

Towards precision in $|V_{cb}|$ and $|V_{ub}|$ measurements

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on behalf of the Belle collaboration

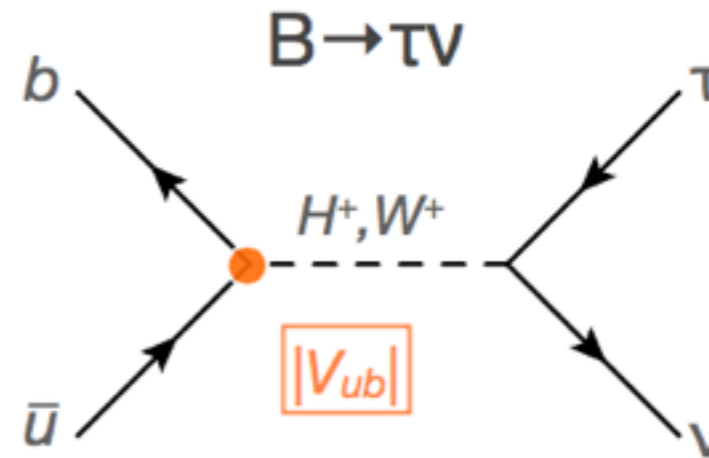
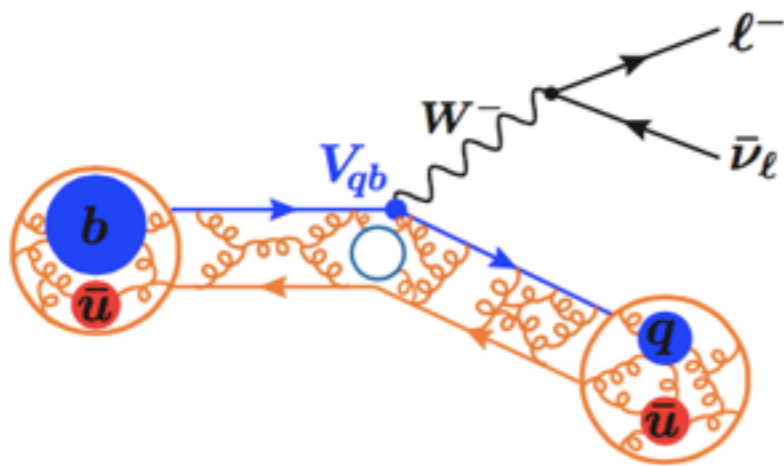
Precision theory for precise measurements at the LHC and future colliders

Quy-Nhon, Vietnam

Sep.25-Oct.1, 2015



$|V_{ub}|$ and $|V_{cb}|$



Experimental measurements

$|V_{cb}|$ Semileptonic

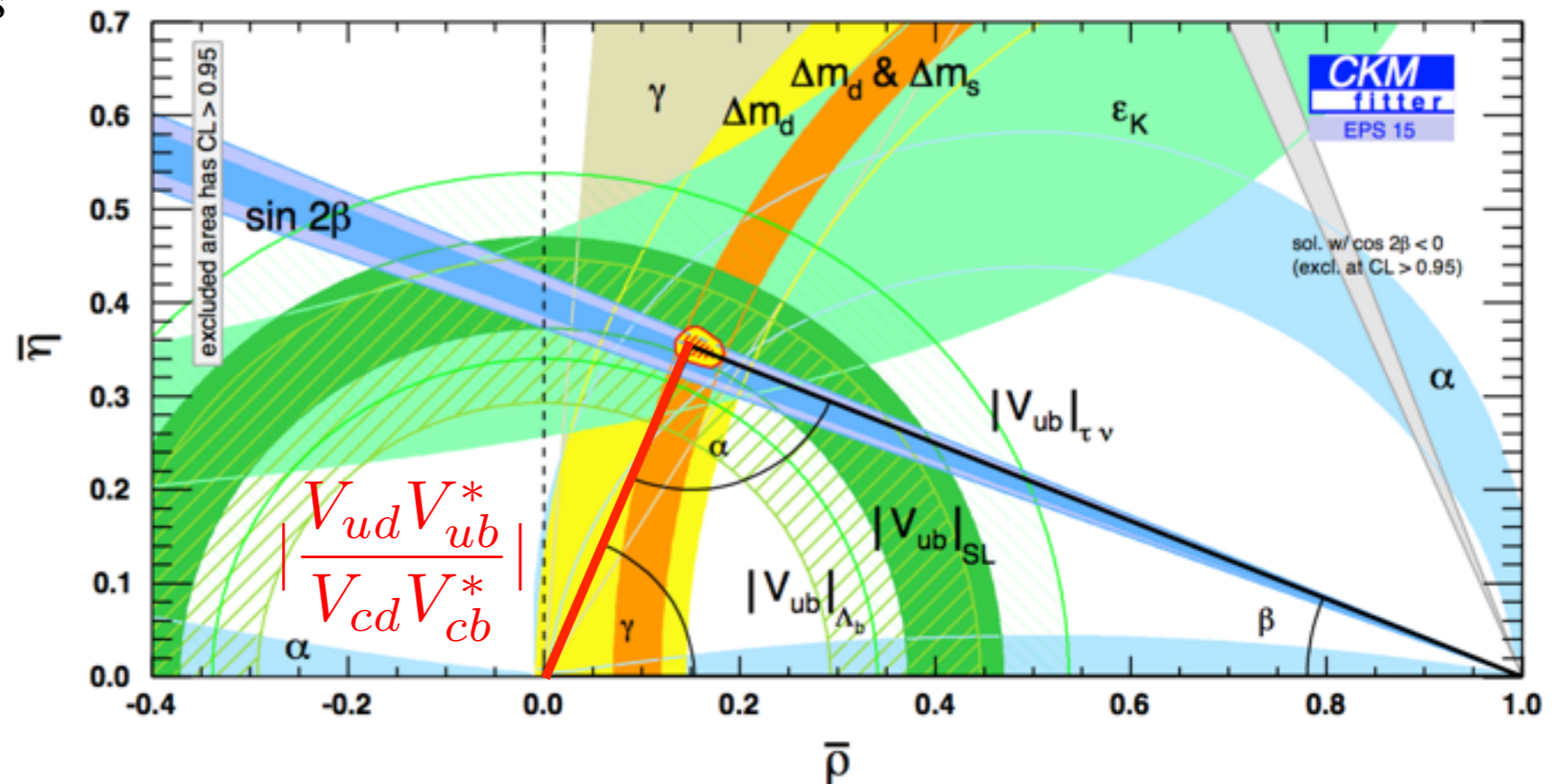
Inclusive: $B \rightarrow X_c l \nu$

Exclusive: $B \rightarrow D^{(*)} l \nu$

$|V_{ub}|$ Semileptonic

Inclusive: $B \rightarrow X_u l \nu$

Exclusive: $B \rightarrow \pi l \nu$



$|V_{cb}|$ determination from inclusive $B \rightarrow X_c \ell \nu$

- Total semileptonic decay width with Operator Product Expansion (OPE)

$$\Gamma_{\text{SL}} = |V_{cb}|^2 \frac{G_F^2 m_b^5(\mu)}{192\pi^3} (1 + A_{\text{ew}}) A_{\text{per}} \times \left[c_0(r) + c_2\left(r, \frac{\mu_\pi^2}{m_b^2}, \frac{\mu_G^2}{m_b^2}\right) + c_3\left(r, \frac{\rho_D^3}{m_b^3}, \frac{\rho_{\text{SL}}^3}{m_b^3}\right) + \dots \right]$$

non-perturbative input

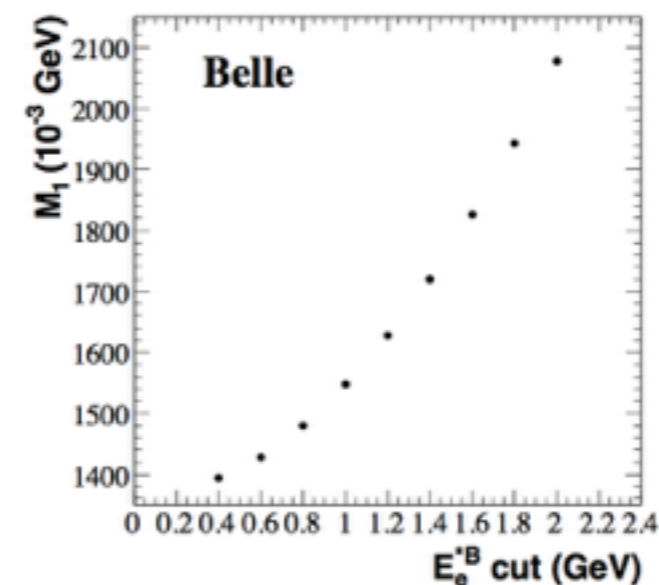
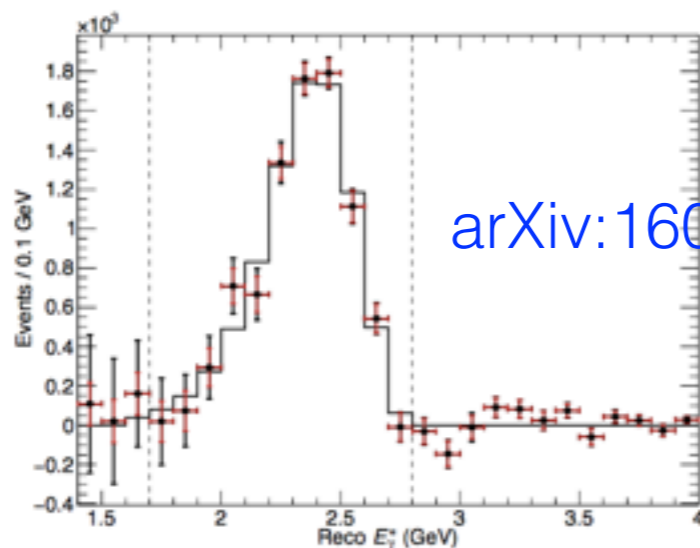
$r = m_c/m_b$, heavy quark parameters $\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{\text{SL}}^3$

- Moments of spectrums **Lepton energy, hadronic mass, and photon-energy** ($B \rightarrow X_s \gamma$)

- n^{th} moment of observables e.g. Lepton energy: $\langle E_\ell^n \rangle = \frac{1}{\Gamma_{E_\ell > E_{\text{cut}}}} \int_{E_\ell > E_{\text{cut}}} E_\ell^n \frac{d\Gamma}{dE_\ell} dE_\ell$

- Similar expansion as Γ_{SL} : $f(E_{\text{cut}}, m_c, m_b, \mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{\text{SL}}^3)$

- Use these moments to extract the quark masses and heavy quark parameters



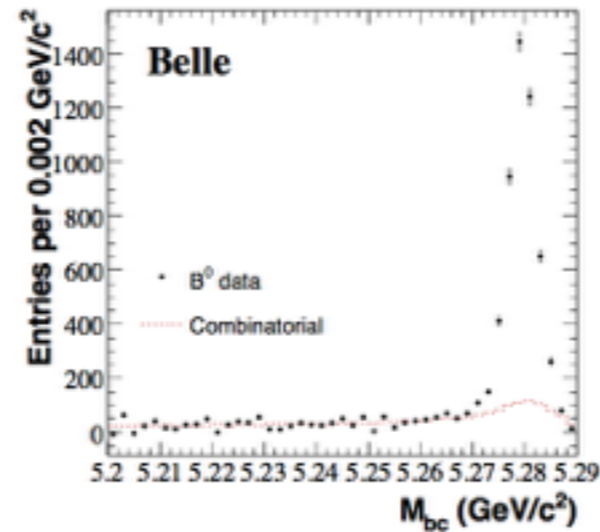
Experimental measurements $B \rightarrow X_c \ell \nu$

- Tag B meson with hadronic decays

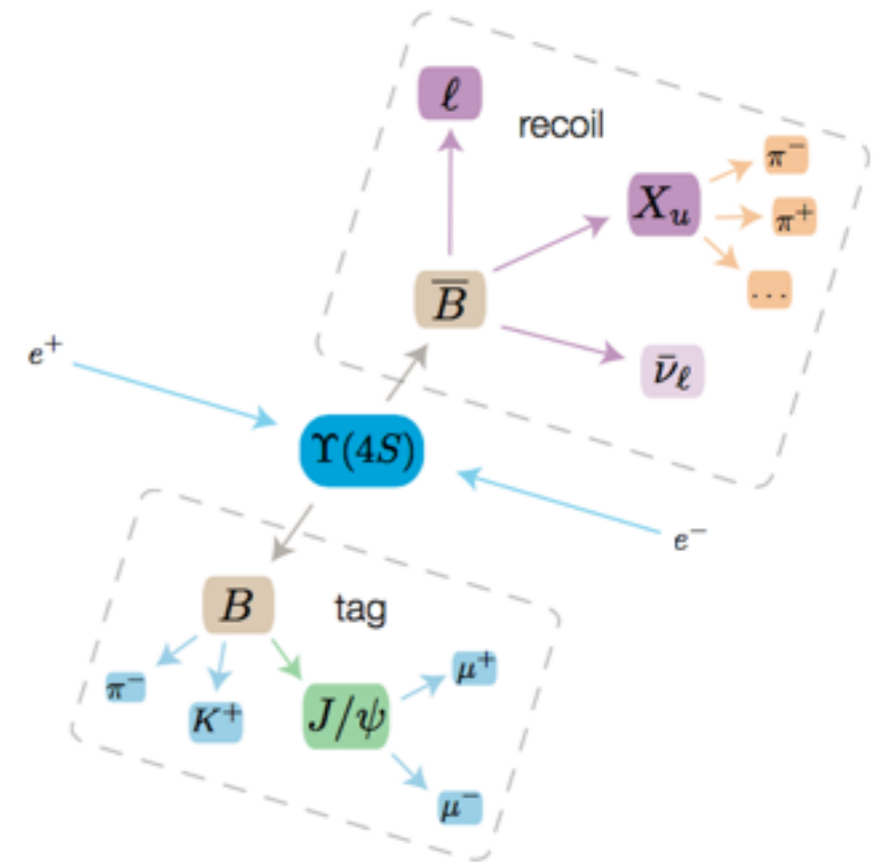
$$B_{\text{tag}} \rightarrow D^{(*)} Y$$

$$M_{bc} = \sqrt{(E_{\text{beam}}^*)^2 - (p_B^*)^2}$$

$$\Delta E = E_B^* - E_{\text{beam}}^*$$



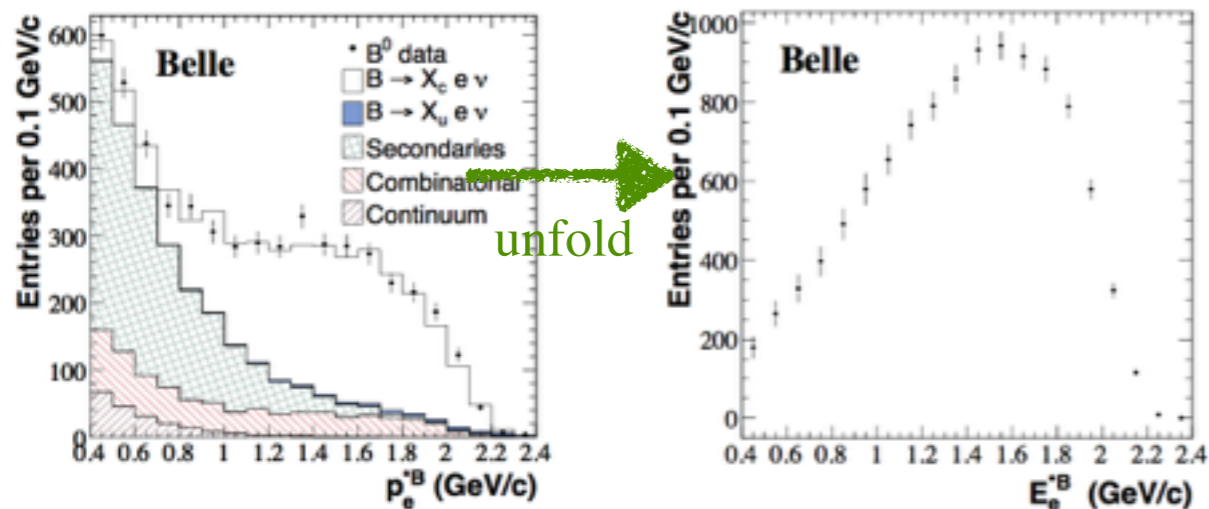
Belle, PRD 75, 032001 (2007)



- Reconstruct X_c with the rest particles not used for B_{tag} and lepton
- Extraction of moments of lepton energy and hadronic mass

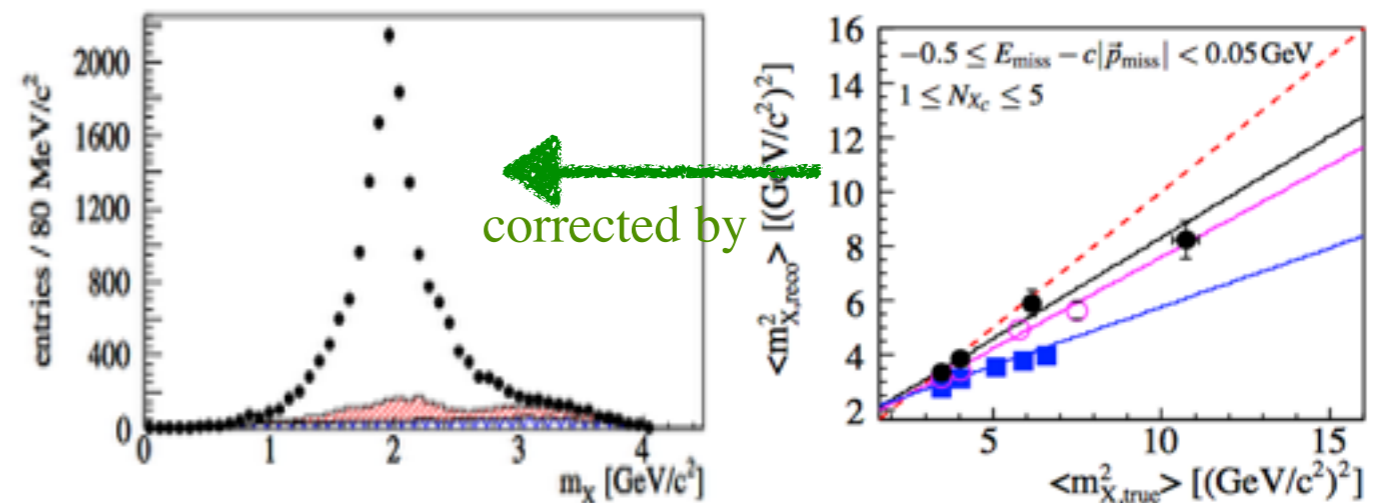
Reconstructed spectrum \rightarrow True spectrum: detectors resolution, selection efficiency, QED radiative effect

Belle: Unfolding



Belle, PRD 75, 032001 (2007)

Babar: Linear correction



Babar, PRD 81, 032003 (2010)

$|V_{cb}|$ determination from $B \rightarrow X_c \ell \nu$

- Global Fitting: fit to the moments of lepton energy and hadronic mass with different energy cut with some parameters constrains.
- Theoretical expressions
 - Kinetic scheme, 1S
 - Fit to the measurements from different experiments
 - Fitting results are consistent

	$ V_{cb} (10^{-3})$	m_b (GeV)	note
Kinetic scheme	42.46 ± 0.88	4.541 ± 0.023	m_c constrain HFAG 2014
	42.21 ± 0.78	4.553 ± 0.020	m_c constrain PRL 114, 061802 (2015) higher order correction $\mathcal{O}(\alpha_s \Lambda_{QCD}^2 / m_b^2)$
1S	41.98 ± 0.45	4.691 ± 0.037	combine $B \rightarrow X_s \gamma$ HFAG 2014

$$|V_{cb}| = (42.21 \pm 0.78) \times 10^{-3}$$

$|V_{cb}|$ determination from exclusive $B \rightarrow D^{(*)} \ell \bar{\nu}_\ell$

- Differential decay ratio in the limit of very small lepton masses :

w : the Lorentz boost of D meson in the B rest frame

$$w = V_B \cdot V_D = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2m_B m_{D^{(*)}}}$$

$$\frac{d\Gamma}{dw}(\bar{B} \rightarrow D^* \ell \bar{\nu}_\ell) = \frac{G_F^2 m_B^5}{48\pi^3} |V_{cb}|^2 (w^2 - 1)^{1/2} P(w) (\eta_{ew} \mathcal{F}(w))^2$$

$$\frac{d\Gamma}{dw}(\bar{B} \rightarrow D \ell \bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 (m_B + m_D)^2 m_D^3 (w^2 - 1)^{3/2} (\eta_{ew} \mathcal{G}(w))^2$$

- Parameterization of the form factors in the framework of Heavy Quark Effective Theory

$$\mathcal{F}(w) : \mathcal{F}(1), \rho^2, R_1(1), R_2(1)$$

$$\mathcal{G}(w) : \mathcal{G}(1), \rho^2$$

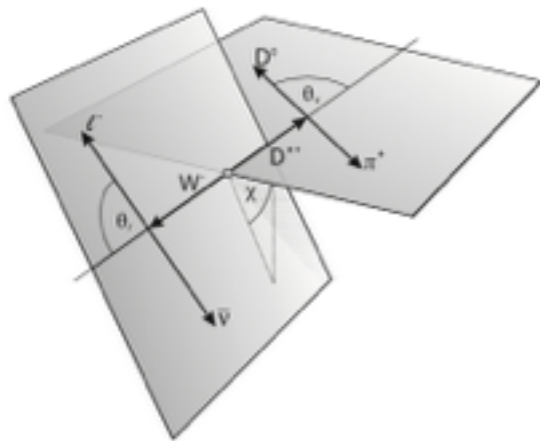
- The form factor normalisation at $w=1$ (zero-recoil) computed by Lattice QCD

$$\mathcal{F}(1) = 0.906 \pm 0.013 \quad \text{PRD.89, 114504(2014)}$$

$$\mathcal{G}(1) = 1.0528 \pm 0.0082 \quad \text{Fermilab/MILC: PR D92, 034506, 2015}$$

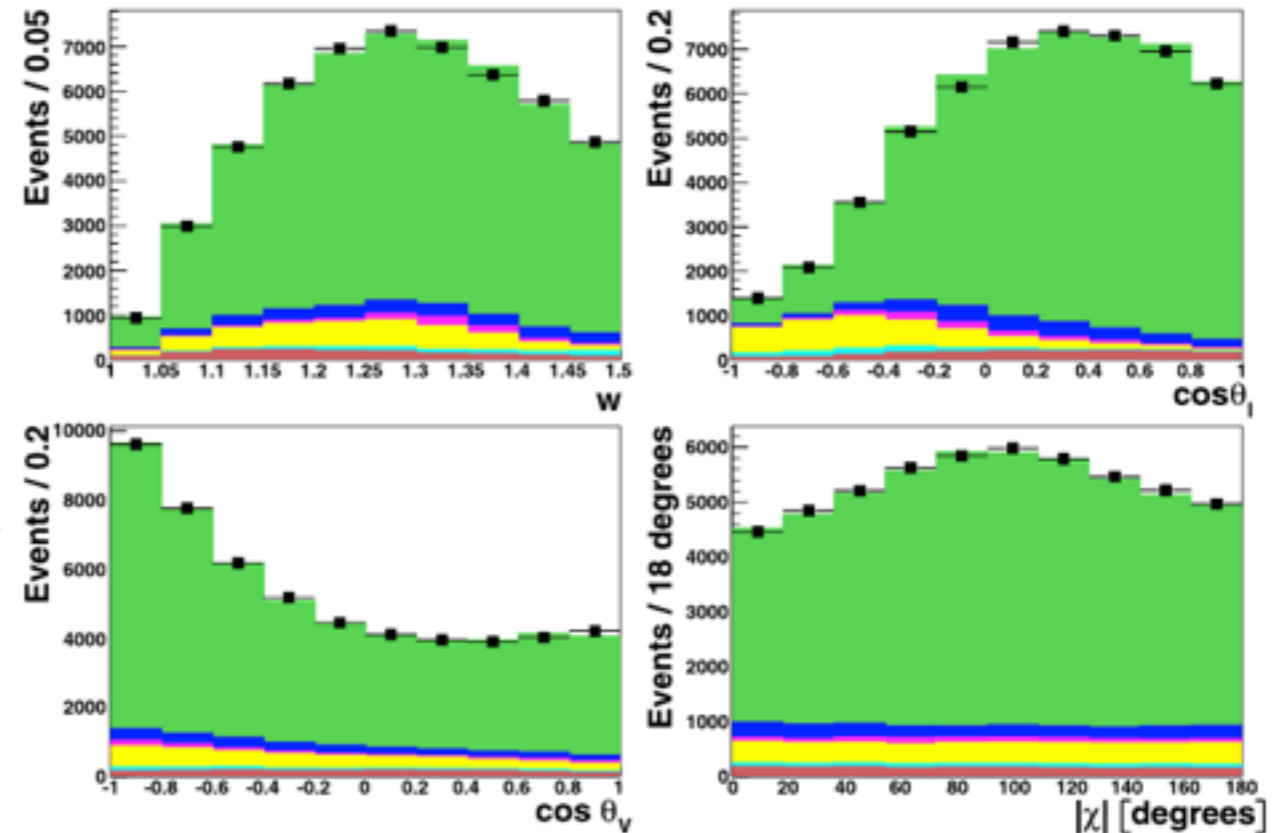
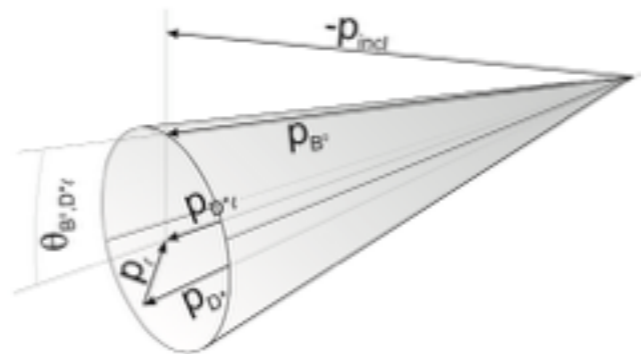
Experimental measurements $\bar{B} \rightarrow D^* \ell \bar{\nu}_\ell$

Belle, PRD82, 112007 (2010)



- Kinematics of the decay is characterized by four variables $\omega, \theta_1, \theta_\nu, |\chi|$
- Fit to $\omega, \cos(\theta_1), \cos(\theta_\nu), |\chi|$

- Untagged (Belle), determine B rest frame with the rest particles based on the feature that B direction lie on a cone around the (D^*) -axis



$B \rightarrow D^* \ell \nu$	\mathcal{B} (%)	$\eta_{EW} \mathcal{F}(1) V_{cb} $ (10^{-3})	$\rho_{D^*}^2$
CLEO untagged (Briere <i>et al.</i> , 2002)	$5.62 \pm 0.18 \pm 0.26$	$39.94 \pm 1.23 \pm 1.63$	$1.37 \pm 0.09 \pm 0.09$
Belle untagged (Dungel <i>et al.</i> , 2010)	$4.56 \pm 0.03 \pm 0.26$	$34.60 \pm 0.17 \pm 1.02$	$1.21 \pm 0.03 \pm 0.01$
BABAR untagged $B^0 \rightarrow D^{*-} \ell^+ \nu$ (Aubert <i>et al.</i> , 2008b)	$4.54 \pm 0.04 \pm 0.25$	$33.94 \pm 0.30 \pm 0.99$	$1.19 \pm 0.05 \pm 0.03$
BABAR untagged $B^+ \rightarrow \bar{D}^{*0} \ell^+ \nu$ (Aubert <i>et al.</i> , 2008d)	$4.97 \pm 0.07 \pm 0.34$	$35.22 \pm 0.59 \pm 1.33$	$1.13 \pm 0.06 \pm 0.06$
BABAR global fit (Aubert <i>et al.</i> , 2009d)	$4.95 \pm 0.02 \pm 0.20$	$35.76 \pm 0.20 \pm 1.10$	$1.19 \pm 0.02 \pm 0.06$
HFAG average (Amhis <i>et al.</i> , 2014)	$4.93 \pm 0.01 \pm 0.11$	$35.81 \pm 0.11 \pm 0.44$	$1.21 \pm 0.02 \pm 0.02$

- Systematic errors are dominant: tracking, particle ID, branching fractions of $D(^*)$ decay...

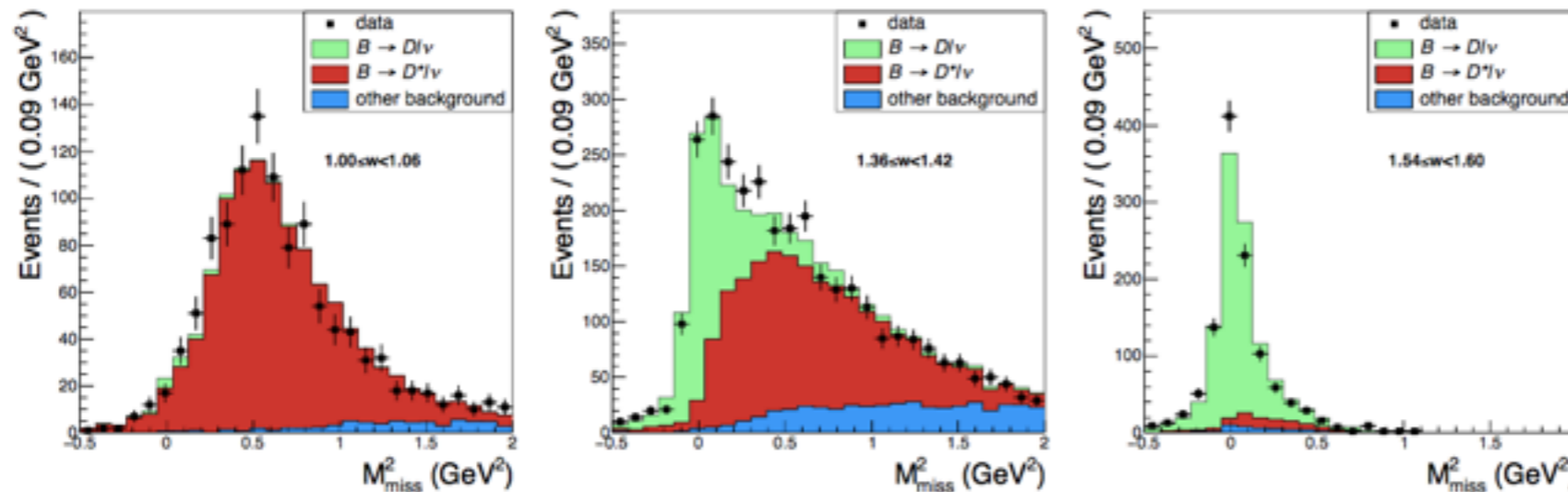
Experimental measurements $\bar{B} \rightarrow D\ell\bar{\nu}_\ell$

- Extract signals

Belle: PRD 93.032006 (2016)

- hadronic tag another B

- identify signal with $m_{miss}^2 = (p(\Upsilon) - p(B_{tag}) - p(D) - p\ell)^2$



- Parameterisation of form factor $\mathcal{G}(w)$

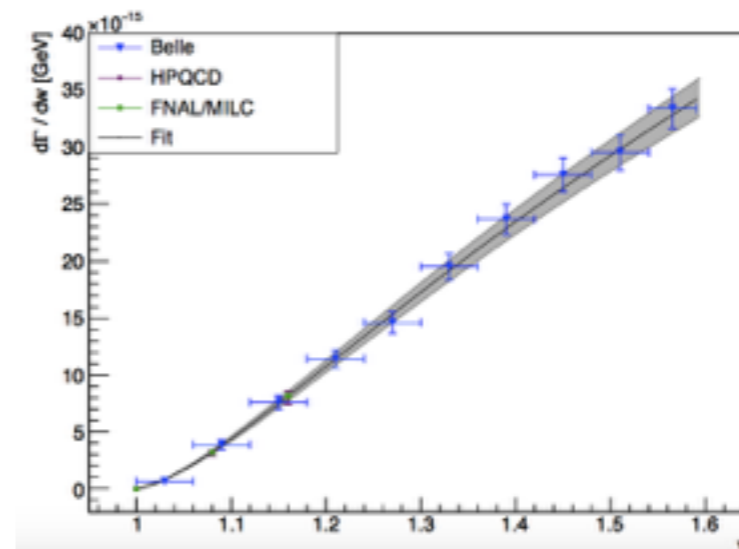
- CLN: $\mathcal{G}(1), \rho^2$

$$\eta_{EW}\mathcal{G}(1)|V_{cb}| = (42.29 \pm 1.37) \times 10^{-3}$$

$$\rho^2 = 1.09 \pm 0.05$$

- Model-independent BGL fit

Combined fit to experimental data and calculation from lattices QCD data (FNAL/MILC and HPQCD)



Lattice data	$\eta_{EW} V_{cb} [10^{-3}]$
FNAL/MILC [15]	40.96 ± 1.23
HPQCD [32]	41.14 ± 1.88
FNAL/MILC & HPQCD [15, 32]	41.10 ± 1.14

Exclusive Average

$$\bar{B} \rightarrow D^* \ell \bar{\nu}_\ell$$

$$|V_{cb}| = (38.9 \pm 0.5 \pm 0.5 \pm 0.2) \times 10^{-3}$$

• HFAG 2014: $\eta_{\text{ew}} \mathcal{F}(1) |V_{cb}| = (35.81 \pm 0.45) \times 10^{-3}$

• LQCD: $\mathcal{F}(1) = 0.906 \pm 0.013$

• Leading EW correction $\eta_{\text{ew}} = 1.015 \pm 0.005$

$$\bar{B} \rightarrow D \ell \bar{\nu}_\ell$$

$$|V_{cb}| = (40.0 \pm 1.4 \pm 0.3 \pm 0.2) \times 10^{-3}$$

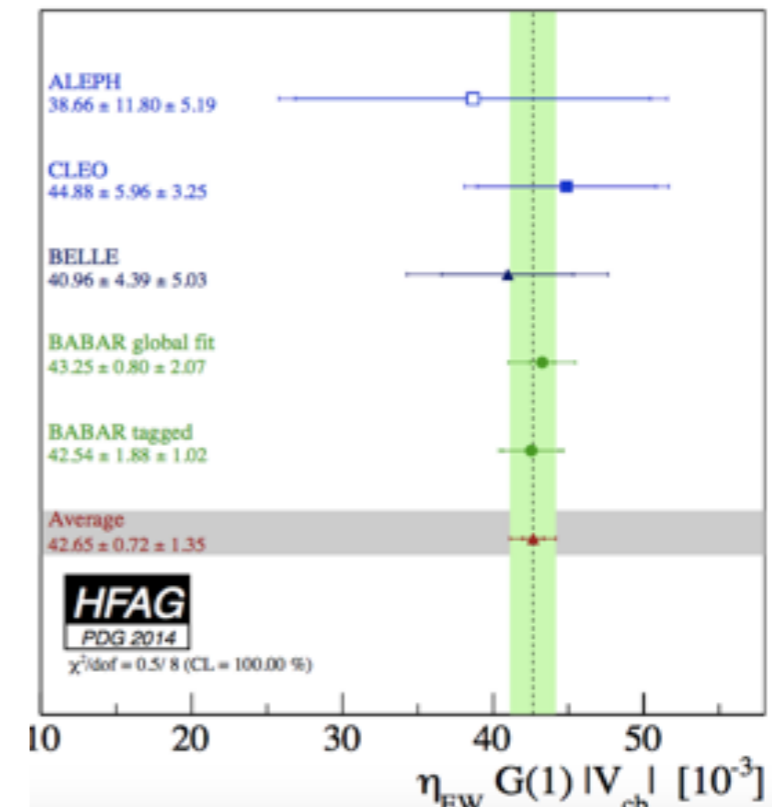
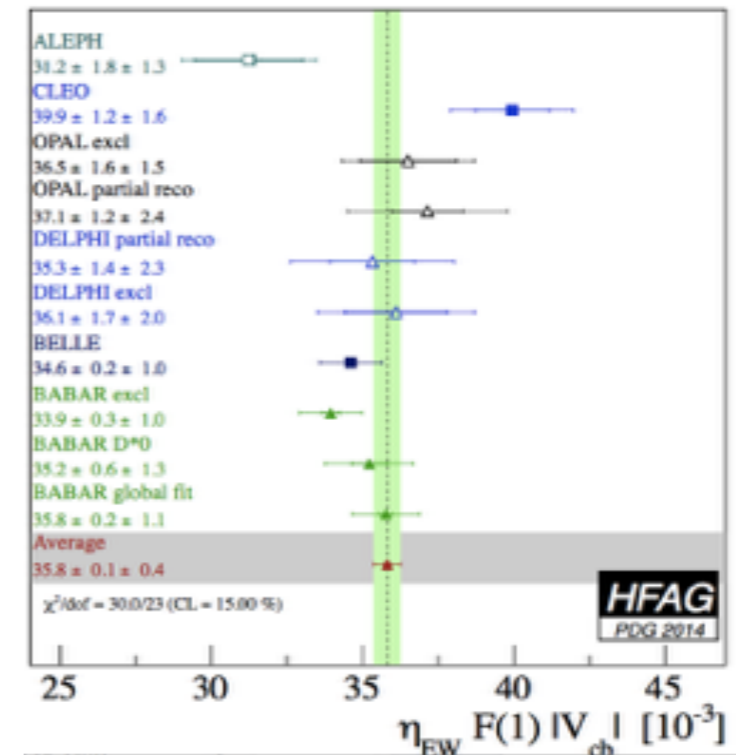
• HFAG 2014: $\eta_{\text{ew}} \mathcal{G}(1) |V_{cb}| = (42.65 \pm 1.53) \times 10^{-3}$

• LQCD: $\mathcal{G}(1) = 1.0528 \pm 0.0082$

• Leading EW correction $\eta_{\text{ew}} = 1.012 \pm 0.005$

PDG 2015: average of the results from $B \rightarrow D \ell \bar{\nu}$ and $B \rightarrow D^* \ell \bar{\nu}$

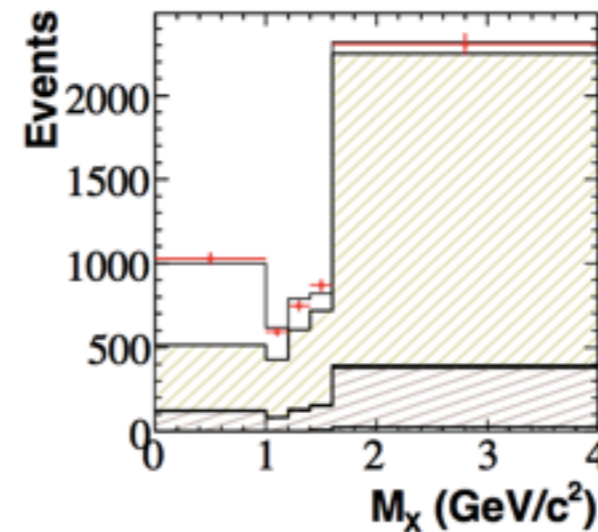
$$|V_{cb}| = (39.2 \pm 0.7) \times 10^{-3}$$



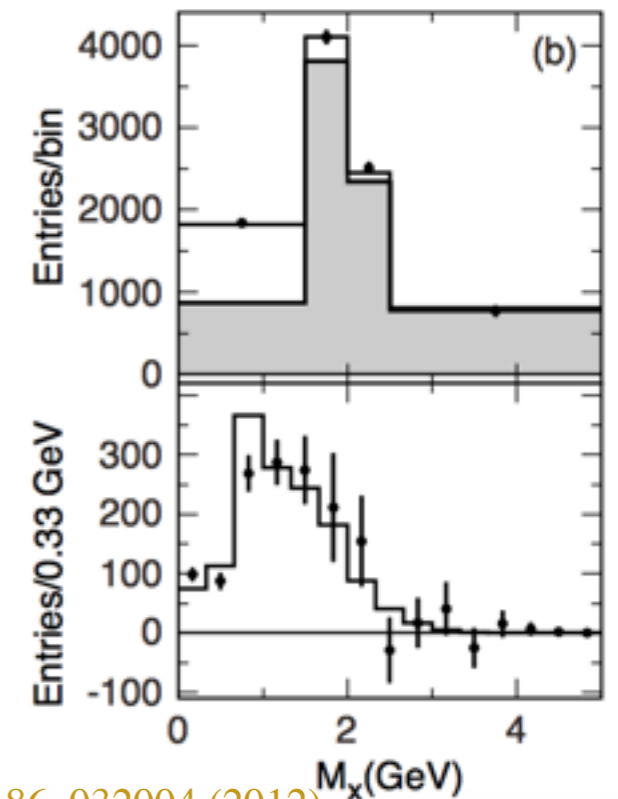
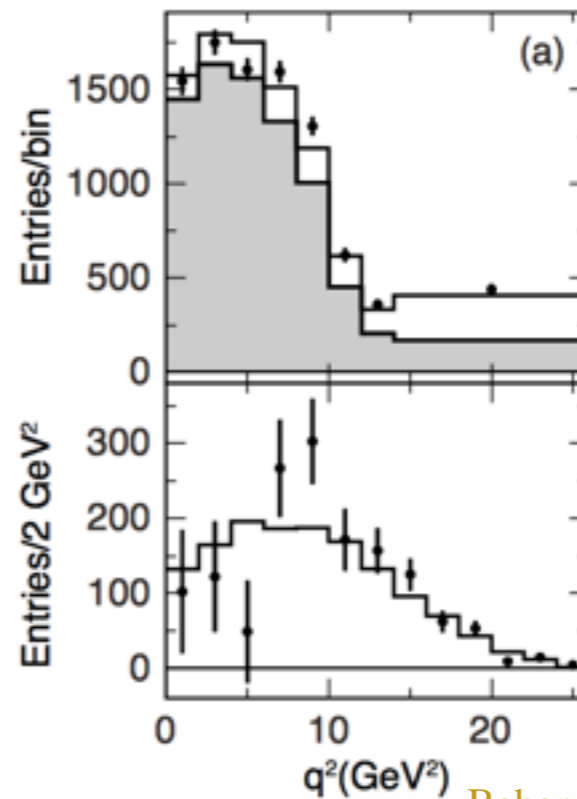
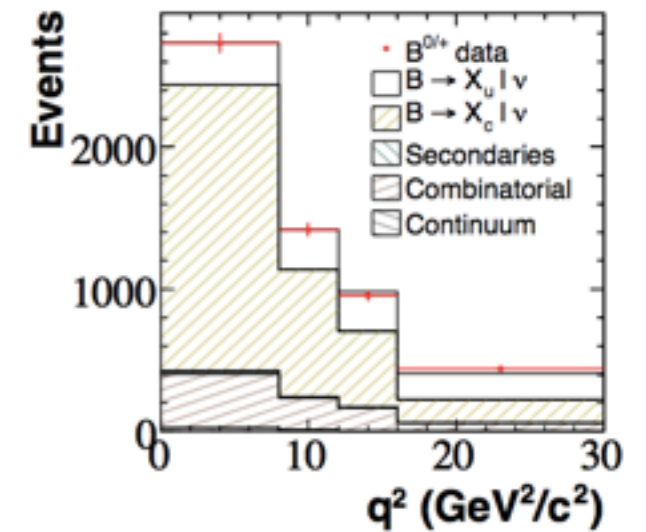
B → X_ulν Experimental measurements

- Tag lepton in signal B side, tag another B meson with Hadronic decays B → D^(*) Y
- Kinematic properties : (M_X, q², p_ℓ^{*}, P₊, MM², etc.)
- Challenge: background from B → X_clν, $\Gamma(b \rightarrow cl\nu) \approx 50 \times \Gamma(b \rightarrow ul\nu)$
- More kinematic variables are used to further suppress background, Belle: BDT, Babar: Cut based selection
- p_ℓ^{*} > 1.0GeV in the B rest frame, cover 90% of B → X_ulν phase space

Dominant Systematic sources	Belle(%)	Babar(%)
B → X _u lν shape function	3.6	5.4
Exclusive B → X _u lν	4.9	1.9
Background simulation (D ^(*))	1.7	2.7
Detector effects	3.1	3.4
BDT	3.1	-



Belle, PRL 104 (2010)



Babar, PRD 86, 032004 (2012)

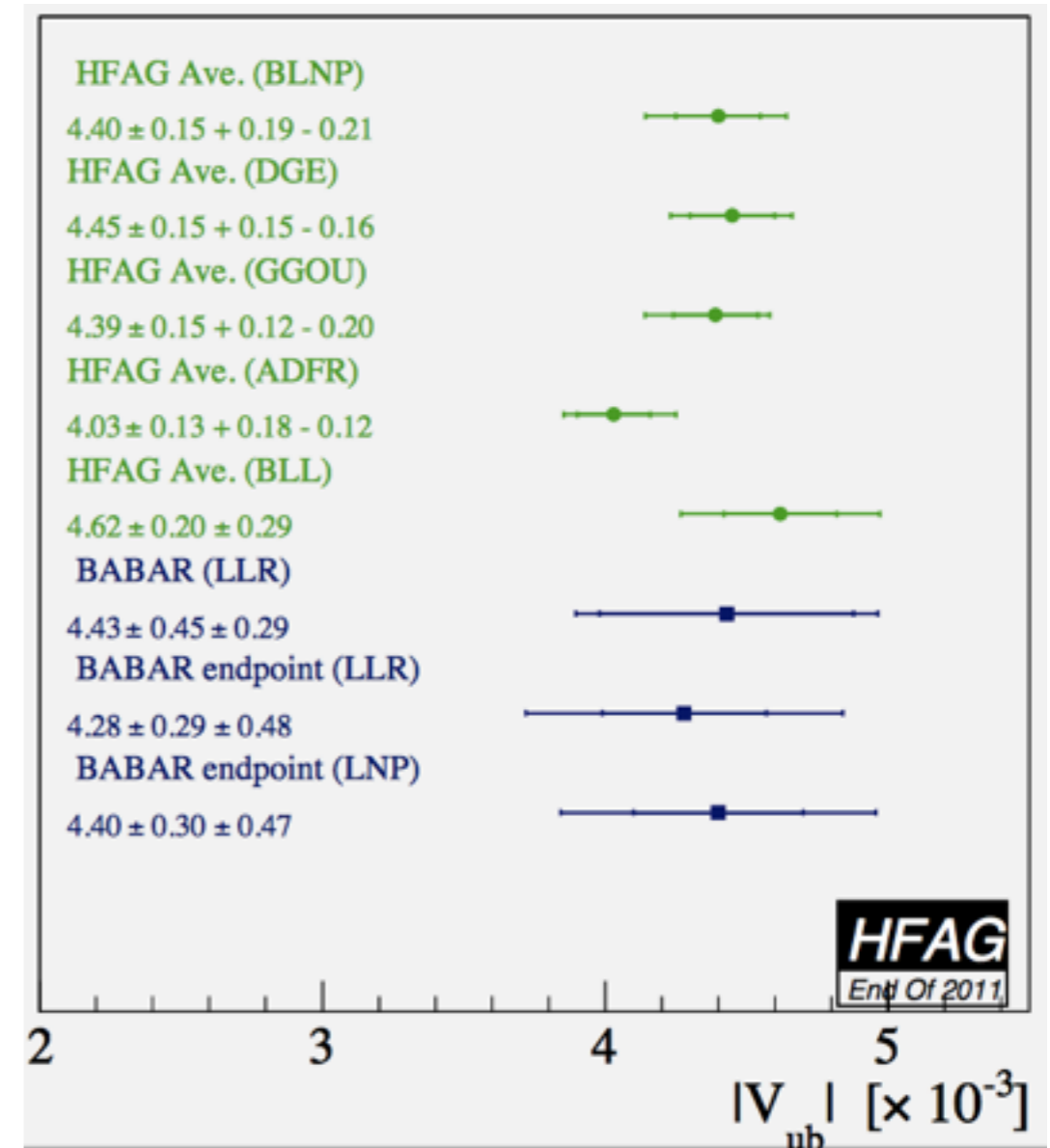
$|V_{ub}|$ determination from $B \rightarrow X_u \ell \bar{\nu}$

$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(\bar{B} \rightarrow X_u \ell \bar{\nu})}{\tau_B \Delta\Gamma_{\text{theory}}}}$$

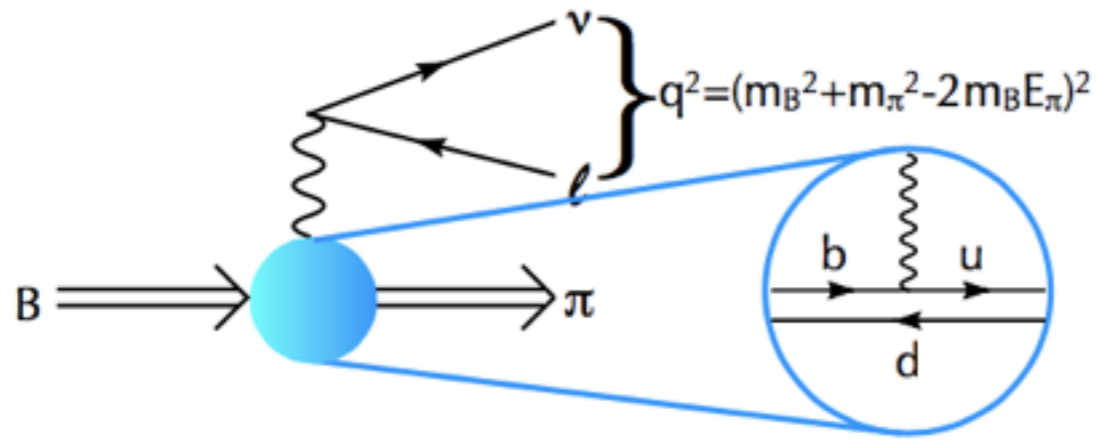
- $\Delta\Gamma_{\text{theory}}$ is the predicated $B \rightarrow X_u \ell \bar{\nu}$ partial rate in the given phase space region
- Theoretical calculations from BLNP, DGE, ADFR, GGOU...
- Agreement between different calculations

PDG 2015

$$|V_{ub}| = (4.49 \pm 0.16_{\text{exp}} \begin{matrix} +0.16 \\ -0.18 \end{matrix}_{\text{theo}}) \times 10^{-3}$$



$|V_{ub}|$ determination from $B \rightarrow \pi \ell \nu$



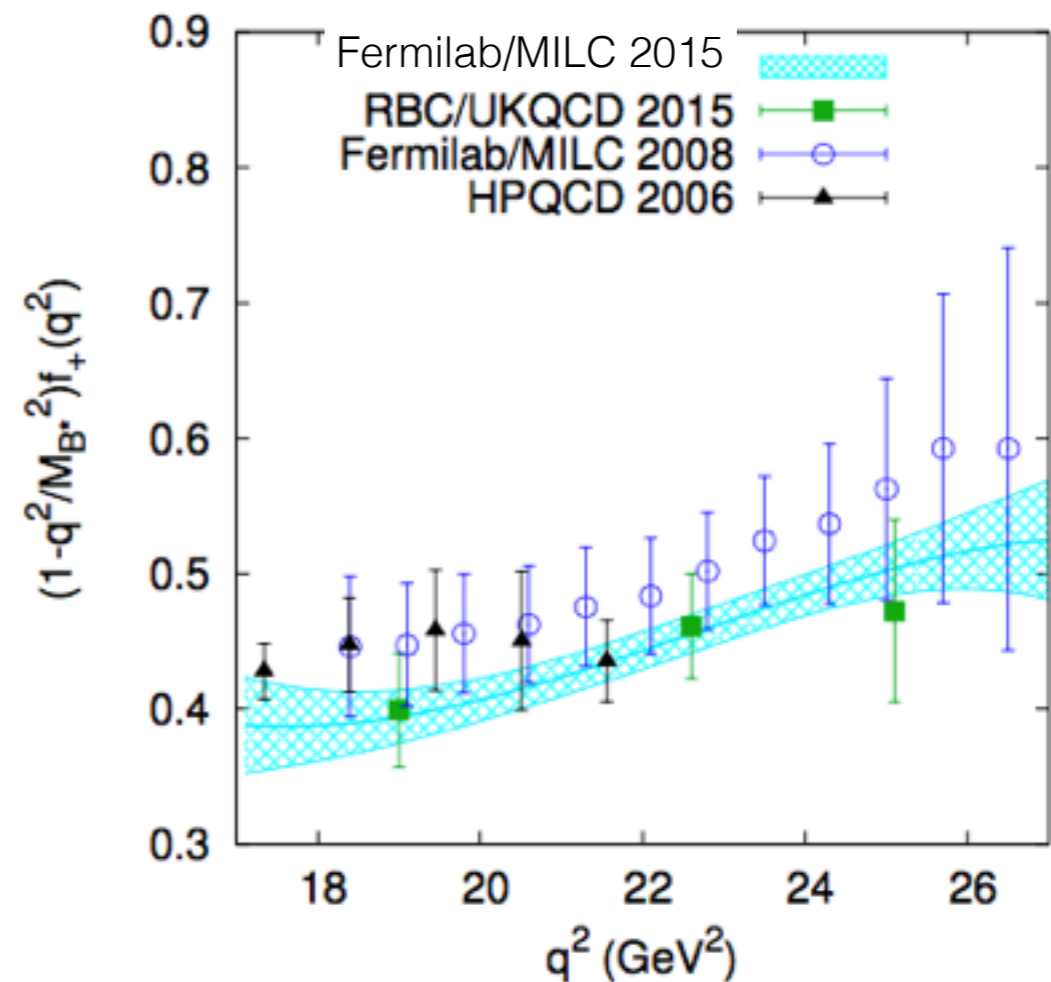
$$\frac{d\Gamma(B \rightarrow \pi \ell \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |\mathbf{p}_\pi|^3 |f_+(q^2)|^2$$

$$|V_{ub}| = \sqrt{\frac{C_v \Delta \mathcal{B}}{\tau_B \Delta \zeta}} \quad \Delta \zeta = \int d\Gamma / |V_{ub}|^2$$

- Form factor calculation

- Lattice QCD: FNAL/MILC, HPQCD, RBC/UKQCD

- Light Cone Sum Rules (LCSR): Ball/Zwicky, Bharucha

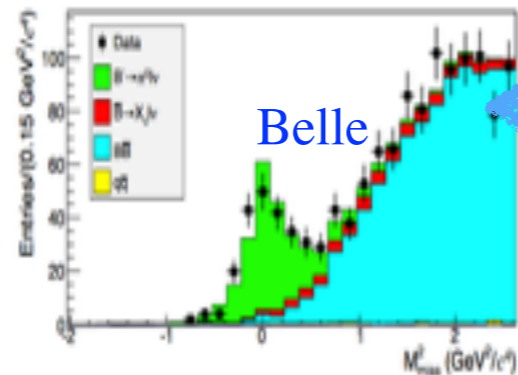


B → π l ν Experimental Measurements

Three methods of identifying signals: *Untagged, Hadronic tag, Semileptonic tag*

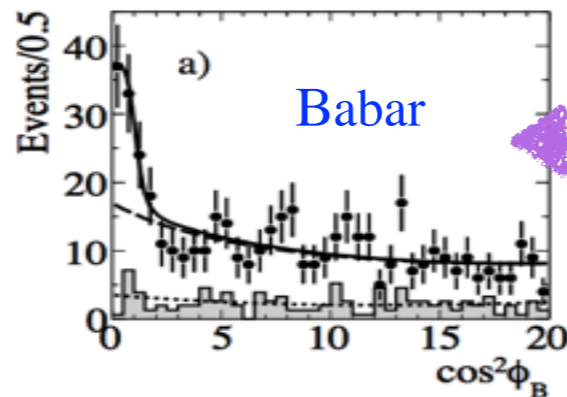
- *Had. tag*

- Hadronic decays
- Missing mass
- efficiency: Belle: ~0.2%, Babar: B⁰~0.3%, B⁺~0.5%
- Dominant Uncertainty: Tag calibration



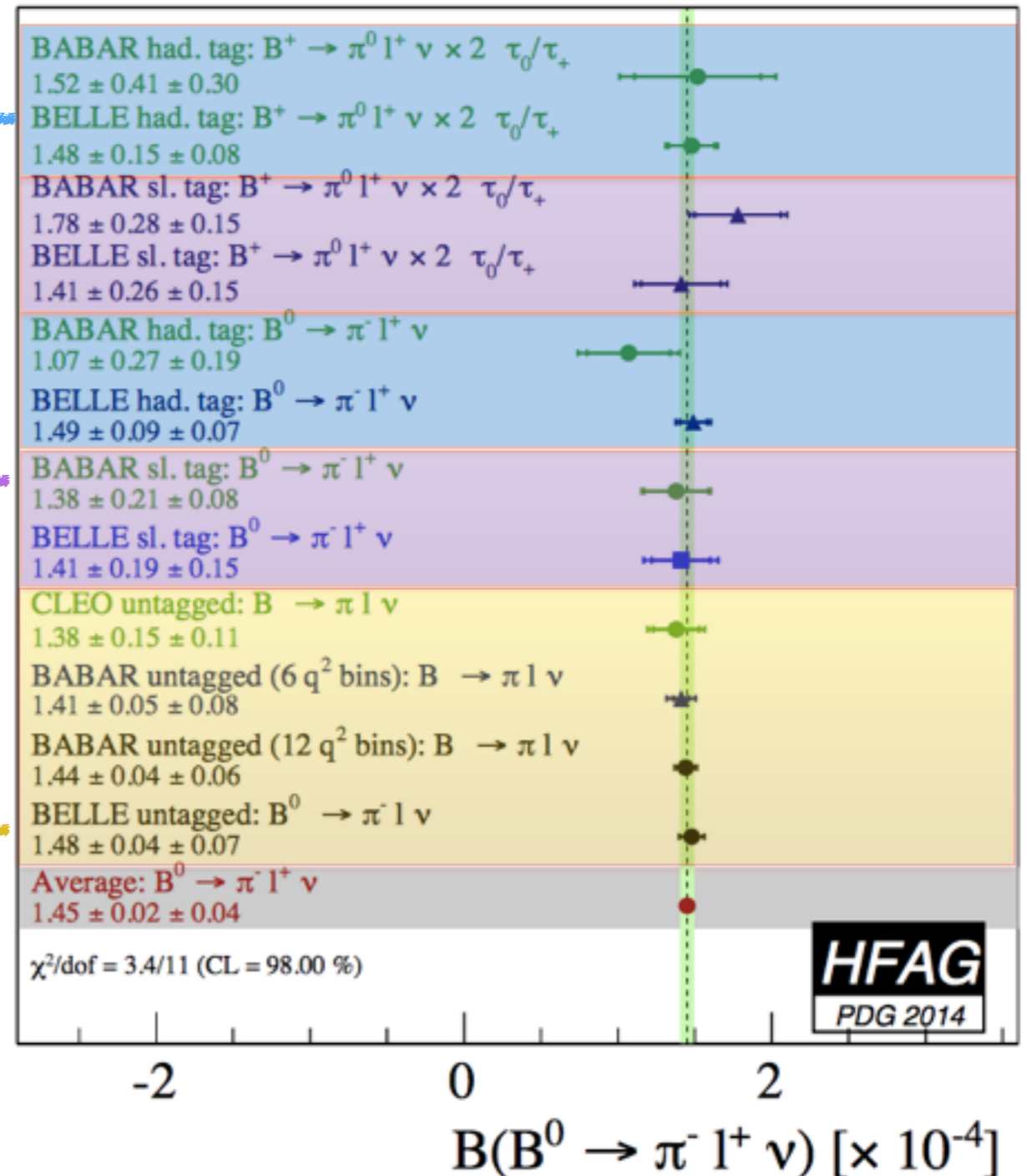
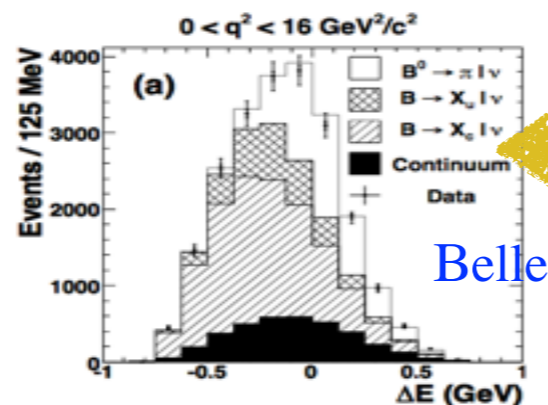
- *S.L. tag*

- tag B → D^(*) l ν
- extract signal yield by using the kinematic feature of the double semileptonic decay
- efficiency: Belle~0.2%, Babar: B⁰(B⁺) ~0.1%(0.3%)



- *Untagged*

- determine neutrino four-momentum with the momenta of all particles and beam particles
- efficiency: Belle~9%, Babar: B⁰(B⁺) ~7%(5%)
- Dominant uncertainty: Detector effects, b → ulν background



$|V_{ub}|$ determination from $B \rightarrow \pi l \nu$

- Combined fit to experimental partial rates and theoretical calculation versus q^2

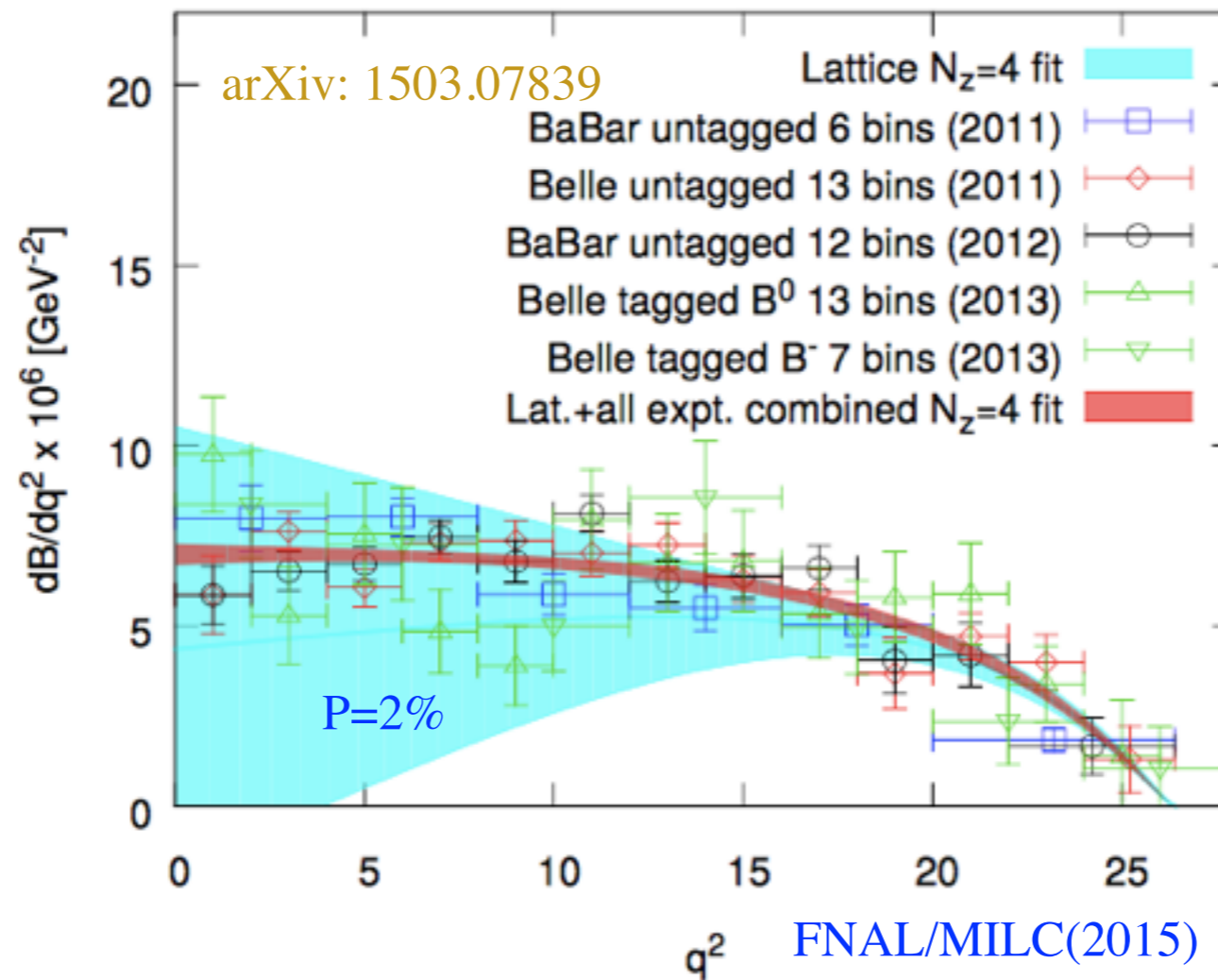
- different experimental measurements

- calculation from Lattice QCD

- fit to **full** q^2 region

- most precise determination $\sigma \sim 4\%$, $\sigma(\text{experimental}) \sim \sigma(\text{QCD})$

$$|V_{ub}| = (3.72 \pm 0.16) \times 10^{-3}$$



$|V_{ub}|$ determination from baryonic decay $\Lambda_b \rightarrow p\mu\nu_\mu$

$\Lambda_b \rightarrow p\mu\nu_\mu$ @LHCb

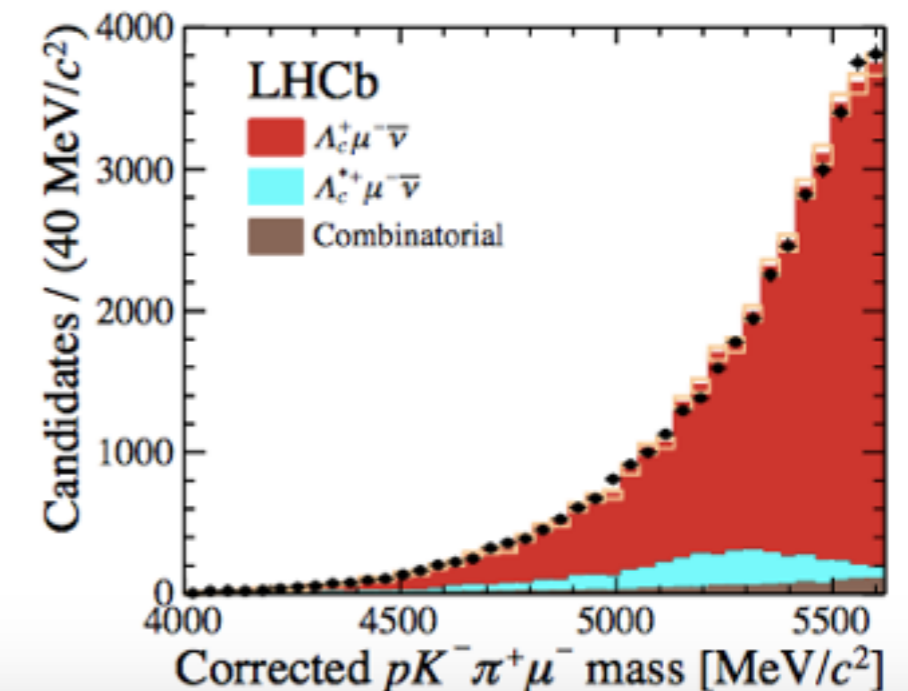
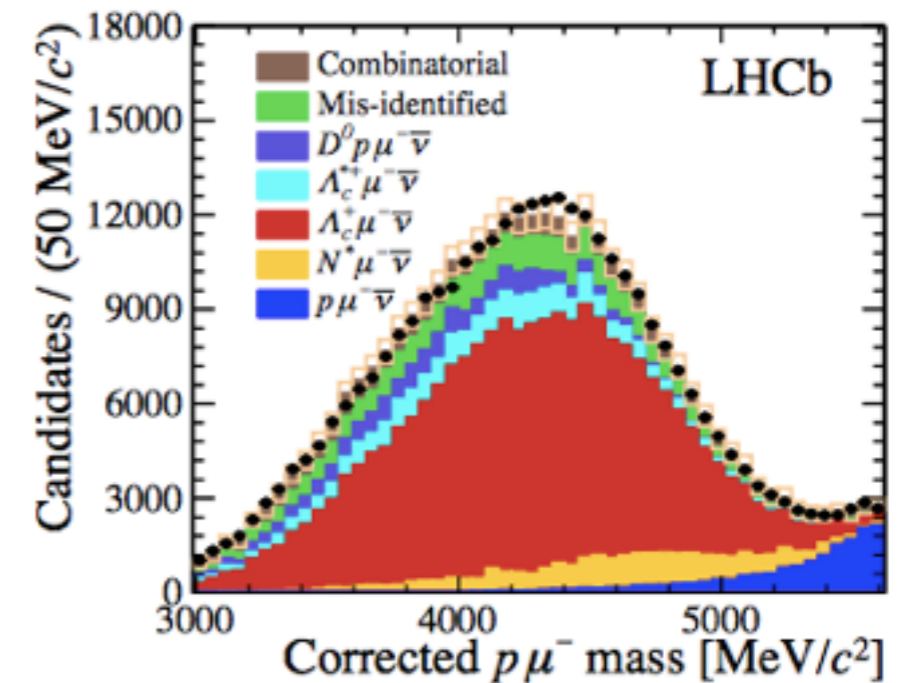
- Measure $|V_{ub}|/|V_{cb}|$ with $\Lambda_b \rightarrow p\mu\nu_\mu$,
 $\Lambda_b \rightarrow (\Lambda_c \rightarrow pK\pi)\mu\nu_\mu$.
- Cancel some of experimental uncertainties.
- Determine $|V_{ub}|$ with $|V_{cb}|$ from experiment

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.004 \pm 0.004$$

$$|V_{cb}| = (39.5 \pm 0.8) \times 10^{-3} \text{ (PDG 2014)}$$

$$|V_{ub}| = (3.27 \pm 0.15(\text{exp}) \pm 0.16(\text{LQCD}) \pm 0.06(\text{norm})) \times 10^{-3}$$

- Consistent with $|V_{ub}|$ determined by exclusive B decays



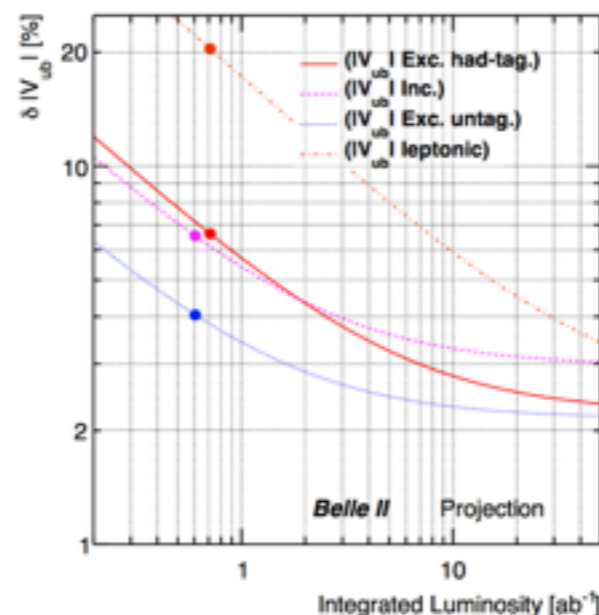
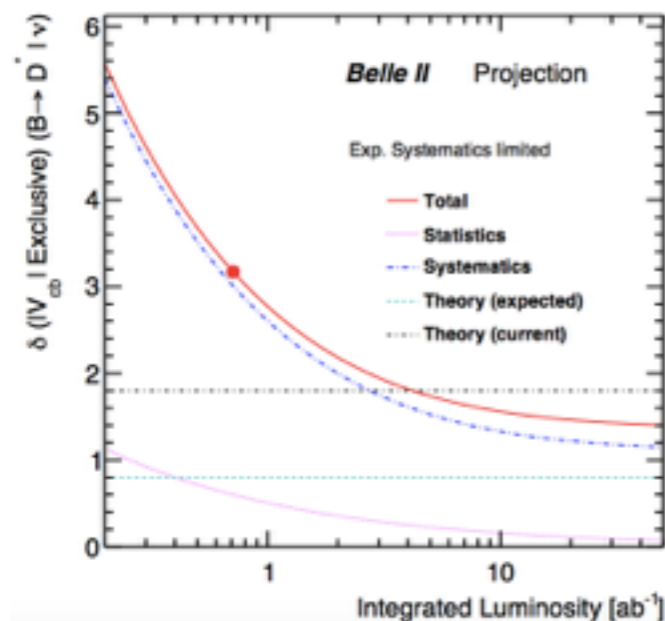
LHCb, arXiv: 1504.01568

Summary

- $|V_{cb}|$ and $|V_{ub}|$ are measured with different methods
 - Inclusive $B \rightarrow X_c l \nu$, $B \rightarrow X_u l \nu$, Hadronic tag, Semileptonic tag, Untagged.
 - Exclusive $B \rightarrow D^{(*)} l \nu$, $B \rightarrow \pi l \nu$
- Inclusive-Exclusive tension in both $|V_{cb}|$ and $|V_{ub}|$ still exist.

	Measurements (10^{-3})	$\delta V/V$
2.8σ $\left\{ \begin{array}{l} V_{ub} \text{ Inc.} \\ V_{ub} \text{ Exc.} \end{array} \right.$	4.49 ± 0.23	6%
	3.72 ± 0.16	4%
2.9σ $\left\{ \begin{array}{l} V_{cb} \text{ Inc.} \\ V_{cb} \text{ Exc.} \end{array} \right.$	42.21 ± 0.78	2%
	39.2 ± 0.7	2%

- Belle II will provide more precise measurements.

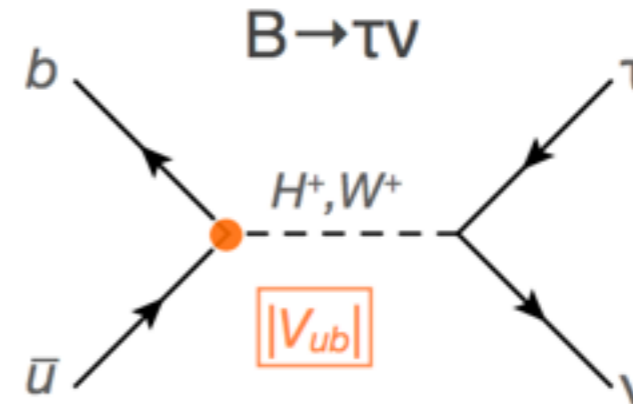


Back up

Experimental measurement $B \rightarrow \tau \nu_\tau$

$B \rightarrow \tau \nu_\tau$

- SM: $\mathcal{B}(B \rightarrow \tau \nu_\tau) \propto (f_B |V_{ub}|)^2$
- Sensitive to New Physics: H^\pm
- \mathcal{B}_{sig} : $\tau \rightarrow (e, \mu) \nu \nu$, $(\pi, \rho) \nu$, Hadronic tag B_{tag}
- Evidence: Belle $\sim 3.0\sigma$, Babar $\sim 3.8\sigma$

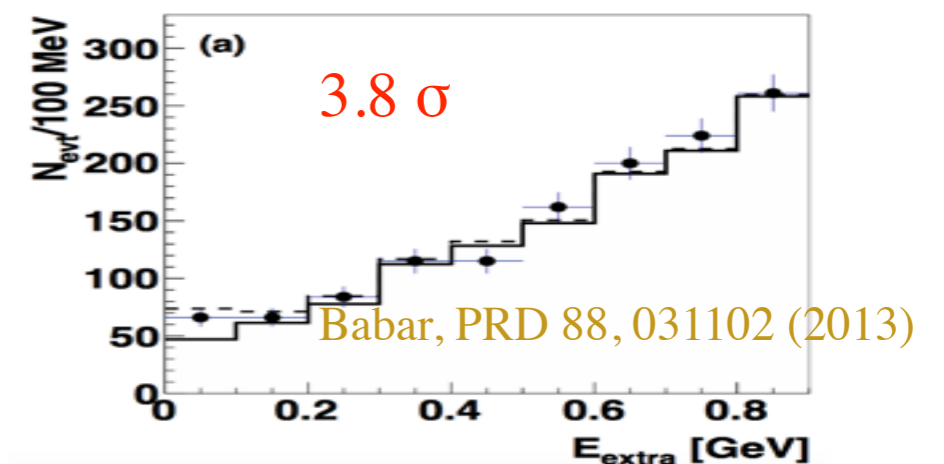
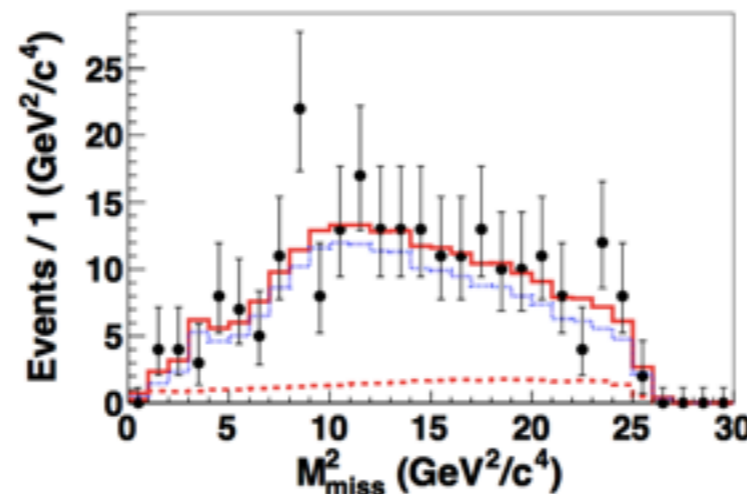
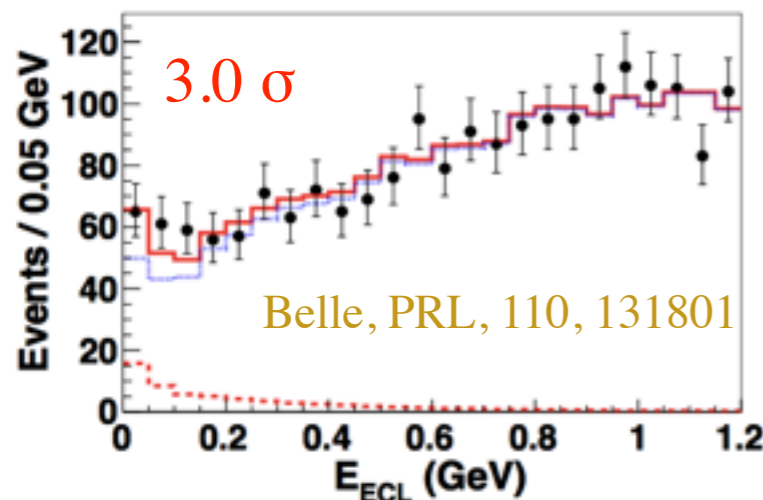


Belle

$$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau) = [0.72_{-0.25}^{+0.27}(\text{stat}) \pm 0.11(\text{syst})] \times 10^{-4}$$

Babar

$$\mathcal{B}(B^+ \rightarrow \tau^+ \nu) = (1.83_{-0.49}^{+0.53}(\text{stat}) \pm 0.24(\text{syst})) \times 10^{-4}$$



Measurements from $B \rightarrow (\rho, \omega, \eta, \eta') l \nu$

Measurement:

