b$ corrections

- $\Lambda_{\rm QCD}/m_b \approx 13\%$ corrections to QCDF not known, only partially (contain endpoint divergences)
  [Kagan/Neubert hep-ph/0110078, Feldmann/Matias hep-ph/0212158, Beneke/Feldmann/Seidel hep-ph/0412400]

- $1/m_b$ corrections $\Rightarrow$ ruin "optimised observables" (some more, others less):

  A) due to use of FF-relations (factorisable) and B) due to QCDF to $B \to K^{(*)}\bar{\ell}\ell$ amplitude (non-factorisable)

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structure of vector ($\propto [\bar{\ell}\gamma_\mu\ell]$)-amplitudes (omitted helicity index $\lambda = 0, \pm 1$)

$$\mathcal{A} \propto \left(\xi_i + \Delta F_9^{\alpha_s} + \Delta F_9^{1/m_b}\right) C_9 + \left(\xi_i + \Delta F_7^{\alpha_s} + \Delta F_7^{1/m_b}\right) C_7 + \Delta^{\rm non-fac} + \Delta^{\bar{c}c}$$

- FF-relation breaking from α_s = known [Beneke/Feldmann hep-ph/0008255]
  $1/m_b$ = "unknown" (LCSR predictions of FF's account for some)
  ad-hoc parameterisation: $\Delta F_i^{1/m_b} = a_i + b_i \frac{q^2}{m_B^2} + c_i \frac{q^4}{m_B^4} + \ldots$ [Jäger/Martin-Camalich 1212.2263]

- $1/m_b$ corrections to non-factorisable parts (resonant, spectator scattering, WA)
  similarly $\Delta^{\rm non-fac} = \left(1 + A\, e^{i\phi_A} + B\, e^{i\phi_B} \frac{q^2}{m_B^2} + C\, e^{i\phi_C} \frac{q^4}{m_B^4}\right) \mathcal{A}^{\rm hadr}$ [Descotes-Genon et al. 1407.8526]
  with $A, B, C \in [0,\ 0.1]$, arbitrary $\phi_{A,B,C}$

- soft gluons to $\bar{c}c$ resonant-contributions [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945]

Global fits of $b \to s + (\gamma, \ell\bar{\ell})$

$B \to X_s\gamma$	(Br)	$C_7^{(\prime)}$
$B \to K^*\gamma$	(Br, S, C, A_I)	$C_7^{(\prime)}$
$B_s \to \ell\bar{\ell}$	(Br)	$C_{10}^{(\prime)}$
$B \to K\ell\bar{\ell}$	(dBr/dq^2)	$C_{7,9,10}^{(\prime)}$
$B \to X_s\ell\bar{\ell}$	(dBr/dq^2)	$C_{7,9,10}^{(\prime)}$
$B_s \to \phi\ell\bar{\ell}$	(dBr/dq^2, ang. obs.)	$C_{7,9,10}^{(\prime)}$
$B \to K^*\ell\bar{\ell}$	(dBr/dq^2, (opt.) ang. obs.)	$C_{7,9,10}^{(\prime)}$

- $\mathcal{O}(100)$ measurements
- $1 - 6$ parameters of interest (MFV scenario)
- $\mathcal{O}(200)$ nuisance parameters

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$B \to K^*\gamma$	$(Br,\ S,\ C,\ A_I)$	$C_7^{(\prime)}$
$B_s \to \ell\bar{\ell}$	(Br)	$C_{10}^{(\prime)}$
$B \to K\ell\bar{\ell}$	(dBr/dq^2)	$C_{7,9,10}^{(\prime)}$
$B \to X_s\ell\bar{\ell}$	(dBr/dq^2)	$C_{7,9,10}^{(\prime)}$
$B_s \to \phi\ell\bar{\ell}$	$(dBr/dq^2$, ang. obs.)	$C_{7,9,10}^{(\prime)}$
$B \to K^*\ell\bar{\ell}$	$(dBr/dq^2$, (opt.) ang. obs.)	$C_{7,9,10}^{(\prime)}$

- $\mathcal{O}(100)$ measurements
- $1-6$ parameters of interest (MFV scenario)
- $\mathcal{O}(200)$ nuisance parameters

Most recent fits use: **"$\Delta\chi^2$"** = approximation to avoid numerical efforts of Frequentist/Bayesian approaches

[Altmannshofer/Straub 1411.3161, Descotes-Genon/Hofer/Matias/Virto 1510.04239, Hurth/Mahmoudi/Neshatpour 1603.00865]

⇒ assuming gaussian experimental and theoretical errors

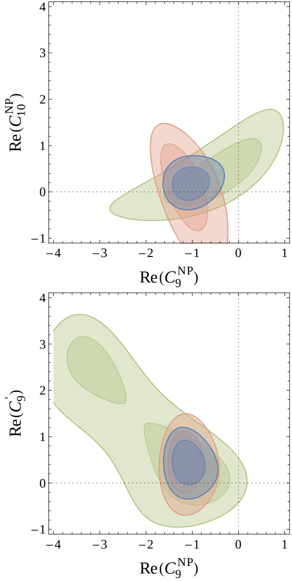
$$-2\ln\mathcal{L}(\vec{\theta}) \equiv \chi^2(\vec{\theta}) \simeq \left[\vec{O}_{\rm exp} - \vec{O}_{\rm th}(\vec{\theta})\right]^T \cdot \left[\widehat{C}_{\rm exp} + \widehat{C}_{\rm th}[\vec{\nu}](\vec{\theta} = \vec{\theta}_{\rm SM})\right]^{-1} \cdot \left[\vec{O}_{\rm exp} - \vec{O}_{\rm th}(\vec{\theta})\right]$$

"theory" covariance matrix $\widehat{C}_{\rm th}[\vec{\nu}](\vec{\theta} = \vec{\theta}_{\rm SM})$ found by — usually $\mathcal{O}(50-100)$ nuisance pmr's

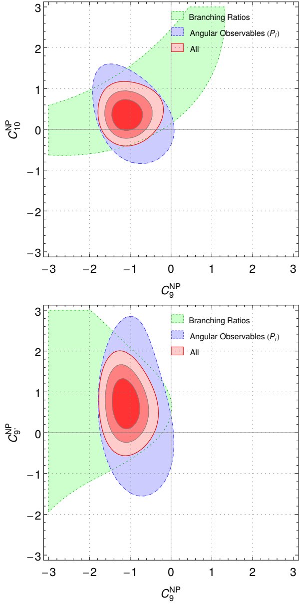
- fix $\vec{\theta} = \vec{\theta}_{\rm SM}$ to SM values *or* some other "reference point"
- generate samples of $(\vec{\nu})_i$ from some "prior" distr. (⇐ bayesian), including correlations
- calculate observables with this sample
- extract a "theory covariance" of observables at $\vec{\theta}_{\rm SM}$ ⇒ neglect non-gauss. from $\vec{\nu}$-pmr's
- use it in fits, assuming it is valid for $\vec{\theta} \neq \vec{\theta}_{\rm SM}$

Latest fits: angular obs's vs BR's — depends on scenario

[Altmannshofer/Straub 1411.3161 & 1503.06199]



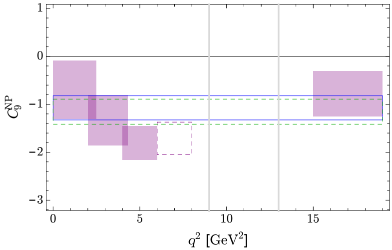
[Descotes-Genon/Hofer/Matias/Virto 1510.04239]



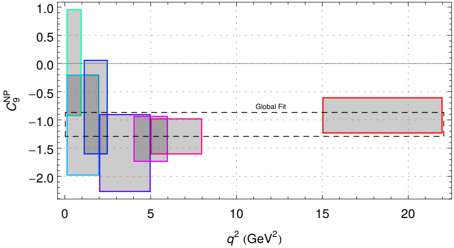
angular obs's (S_i) branching ratios

Latest fits: restricting fit to particular q^2-bins

[Altmannshofer/Straub 1503.06199]



[Descotes-Genon/Hofer/Matias/Virto 1510.04239]

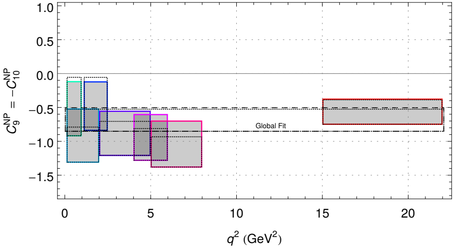


68% intervals from fits to particular q^2-bins

[upper] scenario real-valued C_9^{NP}

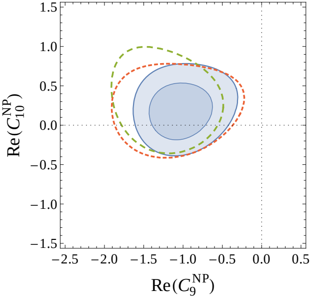
[lower] scenario real-valued $C_9^{NP} = -C_{10}^{NP}$

$\Rightarrow$ also high-q^2 prefers NP in C_9



Influence of $\vec{\nu}$-pmr's — How stable are fits under choice of ranges for $\vec{\nu}$-pmr's ???

[Altmannshofer/Straub 1411.3161 & 1503.06199]



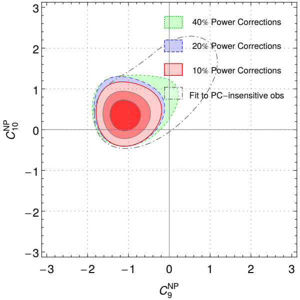
short-dashed = 2 × power corrections

long-dashed = 2 × form factor uncertainties

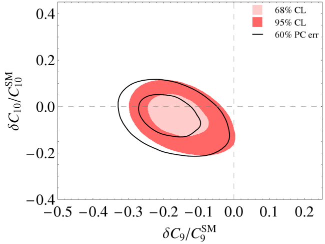
All groups

- use similar parameterisation of form factors and power corrections
- account for duality violation @ high q^2
- choice of $\vec{\theta}^* \neq \vec{\theta}_{SM}$ in theory covariance $\widehat{C}_{th}[\vec{\nu}](\vec{\theta}^*)$ has NO significant influence on the fit

[Descotes-Genon/Hofer/Matias/Virto 1510.04239]



[Hurth/Mahmoudi/Neshatpour 1603.00865]



solid lines 60% power corr's instead of 10% (red contours)

Precision on Wilson coefficients ...

$C_i(m_b) = C_i^{\text{SM}} + C_i^{\text{NP}}$ with $C_7^{\text{SM}} = -0.3$, $C_9^{\text{SM}} = 4.2$, $C_{10}^{\text{SM}} = -4.2$, $C_{7'}^{\text{SM}} = \frac{m_s}{m_b} C_7^{\text{SM}}$, $C_{9',10'}^{\text{SM}} = 0$

[Descotes-Genon et al. 1510.04239]

1D fit = 1 Wilson coeff. at the time

C_i^{NP}	BFP	$1\,\sigma$	$3\,\sigma$
C_7^{NP}	−0.02	[−0.04, 0.00]	[−0.07, 0.03]
C_9^{NP}	−1.11	[−1.31, −0.90]	[−1.67, −0.46]
C_{10}^{NP}	+0.61	[−0.40, 0.84]	[−0.01, 1.34]
$C_{7'}^{\text{NP}}$	+0.02	[−0.00, 0.04]	[−0.05, −0.09]
$C_{9'}^{\text{NP}}$	+0.15	[−0.09, 0.38]	[−0.56, 0.85]
$C_{10'}^{\text{NP}}$	−0.09	[−0.26, 0.08]	[−0.60, 0.42]

6D fit = 6 Wilson coeff. simultaneously

C_i^{NP}	$1\,\sigma$	$3\,\sigma$
C_7^{NP}	[−0.02, 0.03]	[−0.05, 0.08]
C_9^{NP}	[−1.4, −1.0]	[−2.2, −0.4]
C_{10}^{NP}	[−0.0, 0.9]	[−0.5, 2.0]
$C_{7'}^{\text{NP}}$	[−0.02, 0.03]	[−0.06, −0.07]
$C_{9'}^{\text{NP}}$	[0.3, 1.8]	[−1.3, 3.7]
$C_{10'}^{\text{NP}}$	[−0.3, 0.9]	[−1.0, 1.6]

- for 1D fits
  $\text{Pull}_{SM} = (1.2,\ 4.9,\ 3.0,\ 1.0,\ 0.6,\ 0.5)\,\sigma$
- for 6D fit $\text{Pull}_{SM} = 3.6\,\sigma$
- consistency with SM for
  C_9 above $3\,\sigma$, $C_{9'}$ at $2\,\sigma$

⇒ global fits can reach precision on Wilson coefficients of about 10% of SM values at $1\,\sigma$

see also [Altmannshofer/Straub 1411.3161 & 1503.06199, Hurth/Mahmoudi/Neshatpour 1603.00865]

Prospects from inclusive $B \to X_s \ell\bar{\ell}$

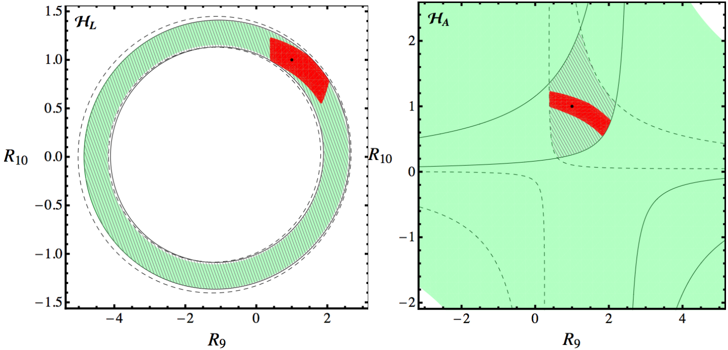
[Huber/Hurth/Lunghi 1503.04849]

!!! complementary to exclusive decays $\Rightarrow$ different treatment of subleading $1/m_b$ corrections

3 observables in angular analysis + 2 more at NLO QED $\quad Br \propto (H_L + H_T)$ and $A_{\rm FB} \propto H_A$

$$\frac{8}{3}\frac{d^2\Gamma}{dq^2\, d\cos\theta_\ell} = \left(1+\cos^2\theta_\ell\right) H_T(q^2) + 2\left(1-\cos^2\theta_\ell\right) H_L(q^2) + 2\cos\theta_\ell H_A(q^2)$$

Projected uncertainty from Belle II 50 ab^{-1} $\quad R_i \equiv C_i(\mu_0)/C_i^{\rm SM}(\mu_0)$



comb. of $H_{A,L,T}$ in 3 q^2-bins

assuming SM central values

uncertainty @ 95 % CL $\quad R_9 \in [0.5, 2.0]$, $R_{10} \in [0.6, 1.2]$ $\quad \Rightarrow$ better for C_{10} (40%), C_9 (50+%)

Summary

- progress @ LHCb and Belle II will improve **experimental accuracy** (2018 – 2024)

- **lattice** community picks up momentum $\Rightarrow$ increasing precision on **"simple" quantities:**
  hadr. ME's of local operators between (stable) meson states:

  decay constants, form factors, $\Delta B = 2$

  $\Rightarrow$ high precision ($\lesssim 1\%$) on Wilson coefficients (w.r.t. SM) feasible in

  - charged-current decays $d_i \to u_j \ell \nu_\ell$
  - leptonic FCNC $B_q \to \ell \bar{\ell}$ and semileptonic LFV $d_i \to d_j \ell \bar{\ell}'$ with $\ell \neq \ell'$
  - semi-neutrino FCNC $d_i \to d_j \nu \bar{\nu}$
  - $\Delta B = 2$

  !!! limited by lattice and ignorance of QED-structure dependent corrections

- harder for **exclusive** decays $b \to s + (\gamma, \ell\bar{\ell})$
  $\Rightarrow$ requires theory progress on power corrections
  current global fits reach 10% sensitivity of SM size

- high precision (few %) from **inclusive** $B \to X_s + \gamma$
  less precision of (20 – 25 %) @ 1 σ in $B \to X_s \ell\bar{\ell}$

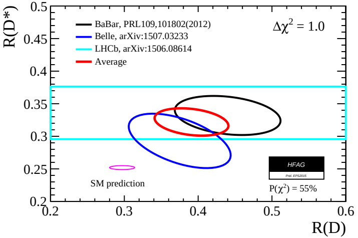
Backup Slides

... some minor deviations from SM ... some deviations in B sector from SM

- **CP-violation:** like-sign dimuon asymmetry by DØ at 3σ [DØ1310.0447]
  from SM "interpretation" [Borissov/Hoeneisen 1303.0175]
- tensions between **exclusive** and **inclusive** determinations of V_{ub} and V_{cb}
- **breaking** of lepton flavour universality (LFU) at **"tree"** and **"loop"** level ?

tree) $R(D^{(*)}) \equiv \dfrac{\mathcal{B}[B \to D^{(*)}\tau\bar{\nu}_\tau]}{\mathcal{B}[B \to D^{(*)}\ell\bar{\nu}_\ell]}$ $(\ell = e, \mu)$

combination of Babar, Belle, LHCb at 3.9σ



loop) $R_K \equiv \dfrac{\mathcal{B}[B^+ \to K^+\bar{\mu}\mu]}{\mathcal{B}[B^+ \to K^+\bar{e}e]} = 0.745^{+0.097}_{-0.082}$

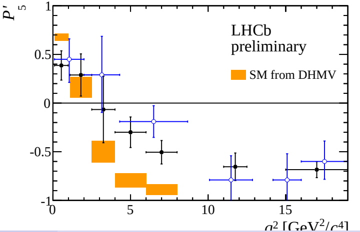
from $R_K|_{\rm SM} \approx 1$ at 2.6σ [LHCb 3/fb 1406.6482]

- $b \to s(\gamma, \bar{\ell}\ell)$ **global fits** for $\ell = \mu$

  with/without $\ell = e$ (i.e. R_K)
  (also known as "$B \to K^*\bar{\mu}\mu$ anomaly" in angular observable P_5')

  prefer non-SM value of eff. coupling

  $C_9 = C_9^{\rm SM} + \Delta C_9$ with $C_9^{\rm SM} \approx +4$ and $\Delta C_9 \approx -1$



Uncertainties @ low-q^2: $1/m_b$ corrections

- $\Lambda_{\rm QCD}/m_b \approx 13\%$ corrections to QCDF not known, only partially (contain endpoint divergences)
  [Kagan/Neubert hep-ph/0110078, Feldmann/Matias hep-ph/0212158, Beneke/Feldmann/Seidel hep-ph/0412400]
- $1/m_b$ corrections ⇒ ruin "optimised observables" (some more, others less):
  A) due to use of FF-relations (factorisable) and B) due to QCDF to $B \to K^{(*)}\bar{\ell}\ell$ amplitude (non-factorisable)

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structure of vector($\propto [\bar\ell\gamma_\mu\ell]$)-amplitudes (omitted helicity index $\lambda = 0, \pm 1$)

$$\mathcal{A} \propto \left(\xi_i + \Delta F_9^{\alpha_s} + \Delta F_9^{1/m_b}\right) C_9 + \left(\xi_i + \Delta F_7^{\alpha_s} + \Delta F_7^{1/m_b}\right) C_7 + \Delta^{\rm non-fac} + \Delta^{\bar cc}$$

- FF-relation breaking from α_s = known [Beneke/Feldmann hep-ph/0008255]
  $1/m_b$ = "unknown" (LCSR predictions of FF's account for some)

  ad-hoc parameterisation: $\Delta F_i^{1/m_b} = a_i + b_i \frac{q^2}{m_B^2} + c_i \frac{q^4}{m_B^4} + \ldots$ [Jäger/Martin-Camalich 1212.2263]

- $1/m_b$ corrections to non-factorisable parts (resonant, spectator scattering, WA)

  similarly $\Delta^{\rm non-fac} = \left(1 + A\, e^{i\phi_A} + B\, e^{i\phi_B} \frac{q^2}{m_B^2} + C\, e^{i\phi_C} \frac{q^4}{m_B^4}\right) \mathcal{A}^{\rm hadr}$ [Descotes-Genon et al. 1407.8526]
  with $A, B, C \in [0,\, 0.1]$, arbitrary $\phi_{A,B,C}$

- soft gluons to $\bar cc$ resonant-contributions [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945]

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How does it affect "optimised" observables ??? Example P_5'

ABSZ and DHMV =

simult. scan of groups of parameters (ABSZ incl. corr. of FF parameters)

$\Rightarrow$ error = linear or quadratic sum of spreads in observable

JMC (68%)=

gaussian priors for parameters

$\Rightarrow$ error = 68% of posterior predictive

JMC (max spread) =

simult. scan of all parameters

$\Rightarrow$ error = max spread in observable

Ref.	$q^2 \in [2.5, 4]$ GeV^2	$q^2 \in [4, 6]$ GeV^2
LHCb (3/fb)	$-0.07^{+0.34}_{-0.36}$	-0.30 ± 0.16
ABSZ (qua)	-0.50 ± 0.10	-0.77 ± 0.07
ABSZ (lin)	-0.50 ± 0.16	-0.77 ± 0.11
DHMV (qua)	$-0.49^{+0.14}_{-0.16}$	$-0.79^{+0.10}_{-0.12}$
DHMV (lin)	$-0.49^{+0.26}_{-0.30}$	$-0.79^{+0.16}_{-0.21}$
JMC (68%)	$-0.28^{+0.14}_{-0.13}$	$-0.71^{+0.11}_{-0.10}$
JMC (max spread)	$-0.28^{+0.54}_{-0.42}$	$-0.70^{+0.49}_{-0.31}$

LHCb = LHCb-CONF-2015-002

ABSZ = 1411.3161 + 1503.05534, DHMV = 1407.8526 + 1503.03328, JMC = 1412.3183 + talk S. Jäger Portoroz '15

Uncertainties @ low-q^2: $1/m_b$ corrections

- $\Lambda_{\rm QCD}/m_b \approx 13\%$ corrections to QCDF not known, only partially (contain endpoint divergences) [Kagan/Neubert hep-ph/0110078, Feldmann/Matias hep-ph/0212158, Beneke/Feldmann/Seidel hep-ph/0412400]
- $1/m_b$ corrections $\Rightarrow$ ruin "optimised observables" (some more, others less):
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How does it affect "optimised" observables ??? Example P_5'

- ABSZ (contrary to DHMV and JMC) uses full QCD FF's from LCSR
  $\Rightarrow$ do not need to consider $1/m_b$ corrections from FF relations, only due to $B \to K^*\bar{\ell}\ell$ amplitudes and $\bar{c}c$ tails
- DHMV try to implement error estimates as closely to JMC
  $\Rightarrow$ same parameterisation of FF-relation breaking corrections
- for linearly added errors: uncertainties of DHMV only half of JMC
- central values of P_5' between DHMV/ABSZ and JMC very different, due to choice of central values of FF-relation breaking corrections: JMC uses heavy quark limit $\Leftrightarrow$ DHMV/ABSZ use LCSR results

ABSZ = 1411.3161 + 1503.05534, DHMV = 1407.8526 + 1503.03328, JMC = 1412.3183 + talk S. Jäger Portoroz '15

Power corrections from $b \to \bar{c}c\,s \to \bar{\ell}\ell\,s$ for $q^2 \lesssim 6$ GeV^2

Parameterisation of power corrections $\lambda = \pm, 0$

$$h_\lambda(q^2) = \frac{\epsilon^*_\mu(\lambda)}{m_B^2} \int \mathrm{d}^4x\, e^{i\,q\cdot x} \left\langle K^{(*)}_\lambda \middle| \mathrm{T}\left\{ j^{\mathrm{em}}_\mu(x),\, \sum_i C_i \mathcal{O}_i(0) \right\} \middle| B(p) \right\rangle$$

$$\approx \underbrace{\left[\text{LO in } 1/m_b\right]}_{\text{QCDF}} + h_\lambda^{(0)} + \frac{q^2}{1\text{GeV}^2} h_\lambda^{(1)} + \frac{q^4}{1\text{GeV}^4} h_\lambda^{(2)}, \qquad h_\lambda^{(0,1,2)} \in \mathbb{C}$$

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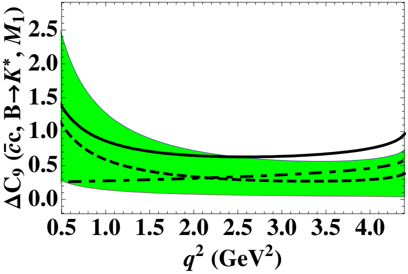
⇒ Soft-gluon emission off $\bar{c}c$-pairs enhanced by tree-level current-current $C_{1,2}$

1) contributions to $h_\lambda(q^2)$ via OPE

- works for $\Lambda_{\mathrm{QCD}} \ll 4m_c^2 - q^2$, also at $q^2 < 0$ GeV^2
- gives q^2-dependent shift to C_9
  $\Delta C_9^1(q^2) = (C_1 + 3C_2) g_{\mathrm{fact}}(q^2) + 2C_1 \tilde{g}_1(q^2)$
  with $\tilde{g}_1(q^2) \propto h_-(q^2) - h_+(q^2)$
- $g_{\mathrm{fact}}(q^2)$ = LO in $1/m_b$ = *dashed*
- soft-gluon emission $\tilde{g}_1(q^2)$ = *dashed-dotted*

[Khodjamirian et al. 1006.4945 + 1211.0234 + 1506.07760]



⇒ power corrections from soft gluons about 20% of C_9 at $1.0 \le q^2 \le 4.0$ GeV^2

2) interpolation up to $q^2 \approx 12$ GeV^2 via dispersion relation

Power corrections from $b \to \bar{c}c\,s \to \bar{\ell}\ell\,s$ for $q^2 \lesssim 6$ GeV^2

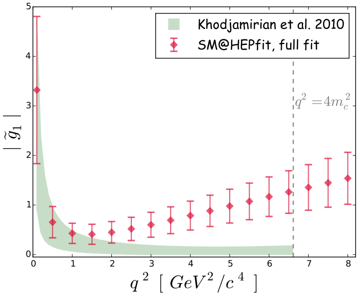
Parameterisation of power corrections $\lambda = \pm, 0$

$$h_\lambda(q^2) = \frac{\epsilon^*_\mu(\lambda)}{m_B^2} \int \mathrm{d}^4x\, e^{i\,q\cdot x} \left\langle K_\lambda^{(*)} \middle| \mathrm{T}\left\{ j_\mu^{\mathrm{em}}(x), \sum_i C_i \mathcal{O}_i(0) \right\} \middle| B(p) \right\rangle$$

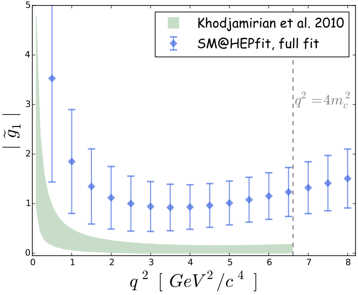
$$\approx \underbrace{[\text{LO in } 1/m_b]}_{\text{QCDF}} + h_\lambda^{(0)} + \frac{q^2}{1\text{GeV}^2} h_\lambda^{(1)} + \frac{q^4}{1\text{GeV}^4} h_\lambda^{(2)}, \qquad h_\lambda^{(0,1,2)} \in \mathbb{C}$$

$\Rightarrow$ Can fit $h_\lambda^{(0,1,2)}$ from data (assuming $C_9^{\mathrm{NP}} = 0$) [Ciuchini et al. 1512.07157]

with OPE-result at $q^2 = 0, 1$ GeV^2



without OPE-result



$\Rightarrow$ leads $(5-10)\times$ larger power corrections than predicted by Khodjamirian et al. for $\tilde{g}$'s

Power corrections from $b \to \bar{c}c\,s \to \bar{\ell}\ell\,s$ for $q^2 \lesssim 6$ GeV^2

Parameterisation of power corrections $\lambda = \pm, 0$

$$h_\lambda(q^2) = \frac{\epsilon_\mu^*(\lambda)}{m_B^2} \int \mathrm{d}^4x\, e^{i\,q\cdot x} \left\langle K_\lambda^{(*)} \middle| \mathrm{T}\left\{ j_\mu^{\rm em}(x), \sum_i C_i \mathcal{O}_i(0) \right\} \middle| B(p) \right\rangle$$

$$\approx \underbrace{\left[\text{LO in } 1/m_b\right]}_{\text{QCDF}} + h_\lambda^{(0)} + \frac{q^2}{1\text{GeV}^2} h_\lambda^{(1)} + \frac{q^4}{1\text{GeV}^4} h_\lambda^{(2)}, \qquad h_\lambda^{(0,1,2)} \in \mathbb{C}$$

- All global fits assume that
  1) power corrections can be parameterised as above
  and 2) size predicted by
  Khodjamirian/Mannel/Pivovarov/Wang 1006.4945
  even allowing for both signs
- large power corrections can not explain R_K measurement
- large power corrections might violate SM-op-basis relation

$$P_2 = \frac{1}{2}\left[P_4' P_5' + \sqrt{(-1 + P_1 + P_4'^2)(-1 - P_1 - P_5'^2)} \right]$$

[Matias/Serra 1402.6855, Matias talk La Thuile 2016]

Inclusive $B \to X_s\gamma$

$$\Gamma(B \to X_q\gamma) = \Gamma(b \to q\gamma)_\mathrm{p} + \delta\Gamma_\mathrm{np}$$

$$\propto (|C_7|^2 + |C_7'|^2)$$

$$O_{7\,(7')} \propto m_b \left[\bar{s}\sigma_{\mu\nu} P_{R(L)} b\right] F^{\mu\nu}$$

- $\Gamma(b \to q\gamma)_\mathrm{p}$ = perturbatively calculable part @ NNLO
- $\delta\Gamma_\mathrm{np}$ = non-perturbative part
  around 5% uncertainty @ $E_\gamma \geq 1.6$ GeV
  [Benzke/Lee/Neubert/Paz arXiv:1003.5012]
- $b \to d u \bar{u} \gamma$ sizeable in $b \to d\gamma$
  [Asatrian/Greub et al. arXiv:1305.6464]

Inclusive $B \to X_s\gamma$

$$\Gamma(B \to X_q\gamma) = \Gamma(b \to q\gamma)_{\rm p} + \delta\Gamma_{\rm np}$$

$$\propto (|C_7|^2 + |C_7'|^2)$$

$$O_{7\,(7')} \propto m_b \left[\bar{s}\sigma_{\mu\nu} P_{R(L)} b\right] F^{\mu\nu}$$

- $\Gamma(b \to q\gamma)_{\rm p}$ = perturbatively calculable part @ NNLO
- $\delta\Gamma_{\rm np}$ = non-perturbative part
  around 5% uncertainty @ $E_\gamma \geq 1.6$ GeV
  [Benzke/Lee/Neubert/Paz arXiv:1003.5012]
- $b \to du\bar{u}\gamma$ sizeable in $b \to d\gamma$
  [Asatrian/Greub et al. arXiv:1305.6464]

Latest SM updates @ NNLO QCD for $E_\gamma \geq 1.6$ GeV [Misiak et al. arXiv:1503.01789]

$$\mathcal{B}(B \to X_s\gamma)|_{\rm SM} = (3.36 \pm 0.23) \times 10^{-4}$$

$$\mathcal{B}(B \to X_d\gamma)|_{\rm SM} = (1.73^{+0.12}_{-0.22}) \times 10^{-5}$$

uncertainty budget due to:

- 5% non-perturbative
- 3% higher order
- 3% interpolation of m_c-dep. in NNLO corr.
- 2% parametric

Better adopted for actual measurement without strange tagging $\Rightarrow X_{s+d}$:

$$R_\gamma \equiv \frac{\mathcal{B}(B \to X_s\gamma) + \mathcal{B}(B \to X_d\gamma)}{\mathcal{B}(B \to X_x\ell\bar{\nu}_\ell)} = (3.31 \pm 0.22) \times 10^{-3}$$

Current world averages

$$\mathcal{B}(B \to X_s\gamma)|_{\rm Exp} = (3.43 \pm 0.22) \times 10^{-4}$$

$$\mathcal{B}(B \to X_d\gamma)|_{\rm Exp} = (1.41 \pm 0.57) \times 10^{-5}$$

$\Rightarrow$ bound on charged Higgs mass in 2HDM (type-II) $m_{H^\pm} > 480$ GeV @ 95% CL

Inclusive $B \to X_s \bar{\ell}\ell$ (at Belle II)

3 observables in angular analysis $\qquad Br \propto (H_L + H_T)$ and $A_{\rm FB} \propto H_A$

$$\frac{8}{3}\frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} = \left(1+\cos^2\theta_\ell\right) H_T(q^2) + 2\left(1-\cos^2\theta_\ell\right) H_L(q^2) + 2\cos\theta_\ell H_A(q^2)$$

different dependence on short-distance $C_{7,9,10}$ – **complementary to $B \to K^{(*)}\bar{\ell}\ell$ at low q^2**

$(\hat{s} = q^2/m_b^2)$

$$H_T \propto \hat{s}(1-\hat{s})^2\left[\left|C_9 + \frac{2}{\hat{s}}C_7\right|^2 + |C_{10}|^2\right] \qquad H_L \propto (1-\hat{s})^2\left[|C_9 + 2C_7|^2 + |C_{10}|^2\right]$$

$$H_A \propto -4\,\hat{s}(1-\hat{s})^2 \mathrm{Re}\left[(C_9 + \frac{2}{\hat{s}}C_7)C_{10}^*\right]$$

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SM predictions @ NNLO QCD and NLO QED [Huber/Hurth/Lunghi arXiv:1503.04849]

- theory unc. for $\mathcal{B}$ and $H_{L,T}$: $6-9\,\%$ in $q^2 \in [1, 3.5], [3.5, 6], [1, 6]$ GeV^2
- theory unc. for H_A: from $5-70\,\%$, depend strongly on q^2-binning around zero-crossing
- zero-crossing of H_A predicted with $\lesssim 4\,\%$
- QED corrections lead to **pronounced differences** for $\ell = e$ and $\ell = \mu$
- at high-q^2 uncertainties larger: $\mathcal{B}$ about $30\,\%$
- PHOTOS gives satisfactory approximation of explicite QED results

Effects of M_{X_s}-cuts analysed in SCET at level of sub-leading shape functions $\Rightarrow$ require combination of $B \to X_s\gamma$, $B \to X_s\bar{\ell}\ell$ and $B \to X_u\ell\bar{\nu}_\ell$ [Lee et al. hep-ph/0511334, 0512191, 0812.0001, Bernlocher et al.1101.3310, Bell et al. 1007.3758]