



To the identification and subsequent stabilization of high-order dynamic systems: observers-based approach

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Plan

I. Identification of dynamic systems

- observers-based approach
- high-dimension systems and systems under uncertainty
- minimal-degree observers
- numerical example

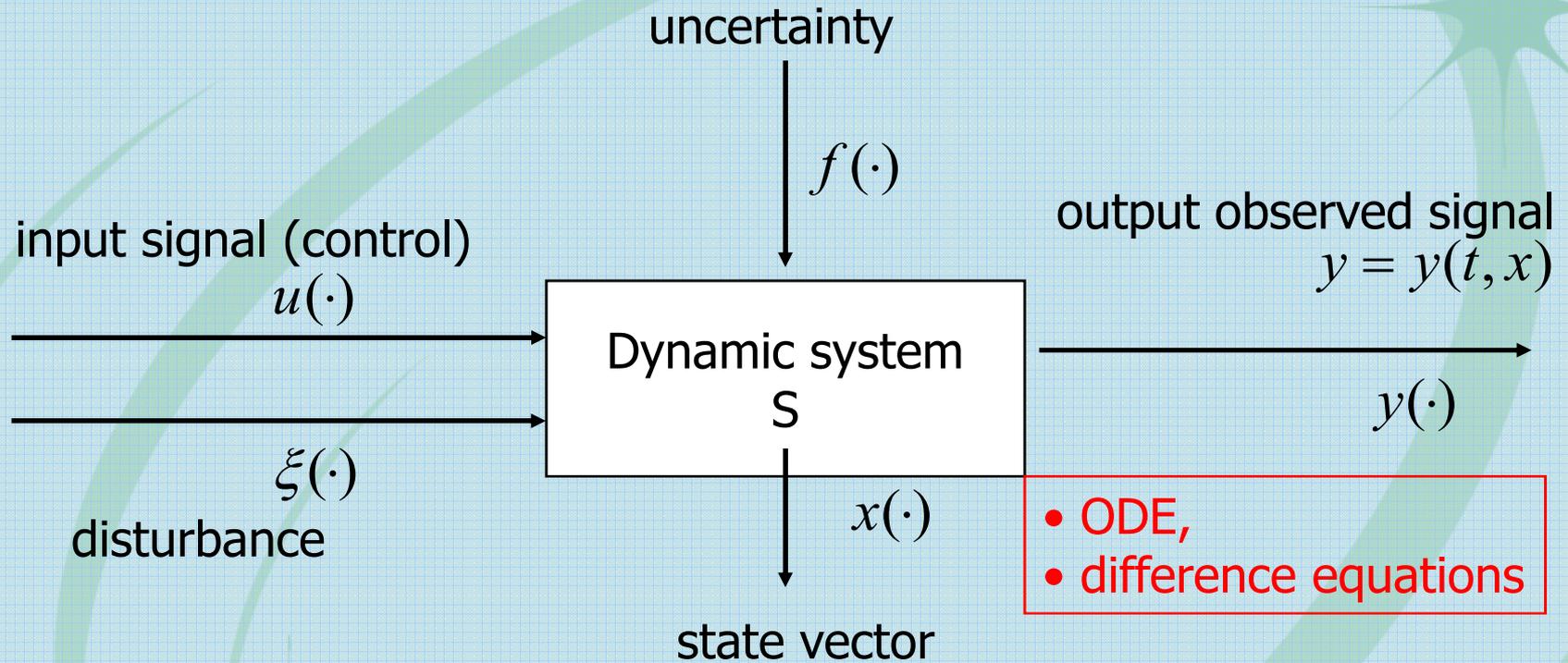
II. Stabilization of dynamic systems

- stabilization under uncertainty
- simultaneous stabilization

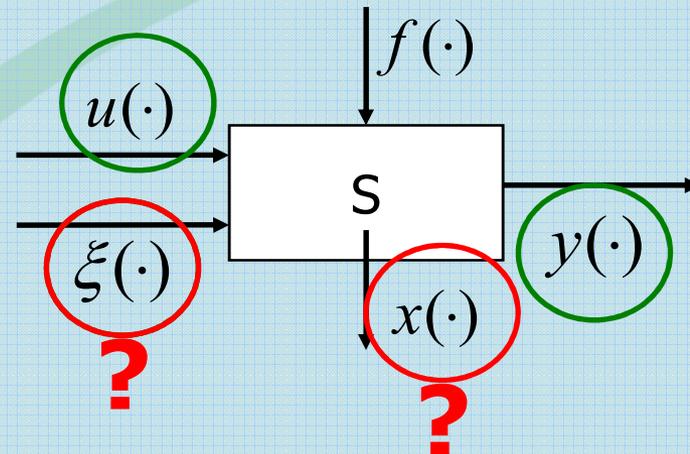


I. To the identification of dynamic systems

Model



Identification: two basic problems



Inverter Problem

Given $u(\cdot)$ and $y(\cdot)$

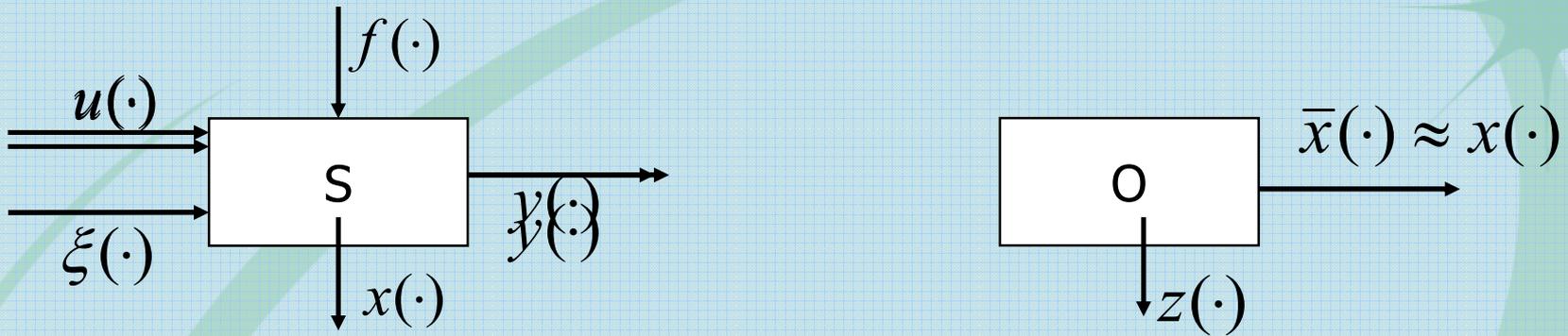
Reconstruct $\xi(\cdot)$

Observer Problem

Given $u(\cdot)$, $\xi(\cdot)$ and $y(\cdot)$

Reconstruct $x(\cdot)$

Observer



$$S: \begin{cases} \dot{x} = \varphi(t, x, u) \\ y = \psi(t, x) \end{cases}$$

$$O: \begin{cases} \dot{z} = q(z, t, y, u) \\ \bar{x} = p(z, y, u) \end{cases}$$

$$\|\bar{x}(\cdot) - x(\cdot)\| \rightarrow 0, \quad t \rightarrow \infty$$

Challenges: high dimension and uncertainty

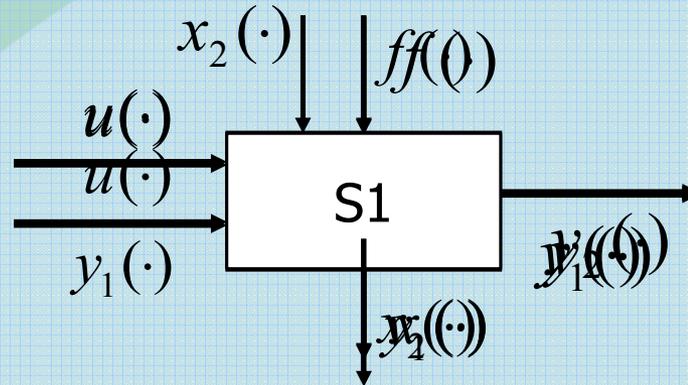
State of the Art

- Luenberger (1963-1967)
 - observers (incl. minimal) for linear stationary systems
- Formann, Williamson (1972), Moore, Ledwich (1975), Roman, Bullok (1973)
 - functional observers (FO) for linear systems,
- Tsui (1996), Darouach (2000)
 - FO of the minimal degree
- Bhattacharyya (1978), Kobayashi, Nakamizo (1982), Hou, Muller (1982), Trihn, Ha (2000), Xiong, Saif (2003)
 - synthesis of an observer for a system under uncertainty
- MSU group – Emel'yanov, Korovin, Iline, Fomichev, Fursov
 - decomposition of systems, observers of the minimal degree

New results

Observer Problem: high dimension and uncertainty

Identification of hyper-input systems via decomposition



$$y(\cdot) = (y_1(\cdot), y_2(\cdot))$$

low-dimension system
with uncertainty

observers for
uncertain systems

high-dimension system
without uncertainty

method of
scalar observers

method of
virtual inputs

New results

Observer Problem: high dimension and uncertainty

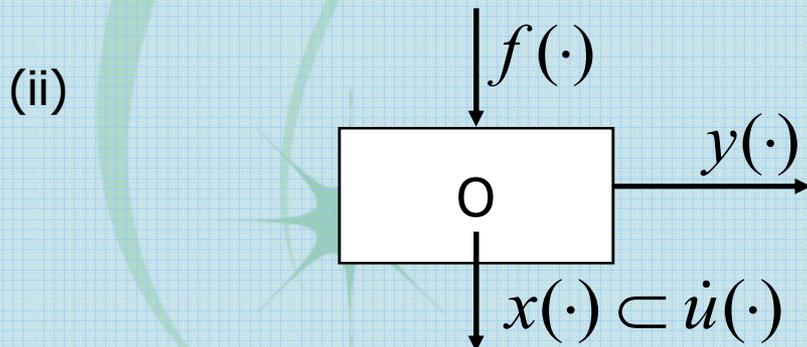
Identification of quadratic systems by the method of hierarchical coefficients

Example: reconstruction of the derivative: given $u(\cdot)$ derive $\dot{u}(\cdot)$

Solution scheme:



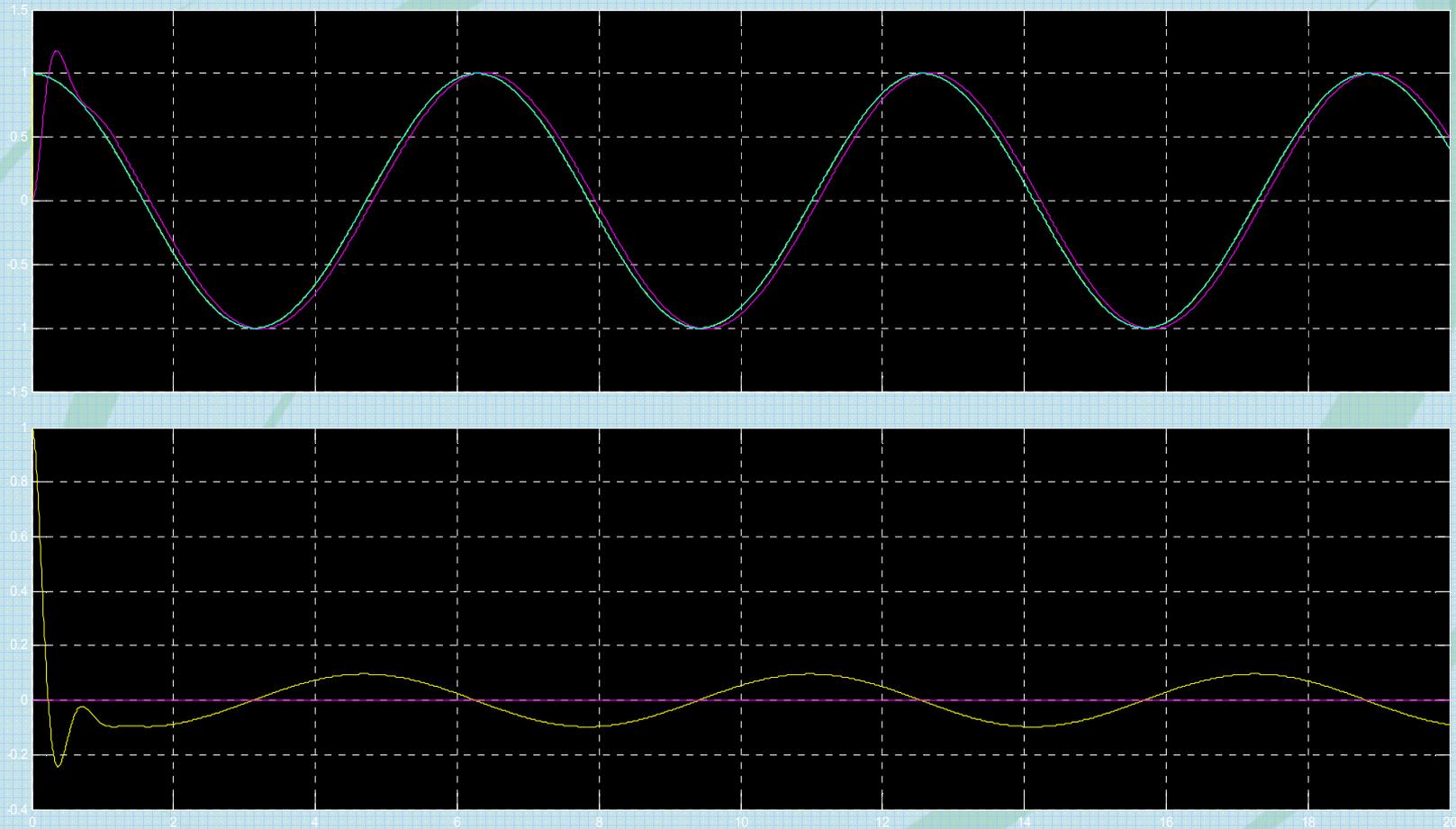
e.g., $u = y$



e.g., $S: \begin{cases} u = x \\ \dot{x} = f \end{cases}$

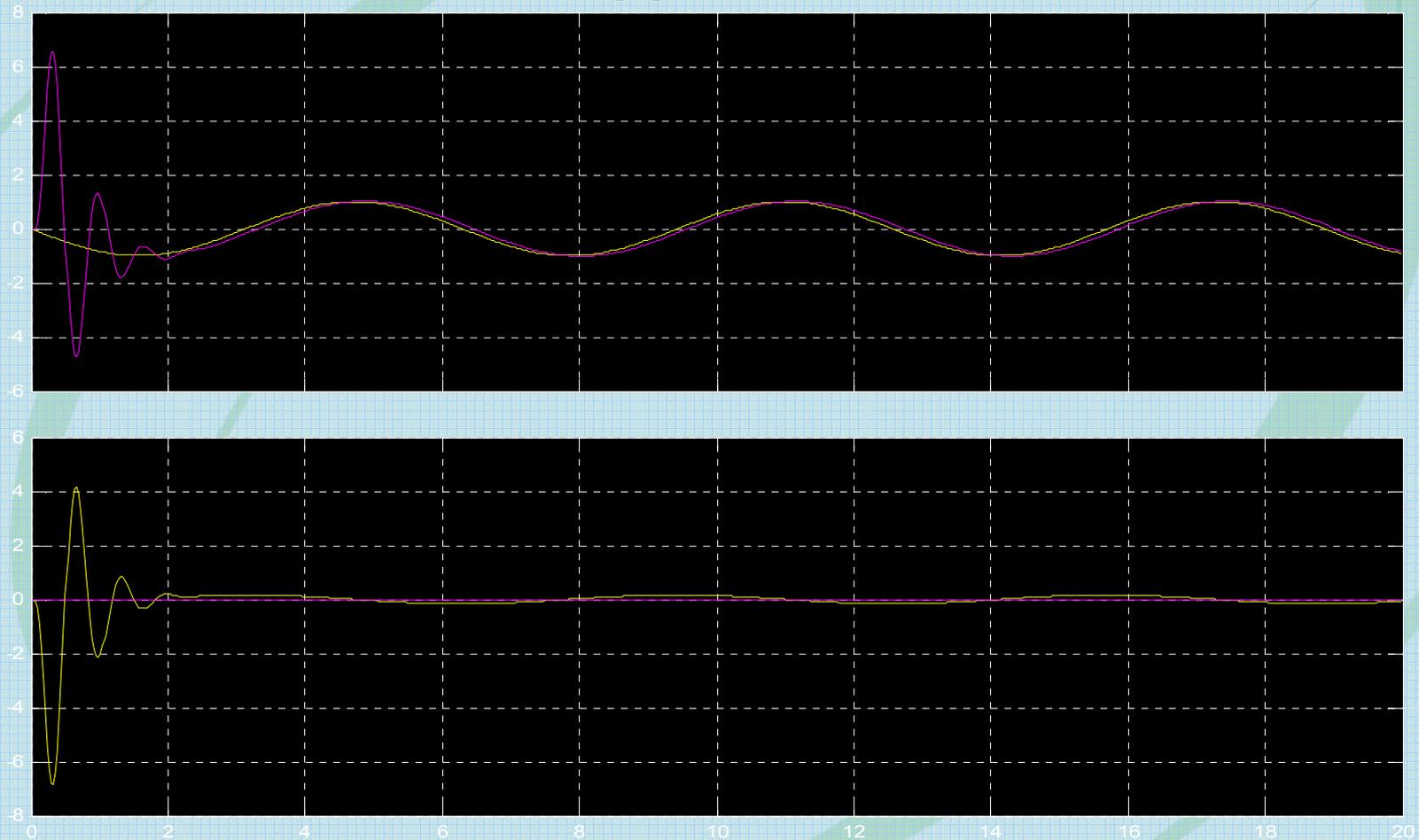
(iii) observers for quadratic systems under uncertainty – the method of hierarchical coefficients

Numerical Experiment: reconstructing the first derivative $u(t)=\sin t$



The output signal and the error with respect to time
($k_1=10$, $k_2=100$)

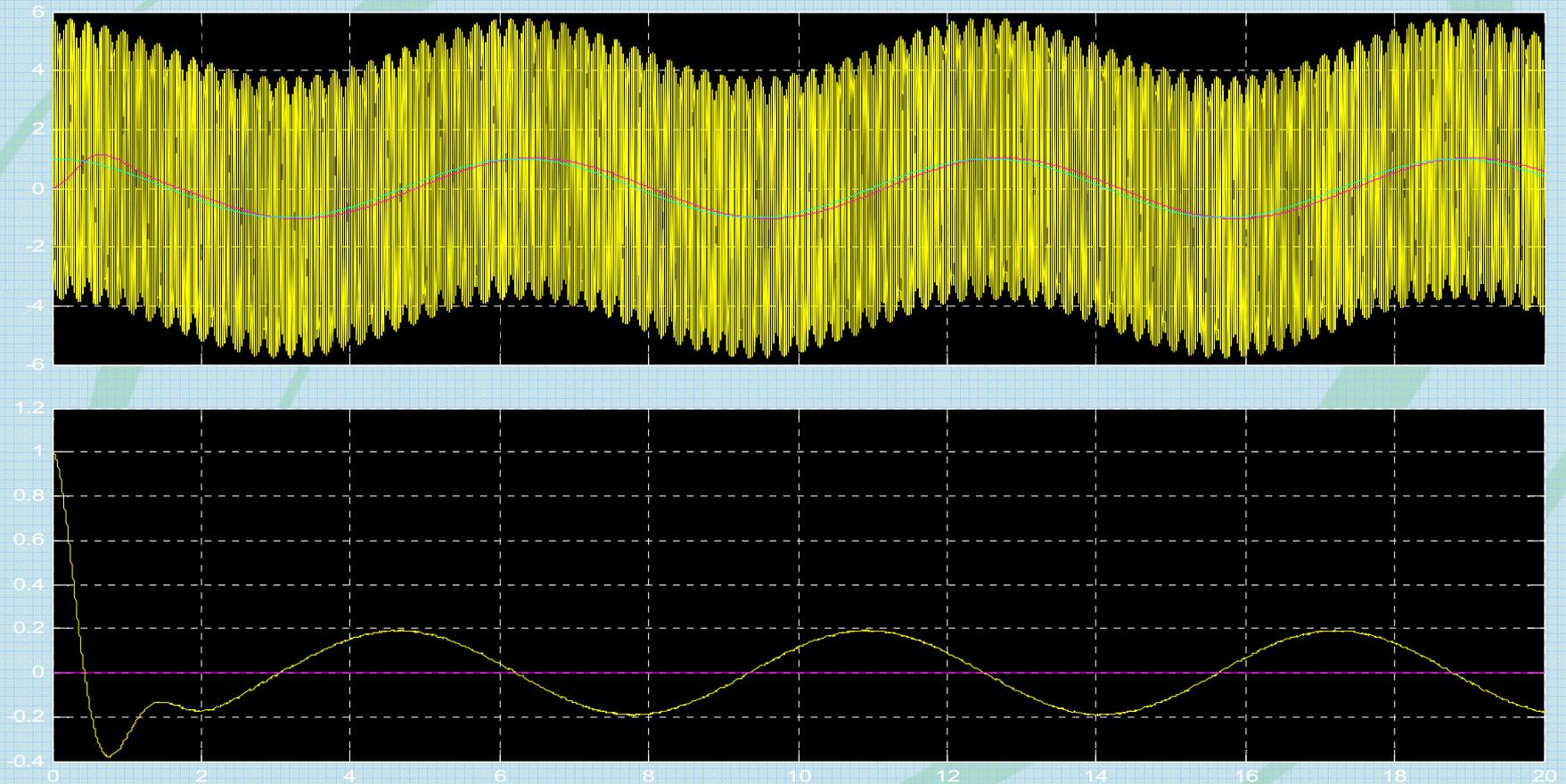
Numerical Experiment: reconstructing the second derivative $u(t)=\sin t$



The output signal and the error with respect to time
($\kappa_1=10, \kappa_2=100$)

Numerical Experiment: reconstructing the first derivative

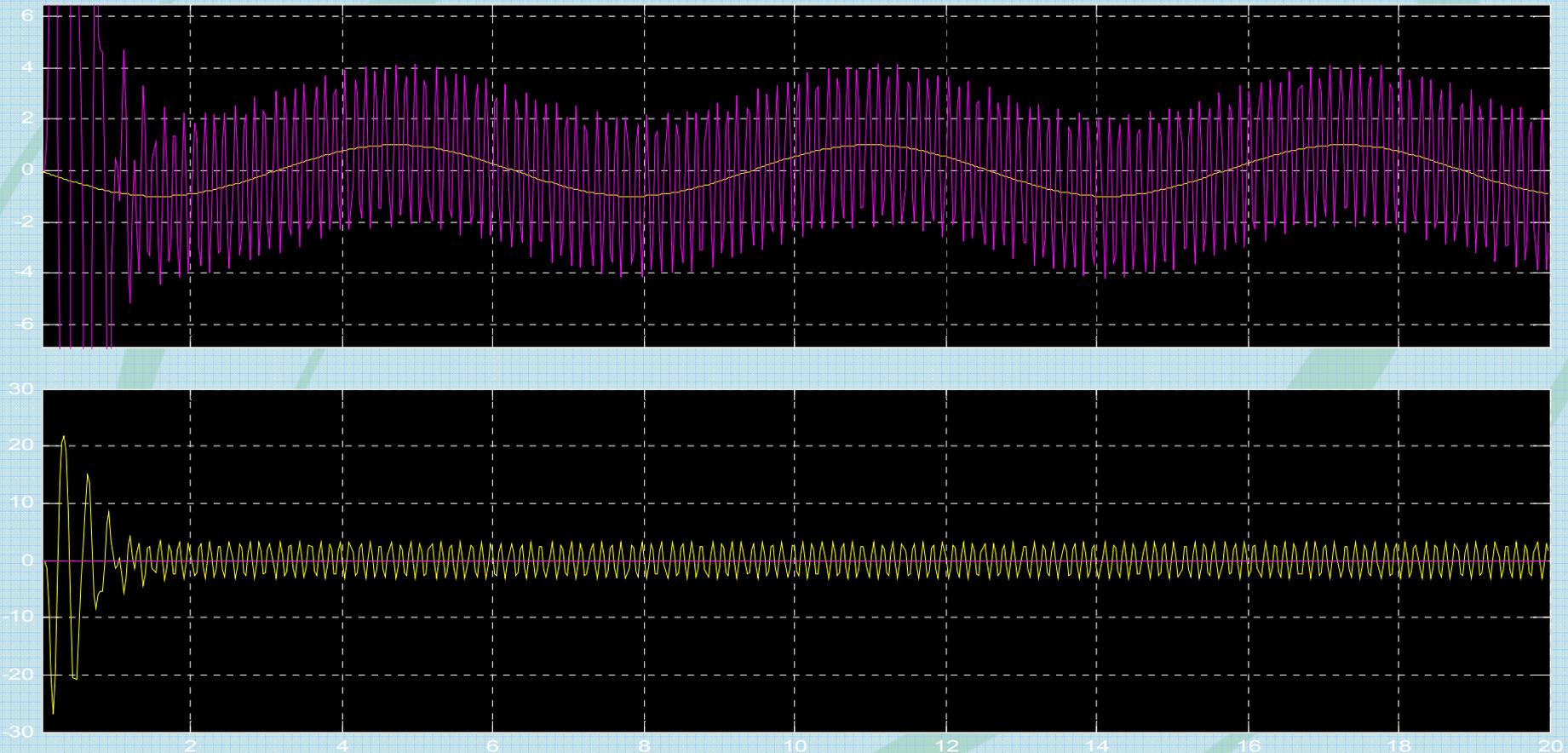
$u(t)=\sin t$ with noise $f(t)=0.02 \sin 1000t$



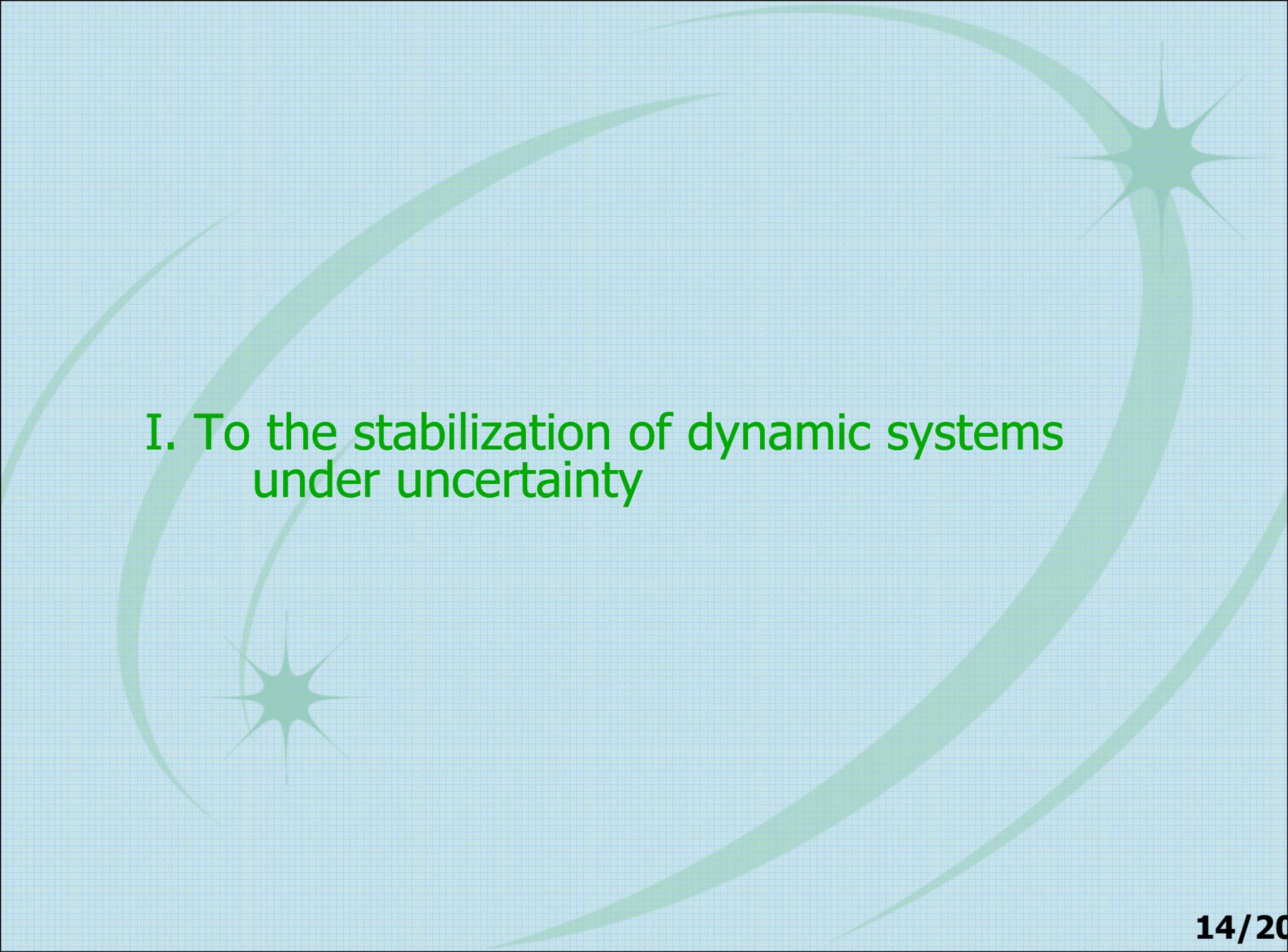
The output signal and the error with respect to time
($k_1=5$ $k_2=25$)

Numerical Experiment: reconstructing the second derivative

$u(t)=\sin t$ with noise $f(t)=0.02 \sin 1000t$



The output signal and the error with respect to time
($\kappa_1=5$ $\kappa_2=25$)



I. To the stabilization of dynamic systems under uncertainty

State of the Art

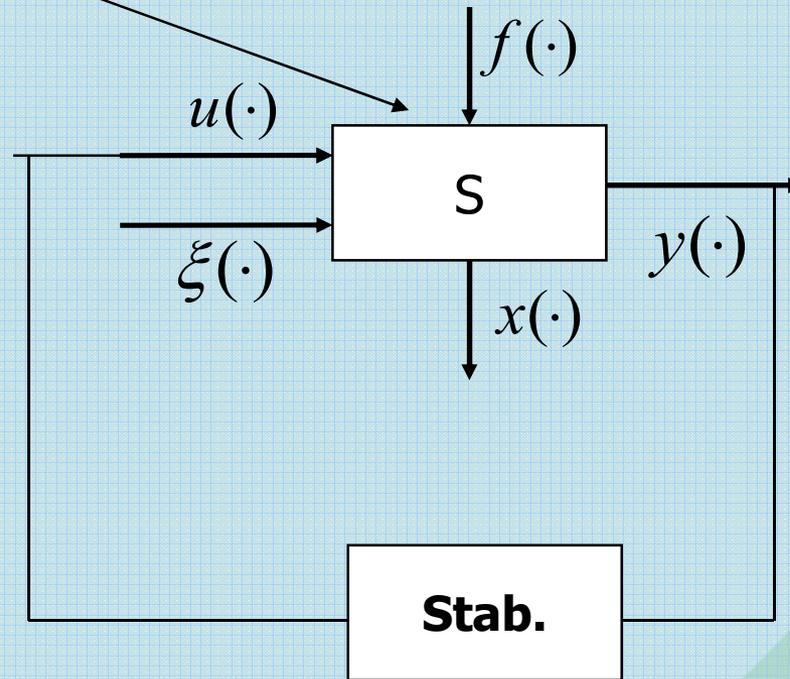
- Krasovskiy, Subbotin (1974)
 - general approach to constructing a feedback control stabilizing systems with uncertain dynamics
- Ledyaev, Sontag (1999), Clarke (2000), Bobylev et al (2002)
 - classical stabilization methods based on constructing appropriate Lyapunov functions
- Kryazhimskiy, Maximov (2004)
 - stabilization algorithms under completely uncertain dynamics
- MSU group – Emel'yanov, Korovin, Iline, Fomichev, Fursov
 - stabilization by means of observers

Stabilization approach

An object working in a few (n) qualitatively different modes

Applications from mechanics:

- a flying object moving with subsonic, sonic and supersonic speed
- a flying object functioning in normal and emergency modes

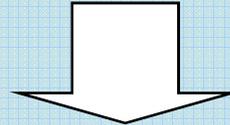


Feedback control

Simultaneous stabilization: New results

Saeks and Muller, 80-s: Simultaneous stabilization of 2 objects

MSU group: Simultaneous stabilization of n objects ($n > 2$):



- Constructive necessary and sufficient conditions for the simultaneous stabilization of n objects by a regulator of a given degree
- A numerical algorithm deriving the simultaneously stabilizing regulator of a given degree for n objects

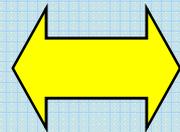
Simultaneous stabilization: algorithm

Let S_1, \dots, S_n be described by transfer functions

$$w_i(t) = \frac{\beta_i(t)}{\alpha_i(t)} \quad \deg(\alpha_i) < \deg(\beta_i) \\ i = 1, \dots, n$$

To construct the regulator

$$R(t) = \frac{p(t)}{q(t)}$$



To construct polynomials

$$p(t), q(t):$$

$$\varphi_i(t) = \beta_i(t)p(t) + \alpha_i(t)q(t)$$

$$i = 1, \dots, n$$

are stable

- localization of a searching area of p- and q-coefficients in a multi-dimensional cube
- finding p- and q-coefficients by means of the method of interval calculations
- synthesis of the regulator

What to read

S.K.Korovin, V.V.Fomichev. *State Observers for Linear Systems under Uncertainty.* Fizmatlit, 2007 (in Russian)

A.V.Il'in, S.K.Korovin and V.V.Fomichev. Positional robust inversion in nonlinear dynamical systems \\ Computational Mathematics and Modeling, 18 (2), 2007

A.V.Il'in, S.K.Korovin and V.V.Fomichev. Methods for constructing observers for linear dynamical systems under uncertainty \\ Proceedings of the Steklov Institute of Mathematics, 262(1), 2008

A.V.Il'in, S.K.Korovin and V.V.Fomichev. Asymptotic observers for bilinear systems with vector output \\ Differential Equations, 44 (5), 2008

S.K.Korovin, A.V.Kraev and V.V.Fomichev. Some inversion algorithms for discrete systems \\ Computational Mathematics and Modeling, 18 (4), 2007.

S.K.Korovin and V.V.Fomichev. Asymptotic observers for n -dimensional bilinear systems \\ Computational Mathematics and Modeling, 18 (2), 2007

*Application of the control theory
to beam dynamics?*

Thank you for your attention!

Example: scalar observer

Full-size observer

dynamic system

$$S: \begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \quad \bar{x} = Fx$$

The diagram shows a dynamic system S with state x of dimension n and output y of dimension l . The system is represented by the equations $\dot{x} = Ax + Bu$ and $y = Cx$. The observer is represented by the equation $\bar{x} = Fx$.

observer

$$\dot{\bar{x}} = A\bar{x} + bu - L(C\bar{x} - y), \quad \bar{x} \in R^n$$

error vector

$$e = x - \bar{x} : \dot{e} = (A - LC)e$$

Theorem: for any observable $\{A, C\}$

there exists a matrix $L : (A - LC)$ is Hurwitz matrix

Theorem: $e \rightarrow 0, t \rightarrow \infty$

Minimal-size observer

Theorem: for any observable $\{A, C\}$

there exists an observer of $n - l$ degree with a given rate of convergence