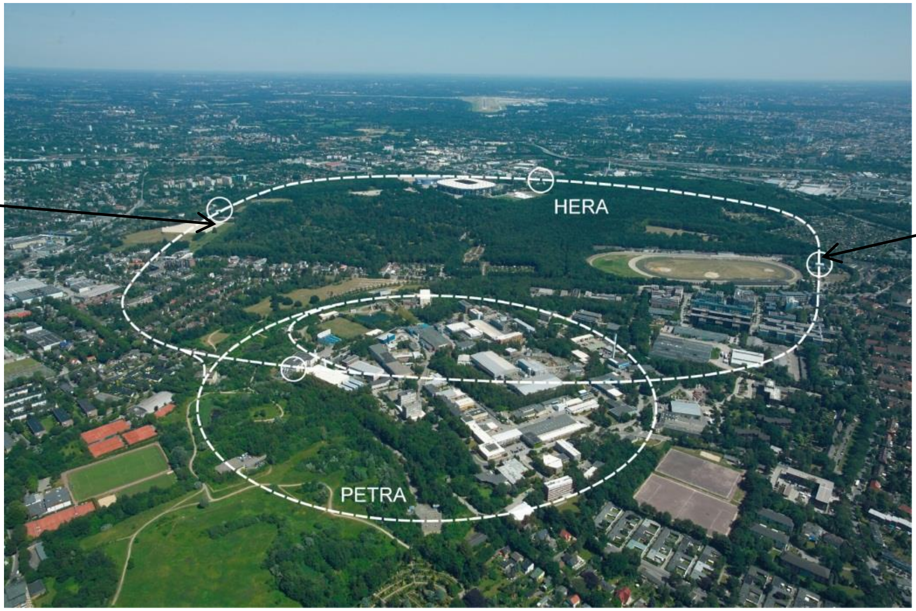
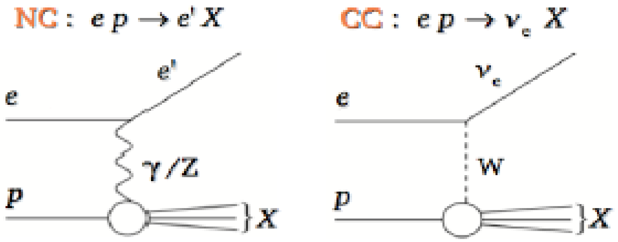


# Proton Structure and Hard QCD

AM Cooper-Sarkar, Oxford

**Phys Rev D93(2016)092002**

# Deep Inelastic Scattering (DIS) is the best tool to probe proton structure



- Kinematic variables:

$Q^2 = -q^2 = -(k-k')^2$ Virtuality of the exchanged boson

$x = \frac{Q^2}{2p \cdot q}$ Bjorken scaling parameter

$y = \frac{p \cdot q}{p \cdot k}$ Inelasticity parameter

$s = (k+p)^2 = \frac{Q^2}{xy}$ Invariant c.o.m.

***Neutral current:***

$$\frac{d^2\sigma^{\pm}_{NC}}{dx dQ^2} = \frac{2\alpha\pi^2}{xQ^4}\left(Y_+ F_2 \mp Y_- xF_3 - y^2 F_L\right)$$

$F_2 \propto \sum_i e_i^2 (xq_i + x\bar{q}_i)$ quark distributions

$xF_3 \propto \sum_i (xq_i - x\bar{q}_i)$ valence quarks

$F_L \propto \alpha_s \times g$ gluon at NLO

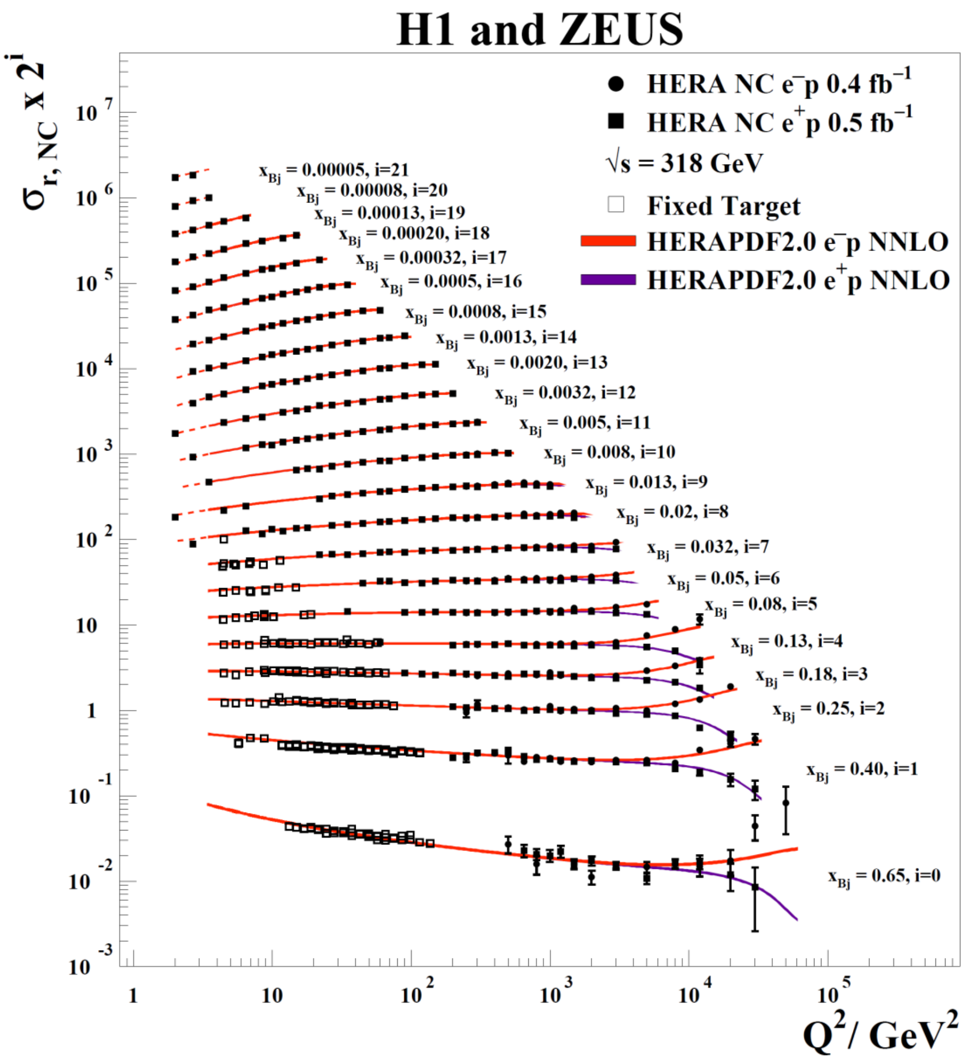
LO expressions

***Charged current:***

$$\frac{d^2\sigma^{-}_{CC}}{dx dQ^2} = \frac{G_F^2}{2\pi}\frac{M_W^2}{M_W^2+Q^2}\left(u+c+(1-y^2)(\bar{d}+\bar{s})\right)$$

$$\frac{d^2\sigma^{+}_{CC}}{dx dQ^2} = \frac{G_F^2}{2\pi}\frac{M_W^2}{M_W^2+Q^2}\left(\bar{u}+\bar{c}+(1-y^2)(d+s)\right)$$

flavour decomposition



Gluon from the scaling violations: DGLAP equations tell us how the partons evolve

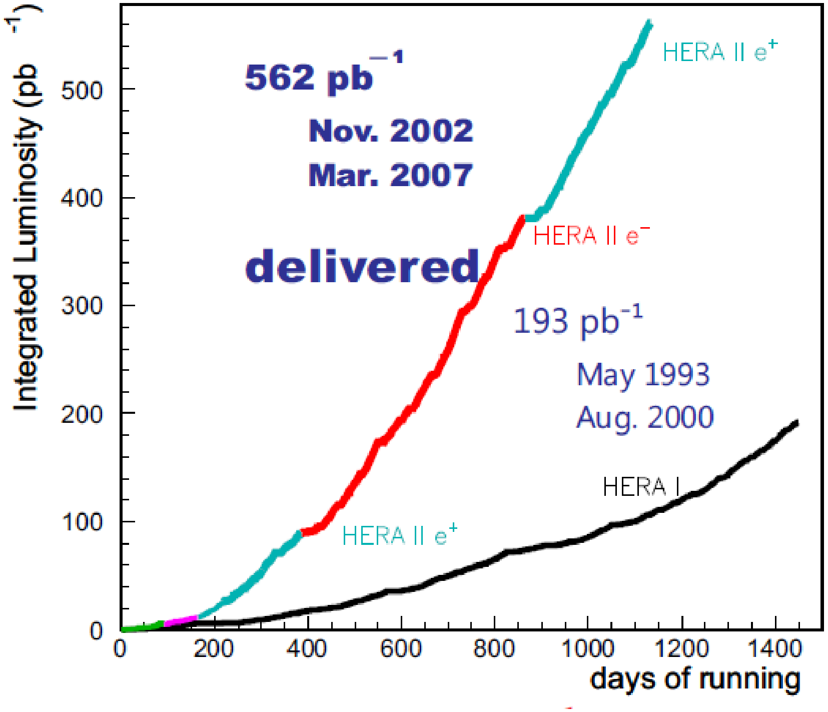
# Final inclusive data combination from all HERA-1+11 running

**~500pb$^{-1}$ per experiment split ~equally between e$^{+}$ and e$^{-}$ beams: DESY-15-039**

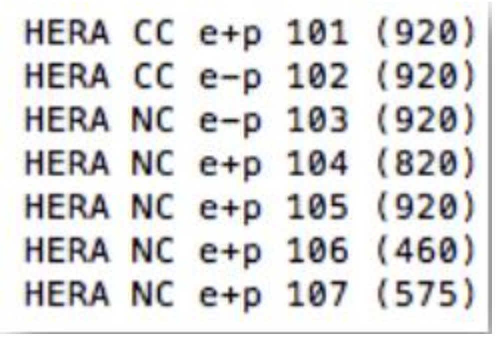
**10 fold increase in e$^{-}$ compared to HERA-I**

**Running at Ep = 920, 820, 575, 460 GeV**
**$\sqrt{s}$ = 320, 300, 251, 225 GeV**

The lower proton beam energies allow a measurement of $F_L$ and thus give more information on the gluon.

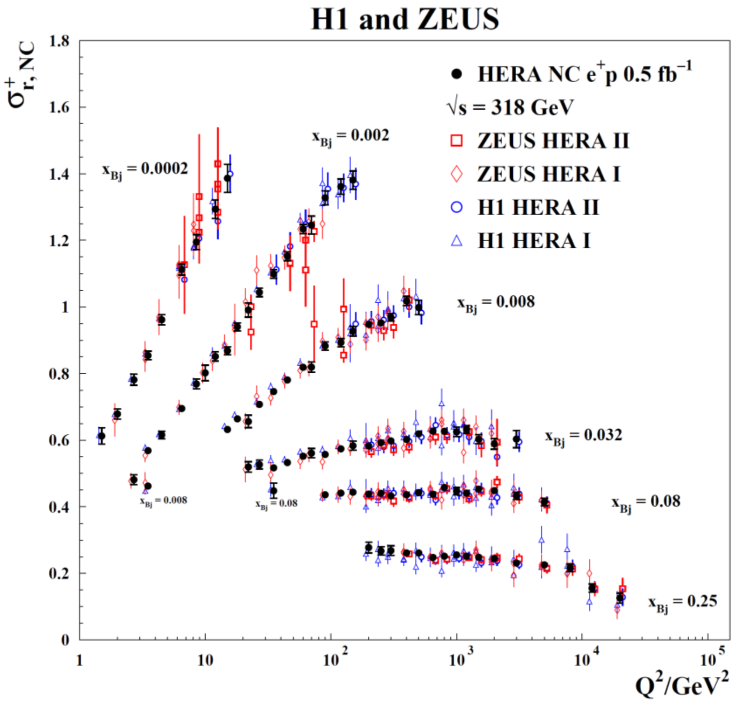


41 input data files to 7 output files with
169 sources of correlated uncertainty

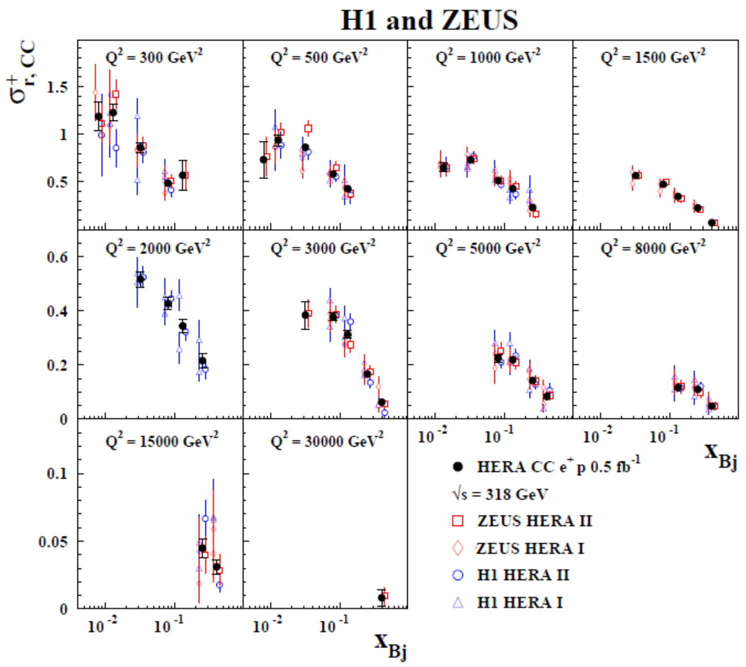


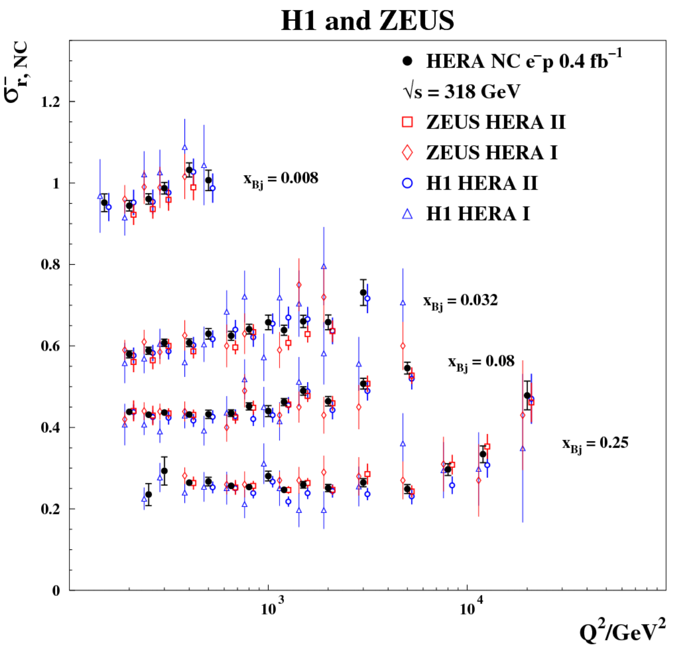
```
HERA CC e+p 101 (920)
HERA CC e-p 102 (920)
HERA NC e-p 103 (920)
HERA NC e+p 104 (820)
HERA NC e+p 105 (920)
HERA NC e+p 106 (460)
HERA NC e+p 107 (575)
```

**$0.045 < Q^2 < 50000$ GeV$^2$ $\quad$ $6.\ 10^{-7} < x_{Bj} < 0.65$**

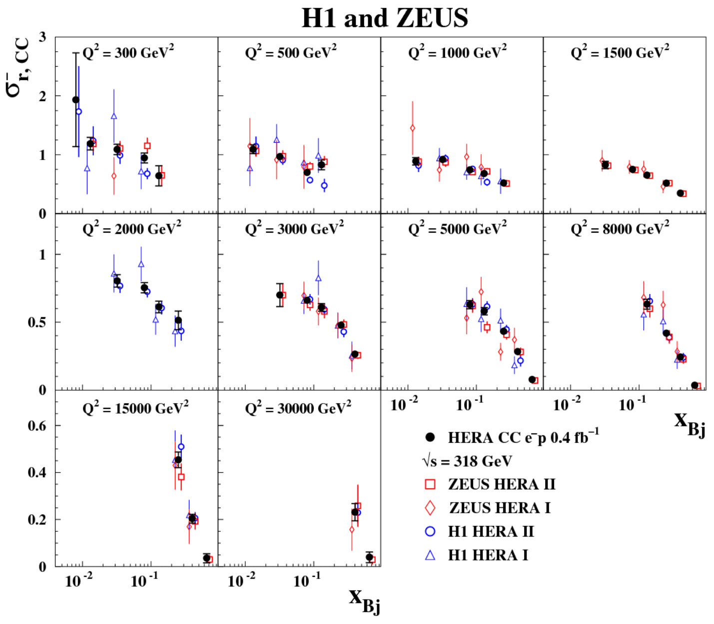


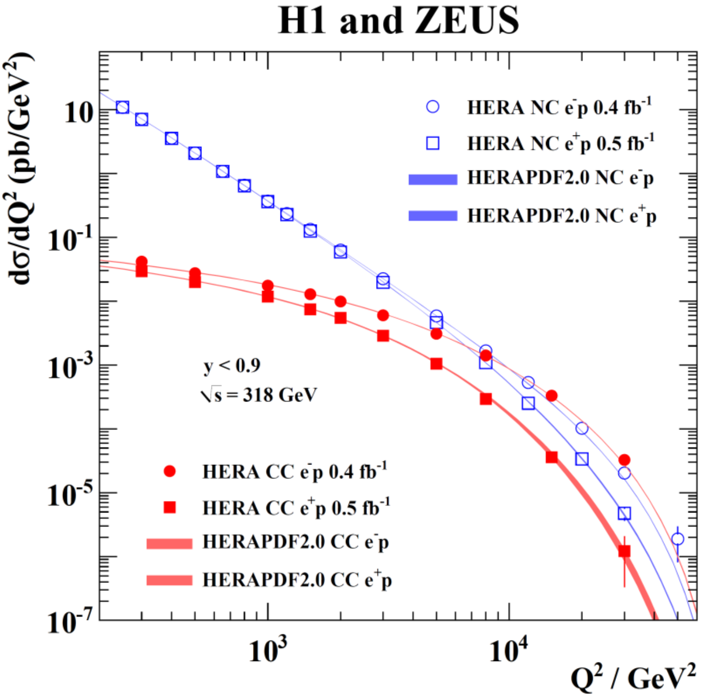
NC and CC e$^+$
vs H1 and
ZEUS inputs





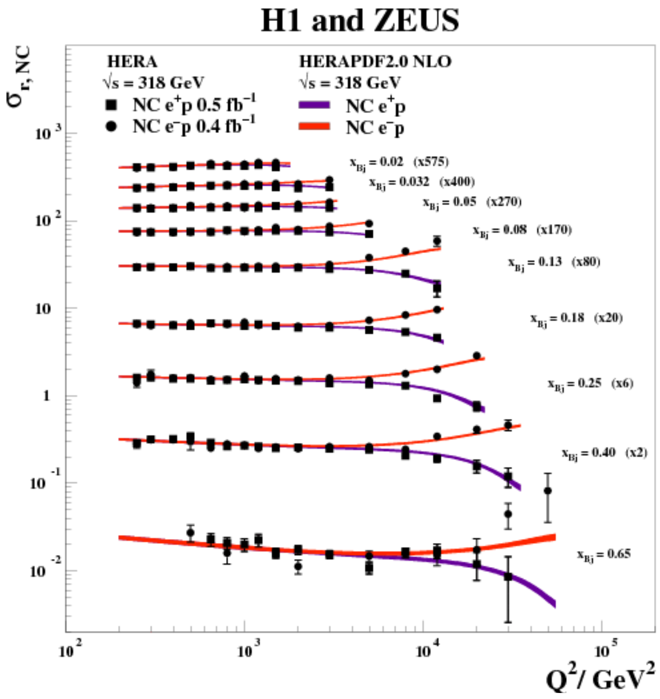
NC and CC e$^-$
vs H1 and
ZEUS inputs
10 fold increase
in e$^-$ statistics
compared to old
HERA-1
combination



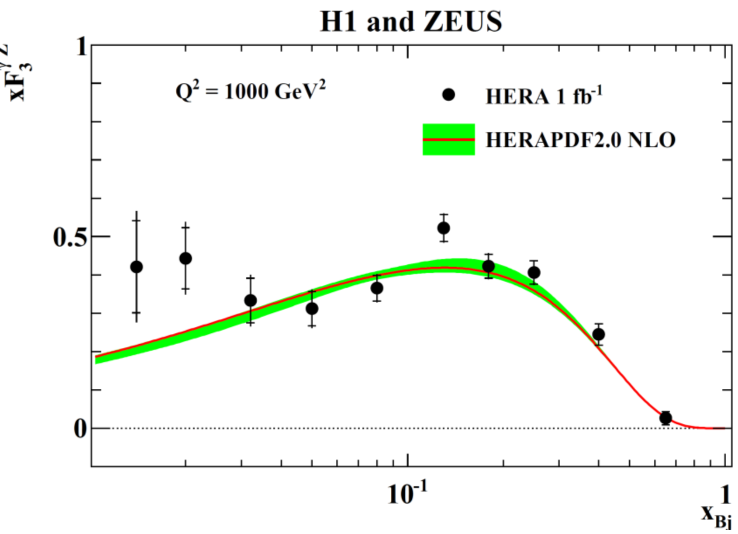


**Electroweak unification**

**NC$e^+$/NC$e^-$ difference at high Q$^2$ due to γZ interference**

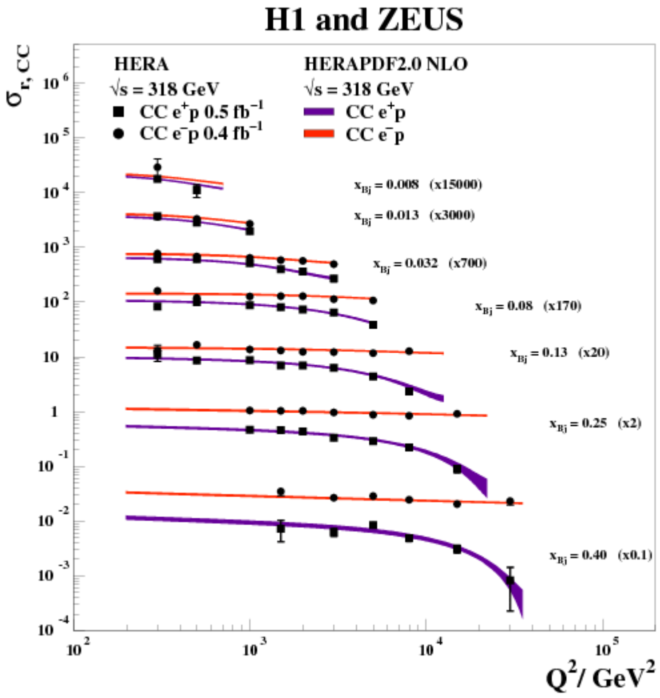


**Enables an extraction of xF3**



**CC$e^+$/CC$e^-$ difference at high Q$^2$ .. due to (1-y)$^2$ suppression at high y**

These plots already show the QCD fit HERAPDF2.0



# The HERAPDF approach uses only HERA data

The combination of the HERA data yields a very accurate and consistent data set for 4 different processes: $e^+p$ and $e^-p$ Neutral and Charged Current reactions and for $e^+p$ Neutral Current at 4 different beam energies

The use of the single consistent data set allows the usage of the conventional χ2 tolerance Δχ2 = 1 when setting 68%CL experimental errors

NOTE the use of a pure proton target means no need for heavy target/deuterium corrections.

d-valence is extracted from CC $e^+p$ without assuming d in proton= u in neutron

All data are at high W (> 15 GeV), so high-x, higher twist effects are negligible.

These are the only PDFs for which this is true

HERAPDF evaluates model uncertainties and parametrisation uncertainties in addition to experimental uncertainties

HERAPDF1.0 was based on the combination of HERA-I data

HERAPDF1.5 included preliminary HERA-II data

HERAPDF2.0 is based on the new final combination of HERA-I and HERA-II data which supersedes the HERA-I combination and supersedes all previous HERAPDFs

# HERAPDF specifications: parameterisation and χ2 definition

For the NLO and NNLO fits the central parametrisation at $Q^2_0$ = 1.9 GeV$^2$ is

$$\begin{aligned}
xg(x) &= A_g x^{B_g}(1-x)^{C_g} - A'_g x^{B'_g}(1-x)^{C'_g},\\
xu_v(x) &= A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}}\left(1+E_{u_v}x^2\right),\\
xd_v(x) &= A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}},\\
x\bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}\,(1+D_{\bar{U}}x),\\
x\bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}}.
\end{aligned}$$

QCD sum-rules constrain $A_g$,$A_{uv}$,$A_{dv}$

$x\bar{s} = f_s x\bar{D}$ sets the size of the strange PDF and the constraints $B_{\bar{U}} = B_{\bar{D}}$ and $A_{\bar{U}} = A_{\bar{D}}(1-f_s)$ ensure $x\bar{u} \to x\bar{d}$ as $x \to 0$.

- There are 14 free parameters in the central fit determined by saturation of the χ2
- $\alpha_S(M_Z)$ = 0.118 for central fits
- PDFs are evolved using the DGLAP equations using QCDNUM and convoluted with coefficient functions to evaluate structure functions and hence measurable cross sections
- Heavy quark coefficient functions are evaluated by the Thorne Roberts Optimized Variable Flavour Number scheme – this is the standard, unless otherwise stated
- Fixed Flavour Number PDFs are also available at NLO
- An LO fit with $\alpha_S(M_Z)$ = 0.130 is also provided with an alternative gluon (AG) parametrisation
- The form of the χ2 accounts for 169 correlated uncertainties, 162 from the input data sets and 7 from the procedure of combination

$$\chi^2_{\rm exp}(\boldsymbol{m},\boldsymbol{s}) = \sum_i \frac{\left[m^i - \sum_j \gamma^i_j m^i s_j - \mu^i\right]^2}{\delta^2_{i,\rm stat}\,\mu^i m^i + \delta^2_{i,\rm uncor}\,(m^i)^2} + \sum_j s_j^2 + \sum_i \ln\frac{\delta^2_{i,\rm stat}\,\mu^i m^i + (\delta_{i,\rm uncor}\, m^i)^2}{(\delta^2_{i,\rm stat} + \delta^2_{i,\rm uncor})(\mu^i)^2}$$

# HERAPDF specifications: sources of uncertainty

**Experimental**

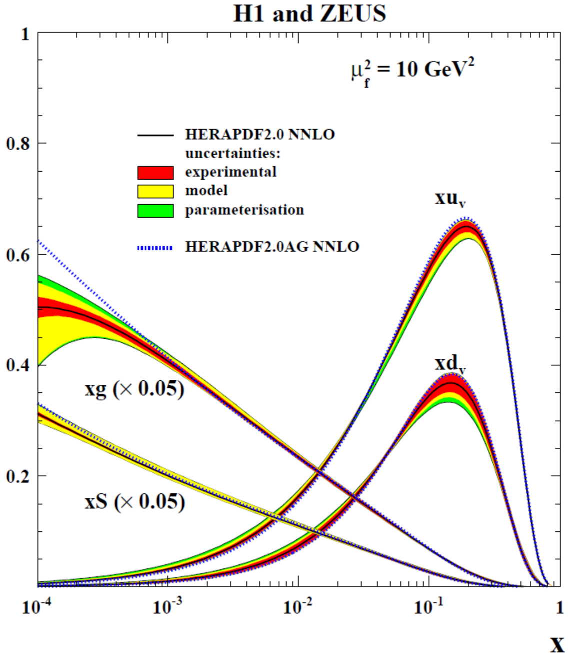
**Hessian uncertainties: 14 eigenvector pairs, evaluated with Δχ2 = 1**

**Cross checked uncertainties evaluated from the r.m.s. of MC replicas**

**Model: Variation of input assumptions**

**Variation of charm mass and beauty mass parameters is restricted using HERA charm and beauty data**

| Variation | central | Upper | lower |
|---|---|---|---|
| $f_s$ size and shape | 0.4 | 0.5 | 0.3 |
| $M_c$ (NLO) GeV | 1.43 | 1.49 | 1.37 |
| $M_c$ (NNLO) GeV | 1.47 | 1.53 | 1.41 |
| $M_b$ GeV | 4.5 | 4.25 | 4.75 |
| $Q^2_{min}$ GeV$^2$ | 3.5 | 2.5 | 5.0 |
| $Q^2_{min}$(HiQ2) | 10.0 | 7.5 | 12.5 |

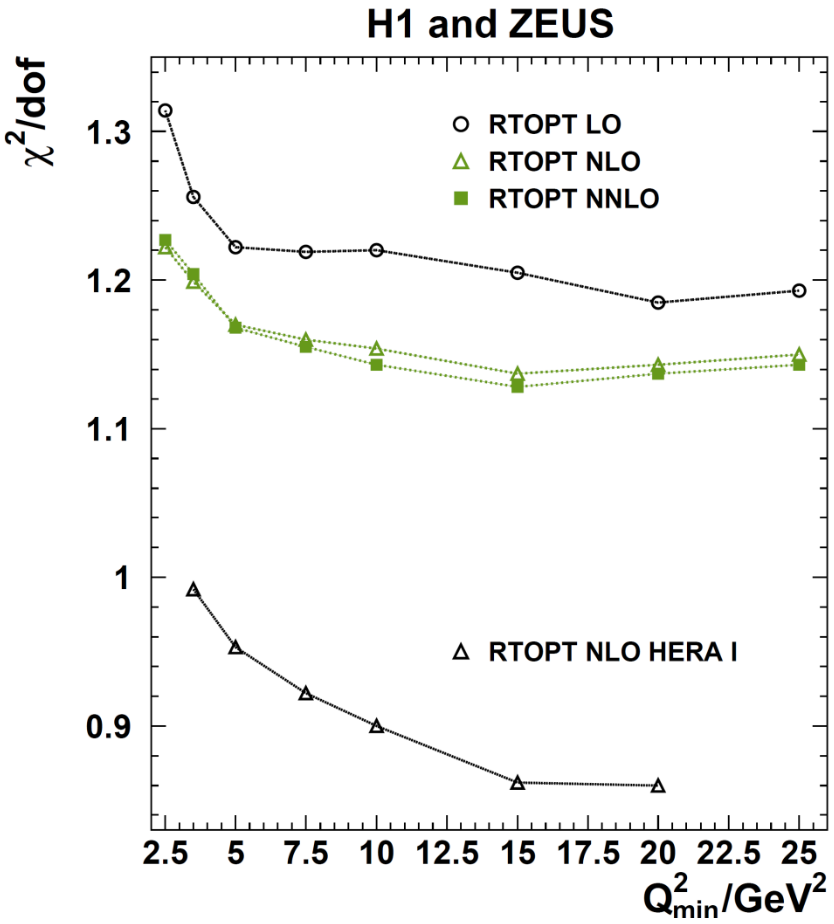


**Parametrisation**

**Variation of $Q^2_0 = 1.9 \pm 0.3$ GeV$^2$ and addition of 15th parameters**

The value of $\alpha_S(M_Z)$ is not treated as an uncertainty. The central value is $\alpha_S(M_Z) = 0.118$
But PDFs are supplied for $\alpha_S(M_Z)$ values from 0.110 to 0.130 in steps of 0.001

# HERAPDF specifications: minimum value of $Q^2$



A minimum value of $Q^2$ for data allowed in the fit is imposed to ensure that pQCD is applicable. For HERAPDF the usual value is $Q^2 > 3.5$ GeV$^2$ but consider the variation of χ2 with this cut

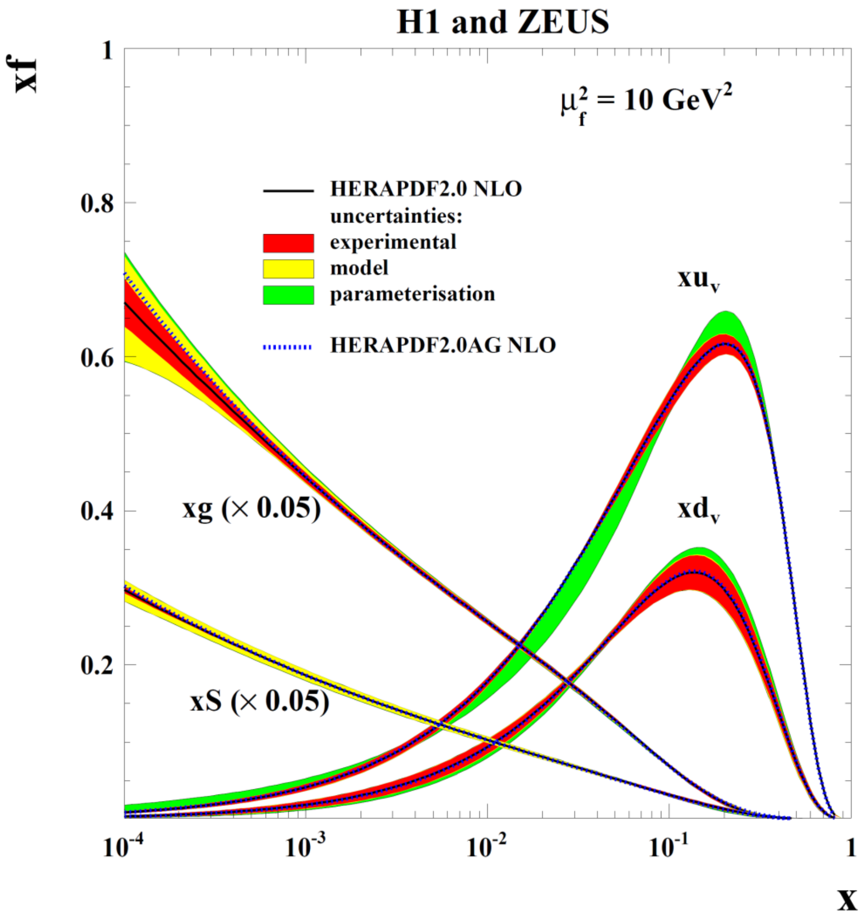
- The χ2 decreases with increase of $Q^2$ minimum until $Q^2_{min} \sim 10$ -15 GeV$^2$
- The same effect was observed in HERA-1 data
- This is independent of heavy flavour scheme
- NLO is obviously better than LO but NNLO is not significantly better than NLO, for RT

Fits for two $Q^2$ cuts were presented: HERAPDF2.0: $Q^2 > 3.5$ and
HERAPDF2.0HiQ2: $Q^2 > 10$ GeV$^2$

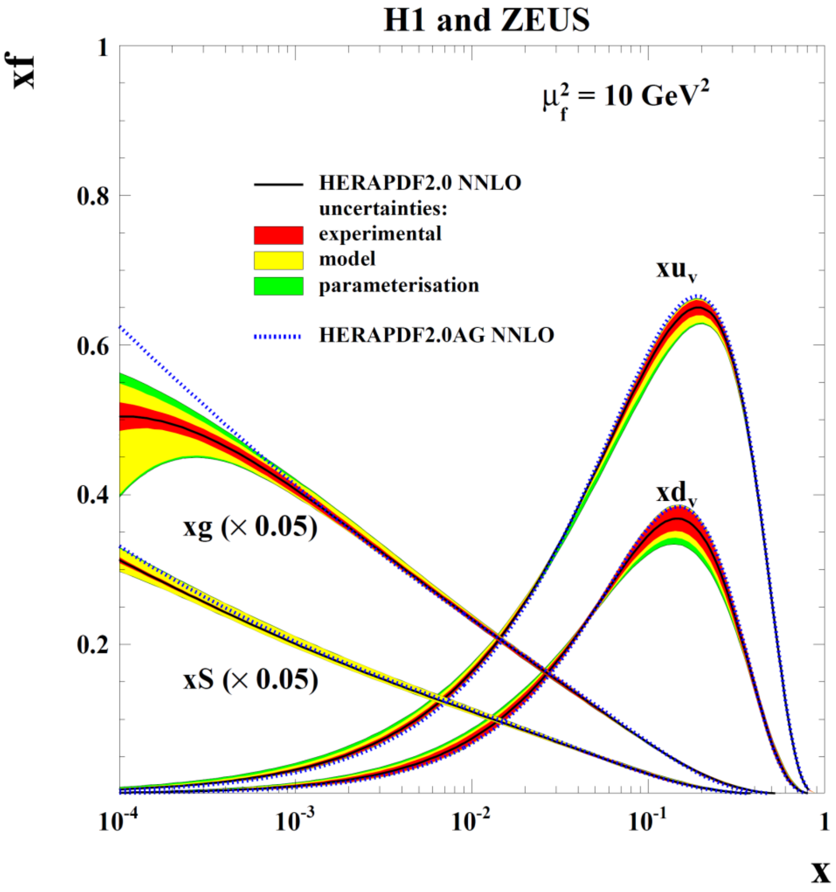
HERA kinematics is such that cutting out low $Q^2$ also cuts the lowest x values, thus HERAPDF2.0HiQ2 is used to assess possible bias in HERAPDF2.0 from including a kinematic region which might require treatment of: non-perturbative effects; ln(1/x) resummation; saturation etc.

# HERAPDF2.0: NLO and NNLO fits

NLO

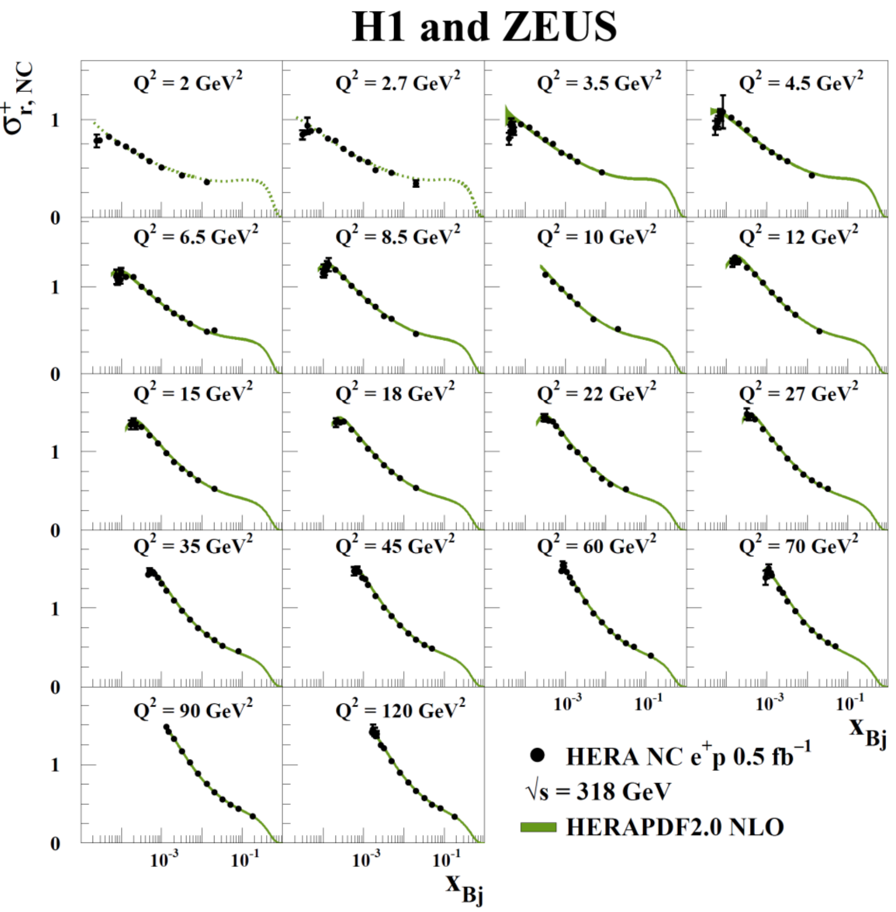


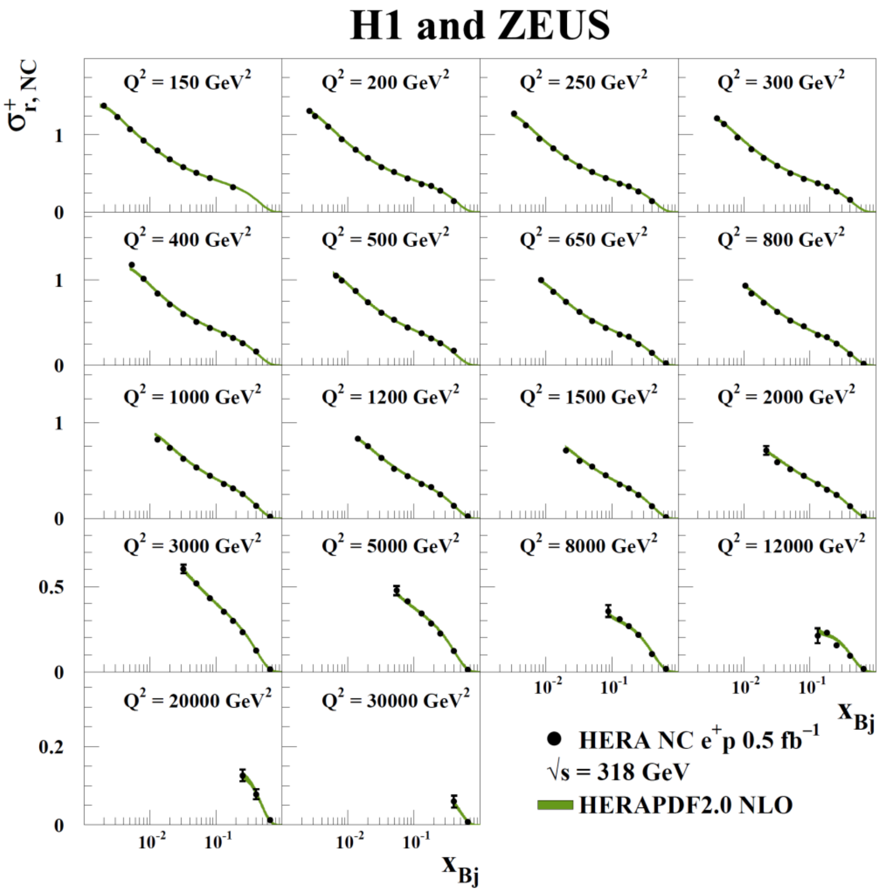
NNLO



The HERAPDF2.0AG is an alternative gluon parametrisation which is positive definite for all x and all $Q^2 > Q^2_0$

# HERAPDF2.0 compared to data

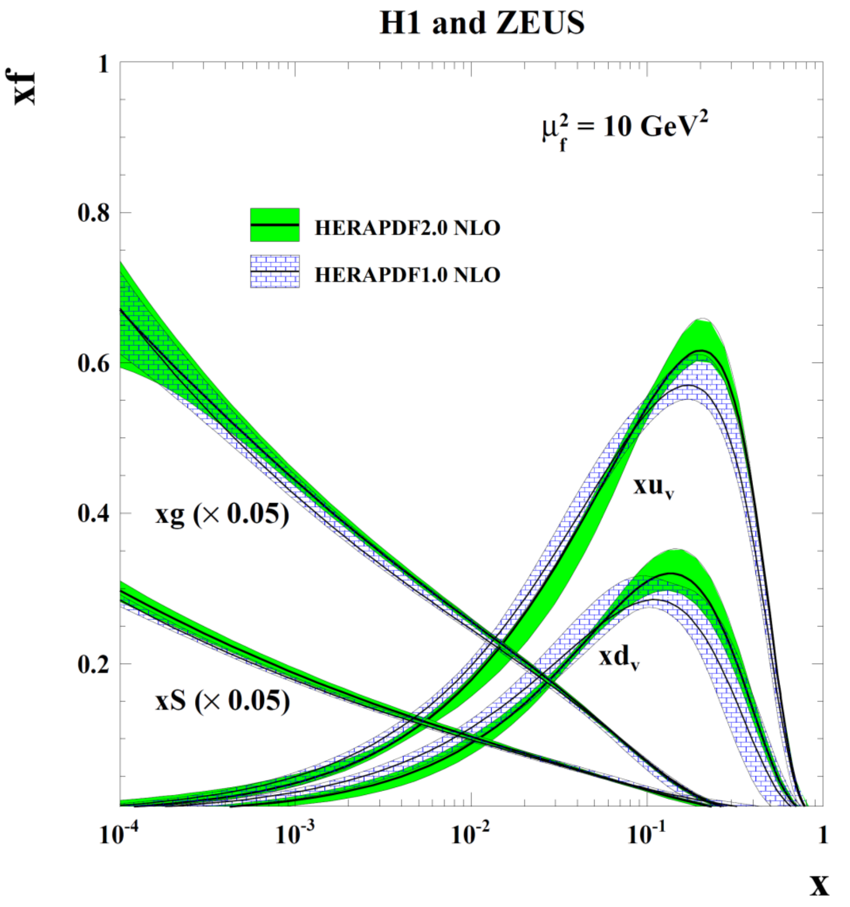


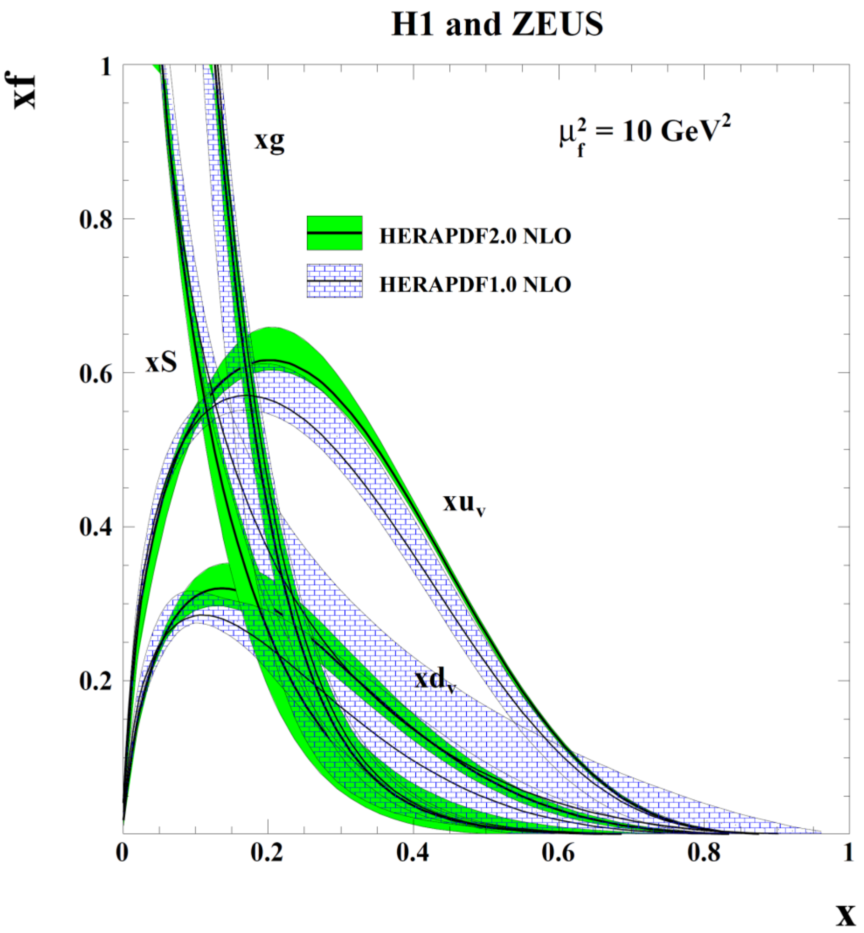


Here is the comparison to the NC e$^+$ data for $2 < Q^2 < 30000$ GeV$^2$

NLO and NNLO fits look very similar

# Compare HERAPDF2.0 to HERAPDF1.0 at NLO

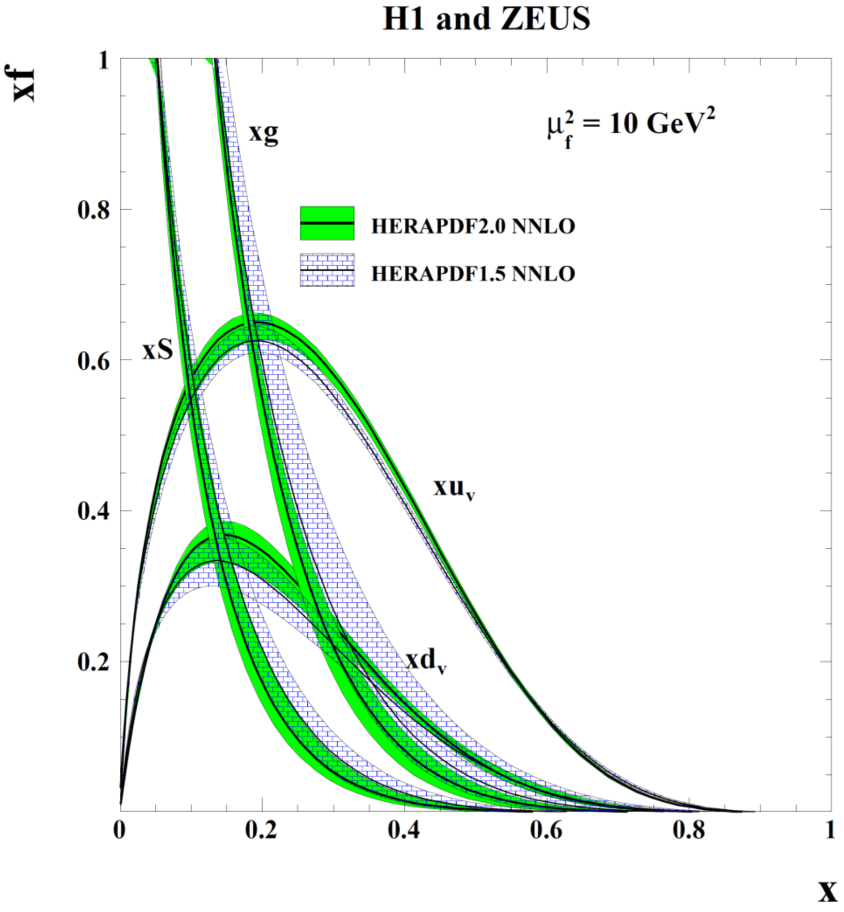


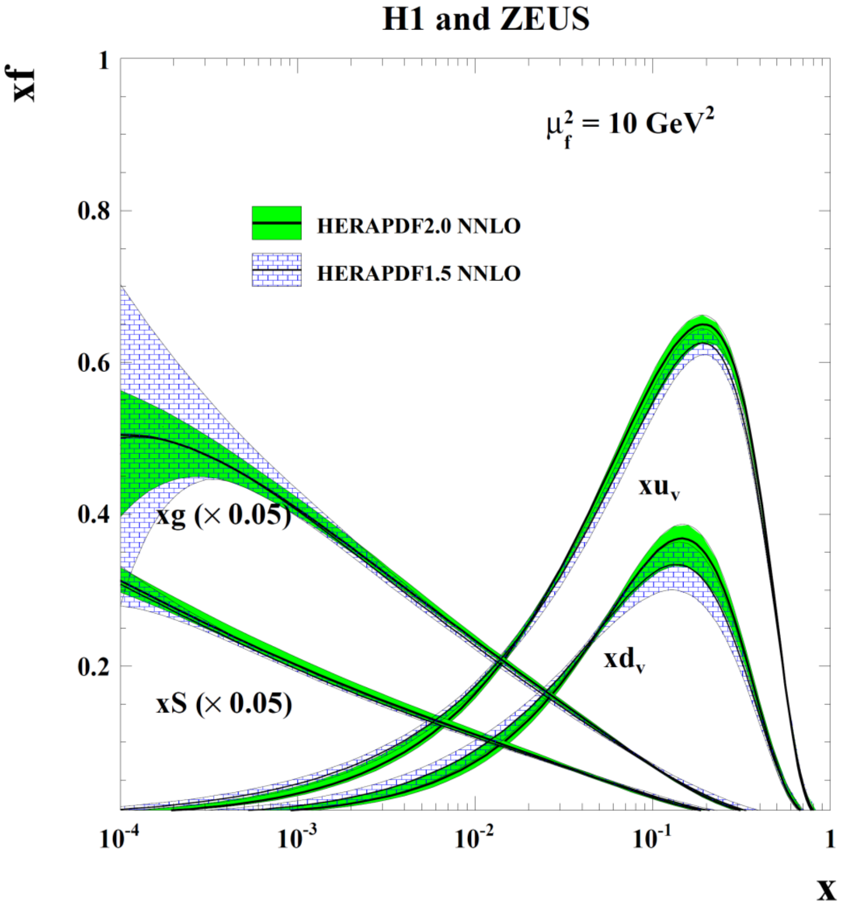


Much more high-x data
Substantial reductions in high-x uncertainty
Some change in valence shape

- HERAPDF1.0 (and 1.5) had rather hard high-x sea, harder than the gluon (within large uncertainties).This is no longer the case and uncertainties are much reduced
- HERAPDF1.0 and 1.5 had a soft high-x gluon this moves to the top of its previous error band- but is still soft (at NLO)

# Compare HERAPDF2.0 to HERAPDF1.5 at NNLO

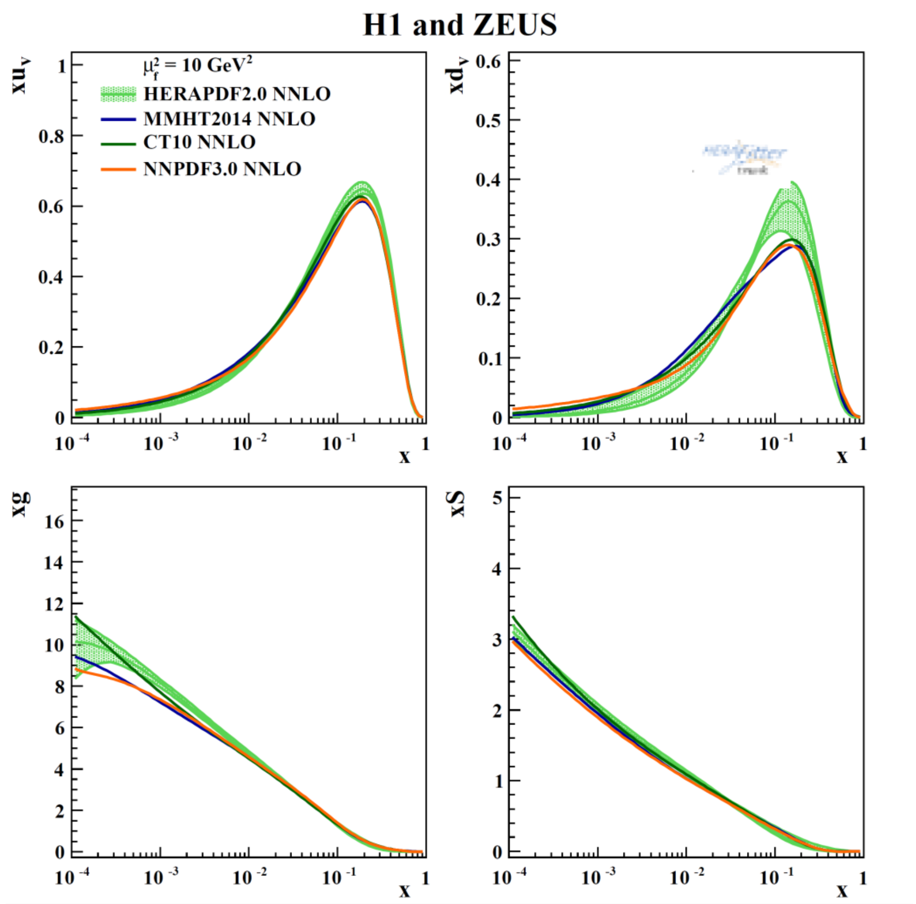


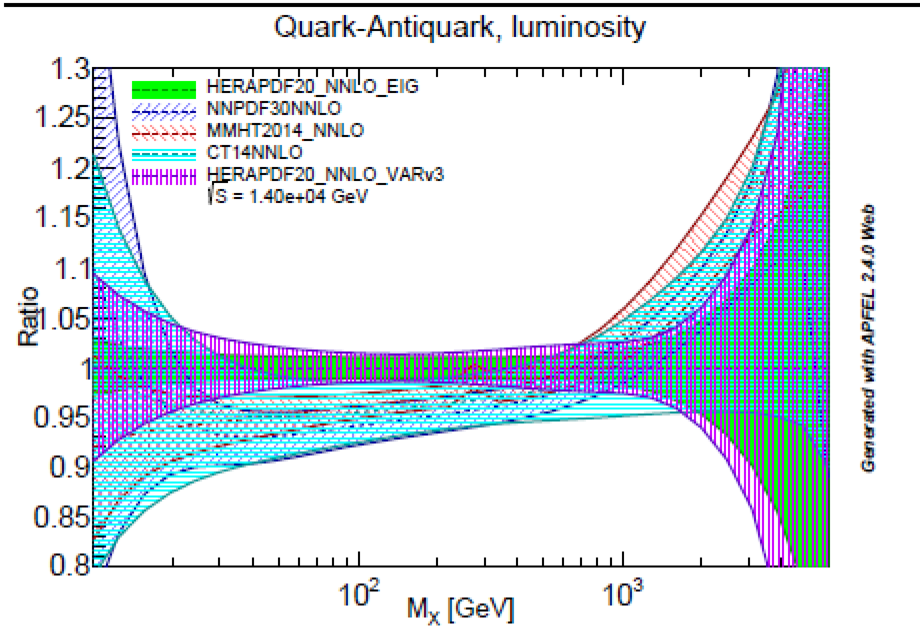


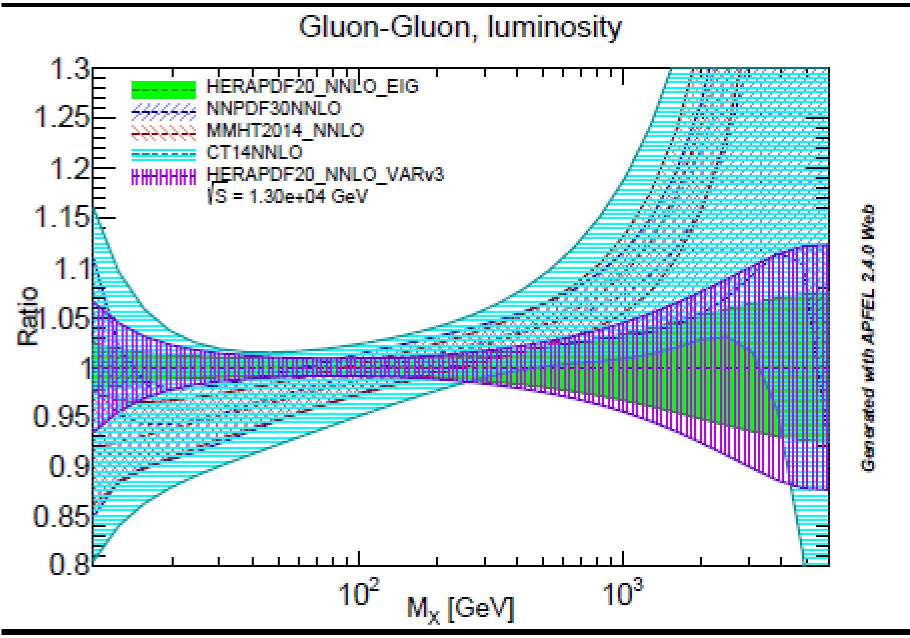
Reduction in gluon uncertainty both at low-x and high-x.
A lot of this reduction is because the model variation due to variation of $Q^2$ cut is not as dramatic now that we have more data.

The HERAPDF1.5 gluon was not soft compared to global PDFs. However it had a large error band.
This uncertainty on the gluon decreases and the central value moves to the lower end of its previous error band

# Compare HERAPDF2.0 to other PDFs at NNLO





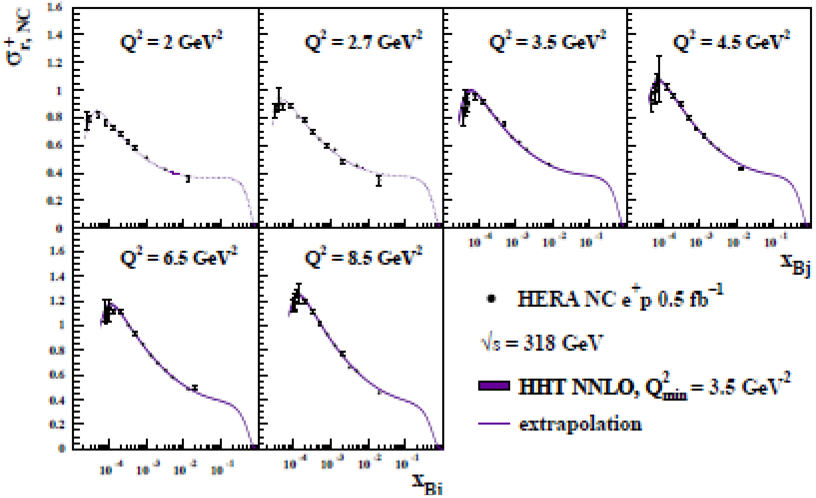


High-x valence shapes somewhat different – new high- x data and use of proton target only. Gluon and Sea are both broadly compatible with other PDFs

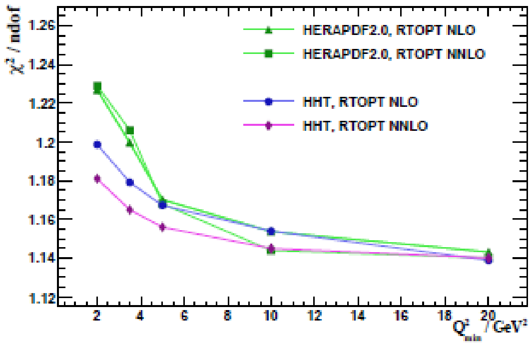
Comparison of q-qbar and glu-glu luminosity at 13 TeV show consequences for LHC

In the original HERAPDF2.0 analysis PDFs were presented for $Q^2 > 10$ GeV$^2$ as well as for $Q^2 > 3.5$ GeV$^2$. This avoids any bias from low-$Q^2$, low-x and results in PDFs which are very similar at LHC scales. However such PDFs cannot be used at low-x, low $Q^2$

$$\sigma^{\pm}_{r,NC} = F_2 - \frac{y^2}{Y_+} F_L$$

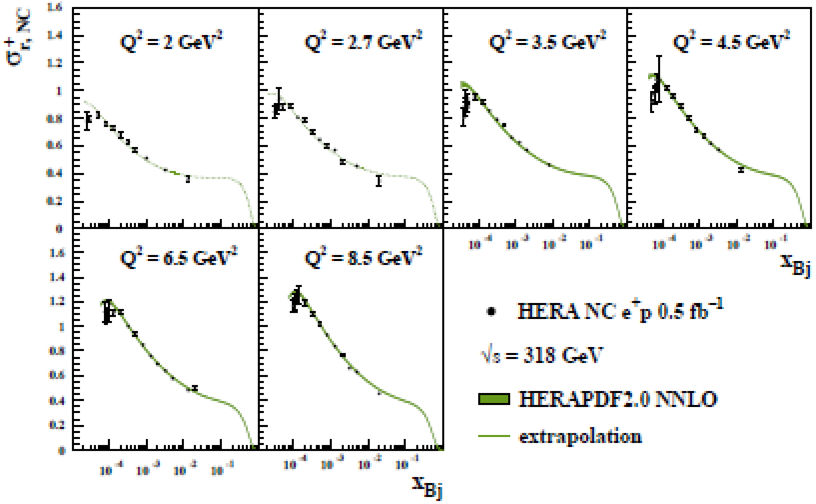


It may be better to consider adding low-x higher twist terms- HHT (arxiv:1604.02299)
The higher twist terms are only significant in $F_L$.
A larger $F_L$ is predicted which fits the high-y turn over of the reduced cross section much better. This reduces the χ2 of the NNLO fit by 47.



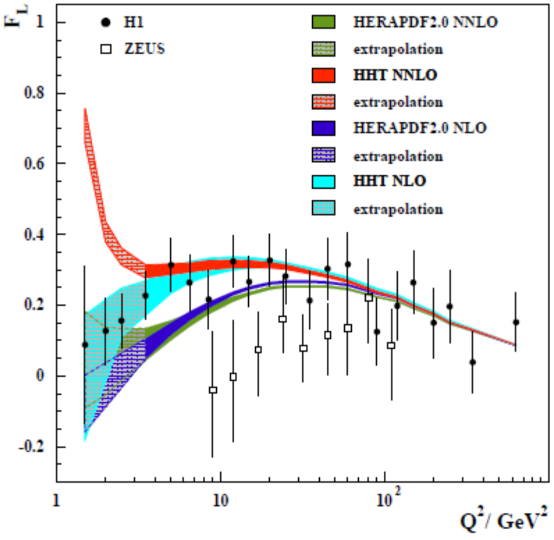
NNLO is now better than NLO

$$F_L^{HT} = F_L^{DGLAP} \left(1 + A_L^{HT}/Q^2\right)$$

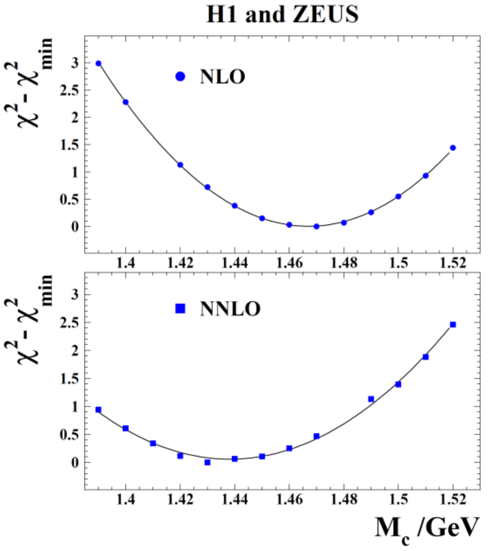


The PDFs from these HHT fits are similar to HERAPDf2.0 at LHC scales

Data can be fitted down to $Q^2 = 2$GeV$^2$ - but lower $Q^2$ cannot be described in such a simple picture



# Adding more data to HERAPDF2.0: heavy flavour data and jet data

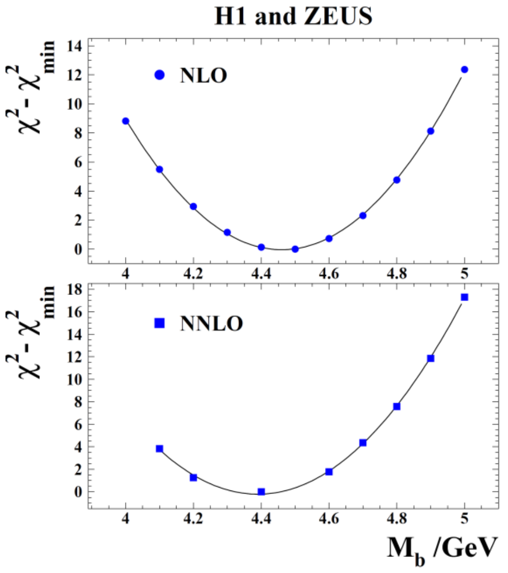


**HERAPDF2.0Jets is based on inclusive + charm + jet data**

The fits with and without jet data and charm data are very compatible for fixed $\alpha_S(M_Z)$

**The main effect of heavy flavour data** is to determine the optimal values of the charm and beauty mass parameters and their variation (as already done in the standard HERAPDF2.0). This variation is much reduced compared to HERAPDF1.0

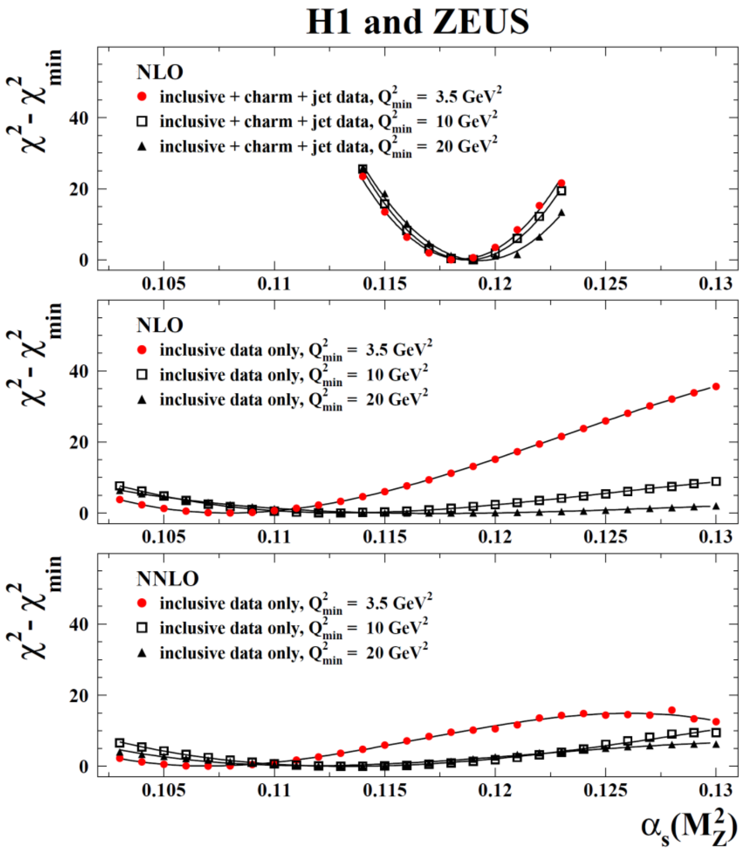
Similarly for beauty data



**The main effect of jet data** is to allow a determination of $\alpha_S(M_Z)$ at NLO. Inclusive data alone cannot give a reliable detrmnation.

When jet data are added one can make a simultaneous fit for PDF parameters and $\alpha_S(M_Z)$ at NLO---

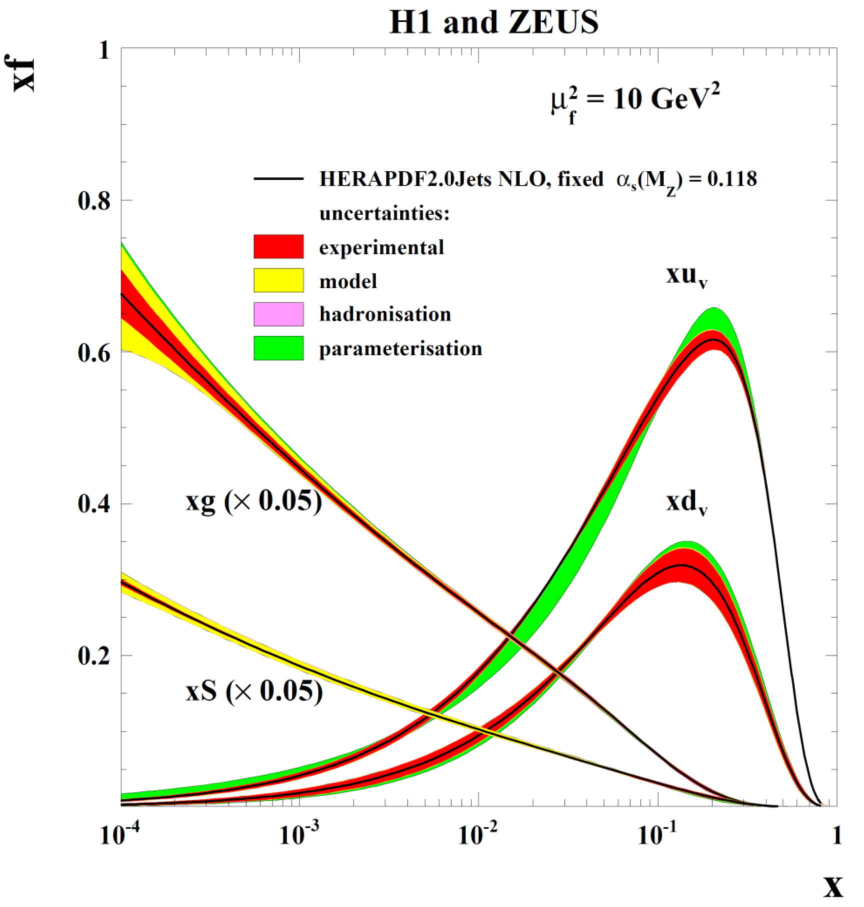
NNLO calculation still not available

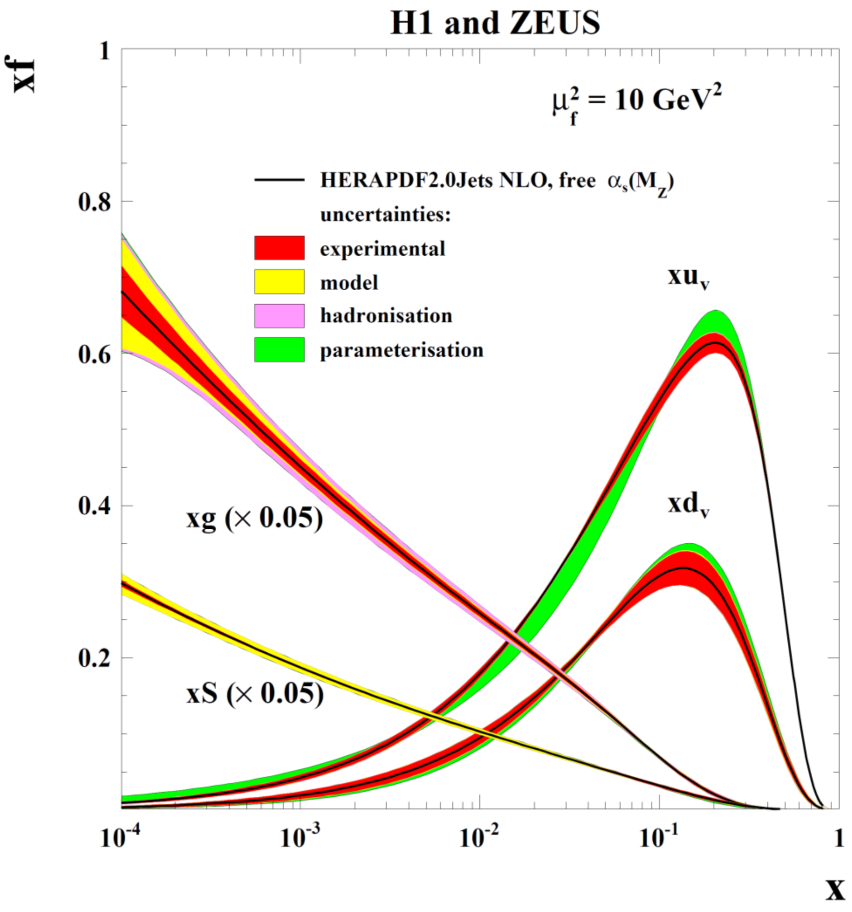


# HERAPDF2.0Jets is based on inclusive + charm + jet data

Fits are made with fixed and free $\alpha_S(M_Z)$

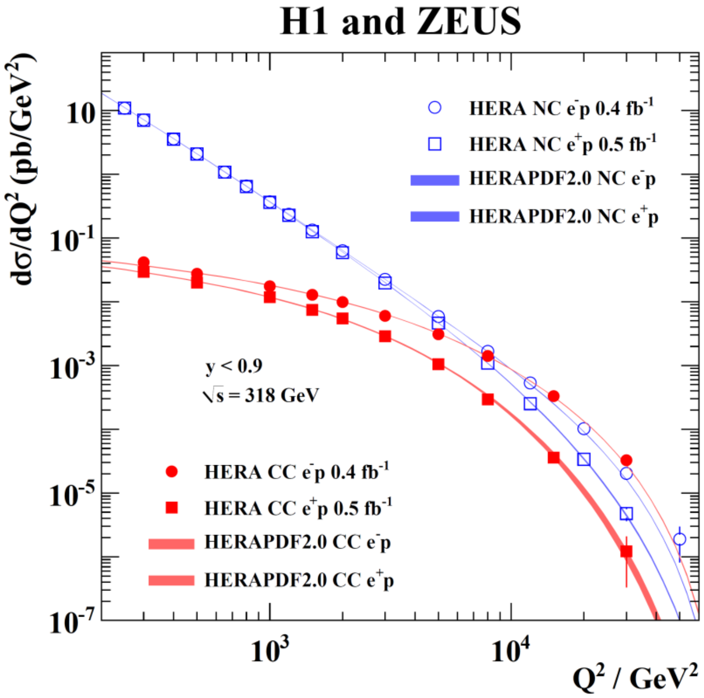
These PDFs are very similar since the fitted value is in agreement with the chosen fixed value. The uncertainties of gluon are not much larger when $\alpha_S(M_Z)$ is free since $\alpha_S(M_Z)$ is well determined. Scale uncertainties are not illustrated on the PDFs



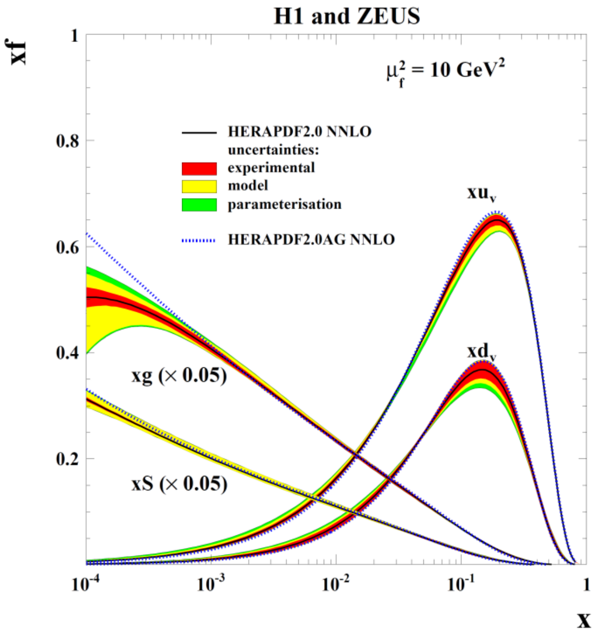


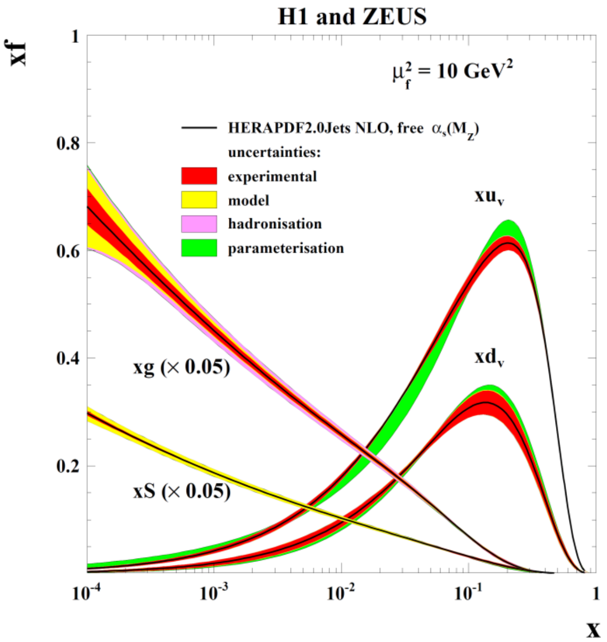
$$\alpha_S(M_Z) = 0.1183 \pm 0.0009_{(exp)} \pm 0.0005_{(model/param)} \pm 0.0012_{(had)} \, {}^{+0.0037}_{-0.0030}(\mathrm{scale})$$

# Summary



**We have the FINAL Inclusive HERA-I and II combination**
**And the HERAPDF2.0 series based upon it**

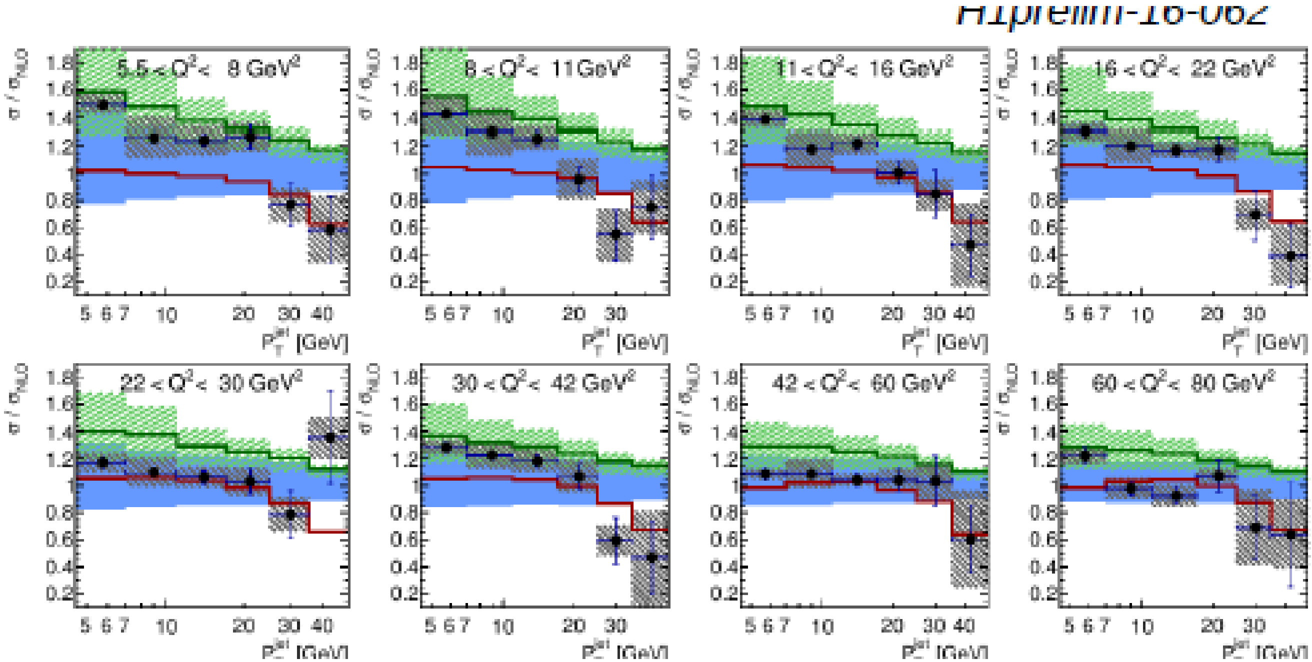




# Outlook

- Forthcoming H1 measurement of normalised inclusive, di-jet and tri-jet cross sections at low Q2 which will complement the data already available at high-Q2
- There are also now complete NNLO predictions for DIS jets which can be compared to even though the technology for fitting is not yet ready

It does look as though NNLO may give a better fit
It will be interesting to see how this affects HERA determinations of $\alpha_S(M_Z)$
The low and the high Q2 jet data together should also impact the gluon from low to high-x



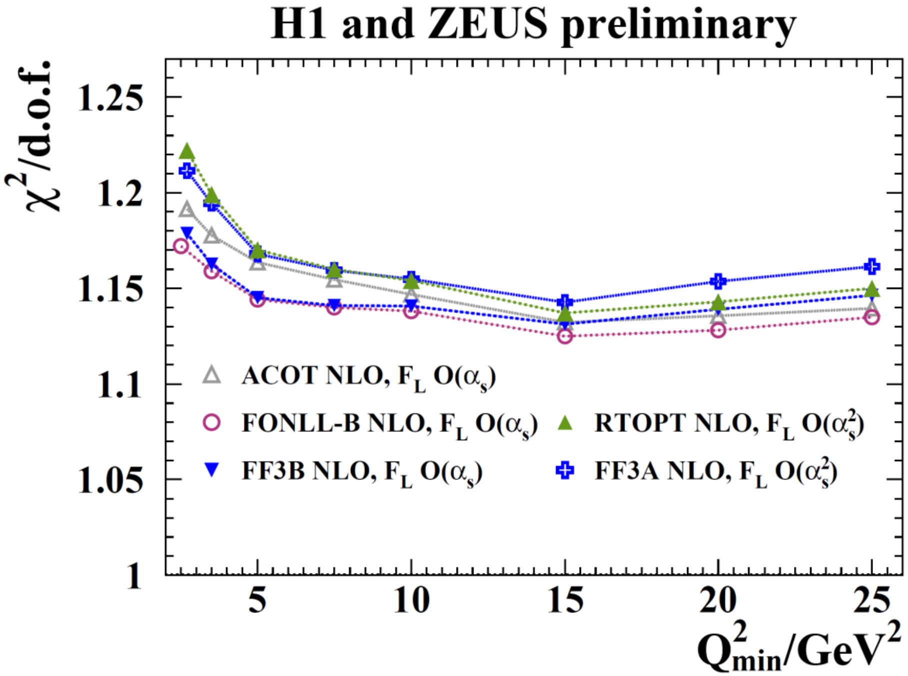
Forthcoming update of the H1 and ZEUS F2c combination with final data plus F2b combination

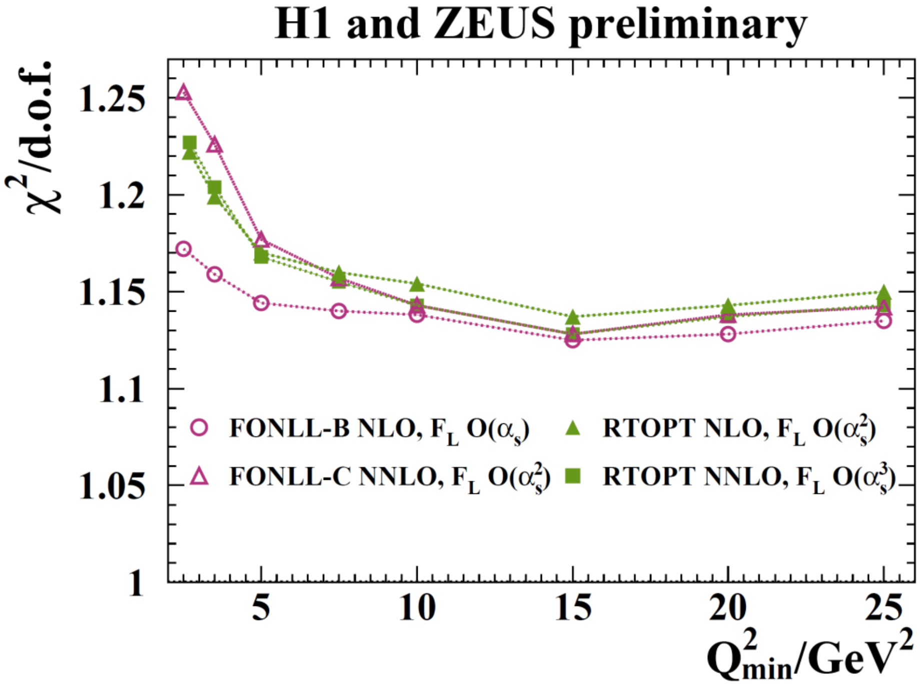
**Back-up**

Cut out FFN PDFs

Cut out the other VFN schemes

# Further remarks on dependence on $Q^2_{min}$
# Compare heavy flavour schemes at NLO and compare NLO to NNLO



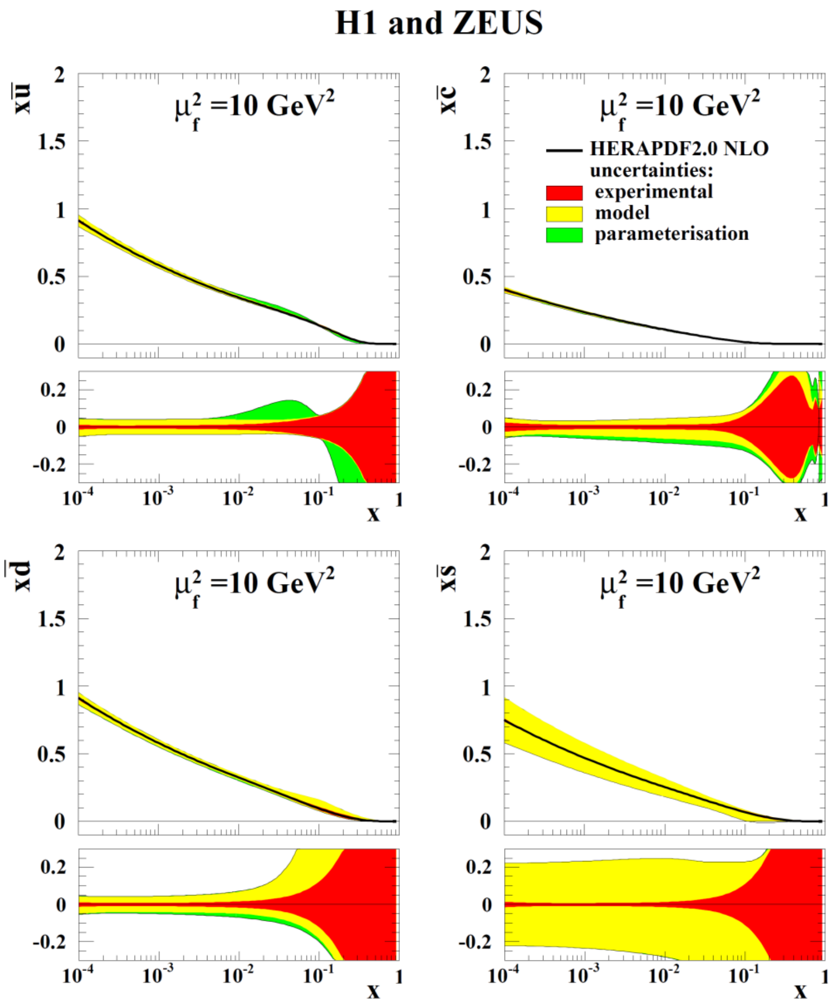


Treating $F_L$ to $O(\alpha_S)$ – the same order as $F_2$ yields better χ2 than treating FL to $O(\alpha_S^2)$ almost independent of heavy flavour scheme

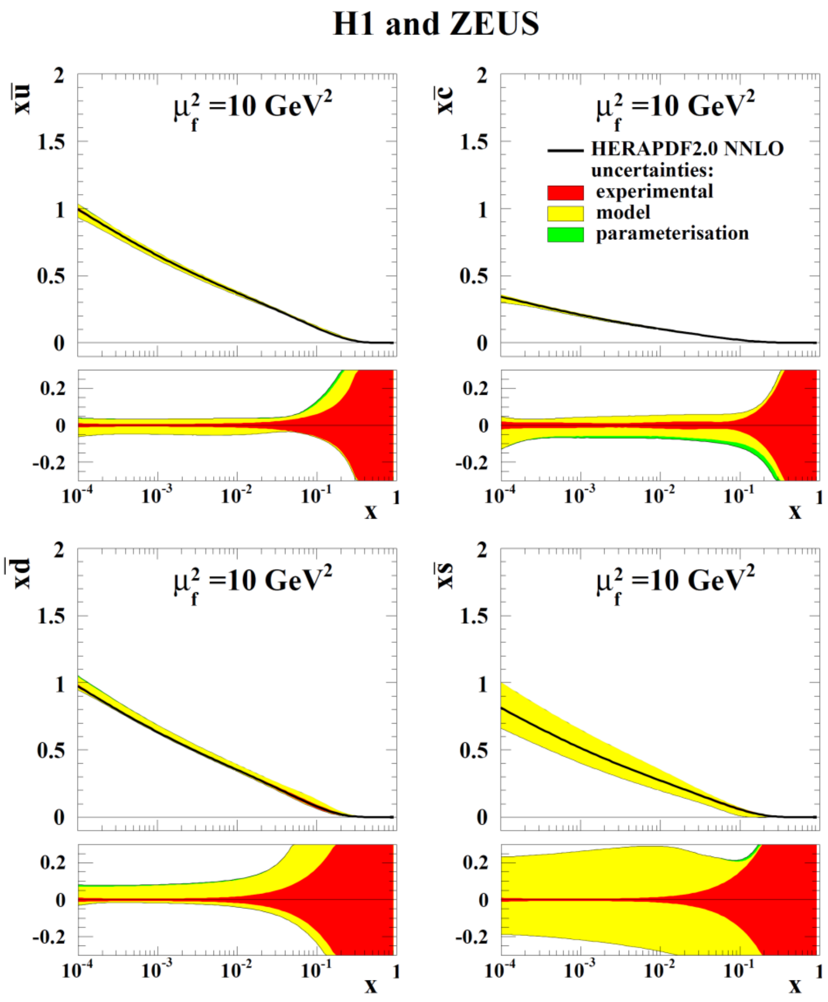
RTOPT NNLO is marginally worse than NLO
FONLL NNLO is a lot worse than NLO

# HERAPDF2.0: NLO and NNLO fits

NLO

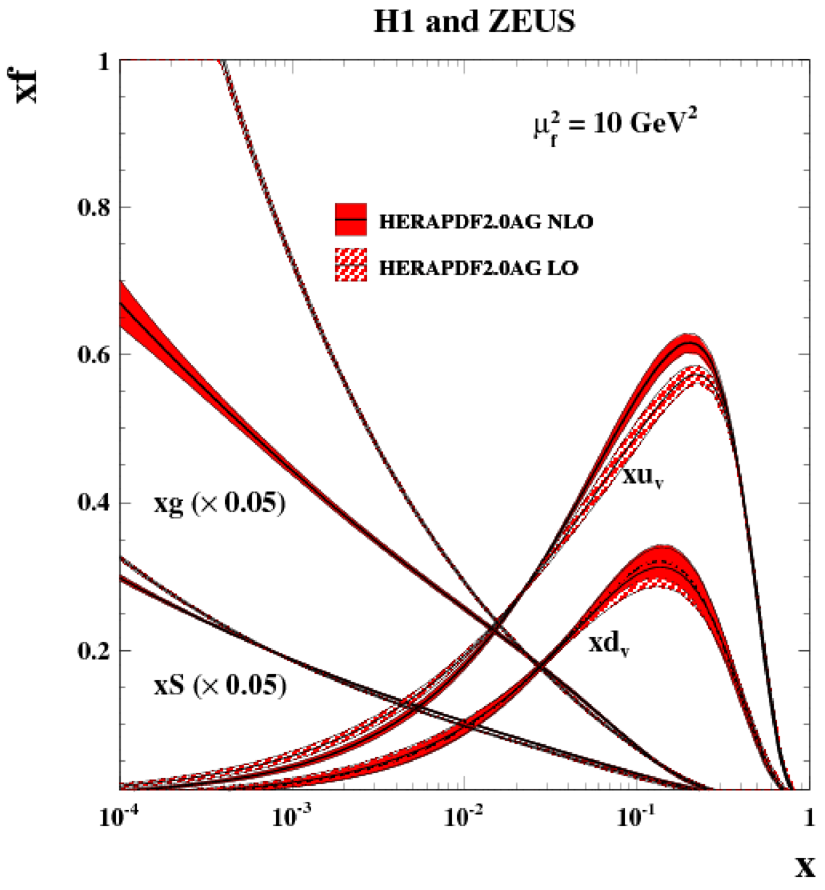


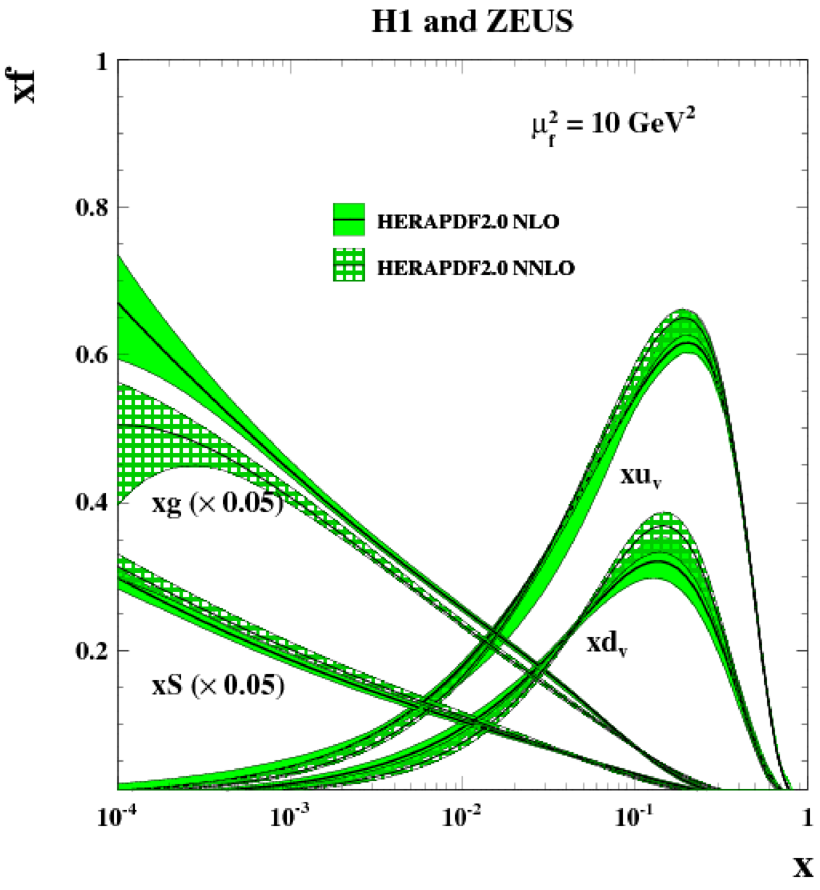
NNLO



Flavour break-up of the sea

# HERAPDF2.0 at LO, NLO and NNLO

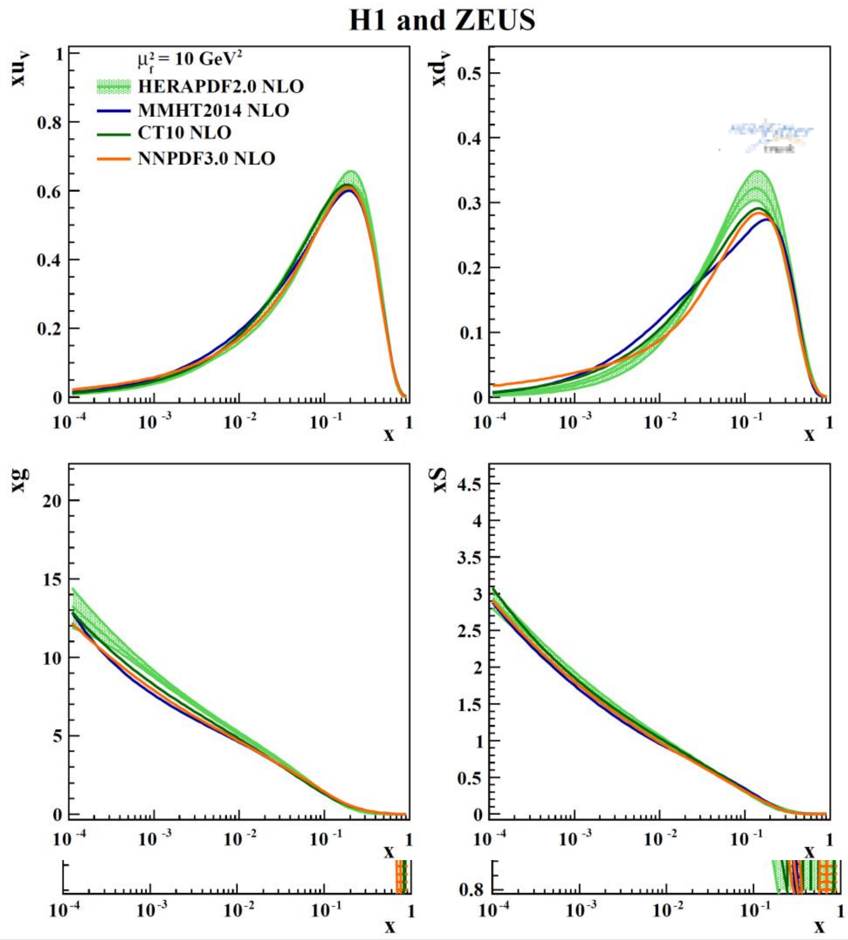


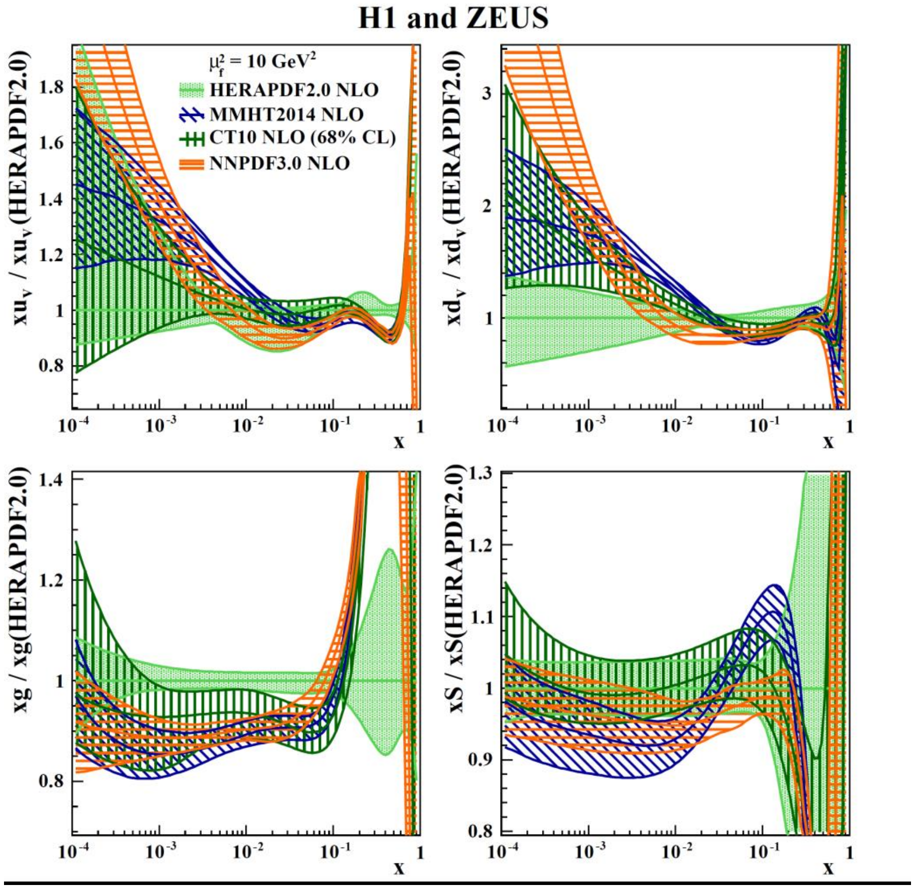


HERAPDF2.0 LO is only available with experimental uncertainties and is here compared to NLO also with experimental uncertainties.
In both cases the alternative gluon parametrisation is used

HERAPDF2.0 NLO and NNLO are compared with full uncertainties.
In both cases a more flexible gluon parametrisation with a term which allows the gluon to be negative at low-x and low $Q^2$ values is used

# Compare HERAPDF2.0 to other PDFs at NLO

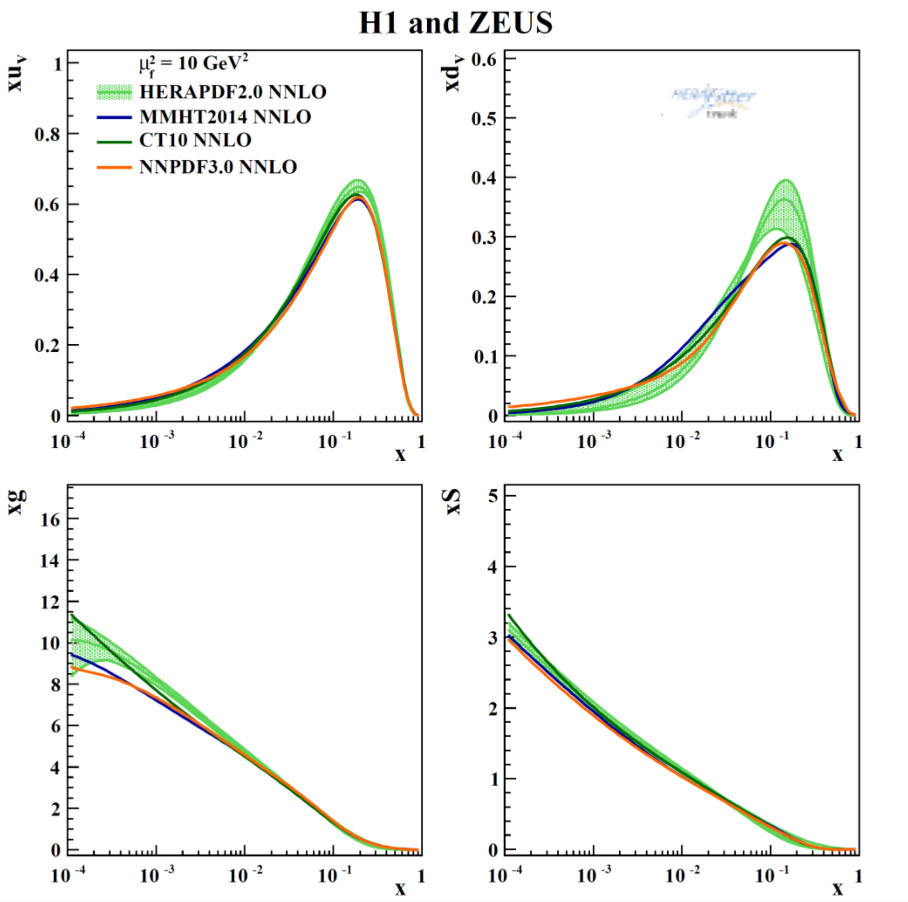


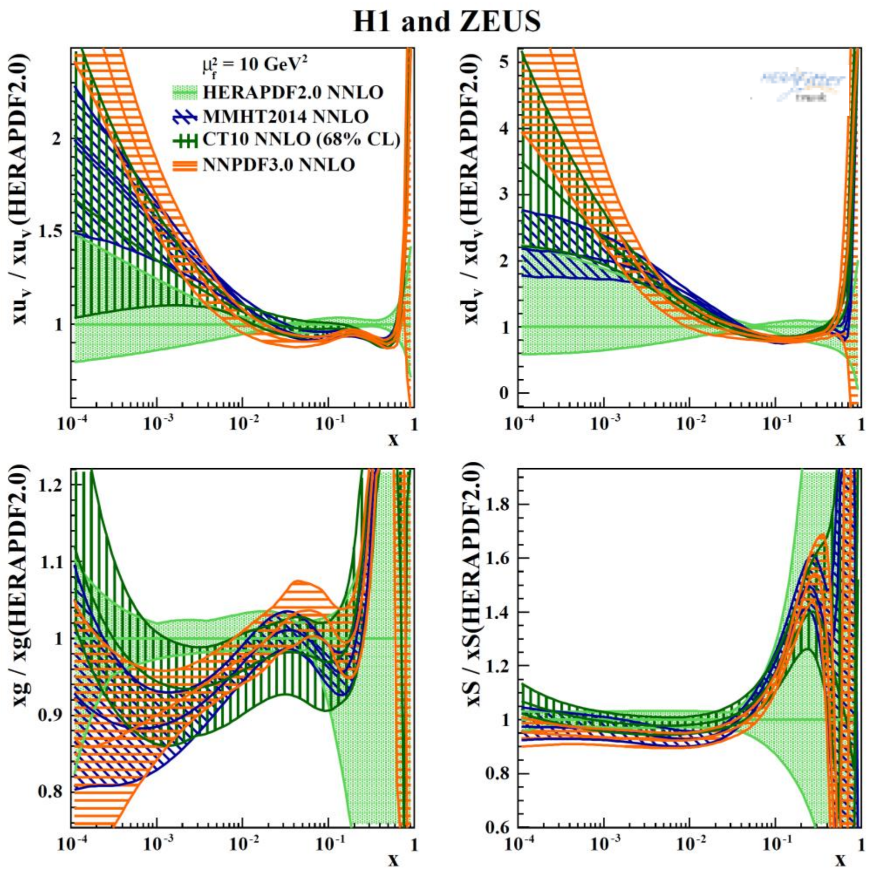


High-x valence shapes somewhat different– new high- x data and use of proton target only

Other PDFs have harder high-x gluon, but Sea is more compatible

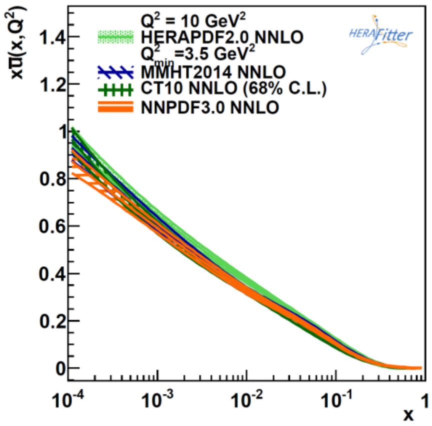
# Compare HERAPDF2.0 to other PDFs at NNLO

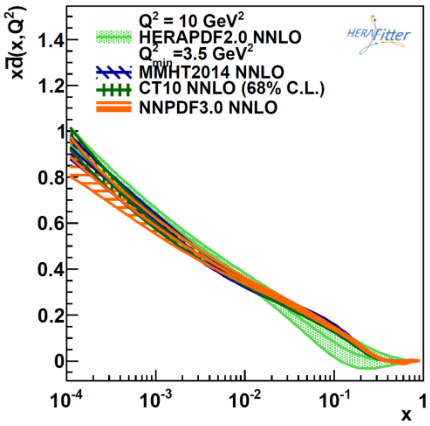


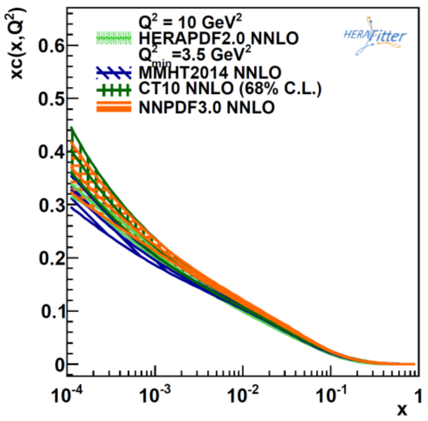


High-x valence shapes somewhat different – new high- x data and use of proton target only

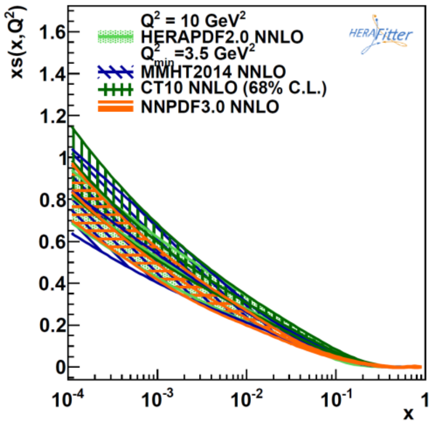
At NNLO gluon and Sea are both compatible with other PDFs



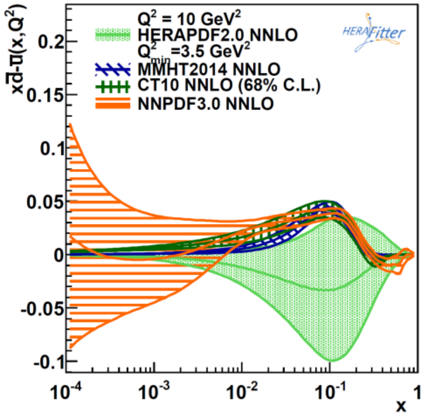


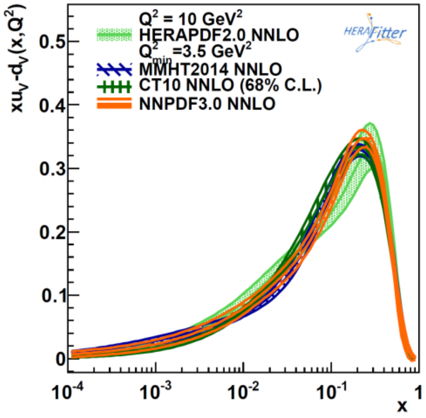


$u_{valence} - d_{valence}$ dominates the W-aymmmetry at Tevatron and LHC

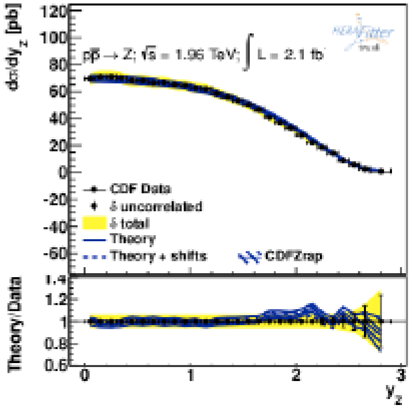


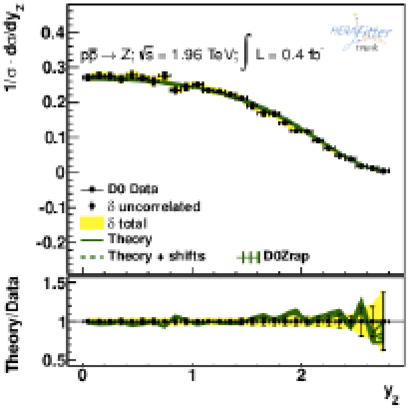
And here are more details on the flavour break up of the Sea
In particular dbar-ubar is negative but with large uncertainties, which cover other PDFs



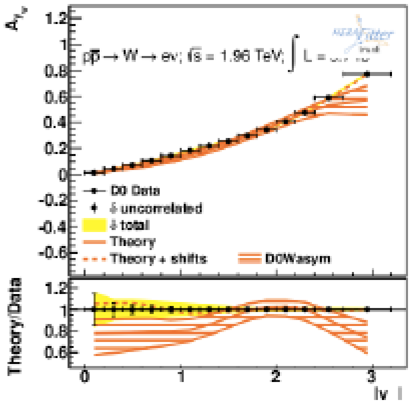


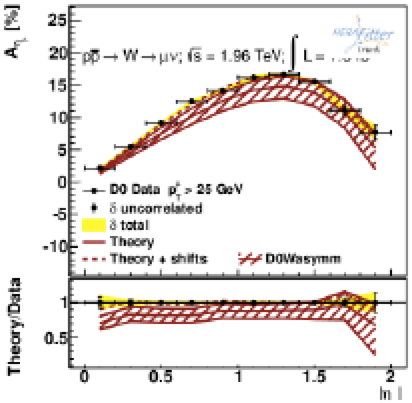
# Compare HERAPDF2.0 to Tevatron and CMS W,Z data

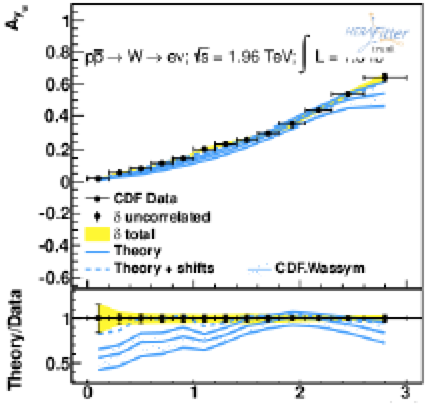


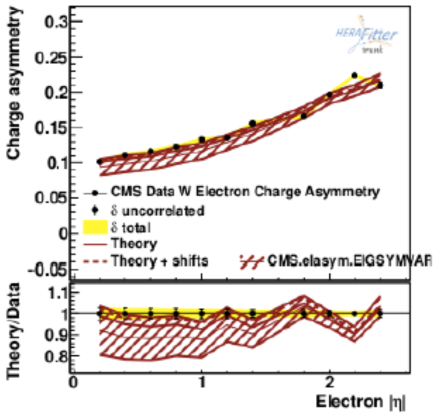


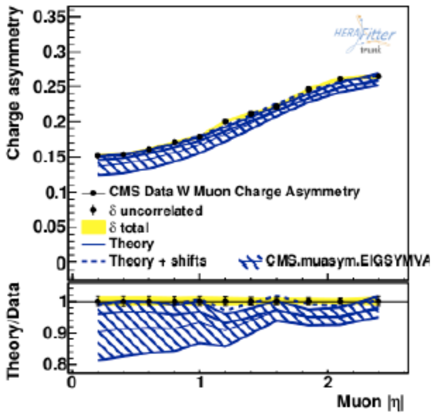
Similar level of agreement as the global PDFs

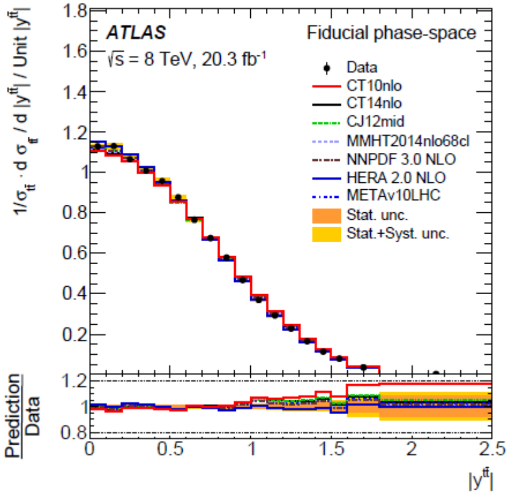






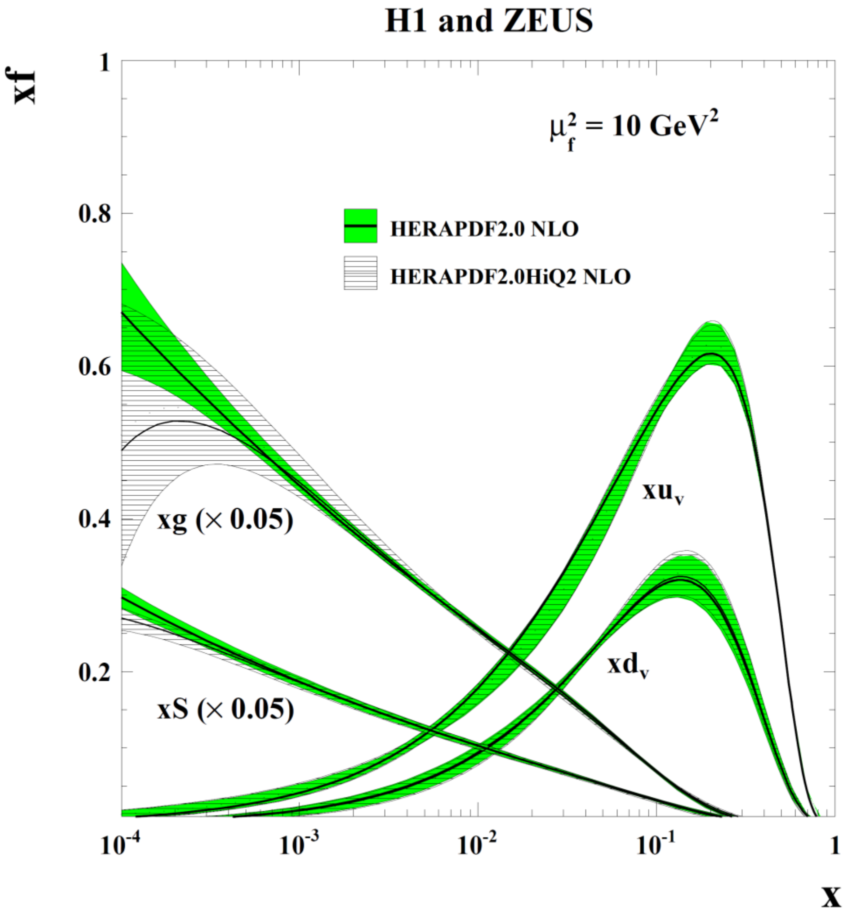


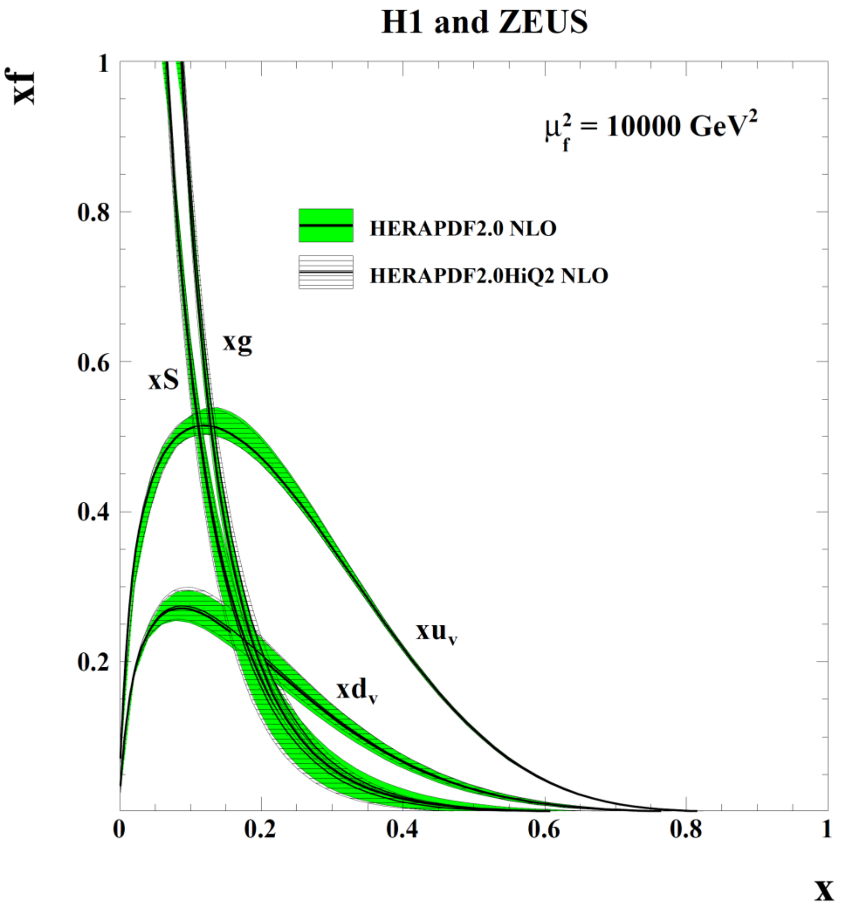




**And ATLAS top data**

# Compare HERAPDF2.0HiQ2, with Q2>10GeV$^2$ , to the standard fit at NLO



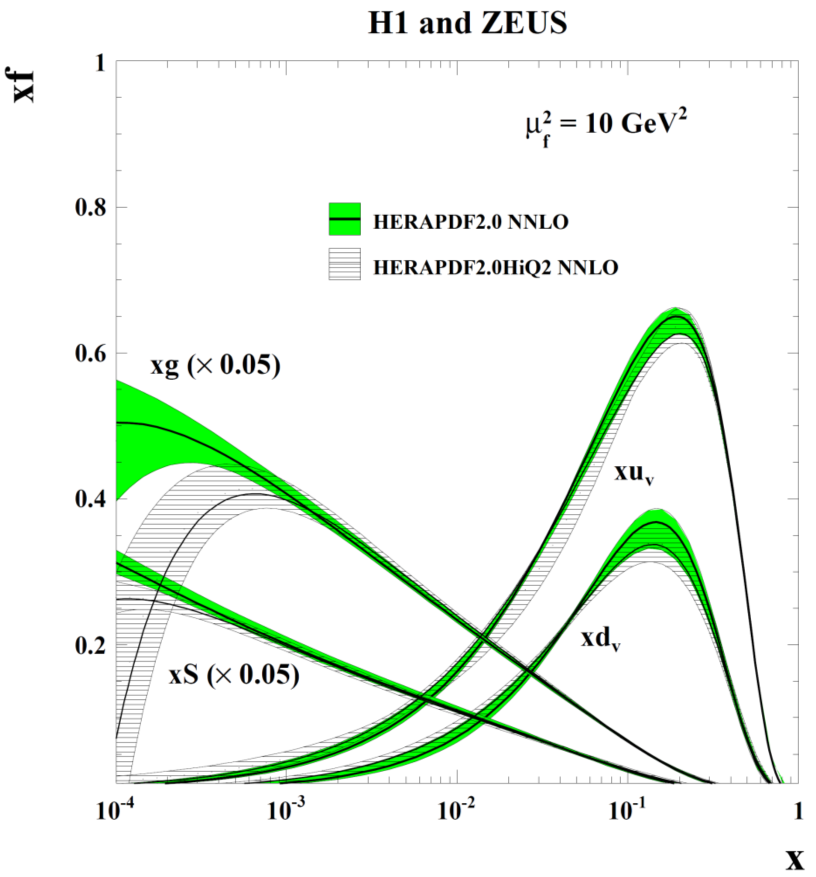


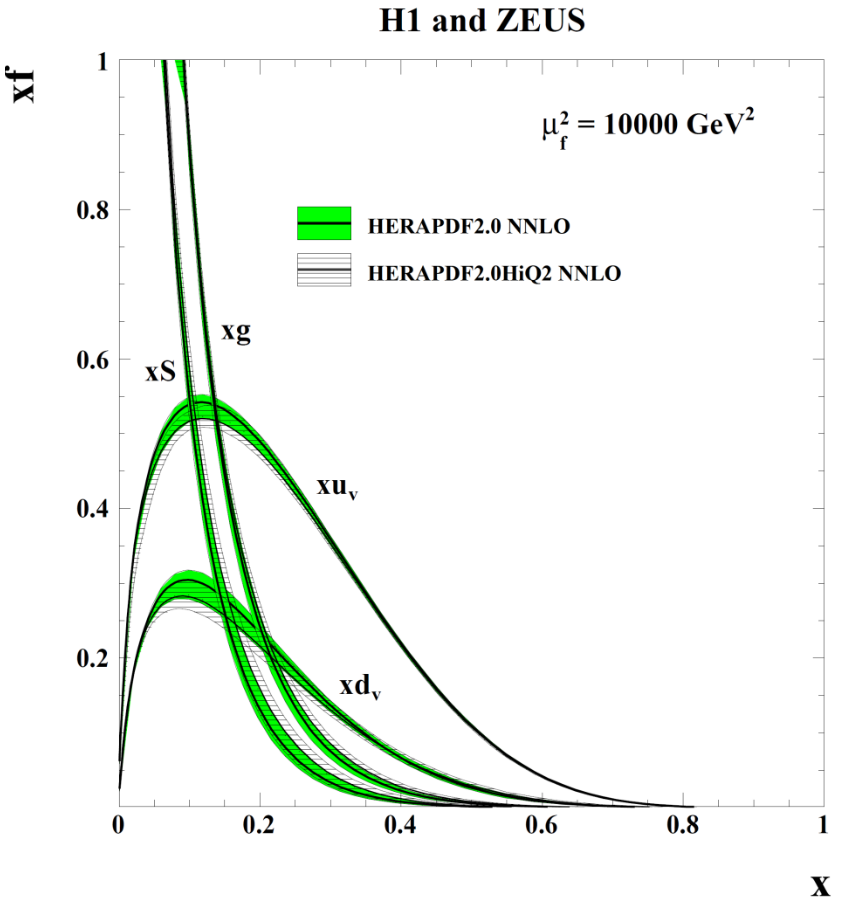
The purpose of this is to check for bias introduced by using low $Q^2$, low-x data in the fit. Fits are compatible. At large x all PDFs are similar for 2.0 and 2.0HiQ2 thus there is no bias at high scale due to the inclusion of the lower $Q^2$, lower x data

This is also true at NNLO.

There is greater uncertainty at low-x for Sea and glue there is some small change of gluon and sea shape at low-x.

# Compare HERAPDF2.0 with Q2>10GeV$^2$ to the standard fit at NNLO





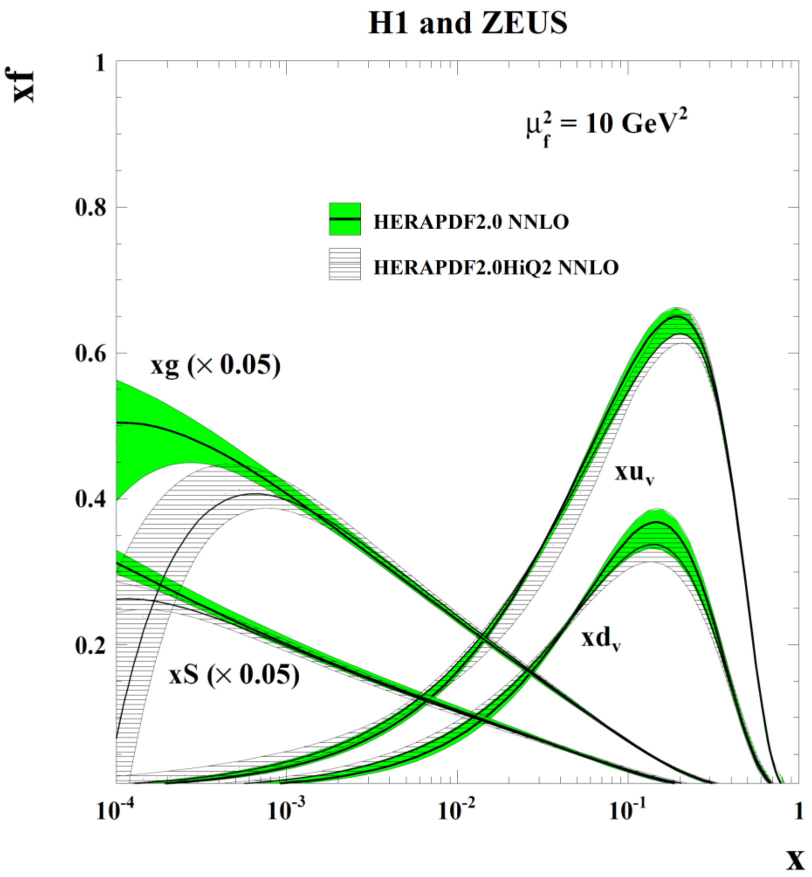
Fits are VERY compatible at high-x ---like in NLO case

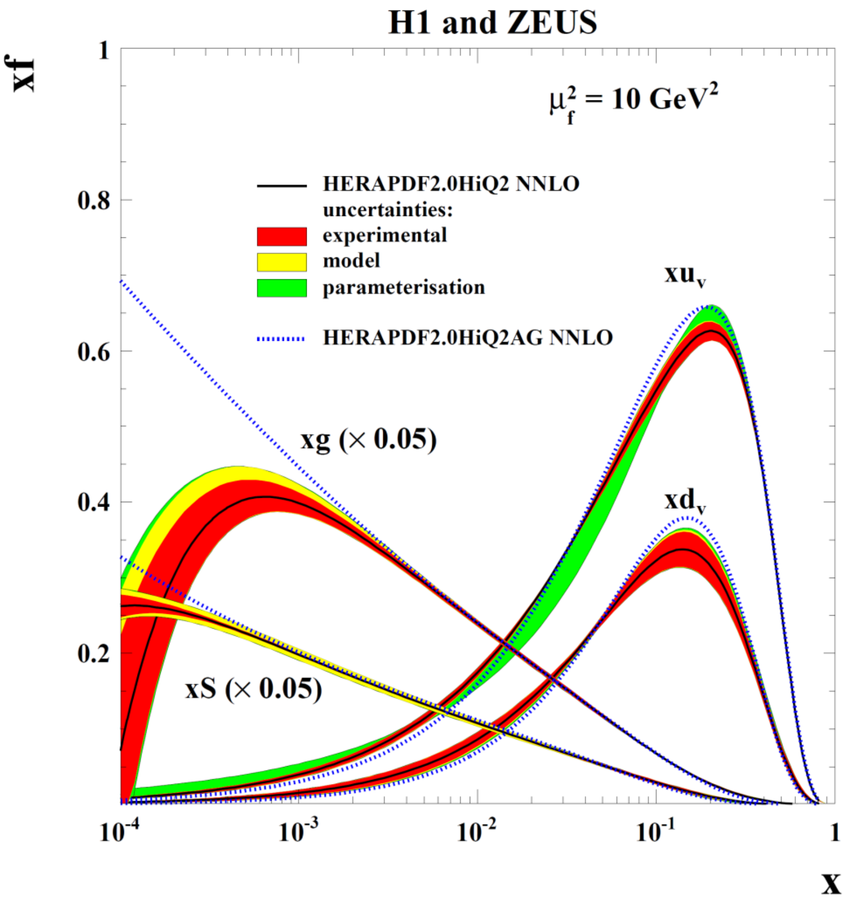
BUT the difference in shape for low-x Sea and gluon– has now become pronounced- fits are no longer compatible

There is still no bias from including the lower Q$^2$, lower x data in the fits if we move to LHC scales ---for the ATLAS,CMS kinematic regimes.

However at very low-x and moderate Q$^2$ --as in LHCb --the NNLOfit for Q$^2_{min}$=10 cannot be used--- the gluon becomes negative and so does the longitudinal cross section

# Compare HERAPDF2.0 with Q2>10GeV$^2$ to the standard fit at NNLO





Fits are VERY compatible at high-x ---like in NLO case

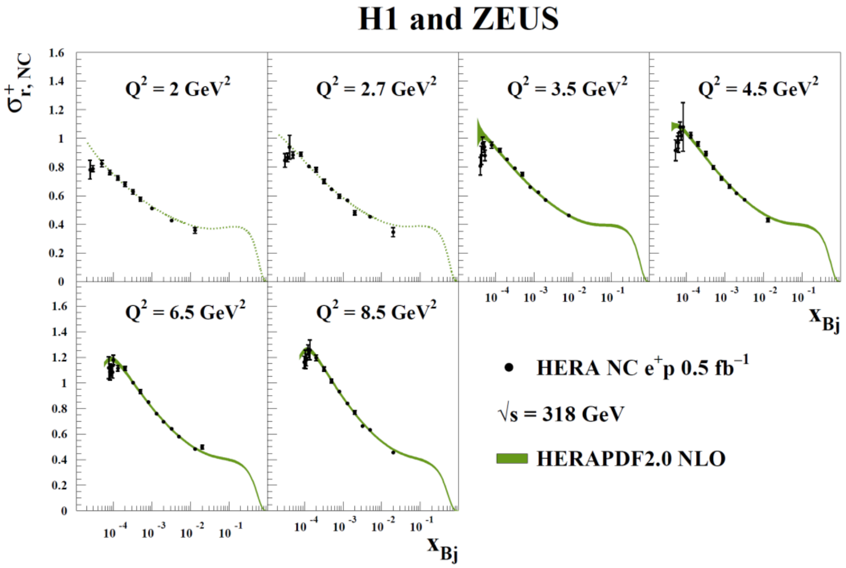
BUT the difference in shape for low-x Sea and gluon– has now become pronounced.

At very low-x and moderate $Q^2$ --as in LHCb --the NNLOfit for $Q^2_{min}$=10 gives a negative gluon and a negative longitudinal cross section, and thus is not fit for purpose.

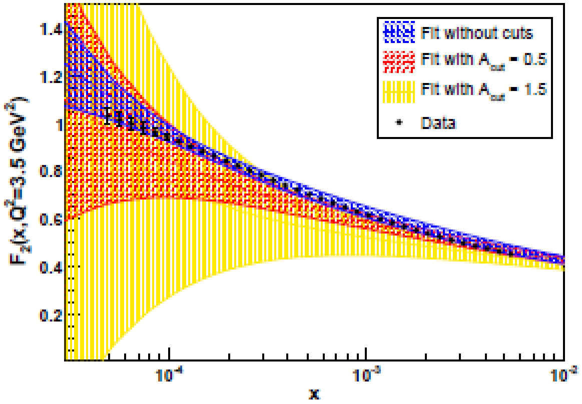
Can use the HERAPDF2.0HiQ2AG– alternative gluon shape—$xg(x) = A_g\, x^{Bg}\, (1-x)^{Cg}\, (1+D_g x)$, which cannot be negative at any x for $Q^2 > Q^2_0$, but fit χ2 is larger by Δχ2~+30

Does this indicate a breakdown of DGLAP at low x?

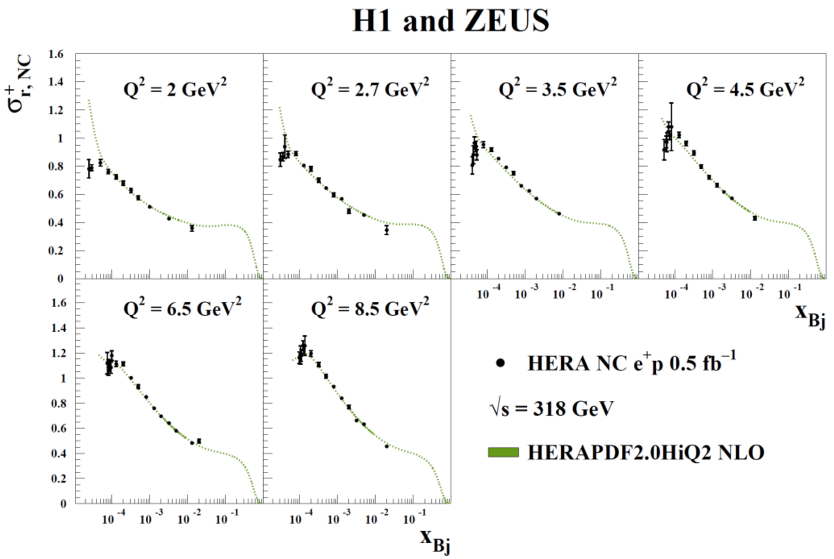
# Low $Q^2$, low-x



NLO
$Q^2$>3.5 GeV$^2$



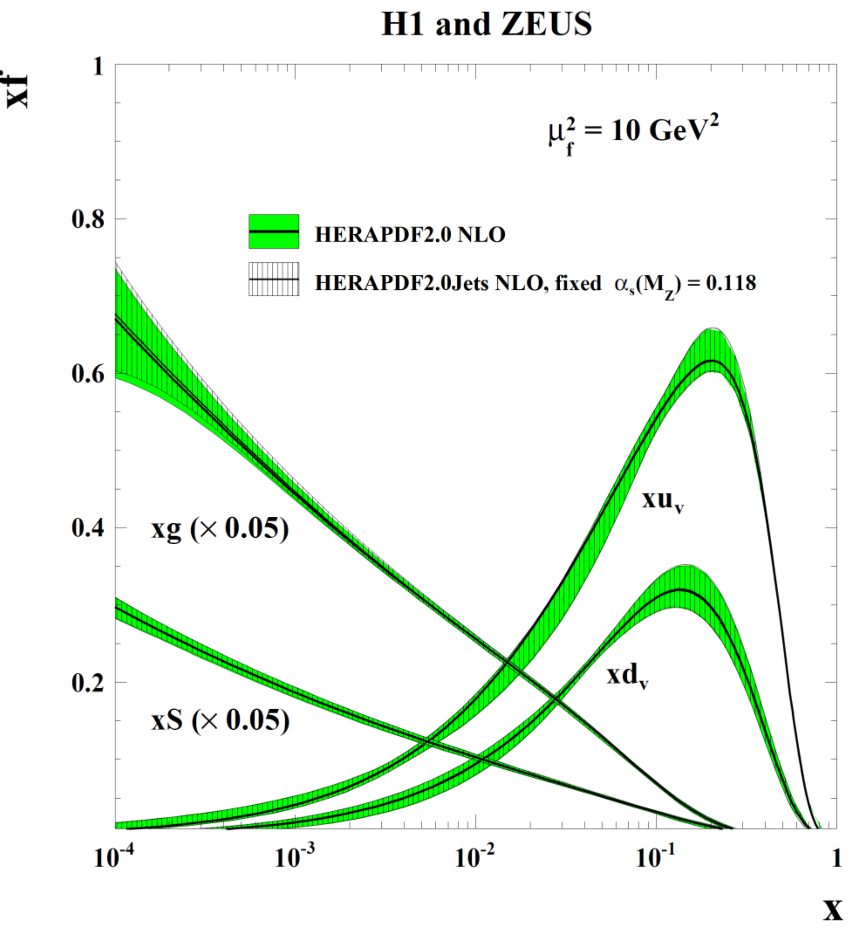
Reminds us of this? arXiv:0910.3143. The fit evolves faster than the data– so going to higher order NNLO does not improve this

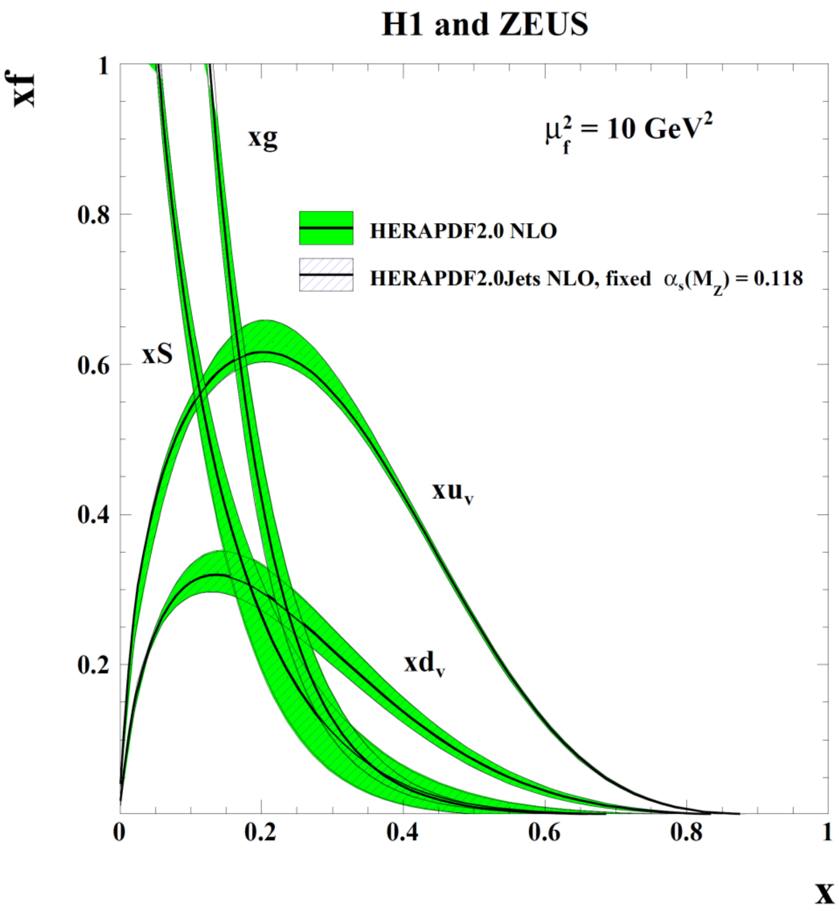


NLO
$Q^2$>10 GeV$^2$

These are the comparisons of the fit to the NC e+p data at low $Q^2$
The fit with $Q^2$>10 misses the lower $Q^2$ data in a systematic matter undershooting the data – worse at low-x and low $Q^2$---and not describing the high-y turn over

# Comparison of HERAPDF2.0Jets to HERAPDF2.0





The fits with and without jet data and charm data are very compatible

The charm and jet data are very well fitted at NLO

There is only marginal further decrease in uncertainty due to these data when $\alpha_S(M_Z)$ is fixed