ILC-CLIC LET Beam Dynamics Workshop (23-25 June 2009)

On the description of ground motion (LHC tunnel)

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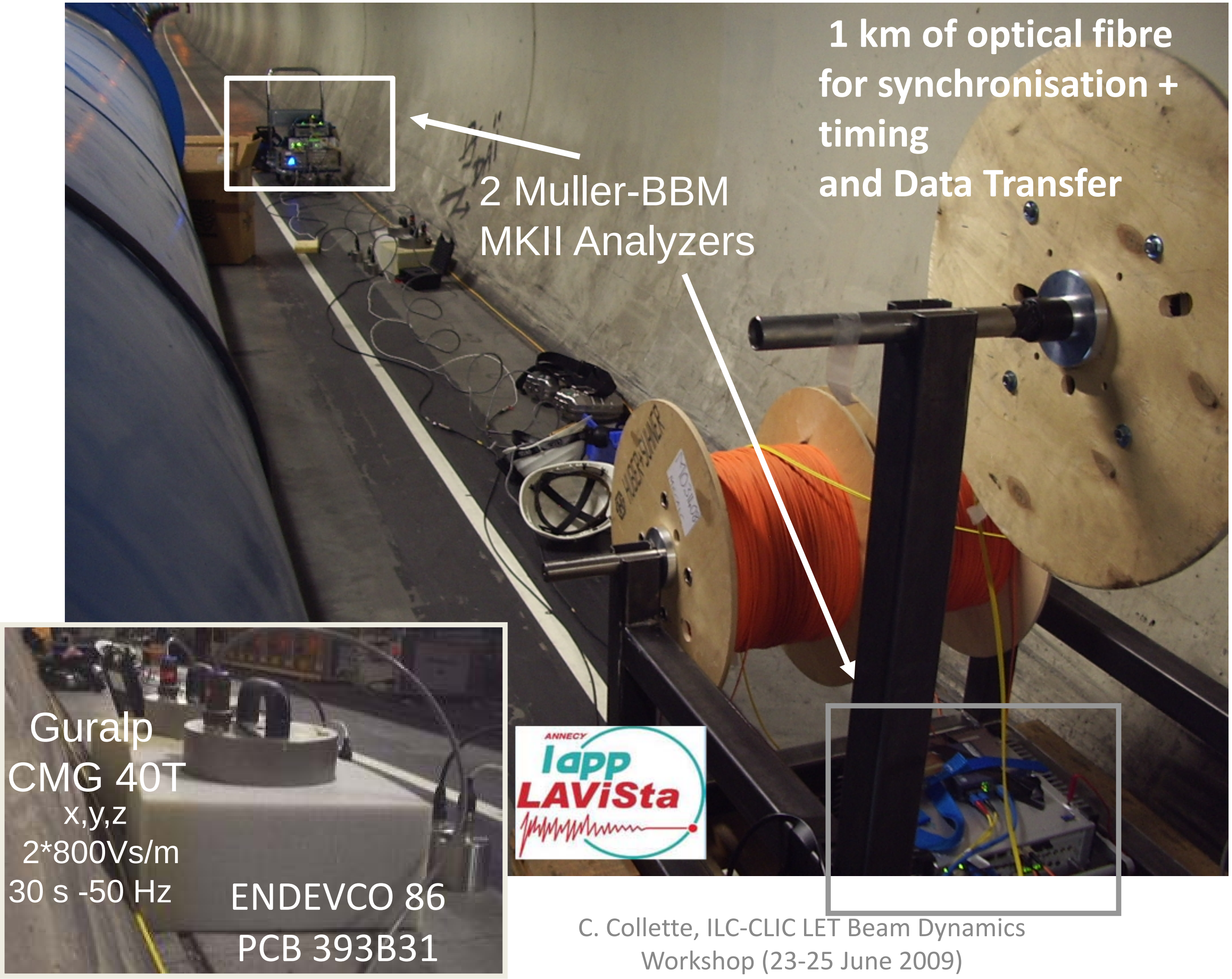
Overview

- Measurements description
- Ground motion modeling
- Lattice excitation
- Example for a single quadrupole

Measurements



<u>Parameters</u>

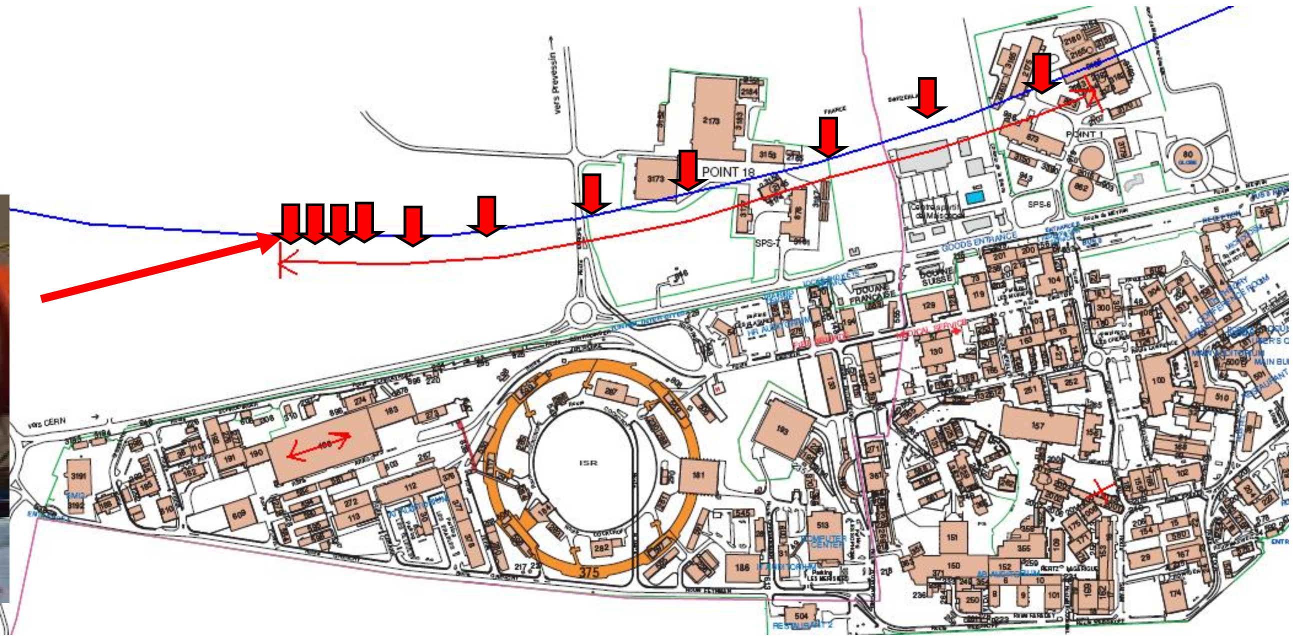
Sampling rate
256 Hz
Block duration
64 s
Average
Lin - 50
Overlap
66.7 %

24 bit on 100 mV

Low ADC noise

LHC Measurements

LHC DCUM 1000
~ 80 m under ground



Measurements: 0 1 2 3 4 5 6 7 8 9 10 12 20 30 38 54 108 198 306 412 509 604 706 960 (m)

Specific features :
- Synchronous measurements
- LHC systems in operation, night time
- Multi-directional

Characteristics

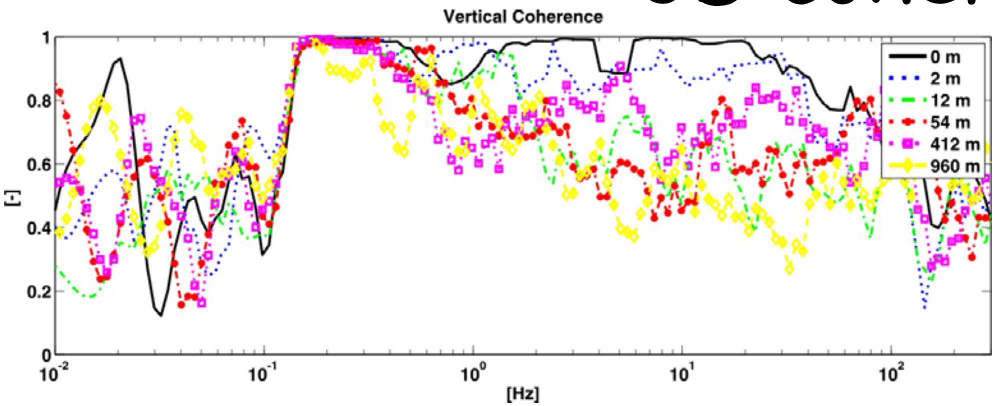
- Correlation function: $$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)y(t+\tau)dt$$

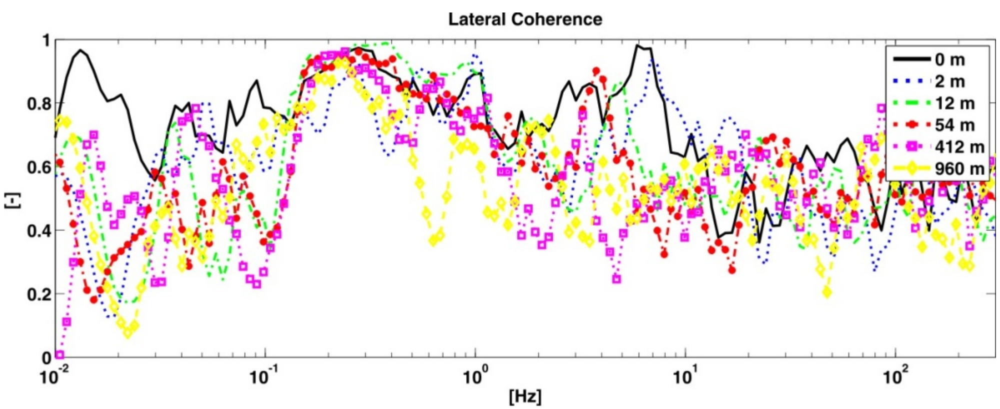
- Cross spectral density: $$\Phi_{xy}(\omega) = \int_{-\infty}^{\infty} R_{xy}(\tau)e^{-i\omega\tau}d\tau$$

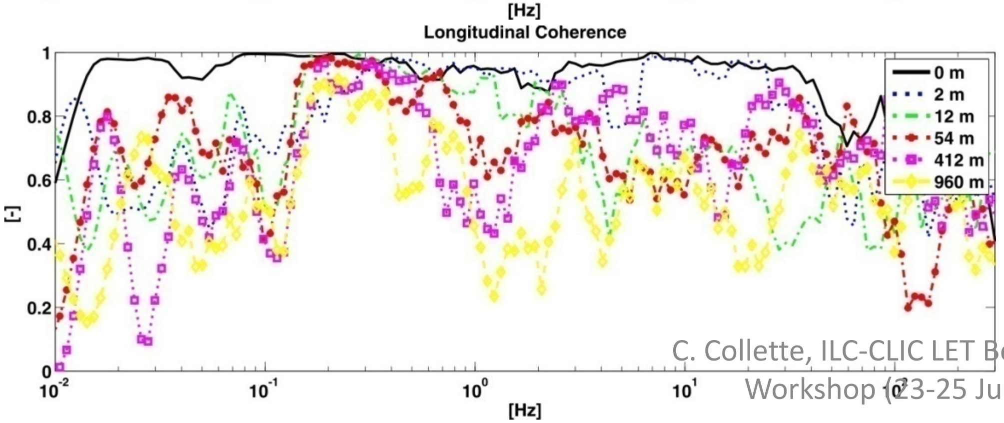
- Normalized spectral density between two measurements x(t) and y(t): $$\gamma_{xy}(\omega) = \frac{\Phi_{xy}(\omega)}{\sqrt{\Phi_{xx}(\omega)\Phi_{yy}(\omega)}}$$

- Power spectral density of the relative motion $d(t)=x_1(t)-x_2(t)$: $$\rho(\omega, L) = \Phi_{xx}(\omega)2\{1 - Re[\gamma_{x_1x_2}(\omega)]\}$$

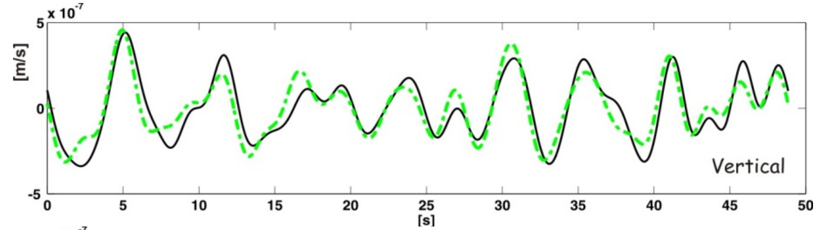
3D coherences

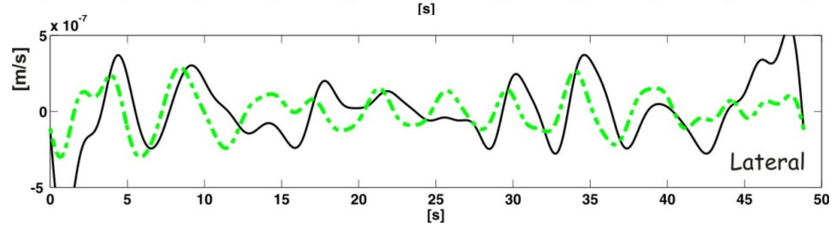


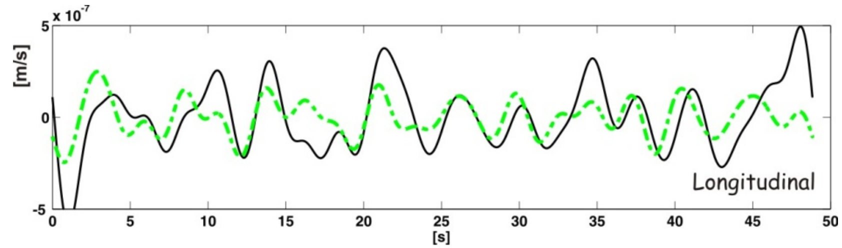




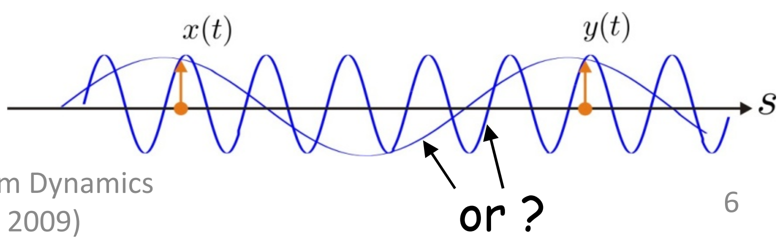
d=960 m; filtered between 0.08 Hz and 0.5 Hz







- Correlation in the low frequency range
- Similar amplitudes in all directions



2D power spectral density (P2D)

$$P(\omega, k) = \lim_{T\to\infty} \lim_{L\to\infty} \frac{1}{T}\frac{1}{L} \left| \int_{-T/2}^{T/2} \int_{-L/2}^{L/2} x(t,s) e^{-i\omega t} e^{-iks} ds \right|^2$$

$$P(\omega, k) = \frac{A}{\omega^2 k^2}[1-\cos(L_0 k)] + D(\omega)U(\omega, k)$$

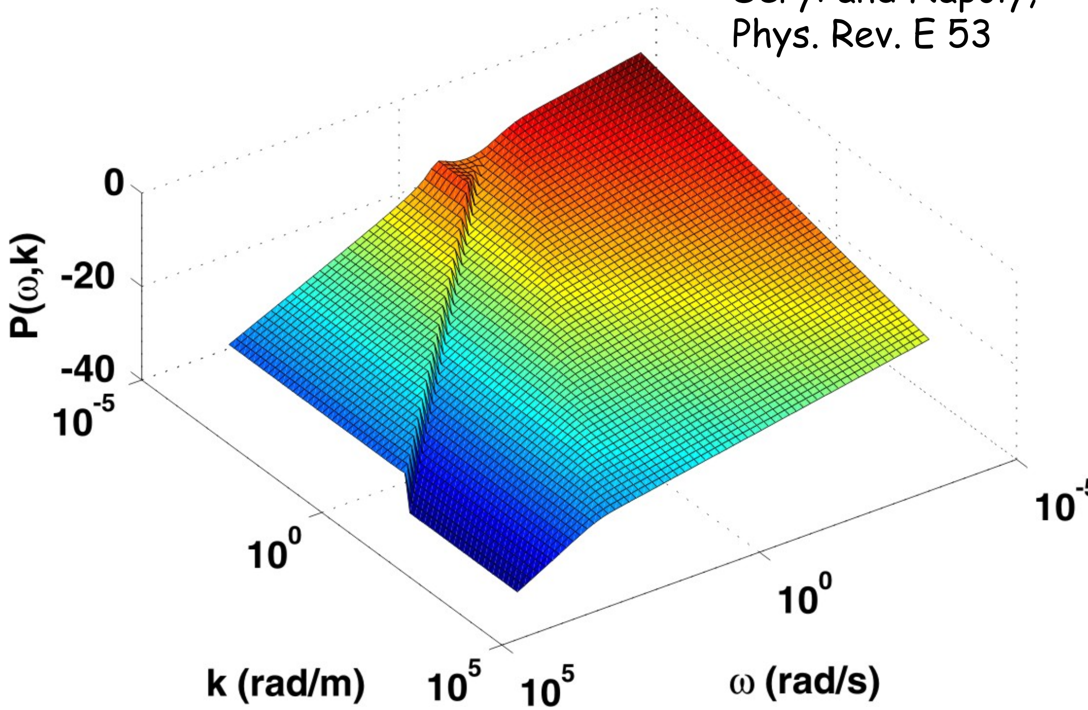
$$U(\omega, k) = \frac{2}{\sqrt{k_{max}^2 - k^2}} \text{ if } \mid \mathrm{k} \mid \leq \mathrm{k_{max}}$$

$$U(\omega, k) = 0 \text{ if } \mid \mathrm{k} \mid > \mathrm{k_{max}}$$

$$D(\omega) = \frac{a_i}{1+[d_i(\omega - \omega_i)/\omega_i]^4}$$

$$L_0 = B/(A\omega^2)$$

Seryi and Napoly, Phys. Rev. E 53



P2D contains all the information BUT does not depend on the position

Calculation of the P2D

1. From spectral densities of the relative measurements

Continuous: $$P(\omega,k)=\int_0^\infty cos(kL)[\rho(\omega,L=\infty)-\rho(\omega,L)]dL$$

Discrete: $$P(\omega,k)=\sum_{j=1}^{m} cos(kL'_j)[\rho(\omega,L_m)-\rho(\omega,L'_j)]\Delta L$$

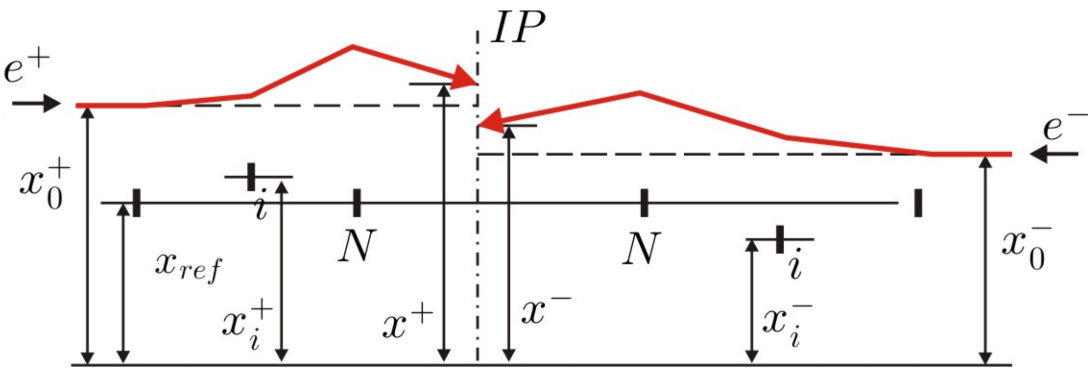
2. From asymptotic behaviour

Zeroth order design report for the next linear collider, pp 1011.

How to use the P2D

1. Direct evaluation of the performances

(Seryi and Napoly, Phys. Rev. E 53)



$$\Delta = x^+ - x^- \qquad \sigma_\Delta^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(\omega, k) 2[1 - cos(\omega T)] G(k) \frac{d\omega}{2\pi}\frac{dk}{2\pi}$$

2. Transform it into the time domain (Seryi et al):

$$x(t,s) = \sum_i \sum_j a_{ij}[\sin(\omega_i t)\sin(k_j s + \phi_{ij}) + (\cos(\omega_i t) - 1)\sin(k_j s + \psi_{ij})]$$

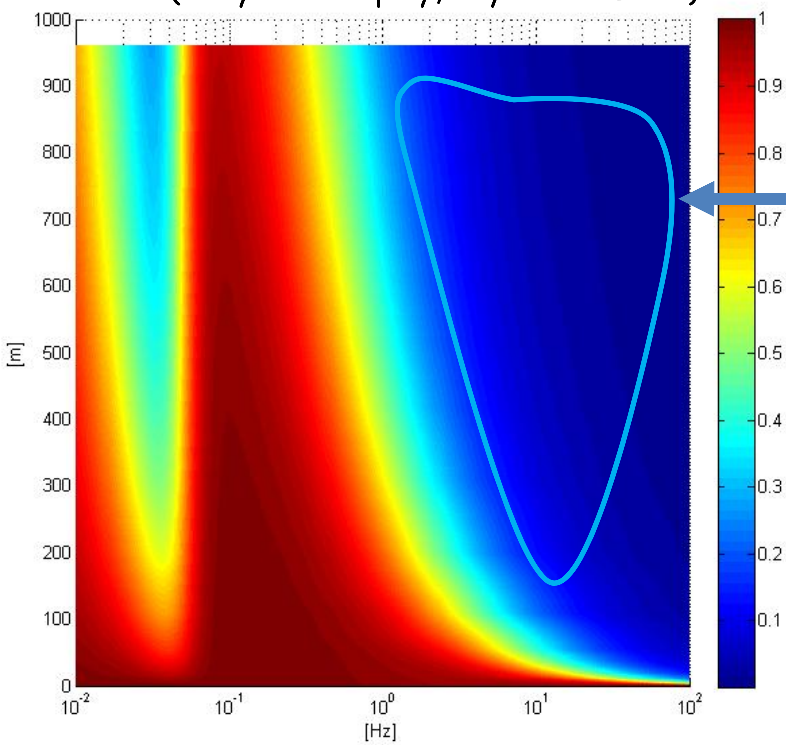
From 2D spectral density

Random phases

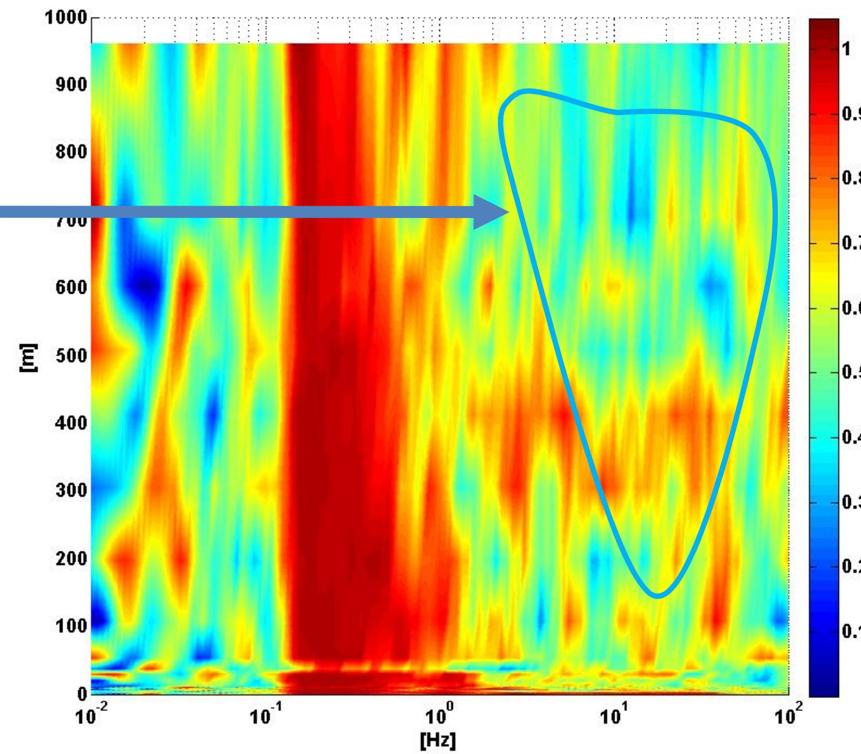
Comparison with measurements

Coherence using a theoretical model

(Seryi and Napoly, Phys. Rev. E 53)



Calculated from measurement



How to account for local dynamics and local technical noise sources ?

Local dynamics

$$\sigma_\Delta^2 = \int\int P(\omega,k) 2[1-\cos(\omega T)] 2[1-\cos(kL)] \frac{d\omega}{2\pi}\frac{dk}{2\pi}$$

If girders amplify by

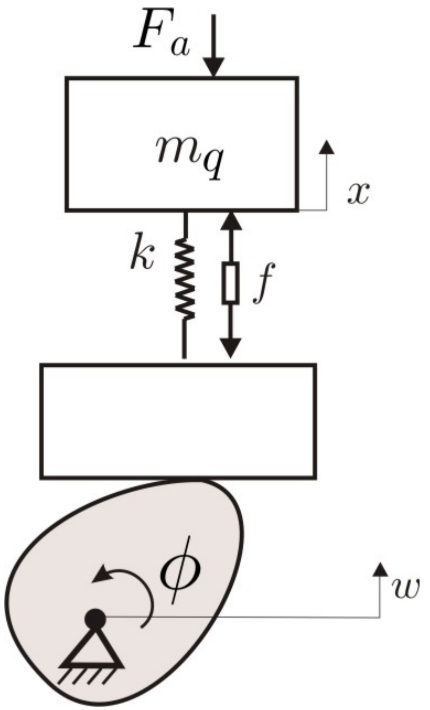
$$R_k(\omega) = r_k(\omega) e^{i\phi_k(\omega)}$$

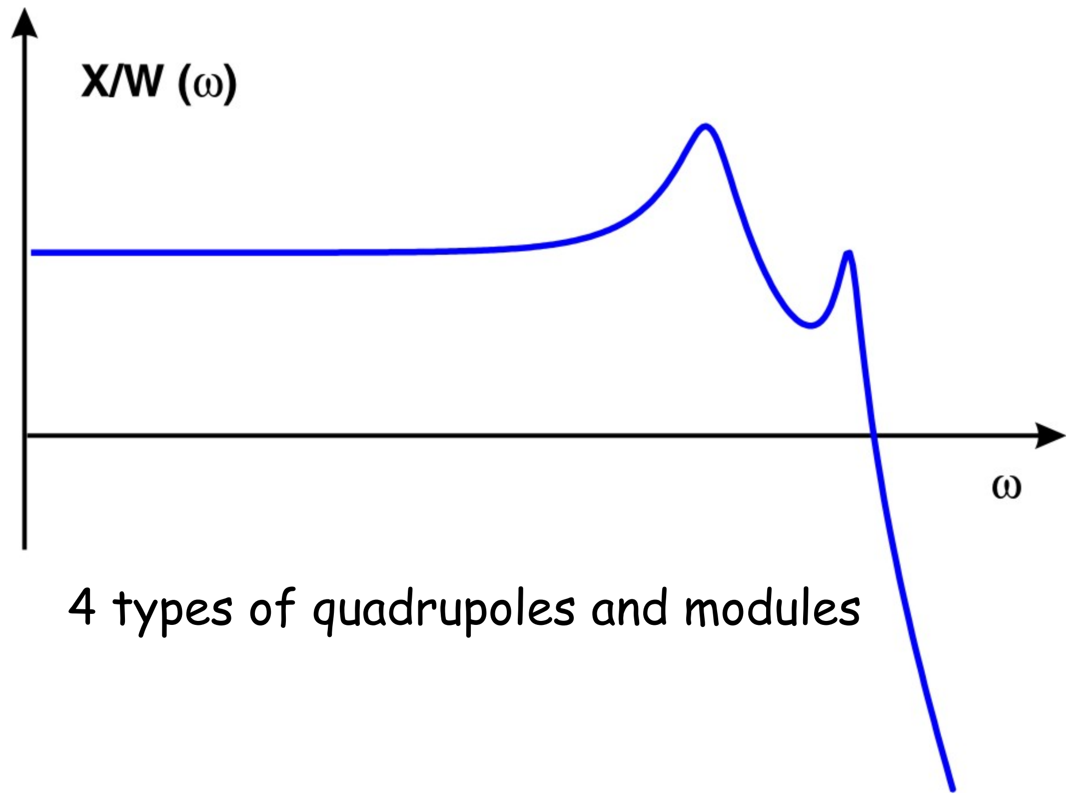
then

Replaced by

(Seryi, 2001)

$$[r_1^2 + r_2^2 - 2 r_1 r_2 \cos(kL + \phi_1 - \phi_2)]$$

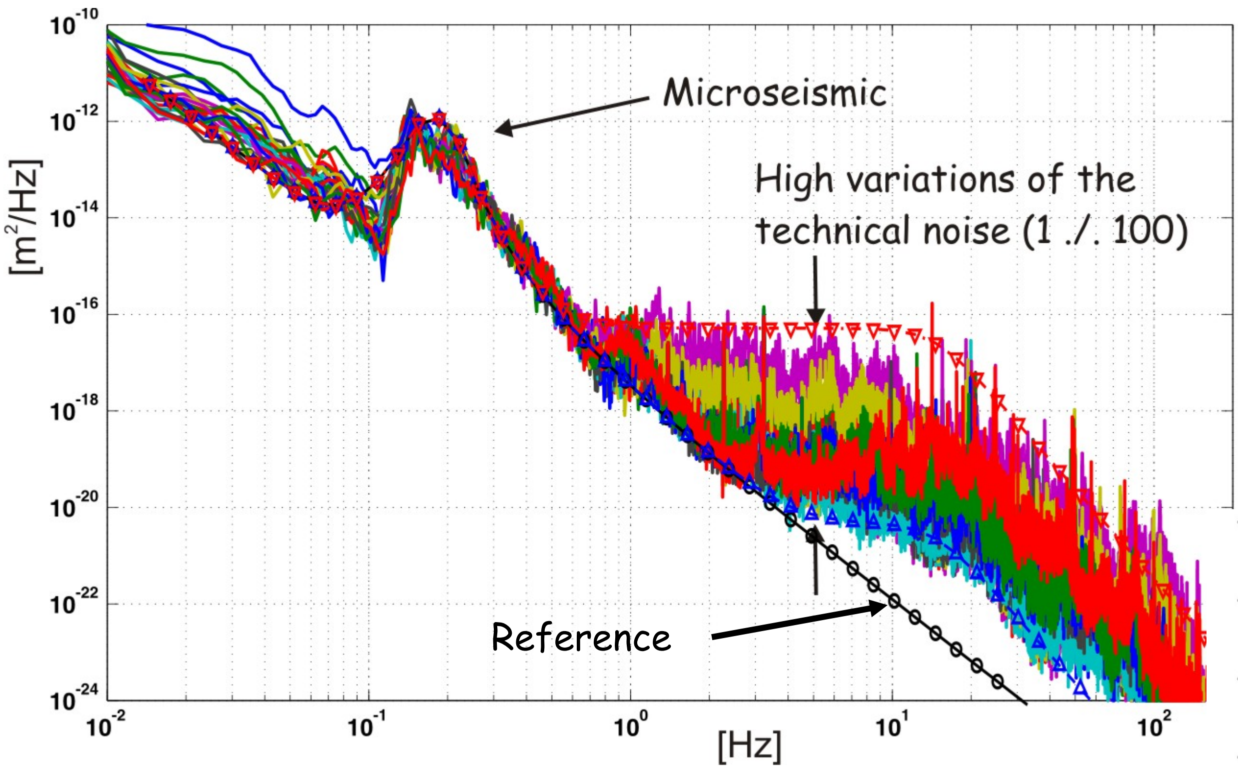




4 types of quadrupoles and modules

Local excitations

Vertical ground motion



Low technical noise: $N_0 = 5 * 10^{-3} (nm^2/Hz)$

High technical noise: $N_0 = 50 (nm^2/Hz)$

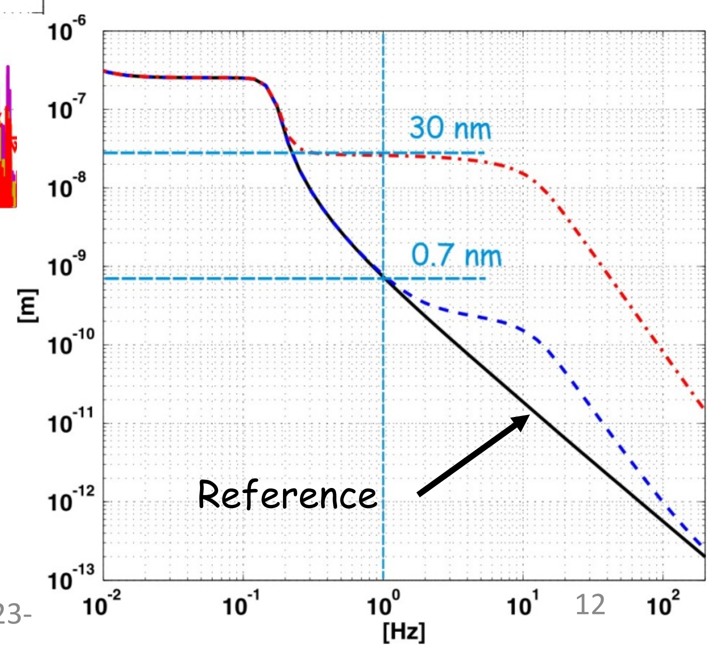
Ref. : $A = 10^{-4}\ (\mu m^2 s^{-1} m^{-1}); B = 10^{-4}\ (\mu m^2 s^{-3});$

$\omega_1 = 2\pi * 0.14\ (rad/s); d_1 = 5; a_1 = 0.1\ (\mu m^2/Hz); v_1 = 1000\ (m/s)$

Additional technical noise:

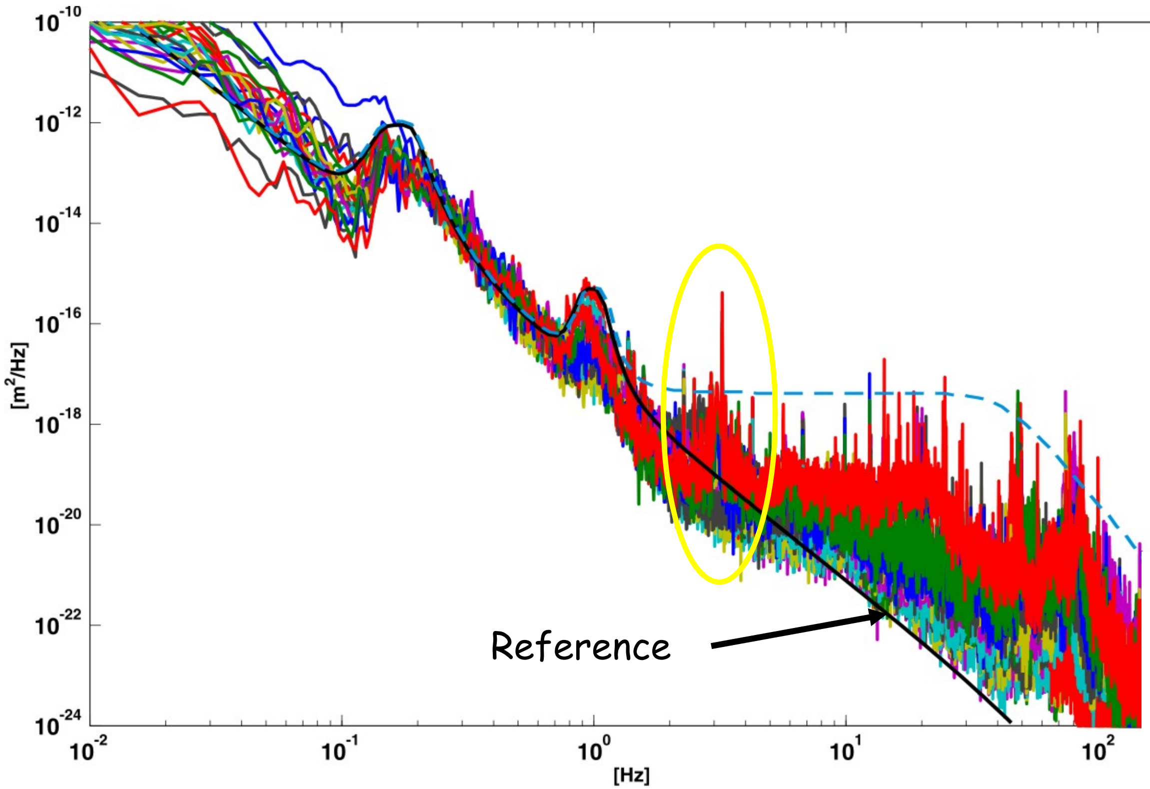
$$N(\omega) = \frac{N_0}{1 + (\frac{\omega}{\omega_0})^6}$$

$$f_0 = 2\pi (Hz)$$



Local excitations

Lateral ground motion

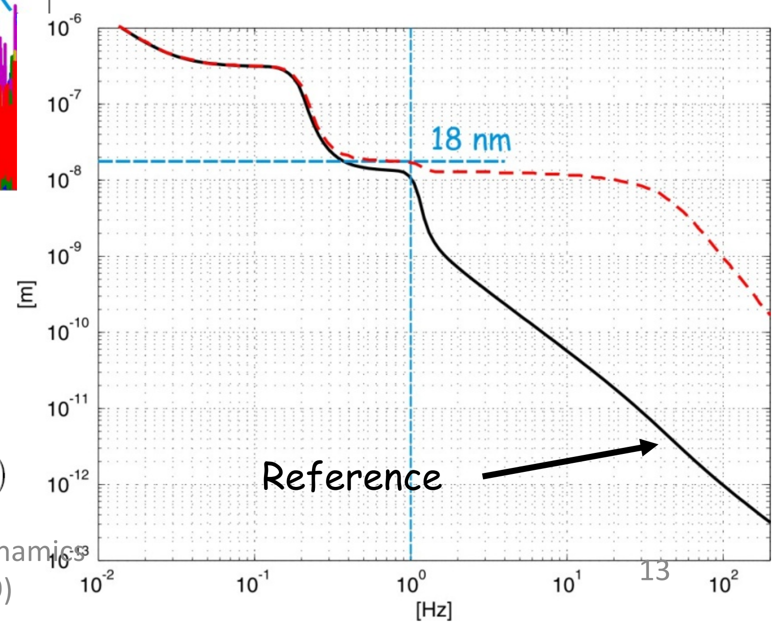


Additional technical noise:

$$N(\omega) = \frac{N_0}{1+(\frac{\omega}{\omega_0})^6}$$

$$N_0 = 0.5(nm^2/Hz)$$

$$f_0 = 40(Hz)$$

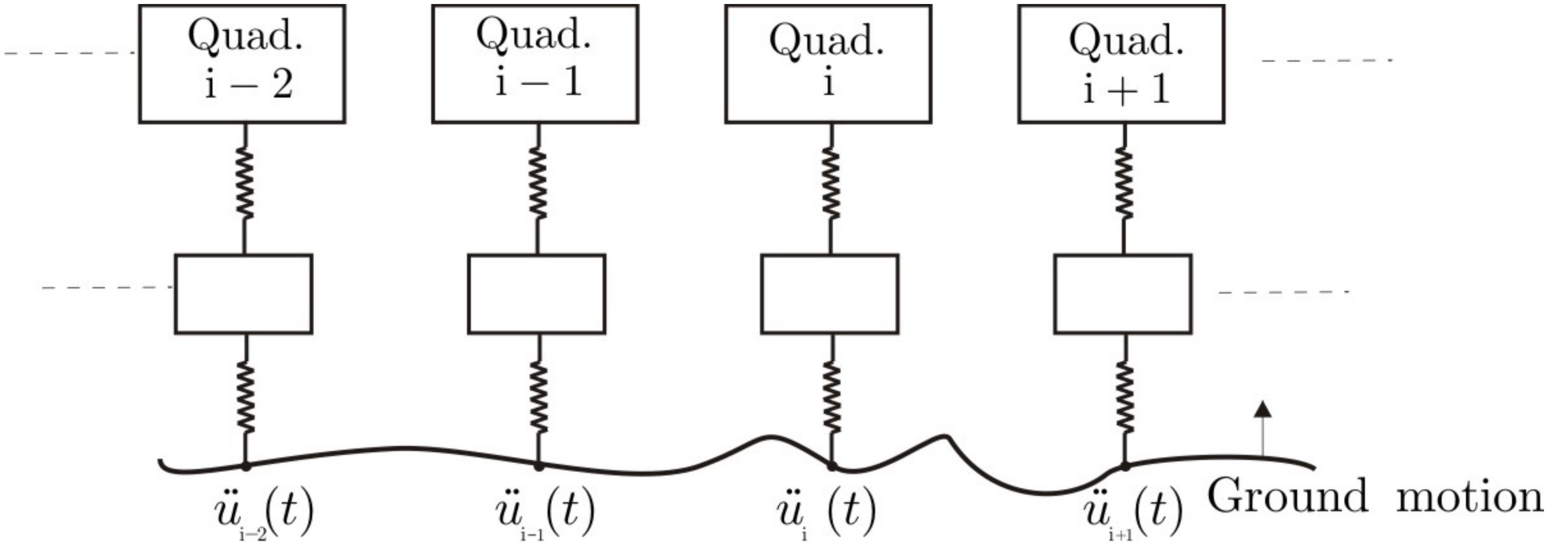


Ref. : $A = 10^{-3}\ (\mu m^2 s^{-1} m^{-1}); B = 10^{-2}\ (\mu m^2 s^{-3});$

$\omega_1 = 2\pi * 0.17\ (rad/s); d_1 = 5; a_1 = 0.5\ (\mu m^2/Hz); v_1 = 1000\ (m/s)$

$\omega_2 = 2\pi * 1\ (rad/s); d_2 = 8; a_2 = 5 * 10^{-4}\ (\mu m^2/Hz); v_2 = 400\ (m/s)$

Explicit formulation



$$\begin{pmatrix} \mathbf{M}_{ll} & \mathbf{M}_{lg} \\ \mathbf{M}_{gl} & \mathbf{M}_{gg} \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{x}} \\ \ddot{\mathbf{u}} \end{pmatrix} + \begin{pmatrix} \mathbf{C}_{ll} & \mathbf{C}_{lg} \\ \mathbf{C}_{gl} & \mathbf{C}_{gg} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{u}} \end{pmatrix} + \begin{pmatrix} \mathbf{K}_{ll} & \mathbf{K}_{lg} \\ \mathbf{K}_{gl} & \mathbf{K}_{gg} \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \mathbf{u} \end{pmatrix} = \begin{pmatrix} 0 \\ \mathbf{f}_g \end{pmatrix}$$

$$\mathbf{X}(\omega) = \left[\frac{\mathbf{A}}{\omega^2} + \mathbf{G}(\omega)\mathbf{\Gamma}\right] \ddot{\mathbf{U}}(\omega)$$

$$\mathbf{A} = -\mathbf{K}_{ll}^{-1}\mathbf{K}_{lg}$$

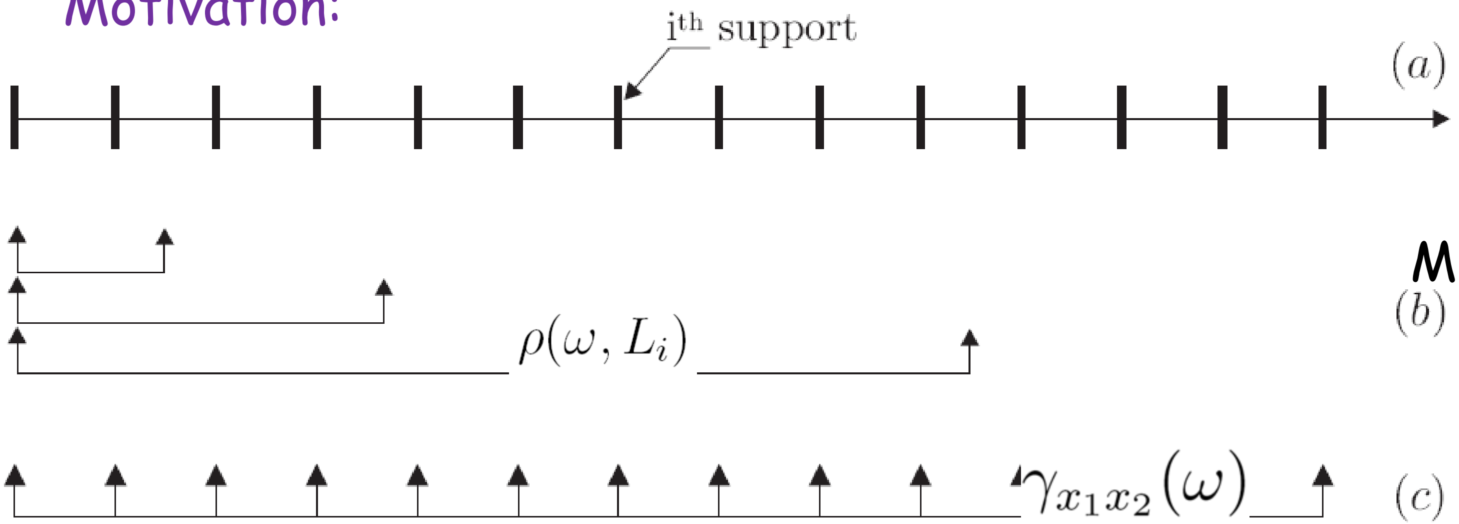
$$\mathbf{\Gamma} = \mathbf{M}_{ll}\mathbf{K}_{ll}^{-1}\mathbf{K}_{lg} - \mathbf{M}_{lg}$$

$$\mathbf{G}(\omega) = [\mathbf{M}_{ll} + \mathbf{C}_{ll} + \mathbf{K}_{ll}]^{-1}$$

Excitation matrix: $\mathbf{S}_{\mathbf{uu}}(\omega)$

Lattice excitation matrix

Motivation:



Support location

Measurements (available)

Excitation matrix (objective)

Procedure:

$\rho(\omega, L_i)$ for a few L_i $\Rightarrow$ $P(\omega, k)$ $\Rightarrow$ $\gamma_{x_1 x_2}(\omega)$ for any x_1 and x_2

$$\rho(\omega, L_i) = \int_{-\infty}^{+\infty} P(\omega, k) 2[1 - cos(kL_i)] \frac{dk}{2\pi}$$

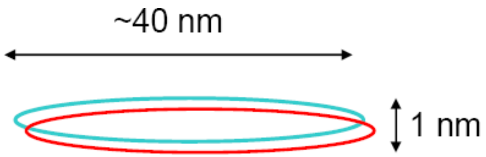
$$\mathbf{S_{\ddot{u}\ddot{u}}}(\omega) = \begin{pmatrix} \Phi_{\ddot{u}_1\ddot{u}_1}(\omega) & \Phi_{\ddot{u}_1\ddot{u}_2}(\omega) & \cdots & \Phi_{\ddot{u}_1\ddot{u}_n}(\omega) \\ \Phi_{\ddot{u}_2\ddot{u}_1}(\omega) & \Phi_{\ddot{u}_2\ddot{u}_2}(\omega) & \cdots & \Phi_{\ddot{u}_2\ddot{u}_n}(\omega) \\ \cdots & \cdots & \cdots & \cdots \\ \Phi_{\ddot{u}_n\ddot{u}_1}(\omega) & \Phi_{\ddot{u}_n\ddot{u}_1}(\omega) & \cdots & \Phi_{\ddot{u}_n\ddot{u}_n}(\omega) \end{pmatrix}$$

$$\mathbf{S_u}(\omega) = \frac{\mathbf{S_{\ddot{u}}}(\omega)}{\omega^4}$$

Quadrupole stabilization

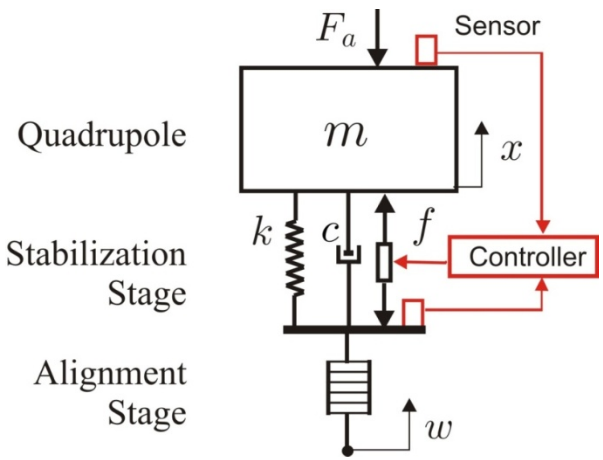
Objective:

Beam size above 1 Hz:



Control strategy:

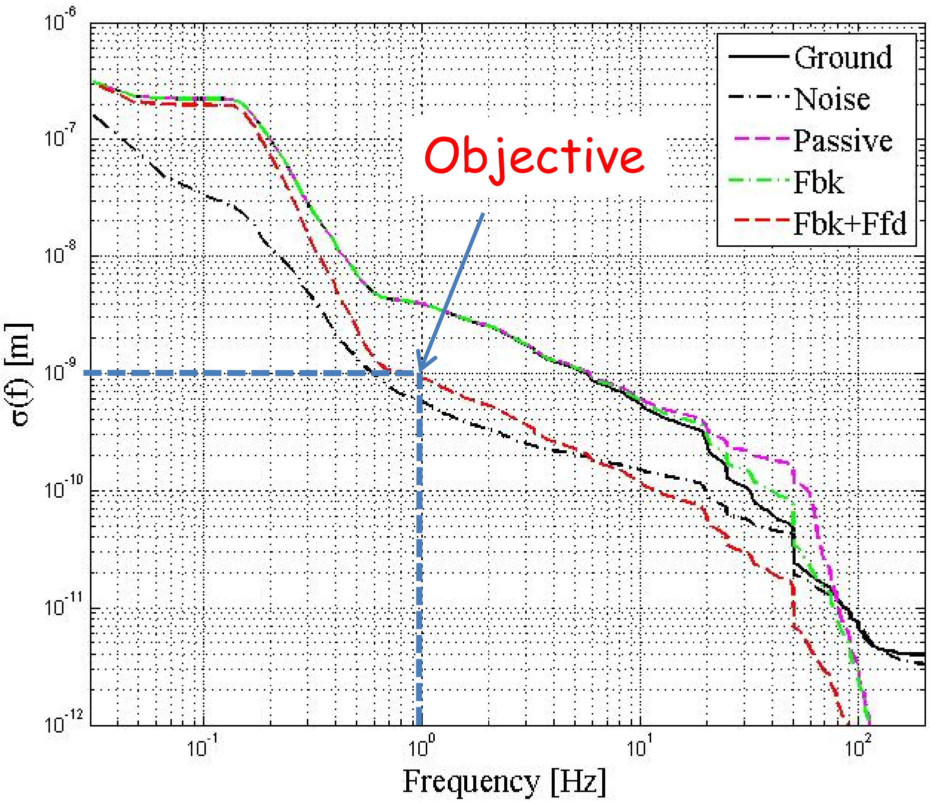
Feedback + Feedforward



$$S_{uu}(\omega) = \Phi_{ww}(\omega)$$

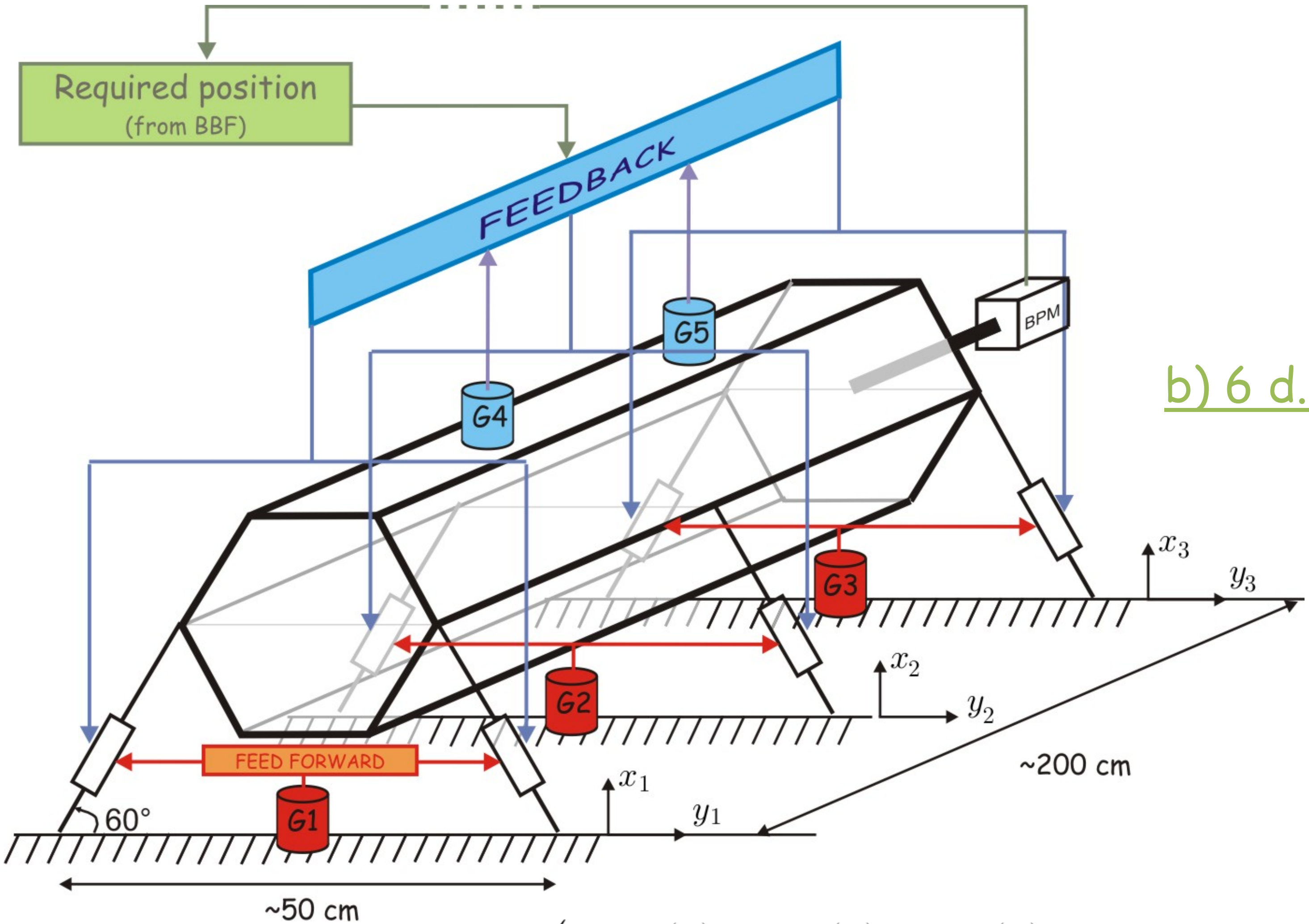
a) 1 d.o.f quadrupole

RMS integrated $\sigma(f) = \sqrt{\int_f^{\infty} \Phi(\nu) d\nu}$



(PAC 2009, Artoos et al)

b) 6 d.o.f quadrupole

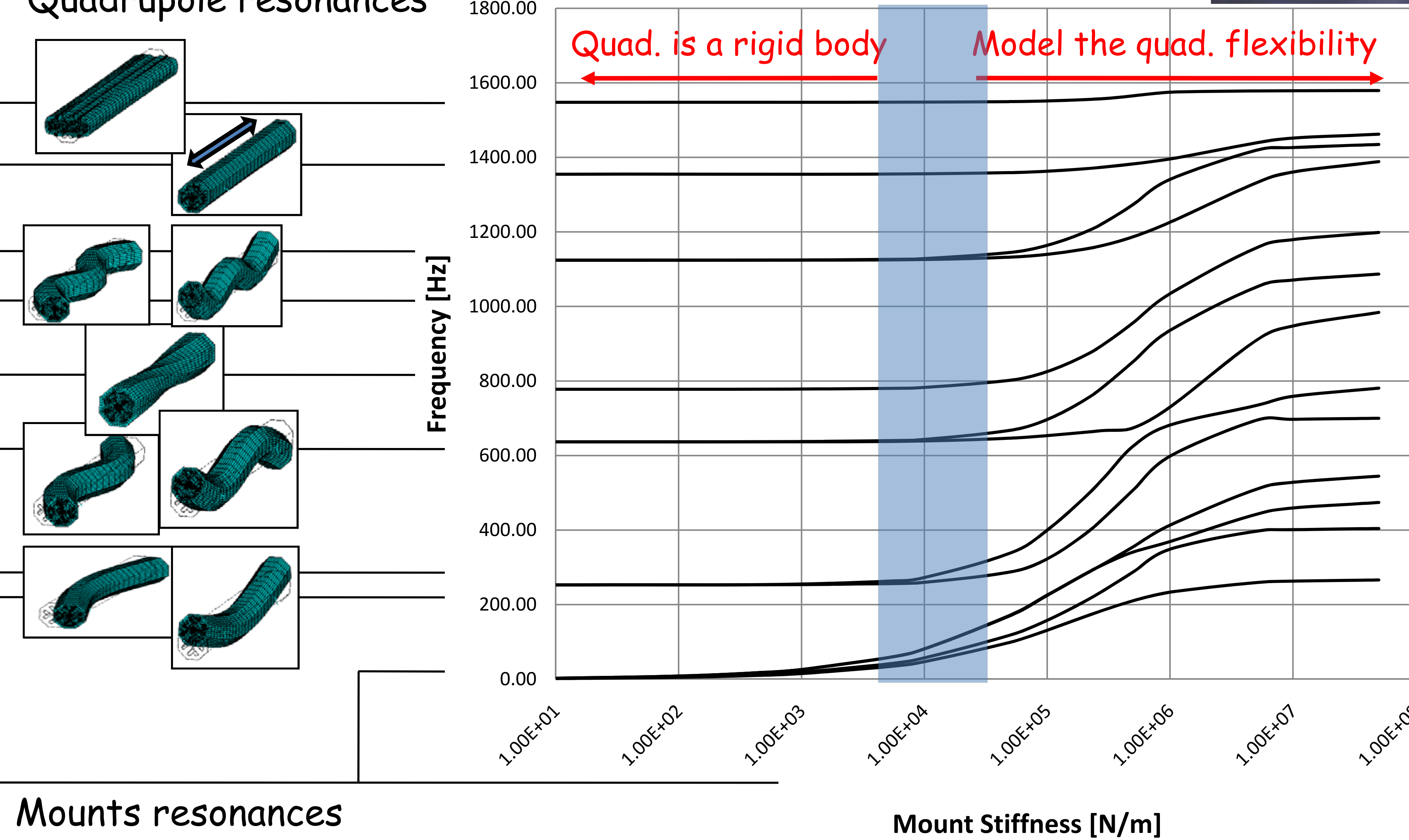


Excitation matrix without x-y coupling

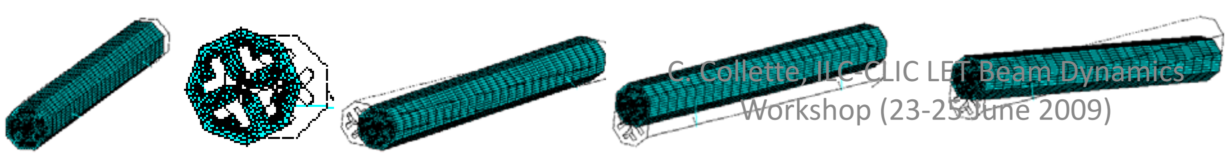
$$\mathbf{S_{uu}}(\omega) = \begin{pmatrix} \Phi_{x_1x_1}(\omega) & \Phi_{x_1x_2}(\omega) & \Phi_{x_1x_3}(\omega) & 0 & 0 & 0 \\ \Phi_{x_2x_1}(\omega) & \Phi_{x_2x_2}(\omega) & \Phi_{x_2x_3}(\omega) & 0 & 0 & 0 \\ \Phi_{x_3x_1}(\omega) & \Phi_{x_3x_2}(\omega) & \Phi_{x_3x_3}(\omega) & 0 & 0 & 0 \\ 0 & 0 & 0 & \Phi_{y_1y_1}(\omega) & \Phi_{y_1y_2}(\omega) & \Phi_{y_1y_3}(\omega) \\ 0 & 0 & 0 & \Phi_{y_2y_1}(\omega) & \Phi_{y_2y_2}(\omega) & \Phi_{y_2y_3}(\omega) \\ 0 & 0 & 0 & \Phi_{y_3y_1}(\omega) & \Phi_{y_3y_2}(\omega) & \Phi_{y_3y_3}(\omega) \end{pmatrix}$$

Modal analysis of the 2m long quad. with 6 mounts

Quadrupole resonances



Mounts resonances



Modal coordinates

$$\mathbf{x} = \mathbf{x}^{d} + \mathbf{x}^{qs} \quad \text{where} \quad \mathbf{x}^{d} = \mathbf{\Phi}\mathbf{z}$$

$$\mathbf{S}_{XX}(\omega) = \left\{\left[\frac{\mathbf{A}}{\omega^2} + \mathbf{\Phi}^{\mathbf{T}}\mathbf{H}(\omega)\mathbf{\Gamma}\right]\ddot{\mathbf{U}}(\omega)\right\}\left\{\left[\frac{\mathbf{A}}{\omega^2} + \mathbf{\Phi}^{\mathbf{T}}\mathbf{H}(\omega)\mathbf{\Gamma}\right]\ddot{\mathbf{U}}(\omega)\right\}^{*}$$

$$= \frac{1}{\omega^4}\sum_{l=1}^{N}\sum_{m=1}^{N} A_{il}A_{im}\Phi_{\ddot{u}_l\ddot{u}_m}(\omega) \longleftarrow \text{Seryi et al}$$

$$+ \sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{N}\sum_{m=1}^{N} \phi_{ij}\phi_{ik}\Gamma_{lj}\Gamma_{mk}H_j(\omega)H_j^{*}(\omega)\Phi_{\ddot{u}_l\ddot{u}_m}(\omega)$$

$$- \frac{1}{\omega^2}\sum_{j=1}^{n}\sum_{l=1}^{N}\sum_{m=1}^{N} \phi_{ij}A_{il}\Gamma_{mj}H_j(\omega)\Phi_{\ddot{u}_l\ddot{u}_m}(\omega)$$

$$= S_{X^{qs}X^{qs}}(\omega) + S_{X^dX^d}(\omega) + S_{X^dX^{qs}}(\omega)$$

Standard calculations in finite element softwares (ANSYS, SAMCEF)

Comparison

Approach	Advantages	Disadvantages
1. Direct	- Compact - Direct use of measurement	- Not easy to include local technical noise and system dynamics
2. Time domain	- Nonlinear system - Transcient conditions	- Double Fourier transform - Time consuming
3. Explicit formulation	- Easy to manipulate (subsystems, parameter variations) - Direct use of measurement - Include flexibility	- Linear system - Steady conditions

Conclusions

- Update of 2D power spectral density for LHC tunnel in the vertical direction

- 2D power spectral density for LHC tunnel in the lateral direction

- Vertical and lateral models of the technical noise

- Explicit formulation of the lattice dynamics in the frequency domain

- Application to a single quadrupole