Status of Calculation of $\alpha_{\rm QED}(M_Z^2)$ and muon $g-2$

Daisuke Nomura (Nat. Inst. Tech., Kagawa)

talk at 'Precision Theory for Precise Measurements'

at Quy Nhon, Vietnam

September 26, 2016

Partially based on:

- K. Hagiwara, A. Keshavarzi, A. D. Martin, DN & T. Teubner, work in progress
- K. Hagiwara, R. Liao, A. D. Martin, DN & T. Teubner **(HLMNT)**, J. Phys. **G38** (2011) 085003

Muon $g-2$: introduction

Lepton magnetic moment $\vec{\mu}$:

$$\boxed{\vec{\mu} = -g\frac{e}{2m}\vec{s}}, \qquad \left(\vec{s} = \frac{1}{2}\vec{\sigma}\ \text{(spin)}, \qquad g = 2 + 2F_2(0)\right)$$

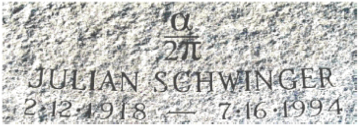
where

$$\overline{u}(p+q)\Gamma^\mu u(p) = \overline{u}(p+q)\left(\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right)u(p)$$

Anomalous magnetic moment: $a \equiv (g-2)/2$ $(= F_2(0))$

Historically,

- ★ $g = 2$ (tree level, Dirac)
- ★ $a = \alpha/(2\pi)$ (1-loop QED, Schwinger)



Today, still important, since...

★ One of the **most precisely measured** quantities:

$$\boxed{a_\mu^{\text{exp}} = 11\ 659\ 208.9(6.3) \times 10^{-10}}\ [0.5\text{ppm}] \qquad \text{(Bennett \textit{et al})}$$

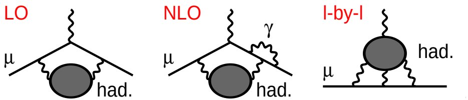
★ **Extremely useful** in **probing/constraining physics beyond the SM**

Introduction: Standard Model prediction for muon $g-2$

QED contribution	11 658 471.808 (0.015)	Kinoshita & Nio, Aoyama et al
EW contribution	15.4 (0.2)	Czarnecki et al
Hadronic contributions		
LO hadronic	**694.9 (4.3)**	HLMNT11
NLO hadronic	−9.8 (0.1)	HLMNT11
light-by-light	10.5 (2.6)	Prades, de Rafael & Vainshtein
Theory TOTAL	**11 659 182.8 (4.9)**	
Experiment	**11 659 208.9 (6.3)**	world avg
Exp − Theory	**26.1 (8.0)**	**3.3 σ discrepancy**

(in units of 10^{-10}. Numbers taken from HLMNT11, arXiv:1105.3149)

n.b.: hadronic contributions:



Improvements in the past few years (1)

- **QED contribution**

  5-loop calculation completed:

  now $a_\mu(\mathrm{QED}) = 11\ 658\ 471.895(08) \times 10^{-10}$

  was $11\ 658\ 471.808(15) \times 10^{-10}$

  (numbers are from Aoyama et al (2012, 2007))

- **EW contribution**

  Higgs-boson mass just fixed:

  now $a_\mu(\mathrm{EW}) = 15.4(1) \times 10^{-10}$ Gnendiger et al '13

  was $15.4(2) \times 10^{-10}$ Czarnecki et al '02

cf: $a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = \begin{cases} (26.1 \pm 8.0) \times 10^{-10} & \text{HLMNT11} \\ (28.7 \pm 8.0) \times 10^{-10} & \text{Davier et al '10} \end{cases}$

Improvements in the past few years (2)

- **NNLO hadronic contribution**

$$a_\mu(\text{had, NNLO}) = (1.24 \pm 0.01) \times 10^{-10}$$

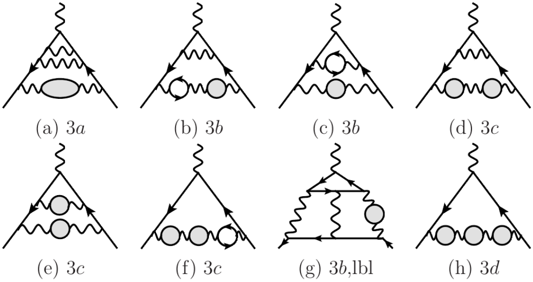


Figure 2: Sample NNLO Feynman diagrams contributing to a_μ^{had}.

Numbers and Figs. from A. Kurz et al, arXiv:1403.6400

3a:	$+0.80 \times 10^{-10}$
3b:	-0.41×10^{-10}
3b,lbl:	$+0.91 \times 10^{-10}$
3c:	-0.06×10^{-10}
3d:	$+0.0005 \times 10^{-10}$

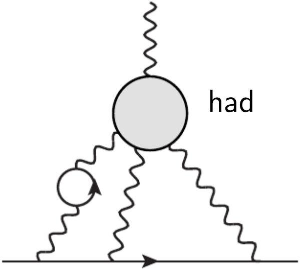
cf: $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = \begin{cases} (26.1 \pm 8.0) \times 10^{-10} & \text{HLMNT11} \\ (28.7 \pm 8.0) \times 10^{-10} & \text{Davier et al '10} \end{cases}$

$$a_\mu(\text{had, NLO}) = (-9.8 \pm 0.1) \times 10^{-10} \text{ HLMNT11}$$

Improvements in the past few years (3)

- **lbyl-NLO hadronic contribution**

$$a_\mu(\text{had, lbyl-NLO}) = (0.3 \pm 0.2) \times 10^{-10}$$



Number and Fig. from Colangelo et al, arXiv:1403.7512

$\Longrightarrow$ **Negligible**. But it is always good to confirm that higher order terms are really negligible.

cf: $$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = \begin{cases} (26.1 \pm 8.0) \times 10^{-10} & \text{HLMNT11} \\ (28.7 \pm 8.0) \times 10^{-10} & \text{Davier et al '10} \end{cases}$$

$$a_\mu^{\text{had,lbyl}} = \begin{cases} (10.5 \pm 2.6) \times 10^{-10} & \text{Prades et al, '09} \\ (11.6 \pm 3.9) \times 10^{-10} & \text{Jegerlehner+Nyffeler, '09} \end{cases}$$

HLbL scattering: Summary of selected results for $a_\mu^{\mathrm{HLbL}} \times 10^{11}$

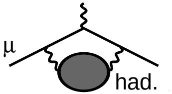
Contribution	BPP	HKS, HK	KN	MV	BP, MdRR	PdRV	N, JN
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	–	114±13	99 ± 16
axial vectors	2.5±1.0	1.7±1.7	–	22±5	–	15±10	22±5
scalars	−6.8±2.0	–	–	–	–	−7±7	−7±2
π, K loops	−19±13	−4.5±8.1	–	–	–	−19±19	−19±13
π, K loops +subl. N_C	–	–	–	0±10	–	–	–
quark loops	21±3	9.7±11.1	–	–	–	2.3 (c-quark)	21±3
Total	83±32	89.6±15.4	80±40	136±25	110±40	105 ± 26	116 ± 39

BPP = Bijnens, Pallante, Prades '95, '96, '02; HKS = Hayakawa, Kinoshita, Sanda '95, '96; HK = Hayakawa, Kinoshita '98, '02; KN = Knecht, AN '02; MV = Melnikov, Vainshtein '04; BP = Bijnens, Prades '07; MdRR = Miller, de Rafael, Roberts '07; PdRV = Prades, de Rafael, Vainshtein '09; N = AN '09, JN = Jegerlehner, AN '09

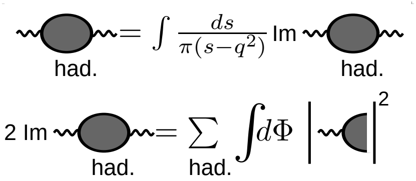
- Pseudoscalar-exchanges dominate numerically. Other contributions not negligible. Cancellation between π, K-loops and quark loops !
- Note that recent reevaluations of axial vector contribution lead to much smaller estimates than in MV: $a_\mu^{\mathrm{HLbL;axial}} = (8 \pm 3) \times 10^{-11}$ (Pauk, Vanderhaeghen '14; Jegerlehner '14, '15). This would shift central values of compilations downwards: $a_\mu^{\mathrm{HLbL}} = (98 \pm 26) \times 10^{-11}$ (PdRV) and $a_\mu^{\mathrm{HLbL}} = (102 \pm 39) \times 10^{-11}$ (N, JN).
- PdRV: Analyzed results obtained by different groups with various models and suggested new estimates for some contributions (shifted central values, enlarged errors). Do not consider dressed light quark loops as separate contribution. Added all errors in quadrature !
- N, JN: New evaluation of pseudoscalar exchange contribution imposing new short-distance constraint on off-shell form factors. Took over most values from BPP, except axial vectors from MV. Added all errors linearly.

A. Nyffeler, talk at Frascati, May 2016

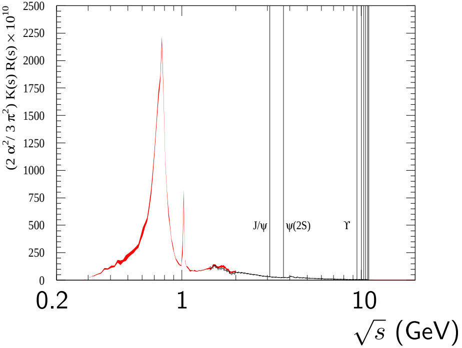
The diagram to be evaluated:



pQCD not useful. Use the dispersion relation and the optical theorem.



$$a_\mu^{\mathrm{had,LO}} = \frac{m_\mu^2}{12\pi^3} \int_{s_{\mathrm{th}}}^{\infty} ds\, \frac{1}{s} \hat{K}(s) \sigma_{\mathrm{had}}(s)$$



- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\mathrm{had,LO}}$

Included Hadronic Final States

Channel	Experiments with References
$\pi^+\pi^-$	OLYA [16, 17, 18], OLYA-TOF [19], NA7 [20], OLYA and CMD [21, 22], DM1 [23], DM2 [24], BCF [25, 26], MEA [27, 28], ORSAY-ACO [29], CMD-2 [10, 11, 30]
$\pi^0\gamma$	SND [31, 32]
$\eta\gamma$	SND [32, 33], CMD-2 [34, 35, 36]
$\pi^+\pi^-\pi^0$	ND [22], DM1 [37], DM2 [38], CMD-2 [10, 13, 34, 39], SND [40, 41], CMD [42]
K^+K^-	MEA [27], OLYA [43], BCF [26], DM1 [44], DM2 [45, 46], CMD [22], CMD-2 [34], SND [47]
$K_S^0K_L^0$	DM1 [48], CMD-2 [10, 14, 49], SND [47]
$\pi^+\pi^-\pi^0\pi^0$	M3N [50], DM2 [51], OLYA [52], CMD-2 [53], SND [54], ORSAY-ACO [55], $\gamma\gamma2$ [56], MEA [57]
$\omega(\to\pi^0\gamma)\pi^0$	ND and ARGUS [22], DM2 [51], CMD-2 [53, 58], SND [59, 60], ND [61]
$\pi^+\pi^-\pi^+\pi^-$	ND [22], M3N [50], CMD [62], DM1 [63, 64], DM2 [51], OLYA [65], $\gamma\gamma2$ [66], CMD-2 [53, 67, 68], SND [54], ORSAY-ACO [55]
$\pi^+\pi^-\pi^+\pi^-\pi^0$	MEA [57], M3N [50], CMD [22, 62], $\gamma\gamma2$ [56]
$\pi^+\pi^-\pi^0\pi^0\pi^0$	M3N [50]
$\omega(\to\pi^0\gamma)\pi^+\pi^-$	DM2 [38], CMD-2 [69], DM1 [70]
$\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$	M3N [50], CMD [62], DM1 [71], DM2 [72]
$\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$	M3N [50], CMD [62], DM2 [72], $\gamma\gamma2$ [56], MEA [57]
$\pi^+\pi^-\pi^0\pi^0\pi^0\pi^0$	isospin-related
$\eta\pi^+\pi^-$	DM2 [73], CMD-2 [69]
$K^+K^-\pi^0$	DM2 [74, 75]
$K_S^0\pi K$	DM1 [76], DM2 [74, 75]
K_S^0X	DM1 [77]
$\pi^+\pi^-K^+K^-$	DM2 [74]
$p\bar{p}$	FENICE [78, 79], DM2 [80, 81], DM1 [82]
$n\bar{n}$	FENICE [78, 83]
incl. (< 2 GeV)	$\gamma\gamma2$ [84], MEA [85], M3N [86], BARYON-ANTIBARYON [87]
incl. (> 2 GeV)	BES [88, 89], Crystal Ball [90, 91, 92], LENA [93], MD-1 [94], DASP [95], CLEO [96], CUSB [97], DHHM [98]

Table 1: Experiments and references for the e^+e^- data sets for the different exclusive and the inclusive channels as used in this analysis. The recent re-analysis from CMD-2 [10] supersedes

channel	inclusive (1.43,2 GeV) $a_\mu^{\rm had,LO}$	inclusive (1.43,2 GeV) $\Delta\alpha_{\rm had}(M_Z^2)$	exclusive (1.43,2 GeV) $a_\mu^{\rm had,LO}$	exclusive (1.43,2 GeV) $\Delta\alpha_{\rm had}(M_Z^2)$
$\pi^0\gamma$ (ChPT)	0.13 ± 0.01	0.00 ± 0.00	0.13 ± 0.01	0.00 ± 0.00
$\pi^0\gamma$ (data)	4.50 ± 0.15	0.36 ± 0.01	4.50 ± 0.15	0.36 ± 0.01
$\pi^+\pi^-$ (ChPT)	2.36 ± 0.05	0.04 ± 0.00	2.36 ± 0.05	0.04 ± 0.00
$\pi^+\pi^-$ (data)	502.78 ± 5.02	34.39 ± 0.29	503.38 ± 5.02	34.59 ± 0.29
$\pi^+\pi^-\pi^0$ (ChPT)	0.01 ± 0.00	0.00 ± 0.00	0.01 ± 0.00	0.00 ± 0.00
$\pi^+\pi^-\pi^0$ (data)	46.43 ± 0.90	4.33 ± 0.08	47.04 ± 0.90	4.52 ± 0.08
$\eta\gamma$ (ChPT)	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00
$\eta\gamma$ (data)	0.73 ± 0.03	0.09 ± 0.00	0.73 ± 0.03	0.09 ± 0.00
K^+K^-	21.62 ± 0.76	3.01 ± 0.11	22.35 ± 0.77	3.23 ± 0.11
$K_S^0K_L^0$	13.16 ± 0.31	1.76 ± 0.04	13.30 ± 0.32	1.80 ± 0.04
$2\pi^+2\pi^-$	6.16 ± 0.32	1.27 ± 0.07	14.77 ± 0.76	4.04 ± 0.21
$\pi^+\pi^-2\pi^0$	9.71 ± 0.63	1.86 ± 0.12	20.55 ± 1.22	5.51 ± 0.35
$2\pi^+2\pi^-\pi^0$	0.26 ± 0.04	0.06 ± 0.01	2.85 ± 0.25	0.99 ± 0.09
$\pi^+\pi^-3\pi^0$	0.09 ± 0.09	0.02 ± 0.02	1.19 ± 0.33	0.41 ± 0.10
$3\pi^+3\pi^-$	0.00 ± 0.00	0.00 ± 0.00	0.22 ± 0.02	0.09 ± 0.01
$2\pi^+2\pi^-2\pi^0$	0.12 ± 0.03	0.03 ± 0.01	3.32 ± 0.29	1.22 ± 0.11
$\pi^+\pi^-4\pi^0$ (isospin)	0.00 ± 0.00	0.00 ± 0.00	0.12 ± 0.12	0.05 ± 0.05
$K^+K^-\pi^0$	0.00 ± 0.00	0.00 ± 0.00	0.29 ± 0.07	0.10 ± 0.03
$K_S^0K_L^0\pi^0$ (isospin)	0.00 ± 0.00	0.00 ± 0.00	0.29 ± 0.07	0.10 ± 0.03
$K_S^0\pi^\mp K^\pm$	0.05 ± 0.02	0.01 ± 0.00	1.00 ± 0.11	0.33 ± 0.04
$K_L^0\pi^\mp K^\pm$ (isospin)	0.05 ± 0.02	0.01 ± 0.00	1.00 ± 0.11	0.33 ± 0.04
$K\bar{K}\pi\pi$ (isospin)	0.00 ± 0.00	0.00 ± 0.00	3.63 ± 1.34	1.33 ± 0.48
$\omega(\to\pi^0\gamma)\pi^0$	0.64 ± 0.02	0.12 ± 0.00	0.83 ± 0.03	0.17 ± 0.01
$\omega(\to\pi^0\gamma)\pi^+\pi^-$	0.01 ± 0.00	0.00 ± 0.00	0.07 ± 0.01	0.02 ± 0.00
$\eta(\to\pi^0\gamma)\pi^+\pi^-$	0.07 ± 0.01	0.02 ± 0.00	0.49 ± 0.07	0.15 ± 0.02
$\phi(\to$ unaccounted$)$	0.06 ± 0.06	0.01 ± 0.01	0.06 ± 0.06	0.01 ± 0.01
$p\bar{p}$	0.00 ± 0.00	0.00 ± 0.00	0.04 ± 0.01	0.02 ± 0.00
$n\bar{n}$	0.00 ± 0.00	0.00 ± 0.00	0.07 ± 0.02	0.03 ± 0.01
$J/\psi, \psi'$	7.30 ± 0.43	8.90 ± 0.51	7.30 ± 0.43	8.90 ± 0.51
$\Upsilon(1S-6S)$	0.10 ± 0.00	1.16 ± 0.04	0.10 ± 0.00	1.16 ± 0.04
inclusive R	73.96 ± 2.68	92.75 ± 1.74	42.05 ± 1.14	81.97 ± 1.53
pQCD	2.11 ± 0.00	125.32 ± 0.15	2.11 ± 0.00	125.32 ± 0.15
sum	692.38 ± 5.88	275.52 ± 1.85	696.15 ± 5.68	276.90 ± 1.77

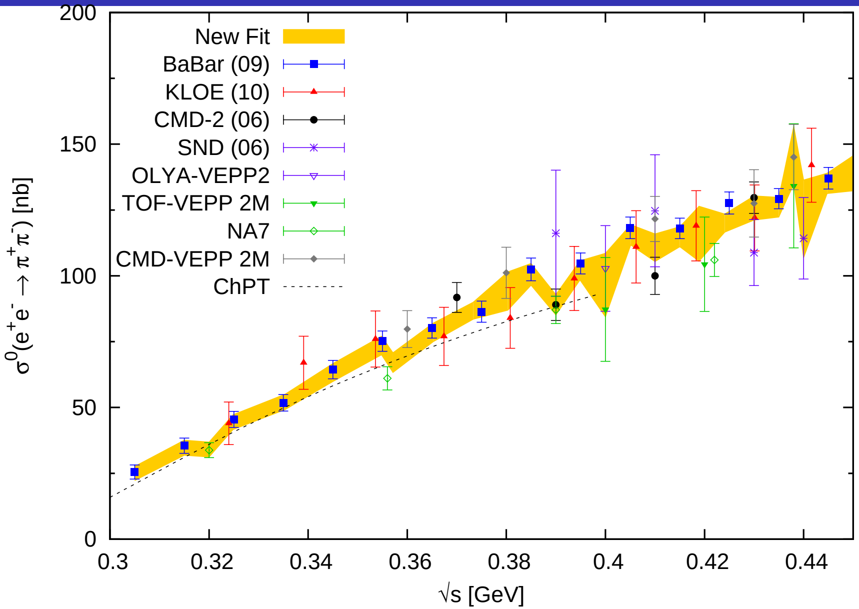
Important Channels

Contributions from various channels to a_μ(LO,had) for $\sqrt{s} < 1.8$GeV:

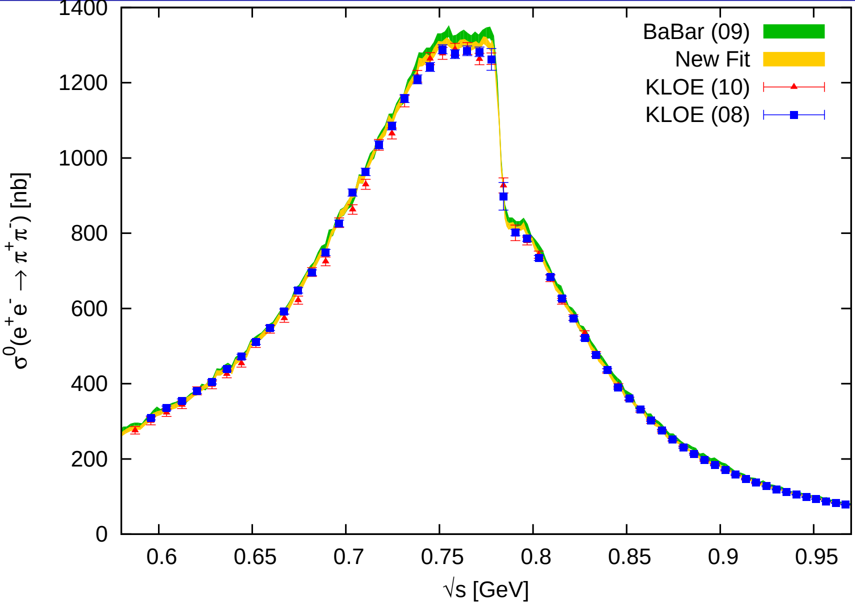
channel	HLMNT11	Davier et al '10	diff
$\pi^+\pi^-$	505.65 ± 3.09	507.80 ± 2.84	-2.15
$\pi^+\pi^-\pi^0$	47.38 ± 0.99	46.00 ± 1.48	1.38
K^+K^-	22.09 ± 0.46	21.63 ± 0.73	0.46
$\pi^+\pi^-2\pi^0$	18.62 ± 1.15	18.01 ± 1.24	0.61
$2\pi^+2\pi^-$	13.50 ± 0.44	13.35 ± 0.53	0.15
$K_S^0K_L^0$	13.32 ± 0.16	12.96 ± 0.39	0.36
$\pi^0\gamma$	4.54 ± 0.14	4.42 ± 0.19	0.12
⋮	⋮	⋮	⋮
Sum	634.28 ± 3.53	633.93 ± 3.61	0.35

table taken from HLMNT11

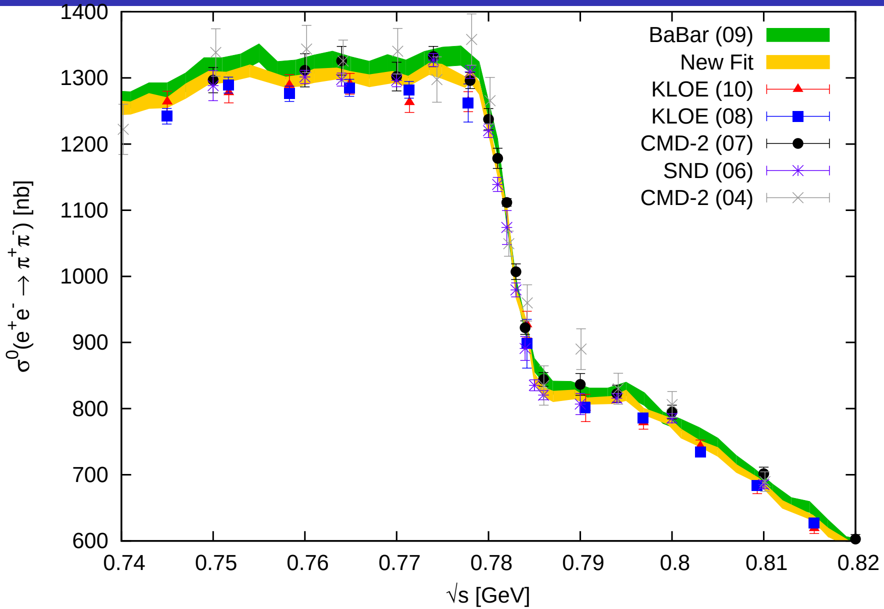
$\pi^+\pi^-$ channel: Low Energy Tail



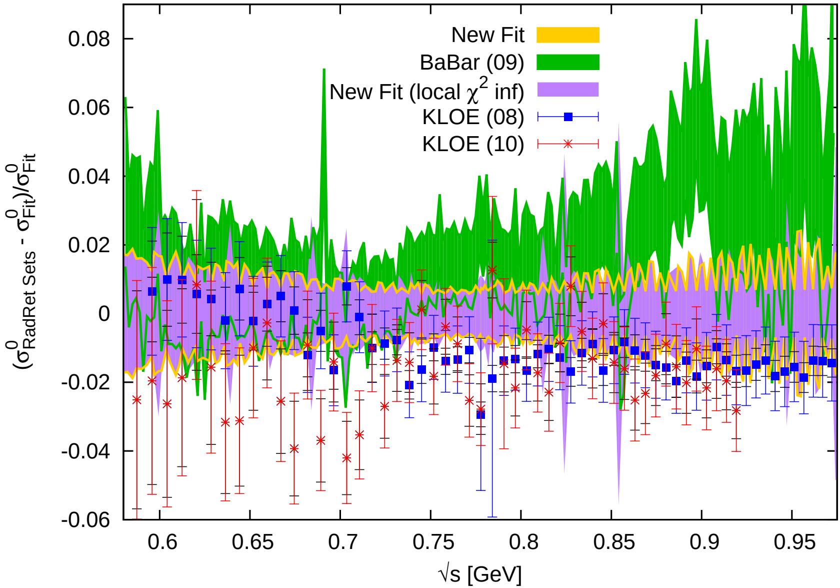
$\pi^+\pi^-$ channel: New Radiative Return Data



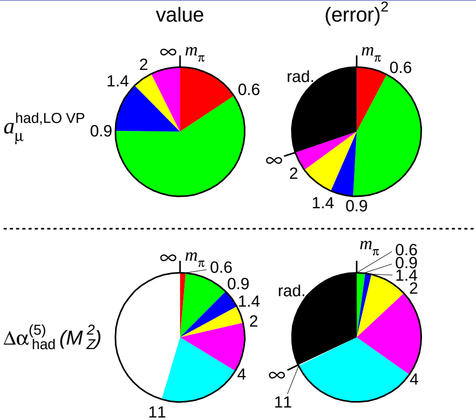
$\pi^+\pi^-$ channel: Zoom-In at ρ-ω Region



Rad. Rtn. Data (for $\pi^+\pi^-$) and Our Combined Result



Results: $a_\mu^{\rm had,LO}$, $a_\mu^{\rm had,NLO}$ and $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$



$$a_\mu^{\rm had,LO} = (694.91 \pm 3.72_{\rm exp} \pm 2.10_{\rm rad}) \times 10^{-10}$$

$$a_\mu^{\rm had,NLO} = (-9.84 \pm 0.06_{\rm exp} \pm 0.04_{\rm rad}) \times 10^{-10}$$

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = (276.26 \pm 1.16_{\rm exp} \pm 0.74_{\rm rad}) \times 10^{-4}$$

Full SM Result and Comparison with Other Groups

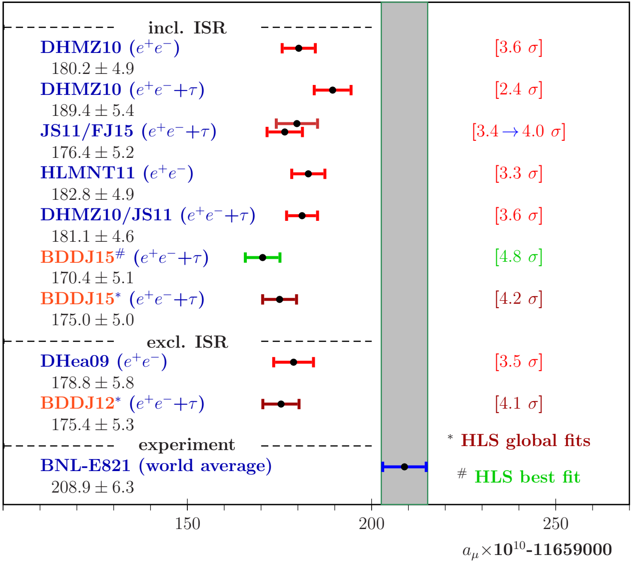
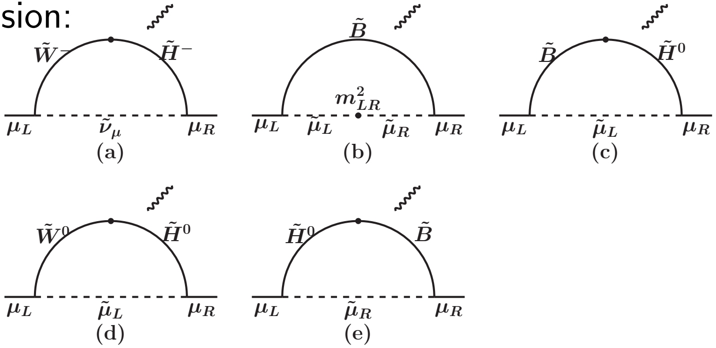


Fig. from F. Jegerlehner, arXiv:1511.04473

SUSY Contributions to Muon $g-2$

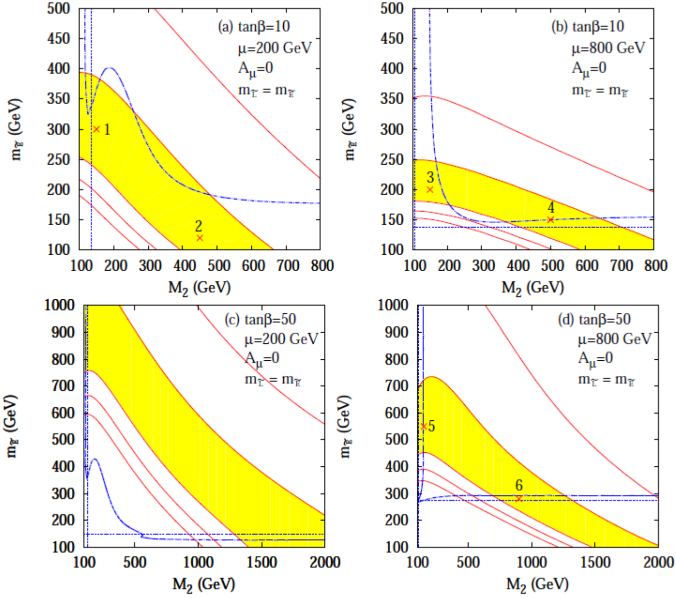
Suppose that the 3.3 σ deviation is due to SUSY...

Leading **SUSY contributions** in the $m_Z/m_{\rm SUSY}$ expansion:



In most cases, the $\tilde{\chi}^{\pm}$-$\tilde{\nu}$ diagram (a) and/or the $\widetilde{B}$-$\widetilde{\mu}_{L/R}$ diagram (b) dominate. (Lopez-Nanopoulos-Wang, Chattopadhyay-Nath, Moroi, $\cdots$)

MSSM Contributions to Muon $g-2$



x-axis: M_2
(gaugino mass)

y-axis: $m_{\tilde{l}}$
(slepton mass)

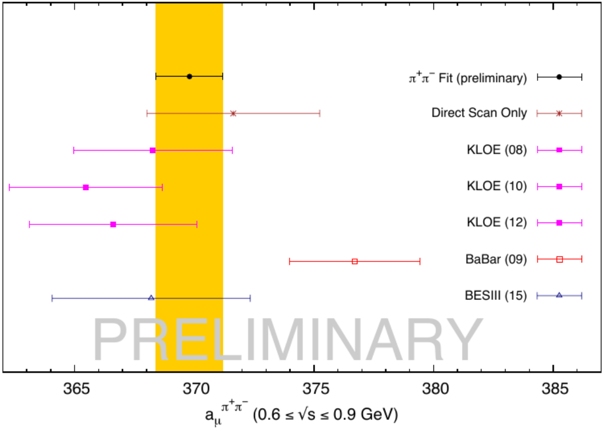
Figs from Cho, Hagiwara, Matsumoto and DN

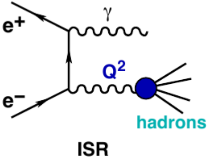
Our preliminary results

HVP: HLMNT -> HKMNT in preparation

$\pi^+\pi^-$ channel: + KLOE12, + BES III from Rad. Ret.:

Prel. HKMNT combination w. full cov.-matrices:

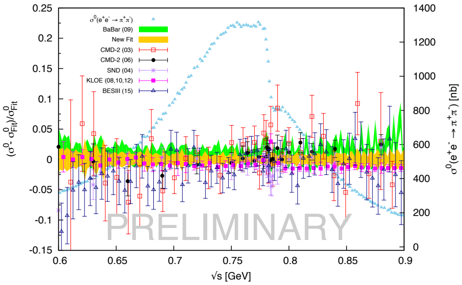




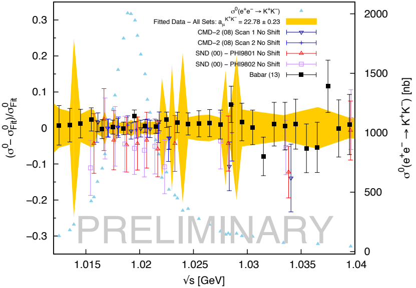
- χ^2_{min}/d.o.f. = 1.4
- further improvements expected from CMD-3, more also from BaBar?

HVP: HLMNT -> HKMNT in preparation

$\pi^+\pi^-$ channel



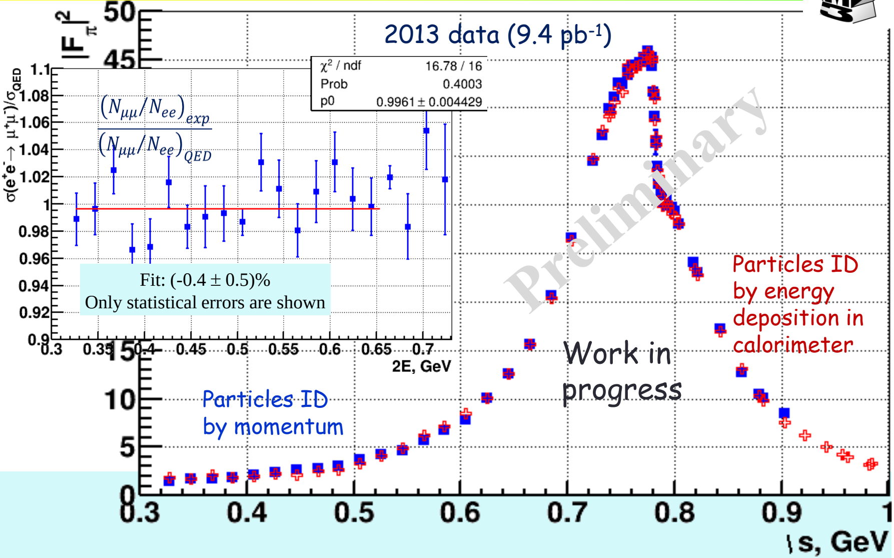
K^+K^- channel with recent BaBar



- Many new data sets and an improved combination algorithm, which takes fully into account all available covariance matrices, give significantly reduced errors and a slightly smaller mean value
- Previously sizeable additional (conservative) error from uncertainty in treatment of radiative corrections (VP + FSR), mainly from older data sets, gets reduced
- More exclusive data in multi-pion and K channels reduce uncertainty from estimate based on Iso-spin correlations

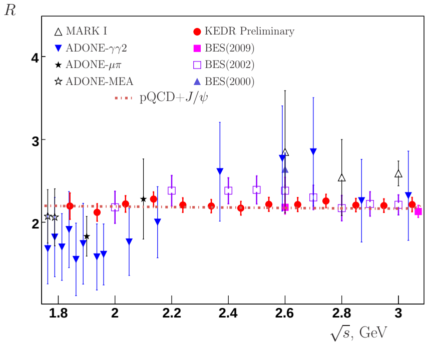
Near future prospects

$e^+e^- \to \pi^+\pi^-$



Slide by S. I. Eidelman (BINP), talk at "Tau 2016", Sept. 2016

R Measurement between 1.84 and 3.05 GeV at KEDR – X



$\overline{R} = 2.209 \pm 0.020 \pm 0.046$ agrees with $R_{\rm pQCD} = 2.18 \pm 0.02$ based on $\alpha_s(m_\tau) = 0.333 \pm 0.013$ derived from hadronic τ decays

S.Eidelman, BINP p.17/18

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Current status and near-future ($\sim$ 5-7 yrs) improvements in δa_μ (in units of 10^{-11})

Error	[20]	[21]	Future
$\delta a_\mu^{\rm SM}$	49	50	35
$\delta a_\mu^{\rm HLO}$	42	43	26
$\delta a_\mu^{\rm HLbL}$	26	26	25
$\delta(a_\mu^{\rm EXP} - a_\mu^{\rm SM})$	80	80	40

[20]: DHMZ
[21]: HLMNT

Near-future improvements in $\delta a_\mu^{\rm HLO}$ mainly from VEPP-2000 and BES-III.

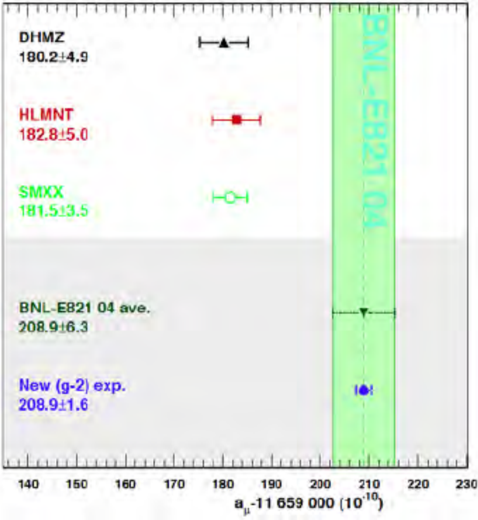


Table and Fig. from T. Blum et al, arXiv:1311.2198

Near Future Prospects, Lattice (Blum et al)

As for the hadronic vacuum polarization (HVP),

"...the lattice-QCD uncertainty on a_μ(HVP), currently at the 5%-level, can be reduced to 1 or 2% within the next few years."

"With increasing experience and computer power, it should be possible to compete with the e^+e^- determination of a_μ(HVP) by the end of the decade, perhaps sooner with additional technical advances."

As for l-by-l,

"... a lattice calculation with even a solid 30% error would already be very interesting. Such a result, ..., is not out of the question during the next 3-5 years."

From T. Blum et al, arXiv:1311.2198

Byproducts

Byproducts: QED coupling at the Z-boson mass

★ $\alpha(M_Z^2)$: the least well known among $\{G_\mu, M_Z, \alpha(M_Z^2)\}$, which are used as **input** to precision electroweak fits.

★ Running of α

$$\alpha(M_Z^2) = \frac{\alpha}{1 - \Delta\alpha_{\rm lep}(M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(M_Z^2) - \Delta\alpha^{\rm top}(M_Z^2)}$$

where $\Delta\alpha_{\rm lep}(M_Z^2) = 0.03149769$ (Steinhauser), $\Delta\alpha^{\rm top}(M_Z^2) = -\ 0.0000728(14)$ and $\alpha = 1/137.035999679(94)$ (PDG10).

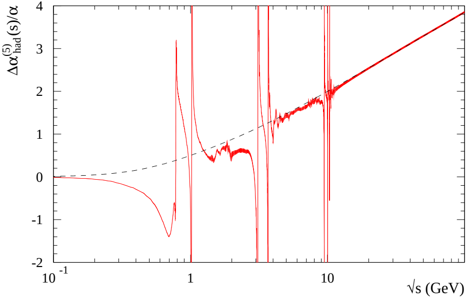
★ Similar dispersion relation: ($\Longrightarrow$ **byproduct** of $a_\mu^{\text{had,LO}}$)

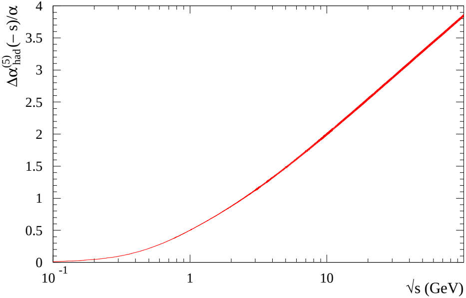
$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha s}{3\pi} \mathrm{P} \int \frac{R(s')ds'}{s'(s'-s)}$$

★ Our results: $\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = (276.3 \pm 1.4) \times 10^{-4}$,

$$\alpha(M_Z^2)^{-1} = 128.944 \pm 0.019.$$

The hadronic contribution $\Delta\alpha_{\rm had}^{(5)}(q^2)$ to the running QED coupling for $q^2 > 0$ (left) and $q^2 < 0$ (right)





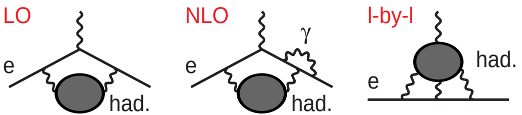
Fortran subroutine to compute the above is available from us upon request

Byproduct: Standard Model prediction for electron $g-2$

QED contribution	115 965 218 00.7 $(0.6)_{4\text{loop}}(0.4)_{5\text{loop}}(7.6)_\alpha$ (using $\alpha(^{87}\text{Rb})$)	Aoyama et al
EW contribution	0.2973 (0.0052)	Czarnecki et al
Hadronic contributions		
LO hadronic	**18.66 (0.11)**	**DN & TT** (was $18.75(0.18)$ Davier et al '98)
NLO hadronic	**$-$2.23 (0.01)**	**DN & TT** (was $-2.25(0.05)$ Krause '97)
light-by-light	0.39 (0.13)	Jegerlehner+Nyffeler
Theory total	**115 965 218 17.8 $(0.6)_{4\text{loop}}(0.4)_{5\text{loop}}(0.2)_{\text{had}}(7.6)_\alpha$**	
Experiment	**115 965 218 07.3 (2.8)**	Gabrielse et al
Theory $-$ Exp	**10.5 (8.1)**	**1.3σ discrepancy**

(in units of 10^{-13}. Numbers taken from Giudice et al, JHEP**11**(2012)113)

n.b.: hadronic contributions:



Summary

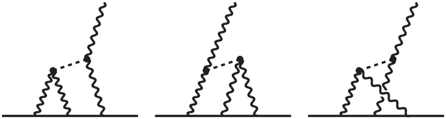
- Hadronic contrib. to the muon $g-2$: key to improve the Standard Model prediction
- $\gtrsim 3\,\sigma$ discrepancy in $(g-2)_\mu$ between experiment and theory
  $\Longrightarrow$ **New physics?** "Light" new particles like $\tilde{\mu}, \mu_{\rm KK}, \ldots$???
  $\Longrightarrow$ Worth studying μ-EDM, $\mu \to e\gamma$, μ-e conv., ...!
  ($\Leftrightarrow$ No new physics seen at the LHC so far. What does this mean?)
- Two new experiments to measure the muon $g-2$ planned at J-PARC and Fermilab.
- To establish this discrepancy more firmly, important to resolve the tension between the KLOE and BaBar data
  $\Longrightarrow$ new data from BES-III appeared recently
  $\Longrightarrow$ new precise data from VEPP-2000 and SuperKEKB strongly awaited.

Backup Slides

Modern evaluation of l-by-l contribution

(Melnikov & Vainshtein)

1. First, use the large N_C expansion to find that the leading contribution is the pion pole contribution.



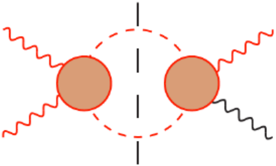
2. Choose the momentum-dependence of the $\pi\gamma\gamma$ coupling (form factor) in such a way that it is consistent with a constraint from QCD (OPE) at the momentum region $q_1^2 \sim q_2^2 \gg q_3^2$. Integrate over the loop momenta.
3. Repeat the above for $\eta, \eta', a_1, \ldots$. Basically that's all for the LO in $1/N_C$.
4. As for NLO in $1/N_C$, it depends on authors which diagram is numerically important.

For example,

$$a_\mu^{\text{lbyl}} = \begin{cases} (10.5 \pm 2.6) \times 10^{-10} & \text{Prades-de Rafael-Vainshtein, arXiv:0901.0306} \\ (11.6 \pm 4.0) \times 10^{-10} & \text{Nyffeler, arXiv:0901.1172} \end{cases}$$

dispersion relation approaches for a_μ (I)

Hoferichter, Colangelo, Procura, Stoffer (2014)



➡ **dispersion formalism** for $\gamma^* \gamma^* \to \gamma^* \gamma$

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\text{FsQED}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \cdots$$

➡ **master formula** for a_μ

$$a_\mu^{\pi\pi} = e^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{I^{\pi\pi}}{q_1^2 q_2^2 s((p+q_1)^2 - m^2)((p-q_2)^2 - m^2)},$$

$$I^{\pi\pi} = \sum_{i \in \{1,2,3,6,14\}} \left(T_{i,s} I_{i,s} + 2 T_{i,u} I_{i,u} \right) + 2 T_{9,s} I_{9,s} + 2 T_{9,u} I_{9,u} + 2 T_{12,u} I_{12,u}$$

with $I_{i,(s,u)}$ dispersive integrals and $T_{i,(s,u)}$ integration kernels

$$I_{1,s} = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'-s} \left(\frac{1}{s'-s} - \frac{s' - q_1^2 - q_2^2}{\lambda(s', q_1^2, q_2^2)} \right) \mathrm{Im}\bar{h}^0_{++,++}(s'; q_1^2, q_2^2; s, 0),$$

$$I_{6,s} = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{(s' - q_1^2 - q_2^2)(s'-s)^2} \mathrm{Im}\bar{h}^2_{+-,+-}(s'; q_1^2, q_2^2; s, 0) \left(\frac{75}{8} \right)$$

Helicity amplitudes contribute up to $J = 2$ (S and D waves)

Slide by M. Vanderhaeghen, talk at "Lepton Moments 2014", July 2014

Exp. inputs for evaluation of a_μ(had, l-by-l)

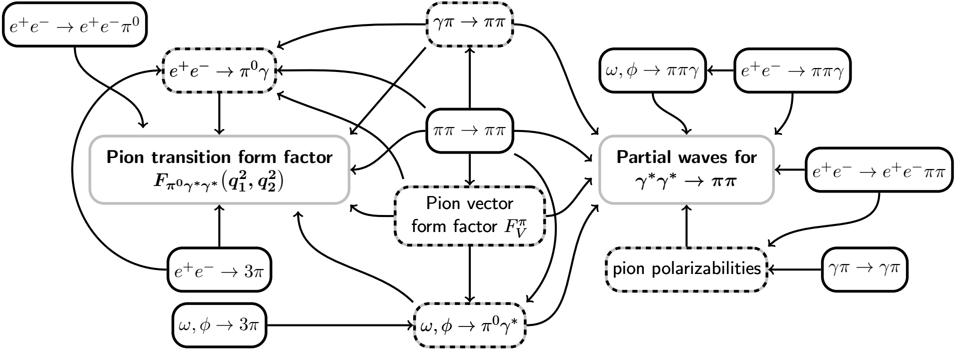
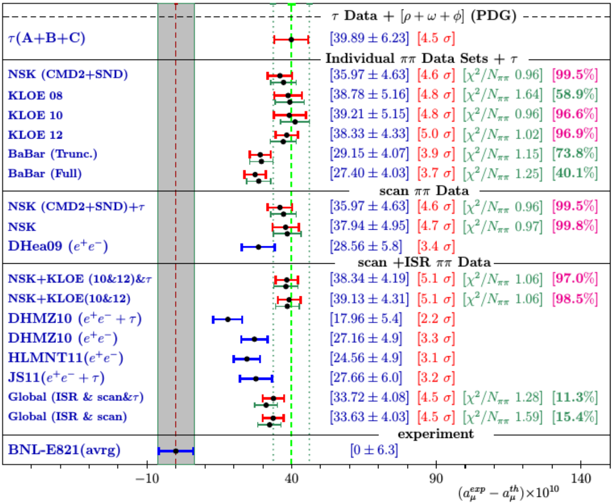


Fig. from G. Colangelo et al, arXiv:1408.2517

Hot from the arXiv: Benayoun et al., 1507.02943



Updated anal. (now with fitted normal. factors) in global fit based on HLS model:

← 'preferred' has 5σ already, from both aggressive errors and shifts due to the HLS model (and the fit?)

Slide by T. Teubner (Liverpool), talk at 'High-precision QCD at low energy,' Aug. '15