Towards the precision of V_{ub} and V_{cb}

Soumitra Nandi

IIT-Guwahati

Outline

- Goal : Unitarity Triangle !
- Measurement Tools:
  - ✓ Inclusive B decays : Theory !!
  - ✓ Exclusive B decays : Theory !!
- State of the Art :
  - ✓ V_{cb} : B-> $D^{(*)} \ell\nu_\ell$
  - ✓ V_{ub} : B-> $\pi \ell\nu_\ell$
  - ✓ From Inclusive Semileptonic decays
    - ✓ B -> X_c l ν
    - ✓ B -> X_u l ν
- OUTLOOK !!

B-Physics: Goal

Quark Mixing

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \hat{V}_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM Phenomenology:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} .$$

Wolfenstein Parametrization

$$V \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix},$$

Unitarity Triangle

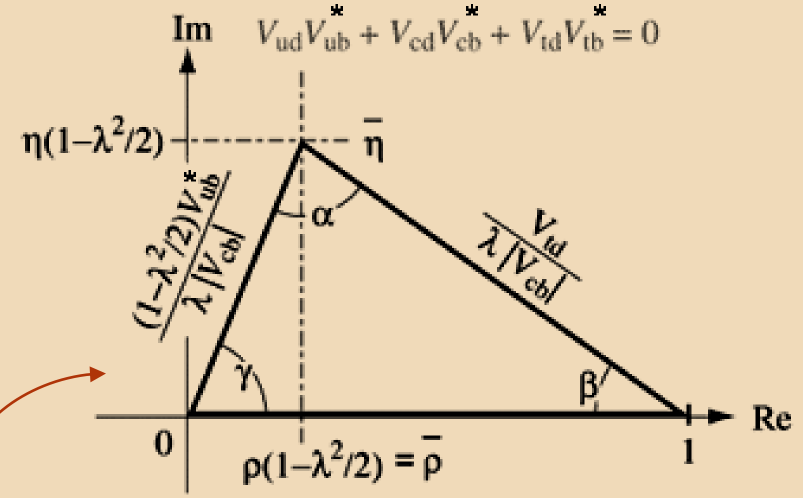


Figure 1. The unitarity relation $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ drawn in the complex $[\bar{\rho}, \bar{\eta}]$ plane.

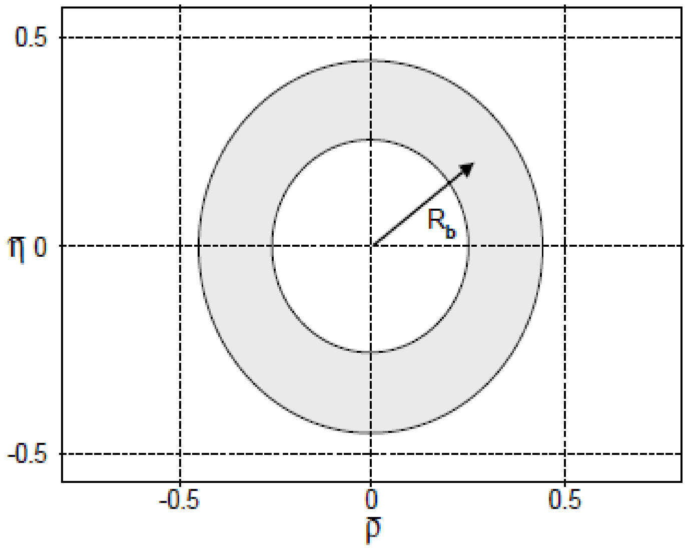
- Consistency check in the SM !!
- Searches for NP evidences !!

Role of $|V_{ub}|$ and $|V_{cb}|$

- ✓ $|V_{ub}|$,$|V_{cb}|$ hence $R_b^2 = V_{ub}^2 + V_{cb}^2$ are determined from tree level decays !

  Expected to be free of NP effects !!

- ✓ They are universal fundamental constants valid in any extension of the SM! !



M. Battaglia et al. arXiv:hep-ph/0304132v2

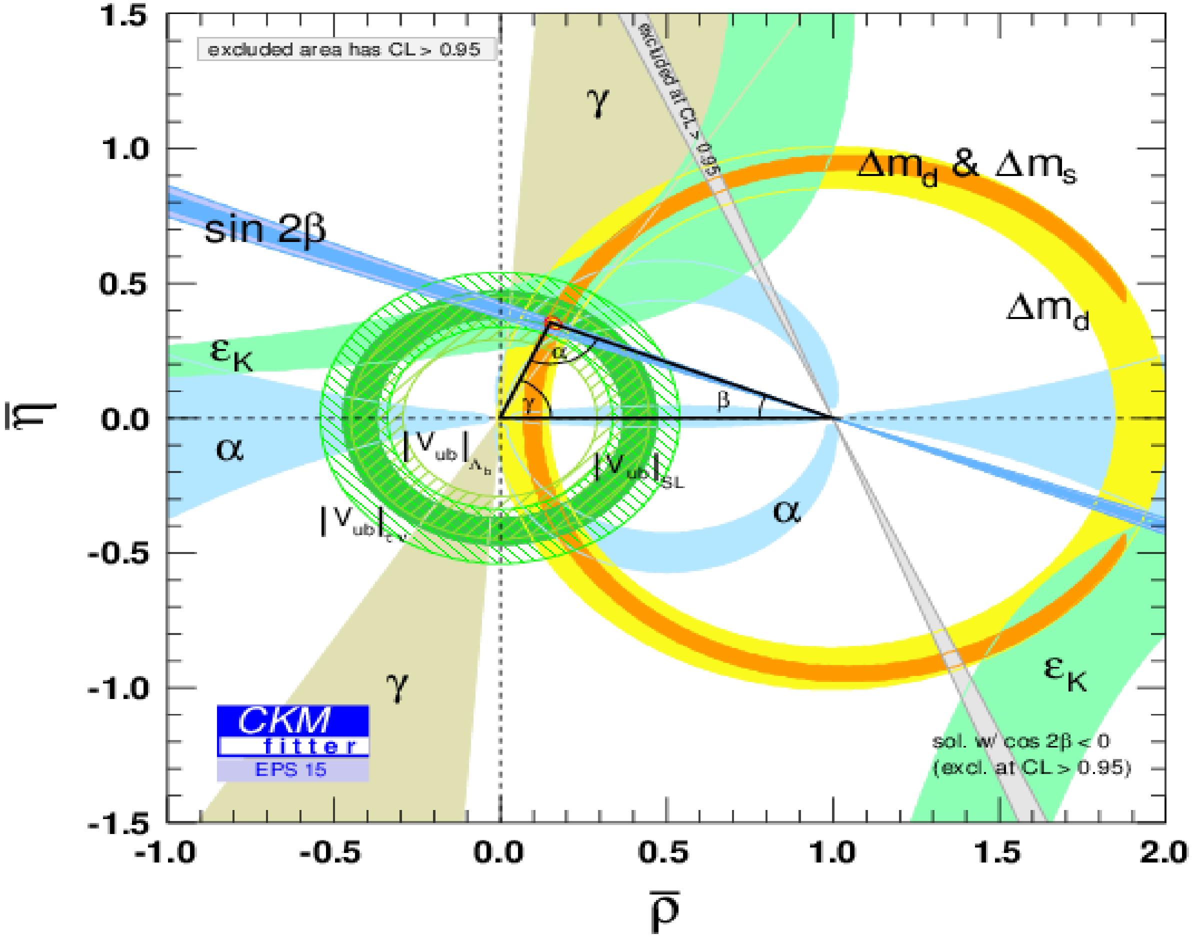
This tells us that the apex of the unitarity triangle lies in the band shown

To find where the apex lies on the UT we have to look at other decays !!

Most promising in this respect are the so-called loop induced decays and CP violating B-decays !!

✓**Precise determination of $|V_{ub}|$, $|V_{cb}|$ is of utmost importance !**

Unitarity Triangle: Fit result



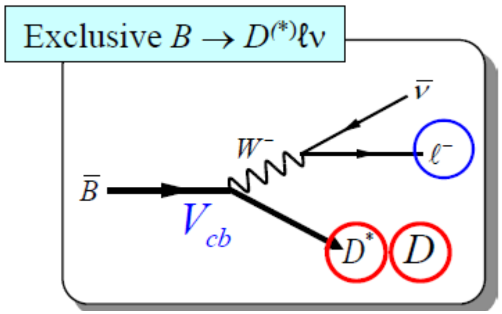
CKM elements: Semileptonic decays

Measurement of $|V_{ub}|$ and $|V_{cb}|$

Semileptonic B-decays provide a clean environment !!

Exclusive Measurement

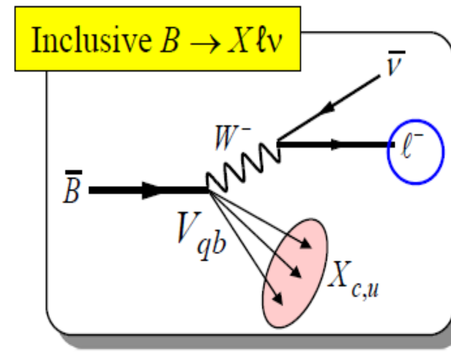
- **B-> D ℓ ν and** B-> D* ℓ ν
- **B-> π ℓ ν**



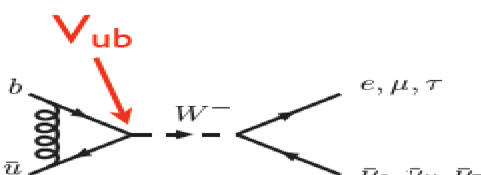
Inclusive Measurement

B-> X_c ℓ ν

B-> X_u ℓ ν

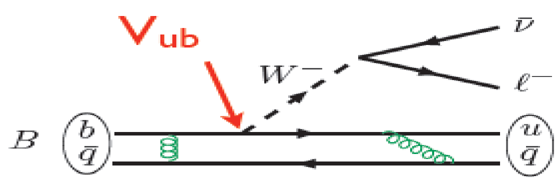


Leptonic | **Semileptonic** | **Hadronic**

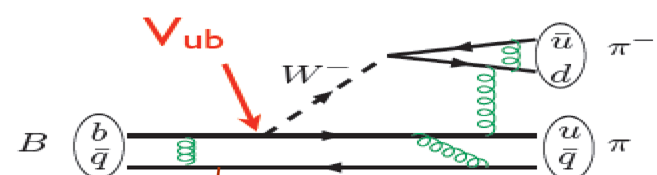


Helicity suppressed

Experimentally difficult



Leptonic and hadronic currents factorize



Complex QCD interactions

Theoretically challenging

Inclusive vs Exclusive

- Tree level semileptonic (s.l.) decays of B mesons are crucial for determining the $|V_{ub}|$ and $|V_{cb}|$ elements of the CKM matrix !
- Inclusive $b \to c(u) l\nu$ decay rates have a solid description via OPE/HQE
- Exclusive s.l. decays have a similarly solid description in terms of heavy-quark effective theory (HQET) !

- Inclusive decays: Non perturbative unknowns can be extracted experimentally!
  ⇒ Experimentally Challenging !!
- Exclusive decays: Non perturbative unknowns have to be calculated !
  ⇒ Major theoretical challenges !!

❖ The inclusive radiative decays of the B meson play a central role in the search for new physics.

❖ A more precise evaluation of the $b \to s\gamma$ photon spectrum will lead to a more precise effective shape function ⇒ Useful for $|V_{ub}|$ measurement !!

V_{cb} :Exclusive decays

$$\frac{d\Gamma(\bar{B} \to D \ell \bar{\nu})}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} (m_B + m_D)^2 m_D^3 (w^2 - 1)^{3/2} |V_1(w)|^2 \, \eta_{EW}^2$$

η_{EW}^2: 1.012 ± 0.005

Zero recoil expansion, HQET

$$\frac{V_1(w)}{V_1(1)} \approx 1 - 8\rho_1^2 z + (51.\rho_1^2 - 10.)z^2 - (252.\rho_1^2 - 84.)z^3$$

$$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

$$1 + \mathcal{O}\left(\frac{m_B - m_D}{m_B + m_D} \frac{\Lambda_{QCD}}{m_c}\right)$$

LQCD:

- Atoui et.al. , 2014: 1.033 ± 0.095
- Fermilab Lattice and MILC , 2015: 1.0528 ± 0.0082
- HPQCD, 2015: 1.035 ± 0.040

η_{EW} V(1) $|V_{cb}|= (42.65 \pm 1.35)\times 10^{-3}$, HFAG 14

$|V_{cb}|= (40.0 \pm 1.4 \pm 0.3 \pm 0.2)\times 10^{-3}$ Fermilab Lattice and MILC , 2015

Experiment — Lattice QCD — QED and Coulomb cor^n

$|V_{cb}| = (40.6 \pm 1.1) \times 10^{-3}$ Belle, 2015

V_{cb} :Exclusive decays

$$\frac{d\Gamma}{dw}(\bar{B} \to D^* \ell \bar{\nu}_\ell) = \frac{G_F^2 m_B^5}{48\pi^3} |V_{cb}|^2 (w^2 - 1)^{1/2} P(w) (\eta_{ew} \mathcal{F}(w))^2$$

1.015 ± 0.005

0.960 ± 0.007

$$\eta_A \left[1 + \delta_{1/m^2} + \cdots\right] \left[1 - 8\rho_{A1}^2 z + (53\rho_{A1}^2 - 15) z^2 - (231\rho_{A1}^2 - 91) z^3\right]$$

$\mathcal{F}(1) = 0.906 \pm 0.013$

LQCD

Fermilab Lattice and MILC , 2014

Includes higher statistics, finer lattice spacing and lighter quark masses than earlier analysis !!

$$|V_{cb}| = (38.9 \pm 0.5 \pm 0.5 \pm 0.2) \times 10^{-3} \; (\bar{B} \to D^* \ell \bar{\nu}_\ell, \text{ LQCD})$$

Experiment

Theory

SUM RULE, Gambino , Mannel and Uraltsev, 2010 and 2012

$\mathcal{F}(1) = 0.86 \pm 0.01 \pm 0.02$

$$|V_{cb}| = (41.0 \pm 0.5 \pm 1.0) \times 10^{-3} \; (\bar{B} \to D^* \ell \bar{\nu}_\ell, \text{ SR})$$

$$|V_{cb}| = (39.2 \pm 0.7) \times 10^{-3} \quad \text{(exclusive)}$$

V_{ub} :Exclusive decays

The decay rate for B->$\pi\ell\nu$ (ℓ = e, μ): $\frac{d\Gamma}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3} p_\pi^3 |f_+^{B\pi}(q^2)|^2$

Precision limited by the uncertainties in the form factor !

Complementary approaches: lattice QCD and light-cone sum rule (LCSR)

Best at high q^2 (> 14 GeV^2)

Applicable at low q^2 (< 12 GeV^2)

Fermilab/MILC, HPQCD, RBC/UKQCD

$|V_{ub}| = (3.72 \pm 0.16) \times 10^{-3}$ PRD, 2015

$|V_{ub}| = 3.61(32) \times 10^{-3}$ PRD, 2015

Non perturbative function: pion distribution amplitudes !

LO: Twist 2 ,3, 4 quark-antiquark and quark-antiquark-gluon !

NLO: Twist – 2 and 3 two particle contribution are known at order α_s

NNLO: Twist-2 contribution at order α_s^2 !

$|V_{ub}| = (3.50^{+0.38}_{-0.33}|_{th.} \pm 0.11|_{exp.}) \times 10^{-3}$

Khodjamirian, PRD 2011

Imsong et.al. JHEP, 2015 $|V_{ub}| = (3.32^{+0.26}_{-0.22}) \cdot 10^{-3}$

First Bayesian analysis of the B-> π form factor

Inclusive Semileptonic

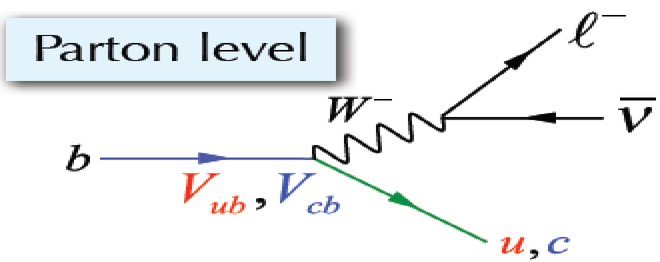
Inclusive channels are relatively clean

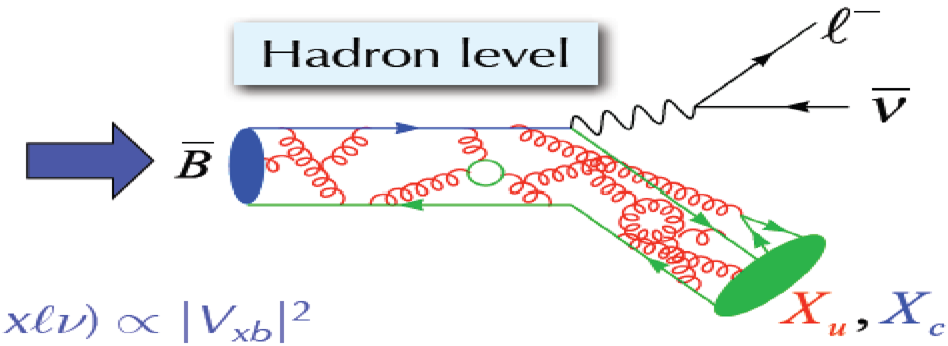
- **Theoretical framework is OPE/HQE !**
- **Analysis of the final state lepton and hadron energy distribution yields:**
  - ✓ **b-quark mass !**
  - ✓ **Non-perturbative QCD parameters !**
  - ✓ **Consistency check of the OPE/HQE and other effective theory approaches !**
- **As per the measurement is concern : small statistical and systematic errors !**
  - ✓ **High sensitivity to the theoretical uncertainties !**

Precise predictions in the SM including reliable uncertainties is possible !!

Decay Width

OPE relates parton to meson decay rate: $1/m_b$ and $\alpha_s(m_b)$





- Decay rate $\Gamma_x \equiv \Gamma(b \to x\ell\nu) \propto |V_{xb}|^2$

$$\Gamma_{SL} = \underbrace{|V_{cb}|^2 \frac{G_F^2 m_b^5}{192\pi^3}}_{\text{free quark decay}} \underbrace{(1+A_{EW})A_{pert}}_{\text{perturbative corrections}} \times \underbrace{[c_0(r) + \frac{0}{m_b} + c_2(r, \frac{\mu_\pi^2}{m_b^2}, \frac{\mu_G^2}{m_b^2}) + c_3(r, \frac{\rho_D^3}{m_b^3}, \frac{\rho_{LS}^3}{m_b^3}) + \ldots]}_{\text{Non-perturbative power corrections}}$$

$r = m_c/m_b$

Main sources of uncertainties : (1) Mass of the b-quark and the mass ratio ‘r’
(3) Higher order QED and QCD radiative corr.
(4) Higher order of the $1/m_b$ corrections !
(5) Extraction of HQE parameters !
(6) Parton Hadron Duality !!

Moments

OPE parameters can be extracted from the moments of the differential distributions

Leptonic Energy Moments: $M_1^\ell = \frac{1}{\Gamma}\int dE_\ell\, E_\ell \frac{d\Gamma}{dE_\ell}; \qquad M_n^\ell = \frac{1}{\Gamma}\int dE_\ell \left(E_\ell - M_1^\ell\right)^n \frac{d\Gamma}{dE_\ell} \quad (n>1),$

Moments of Invariant Hadronic Mass:

$$M_1^X = \frac{1}{\Gamma}\int dM_X^2\,(M_X^2 - \bar{M}_D^2)\frac{d\Gamma}{dM_X^2}; \qquad M_n^X = \frac{1}{\Gamma}\int dM_X^2\,(M_X^2 - \langle M_X^2\rangle)^n \frac{d\Gamma}{dM_X^2} \quad (n>1),$$

$$M_n^\ell = \left(\frac{m_b}{2}\right)^n \left[\varphi_n(r) + \bar{a}_n(r)\frac{\alpha_s}{\pi} + \bar{b}_n(r)\frac{\mu_\pi^2}{m_b^2} + \bar{c}_n(r)\frac{\mu_G^2}{m_b^2} + \bar{d}_n(r)\frac{\rho_D^3}{m_b^3} + \bar{s}_n(r)\frac{\rho_{LS}^3}{m_b^3} + \ldots\right]$$

$$M_n^X = m_b^{2n}\sum_{l=0}\left[\frac{M_B - m_b}{m_b}\right]^l \left(E_{nl}(r) + a_{nl}(r)\frac{\alpha_s}{\pi} + b_{nl}(r)\frac{\mu_\pi^2}{m_b^2} + c_{nl}(r)\frac{\mu_G^2}{m_b^2} + d_{nl}(r)\frac{\rho_D^3}{m_b^3} + s_{nl}(r)\frac{\rho_{LS}^3}{m_b^3} + \ldots\right).$$

arXiv:hep-ph/0304132v2

$\mathbf{M}_n^l$ and $\mathbf{M}_n^X$ are highly sensitive to the quark masses and OPE parameters !

- Global fit to decay rate and moments extracts: $|V_{cb}|$, m_b, m_c, μ_π^2, μ_G^2, ρ_D^3, ρ_{LS}^3

Theory : State of the art !

- ✓ Tree level upto $(1/m_b)^5$ is known ….Mannel, Turczyk, Uraltsev, Gremm, Kapustin, Gambino, Healy…
- ✓ **$\mathcal{O}(\alpha_s)$ corrections** for the partonic rate are fully known !
- ✓ **$\mathcal{O}(\alpha_s^2)$ known** ⟶ Technically challenging ⟶ Numerical calculation,

  Pak and Czarnecki PRL 2008 , Melinkov PLB 2008 analytic results for limiting cases
- ✓ $\mathcal{O}(\alpha_s^2 \beta_0)$ are known !! …Aquila, Gambino, Ridolfi and Uraltsev NPB 2005
- ✓ $\alpha_s \mu_\pi^2 / m_b^2$ only numerically ! ….Becher, Boss and Lunghi, JHEP, 2007
- ✓ $\alpha_s \mu_\pi^2 / m_b^2$ and $\alpha_s \mu_G^2 / m_b^2$ corrections are calculated with analytical expressions!

  Aberti, Ewerth, Gambino, Nandi , NPB(2013)

  Alberti, Gambino, Nandi, JHEP(2014)

  Mannel, Pivovarov, Rosenthal, PRD(2015)

α_s / m_b^3 In progress….Alberti, Gambino, Healy , Nandi

V_{cb} : Inclusive decays

Alberti, Gambino , Healy and Nandi, PRL 2015; Gambino, Healy , Turczyk, June 2016

$$\Gamma_{sl} = \Gamma_0 \left[1 + a^{(1)} \frac{\alpha_s(m_b)}{\pi} + a^{(2,\beta_0)} \beta_0 \left(\frac{\alpha_s}{\pi}\right)^2 + a^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \left(-\frac{1}{2} + p^{(1)} \frac{\alpha_s}{\pi}\right) \frac{\mu_\pi^2}{m_b^2} + \left(g^{(0)} + g^{(1)} \frac{\alpha_s}{\pi}\right) \frac{\mu_G^2(m_b)}{m_b^2} + d^{(0)} \frac{\rho_D^3}{m_b^3} - g^{(0)} \frac{\rho_{LS}^3}{m_b^3} + \text{higher orders} \right]$$

$$\overline{m}_c(3\ \text{GeV}) = 0.986(13)\ \text{GeV}$$

After fitting the parameters with the available data on width and moments :

$$\frac{\Gamma}{z(r)\Gamma_0} = 1 - 0.116_{\alpha_s} - 0.030_{\alpha_s^2} - 0.042_{1/m^2} - 0.002_{\alpha_s/m^2} - 0.030_{1/m^3} + 0.005_{1/m^4} + 0.005_{1/m^5}$$

$$1 - 8r + 8r^3 - r^4 - 12r^2 \ln r$$

$$A_{ew} |V_{cb}^2| G_F^2 m_b^5 / 192\pi^3$$

1.014

$|V_{cb}| = (42.42 \pm 0.86) \times 10^{-3}$ → Fit without (α_s / m_b^2) and ($1/m_b^{4,5}$) and h.o. contributions ,

Gambino and Schwanda, PRD 2014

$|V_{cb}| = (42.21 \pm 0.78) \times 10^{-3}$ → Fit without ($1/m_b^{4,5}$) and h.o. contributions ,

Alberti, Gambino , Healy and Nandi, PRL 2015

$|V_{cb}| = (42.11 \pm 0.74) \times 10^{-3}$ → Fit includes all the known h.o. corrections,

Gambino, Healy , Turczyk, June 2016

Comments on $|V_{ub}|$

- The charmless s.l. decay channel $b \to u\ell^- \nu$ can in principle provide a clean determination of $|V_{ub}|$ along the lines of that of $|V_{cb}|$!!
- The main problem is the large background from $b \to c\ell^- \nu$ decay !!
- Experimental cuts necessary to distinguish the $b \to u$ from the $b \to c$ transitions
  - ⇒ Enhance the sensitivity to the non-perturbative aspects of the decay!

    ⇓

    Complicate the theoretical interpretation of the measurement !!

- The inclusive decay rate $B \to X_u l \nu$ is calculated using the OPE !!
- There are several methods to suppress this background
  - ⇒ Restrict the phase space region where the decay rate is measured!
  - ⇒ Great care must be taken to ensure that the OPE is valid in the relevant phase space region.

Kinematical cuts!

- There are three main kinematical cuts which separate the $b \to u\ell^-\bar{\nu}$ signal from the $b \to c\ell^-\bar{\nu}$ background:

1. A cut on the lepton energy $E_\ell > (M_B^2 - M_D^2)/2M_B$, 10% of the signal selected !
2. A cut on the hadronic invariant mass $q^2 > M_B^2 - M_D^2$, 80% ……………………!
3. A cut on the leptonic invariant mass $M_X < M_D$, 20%……………………!

✓ Forces us into the corner of the phase space …required to introduce shape functions!

$$\frac{d^3\Gamma}{dp_X^+\, dp_X^-\, dE_\ell} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} \int dk\, C(E_\ell, p_X^-, p_X^+, k)\, F(k) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

$$p_X^+ = E_X - |\mathbf{p}_X|, \qquad p_X^- = E_X + |\mathbf{p}_X|,$$

Perturbatively calculable functions

Non-perturbative Shape function

Uncertainties due to unknown higher order corrections !

1. From comparison with B-> $X_s\gamma$ (L.O.)
2. From the knowledge of the moments !
3. Modelling !

Approaches

1) BNLP (Bosch, Lange, Neubert and Paz) => Shape function based !

- Includes corrections upto α_s at leading order in $1/m_b$ expansion, power corrections upto $1/m_b^2$ has taken into account . Corrections at order α_s^2 are not added in the evaluation of V_{ub} !

2) GGOU (Gambino, Giordano, Ossola and Uraltsev) => OPE hard cutoff based !

- Includes all known perturbative and non-perturbative effects through ($\alpha_s^2 \beta_0^2$) and $1/m_b^3$!

3) Dressed gluon approximation (Andersen and Gardi) => Resummation based !

- This approach try to compute the shape function, different from the above two approaches ! Unknown NNLO corrections are the missing pieces !

4) Other approaches : a) SIMBA (Tackmann, Lacker, Ligeti, Stewart…..)

b) Analytic coupling (Aglietti et.al.)

c) Method to avoid shape function (Bauer, Ligeti, Luke…)

V_{ub} : inclusive measurements

In GeV

cut (GeV)	BLNP (SF scheme, m_b=4.569±0.029)	GGOU (kin scheme, m_b=4.541±0.023)	DGE ($\overline{MS}$ scheme, m_b=4.177±0.043)
$E_e > 2.1$	$428 \pm 50 \,^{+31}_{-36}$	$421 \pm 49 \,^{+23}_{-33}$	$390 \pm 45 \,^{+26}_{-28}$
$E_e - q^2$	$453 \pm 22 \,^{+33}_{-38}$	not available	$417 \pm 20 \,^{+28}_{-29}$
$E_e > 2.0$	$454 \pm 26 \,^{+27}_{-33}$	$450 \pm 26 \,^{+18}_{-25}$	$434 \pm 25 \,^{+23}_{-25}$
$E_e > 1.9$	$493 \pm 46 \,^{+27}_{-29}$	$493 \pm 46 \,^{+17}_{-22}$	$485 \pm 45 \,^{+21}_{-25}$
$q^2{>}8$, $m_X{<}1.7$	$430 \pm 23 \,^{+26}_{-28}$	$432 \pm 23 \,^{+27}_{-30}$	$427 \pm 22 \,^{+20}_{-20}$
$P_+ < 0.66$	$415 \pm 25 \,^{+28}_{-27}$	$424 \pm 26 \,^{+32}_{-32}$	$424 \pm 26 \,^{+37}_{-32}$
$m_X < 1.55$	$430 \pm 20 \,^{+28}_{-27}$	$429 \pm 20 \,^{+21}_{-22}$	$453 \pm 21 \,^{+24}_{-22}$
$E_\ell > 1$	$432 \pm 24 \,^{+19}_{-21}$	$442 \pm 24 \,^{+9}_{-11}$	$446 \pm 24 \,^{+13}_{-13}$
$E_\ell > 1$	$449 \pm 27 \,^{+20}_{-22}$	$460 \pm 27 \,^{+10}_{-11}$	$463 \pm 28 \,^{+13}_{-13}$
HFAG average	$445 \pm 16 \,^{+21}_{-22}$	$451 \pm 16 \,^{+12}_{-15}$	$452 \pm 16 \,^{+15}_{-16}$

Sources of errors: Statistical , experimental, $B \to X_c \ell \nu_\ell$ and $B \to X_u \ell \nu_\ell$ modelling, HQE parameters, missing higher order corrections, q^2 modelling , weak annihilation, SF parameterization

OUT LOOK

The onset of SUPER-B (BELLE-II) factory will bring us to a high precision era

- A more precise extraction of the CKM elements are necessary in order to understand SM, QCD, and for an implicit search of NP !
- Considerable progress has been made !!

  ⇨ Much more to do in order to improve precision !!
- Stay tuned for more results !!