Dynamics of Chiral Symmetry Breaking

Recent developments in computing the chiral condensate

and related low energy constants from the lattice

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Outline of the talk

1. Chirality and strong interactions
2. Lattice Quantum Chromodynamics
3. Chiral fermions and the lattice
4. Chiral dynamics on the lattice

Chirality of visible matter

- The Nature is chiral:
  - neutrinos come left handed in beta decays
  - pions are light compare to other hadrons
- The dark matter waiting for discovery at LHC may be chiral:
  - the Higgs particle may well be a light cousin of dark matter hadrons
- $\Rightarrow$ Understand fundamentally the impact of chirality:
  - phenomenology of spontaneous chiral symmetry breaking is well understood
  - no way to understand the dynamics but theoretically
  - $\Rightarrow$ experimental falsification would invalidate the Standard Model

Spontaneous symmetry breaking

- Light quarks make chiral symmetry plausible
- Quarks are not massless
  - $\Rightarrow$ explicit symmetry breaking
  - PCAC requires:

$$M_\pi^2 \sim (m_u + m_d)$$

- However light pions are not parity doublets
- $\Rightarrow$ chiral symmetry is broken spontaneously
- Gell-Mann-Oakes-Renner formula:

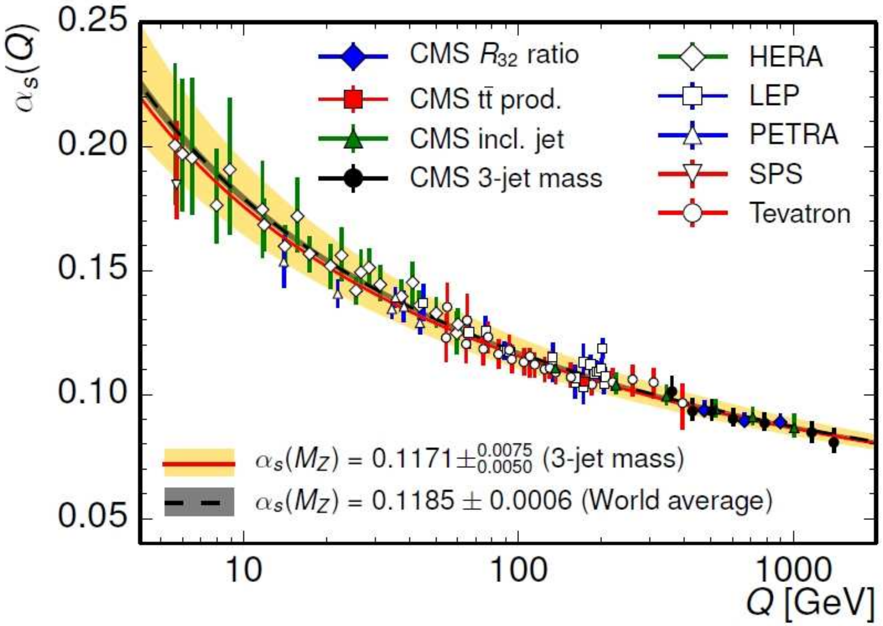
$$M_\pi^2 = (m_u + m_d)\langle 0|\bar{u}u|0\rangle \frac{1}{F_\pi^2}$$

  - Chiral condensate $\Sigma = \langle 0|\bar{u}u|0\rangle$ as order parameter

Strong interactions as a gauge theory

- Deep inelastic lepton-nucleon stattering of SLAC-MIT (1969)
- QCD (Wilczek&Gross,Politzer 1973): effective coupling goes to zero for large momenta:

$$\alpha_S(q) \sim \frac{1}{\ln(q/\Lambda_{QCD})} \,, \qquad \Lambda_{QCD} \sim 200\text{MeV}$$



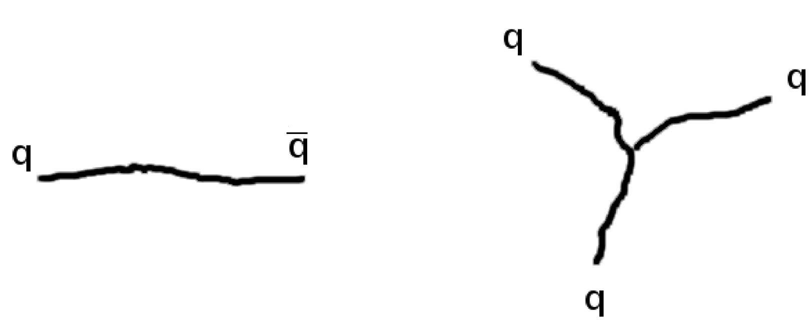
Courtesy CMS collaboration 2015

String picture of hadrons

- QCD on the lattice (K.G. Wilson, Kogut-Sussking 1974)
- Running coupling confirmed on the lattice
- QCD can be studied nonperturbatively at all energy scales
- Quarks are confined:

$$V(R) \sim \frac{\alpha_s}{R} + KR\ , \qquad K = (440\mathrm{MeV})^2$$

- $\Rightarrow$ Mesons ($q\bar{q}$) and baryons (qqq) as strings:



A numerical problem

- Simulate probability amplitude of quarks and gluons:

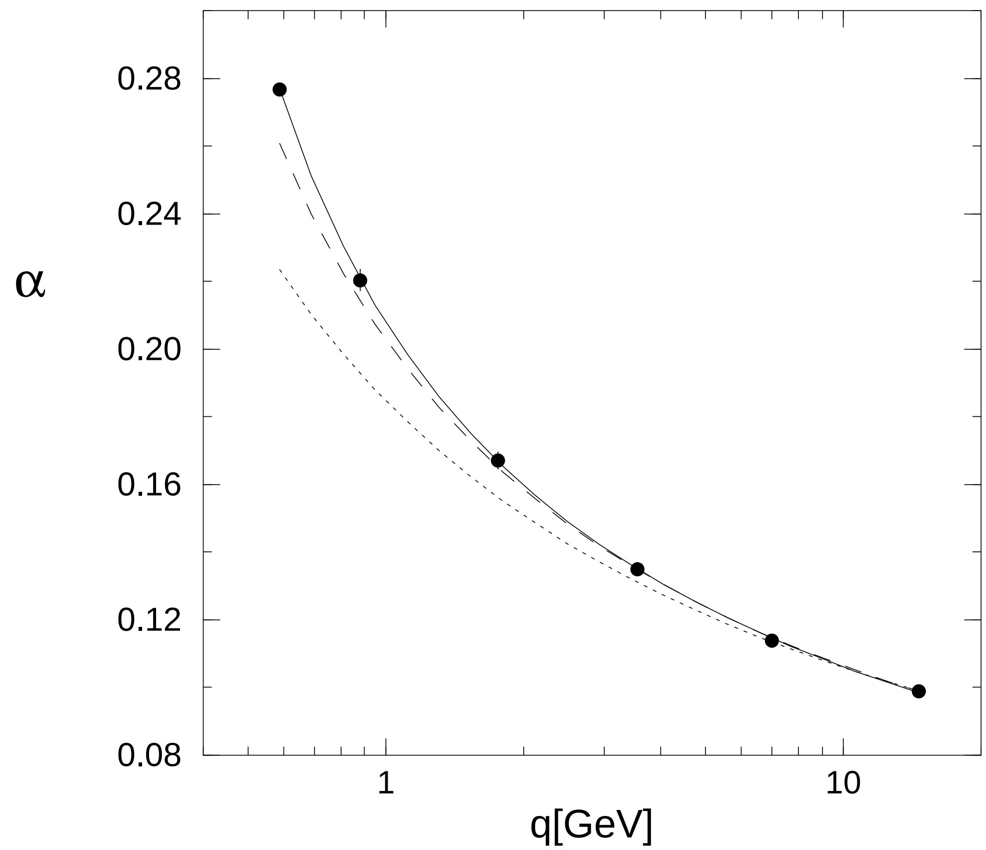
$$Z = \sum_{field\ configuration} e^{-\text{Action}(configuration)}$$

- Evaluate observables on *representative* configurations:

$$\overline{A} = \frac{A(C_1) + A(C_2) + \cdots + A(C_N)}{N}$$

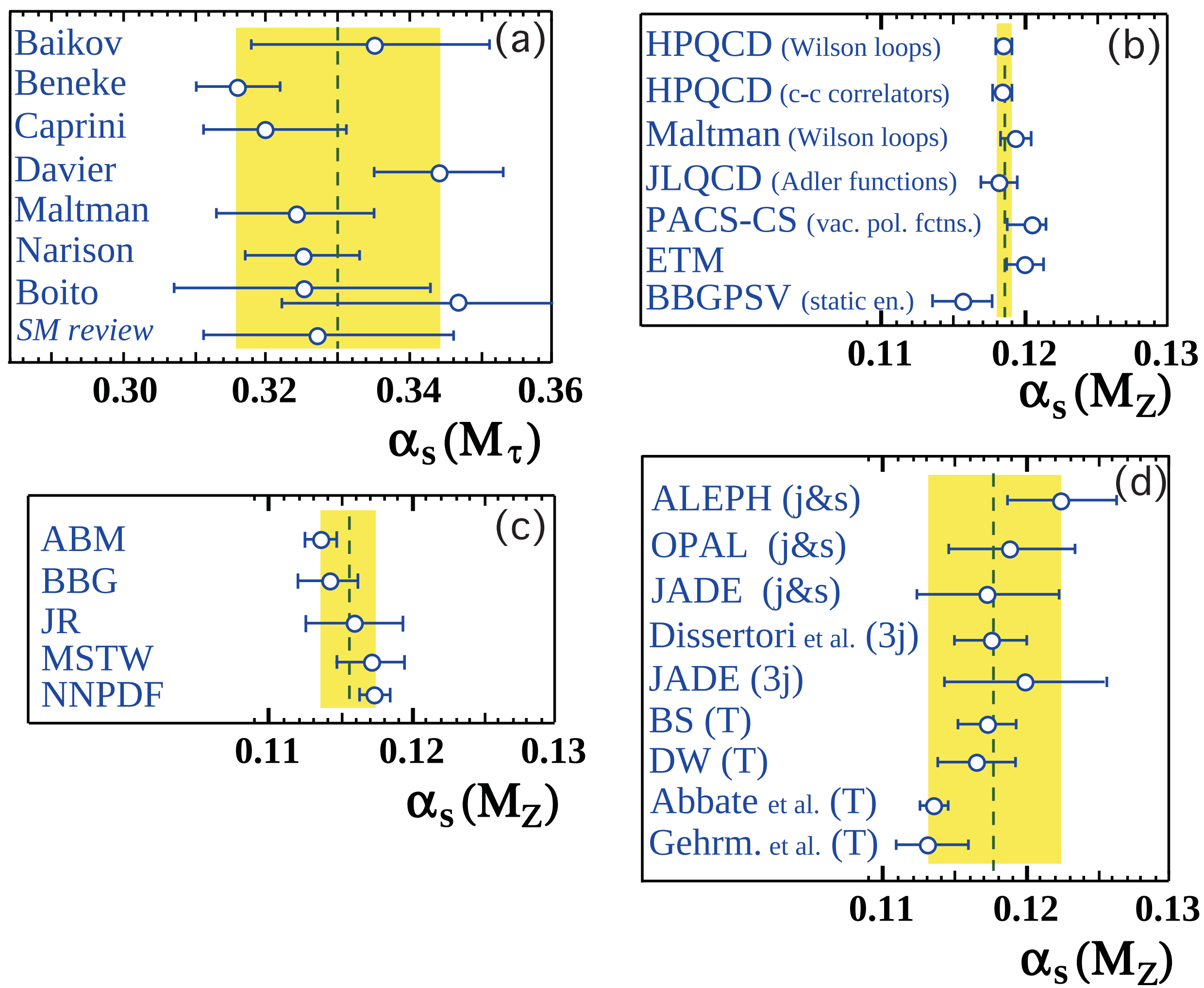
- Effective coupling at all energies
- Hadron spectrum, decay constants, qaurk masses, etc.
- Phases of QCD (talk of de Forcrand)
- Constrain CKM matrix, compute chiral condensate
- ...

The running coupling of $SU(3)$ theory



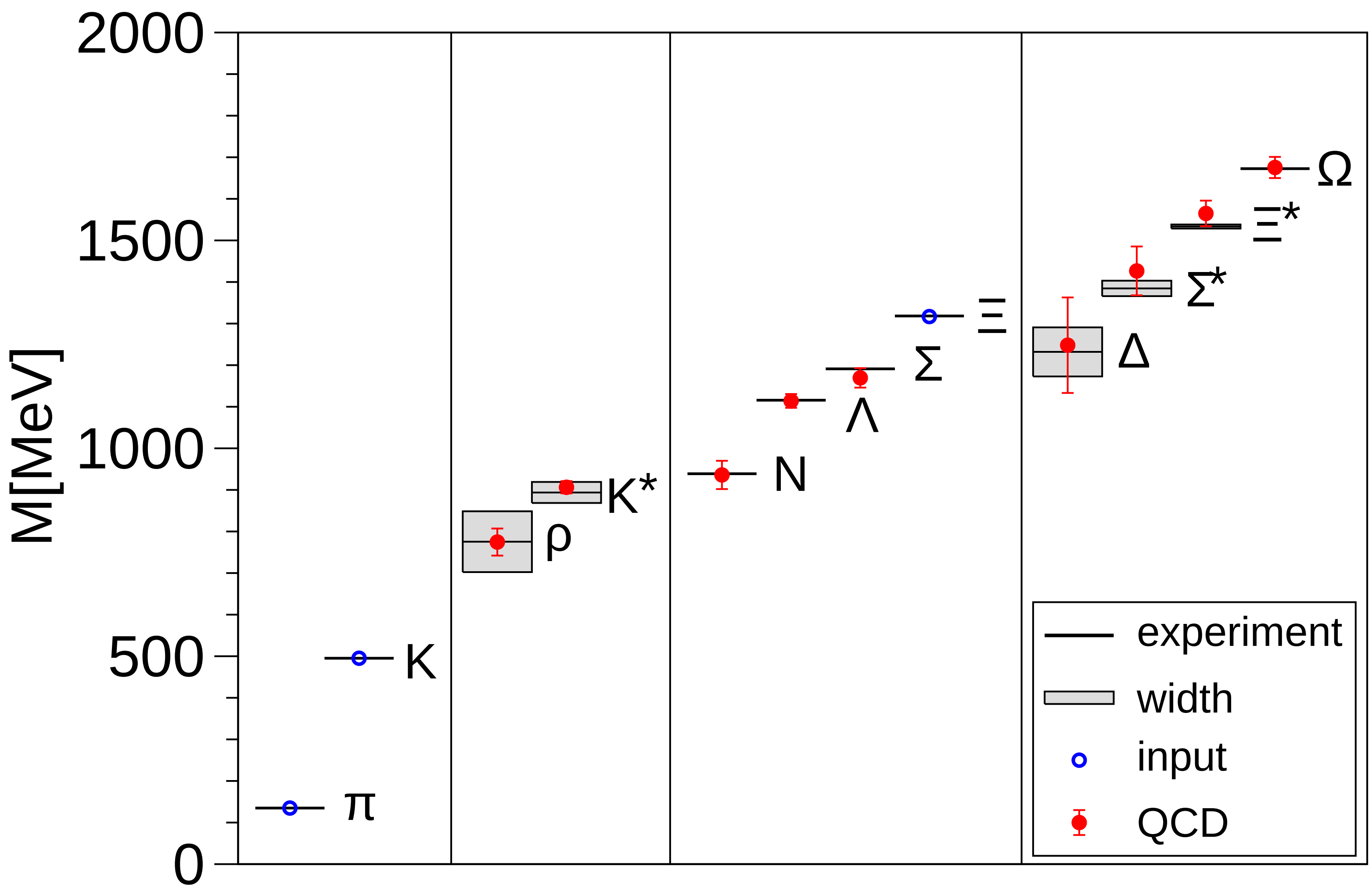
Lattice computations together with the one and two loop results:
(Lüscher, Sommer, Weisz, Wolff 1994)

The running coupling: lattice QCD vs experiment



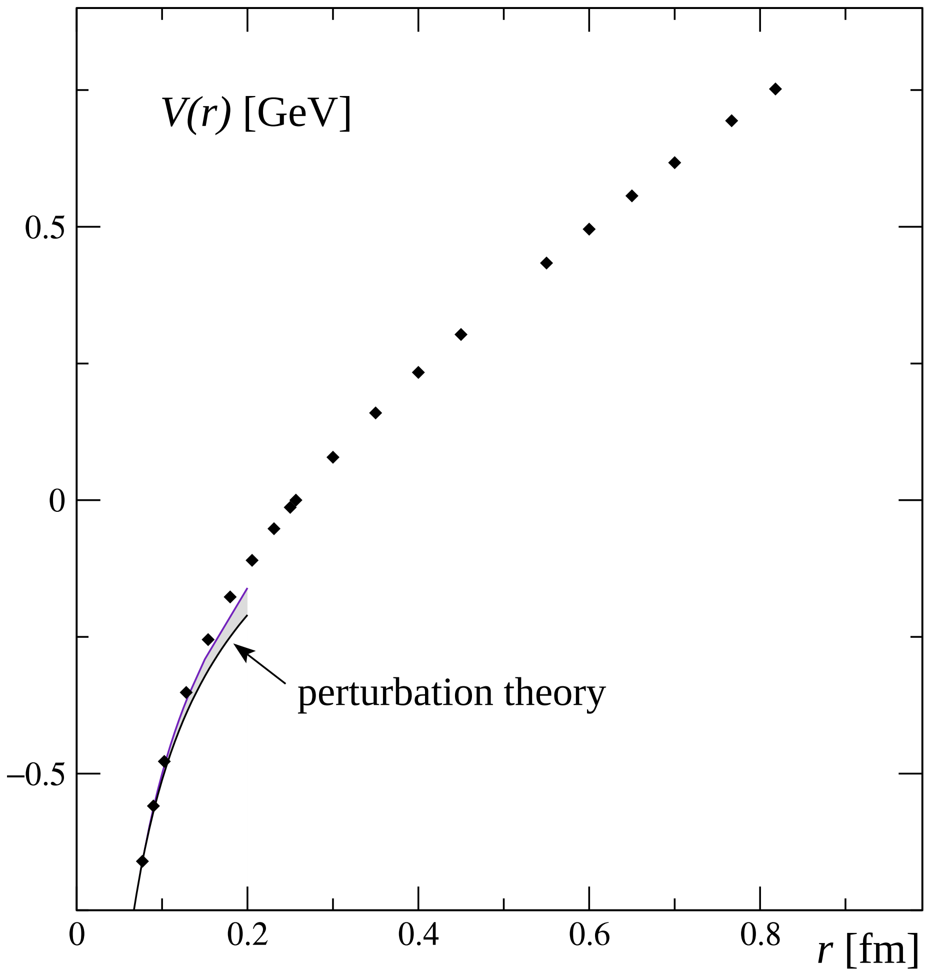
K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 010009 (2014)

Particle spectrum



Dürr et al, Science 322:1224-1227,2008

Quark-antiquark potential



M. Lüscher, P. Weisz, J. High Energy Phys. 07 (2002) 049

Lattice QCD computing requirements

Example: pure gluons on the lattice

- A box $> 3fm$ is needed to have finite size error under control
- A lattice spacing $< 0.1fm$ is needed for reliable continuum limit

  $\Rightarrow$ *a minimum of* 30^4 *lattice sites*
- 4 link plaquette requires 3 matrix multiplications: 324 real `saxpy`-s
- Action evaluation requires 6×30^4 plaquette evaluations

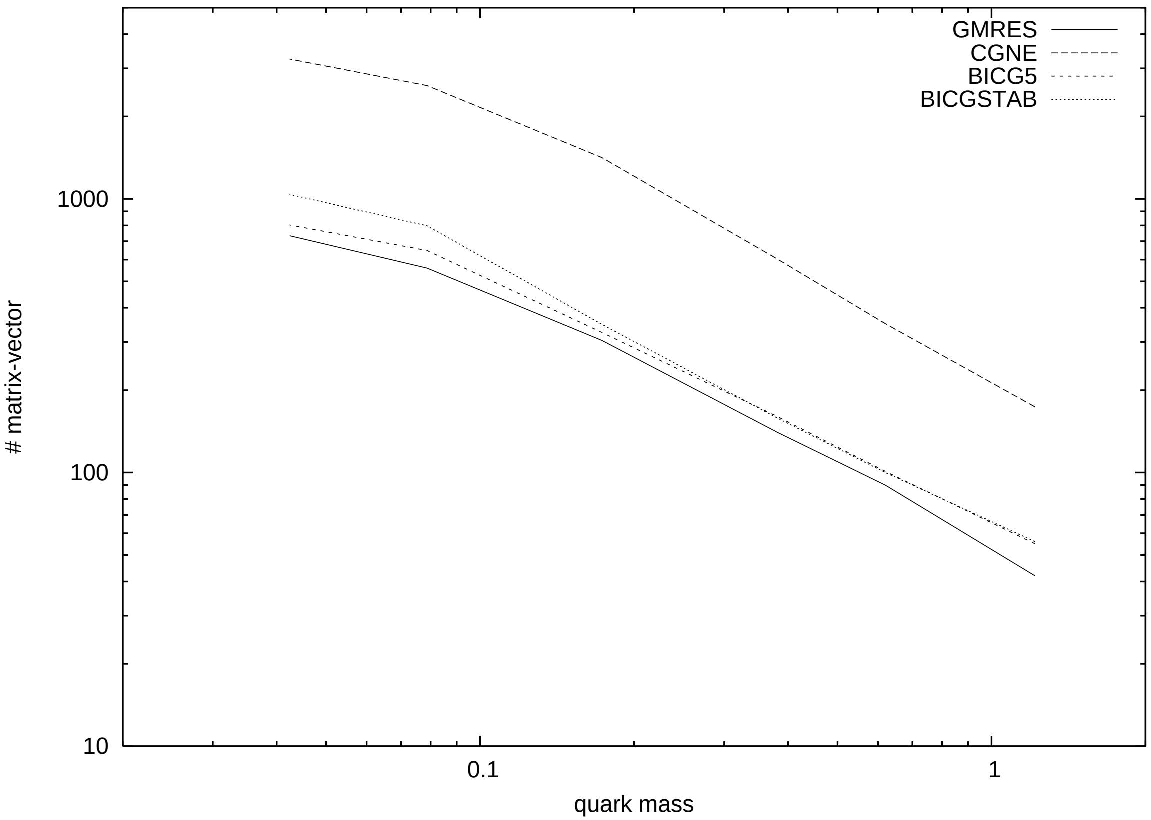
  $\Rightarrow 1574640000 \approx 1.6 \times 10^9$ *real* `saxpy`-*s*
- Assume that a modern processor at $3GHz$ delievers around 10^9 real `saxpy`-s

  $\Rightarrow$ One action evaluation per second: a desktop project today

  $\Rightarrow$ A Cray C90 project in 1990!

Example: Wilson quark propagator

- Solution of large and sparse systems of equations
- Krylov subspace solvers
- Solvers require $O(1000)$ iterations at the physical quark mass



de Forcrand 1996, AB(PhD thesis)

Advances in algorithms

- Low mode factorisation and multiple scale integration (Lüscher 2003,2005)
- Low mode deflation via local coherence (Lüscher 2007)

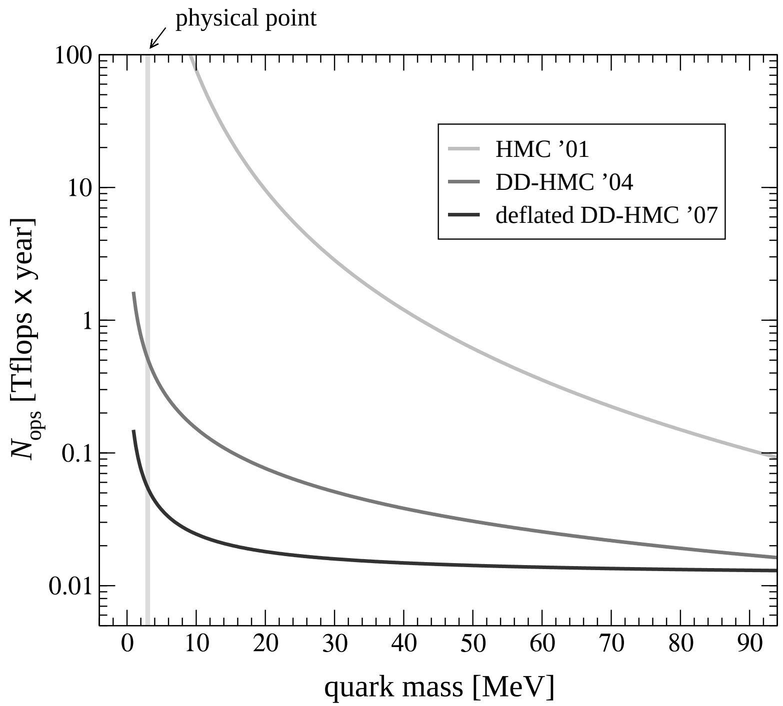


Figure: **saxpy** operations for 100 gauge fields on 64×32^3 lattice at $a = 0.08 fm$

Advances in machines

Parallel computers for lattice QCD (Ukawa 2014)

name	year	authors	CPU	FPU	peak speed
PAX-32*	1980	Hoshino-Kawai	M6800	AM9511	0.5 MFlops
Columbia	1984	Christ-Terrano	PDP11	TRW	–
Columbia-16	1985	Christ et al	Intel 80286	TRW	0.25 GFlops
APE1	1988	Cabibbo-Parisi	3081/E	Weitek	1 GFlops
Columbia-64	1987	Christ et al	Intel 80286	Weitek	1 GFlops
Columbia-256	1989	Christ et al	M68020	Weitek	16 GFlops
ACPMAPS	1991	Mackenzie et al	micro VAX	Weitek	5 Gflops
QCDPAX	1991	Iwasaki-Hoshino	M68020	LSI-logic	14 GFlops
GF11	1992	Weingarten	PC/AT	Weitek	11 GFlops

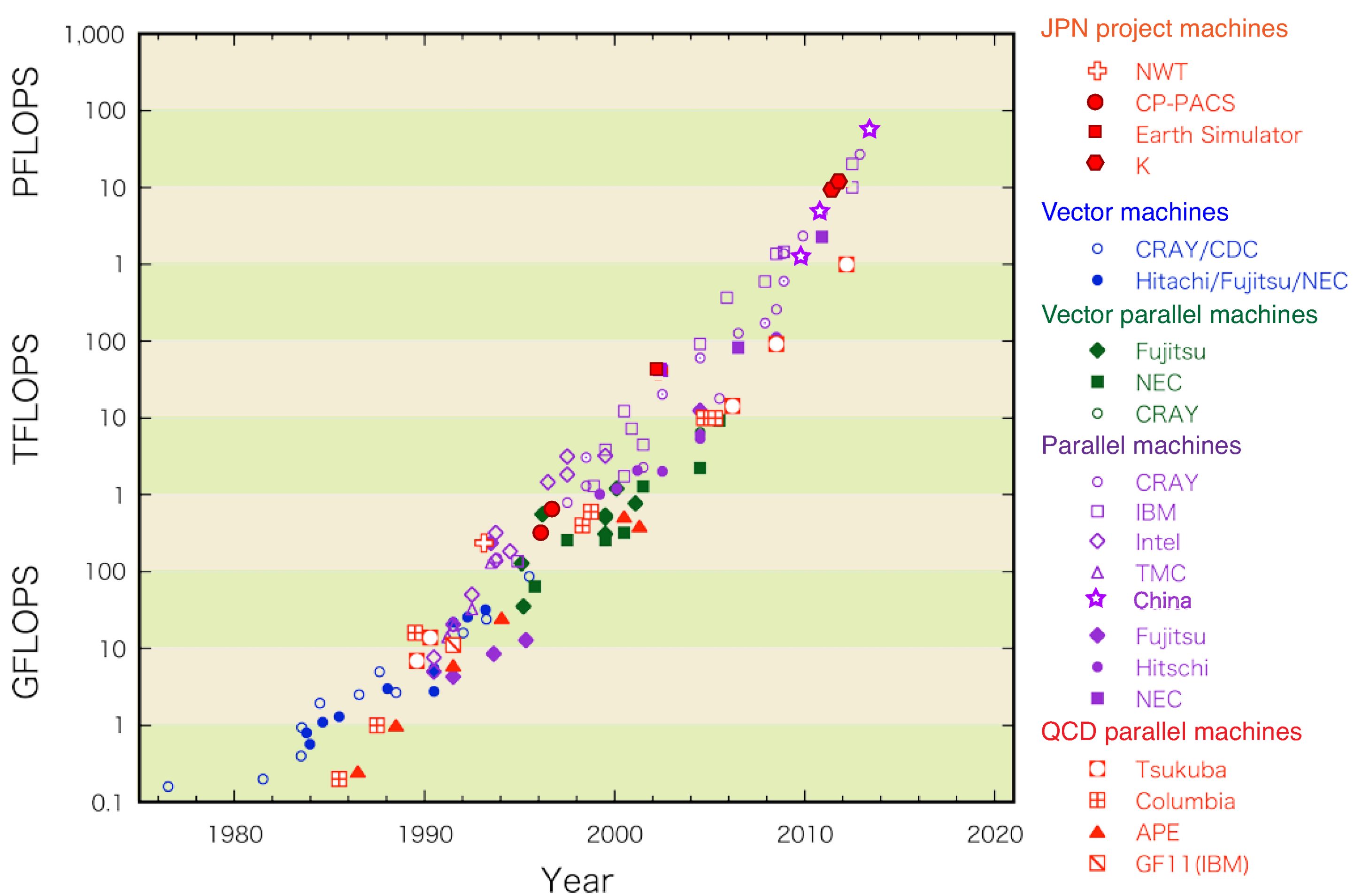
*not for lattice QCD

Advances in machines

Parallel computers for lattice QCD (Ukawa 2014)

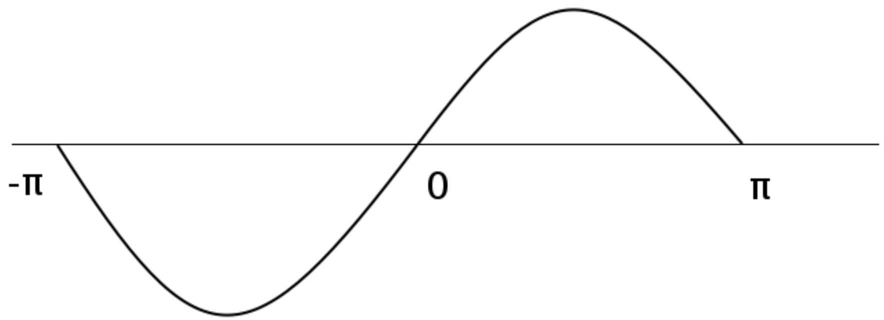
name	year	authors	CPU	vendor	peak speed
APE100	1994	APE Collab.	custom	–	0.1Tflops
CP-PACS	1996	Iwasaki et al	PA-RISC	Hitachi(SR2201)	0.6TFlops
QCDSP	1998	Christ et al	TI DSP	–	0.6TFlops
APEmille	2000	APE Collab.	custom	–	0.8Tflops
QCDOC	2005	Christ et al	PPC-based	IBM(BG/L)	10TFlops
PACS-CS	2006	Ukawa et al	Intel Xeon	Hitachi	14TFlops
QCDCQ	2011	Christ et al	PPC-based	IBM(BG/Q)	500TFlops
QPACE	2012	Wettig et al	PowerXCell	–	200TFlops

Advances in machines



The doubling problem

- Inverse fermion propagator is 2π periodic:



- $\Rightarrow$ 16 number of fermions: opposite chirality partners cannot be avoided
- Wilson (1974): break chiral symmetry and recover in continuum limit
- Kogut-Susskind (1974): reduce the number of doublers to four degenerate quarks
- Karsten-Wilczek (1981), Creutz and AB (2008): reduce the number of doublers to two, break hypercubic invariance and recover in continuum limit (Osmanaj talk)

Chiral fermions on the lattice

A five dimensional world

- Kaplan (1992), Shamir (1995): chiral fermions live in a 4d-domain wall of 5d world
- Neuberger (1998): 4d operator as overlap of two vacua
- Lüscher (1998): the chiral symmetry is exact on the lattice
- Perfect construction with a high price tag: simulate a five dimensional theory
- Special algorithms required (Hyka talk)

The menu of lattice fermions

- Chiral fermions: smart and expensive
- Break a symmetry and recover it in continuum limit: clever and less expensive

The chiral condensate

The mechanism of chiral symmetry breaking

- Isospin breaking is a few percent:

  $\Rightarrow$ almost degenerate u and d quarks

- GMOR relation yields a few MeV quark masses:

  $\Rightarrow$ chiral symmetry broken explicitely by quark masses

- Why then a scalar should condense?
- Quark masses are small $\Rightarrow$ left and right movers are created at will
- QCD strings 

break chiral symmetry
- $\Rightarrow$ only 4 states condense instead of 8:

$$\pi^+ = u\bar{d}\ , \quad \pi^- = d\bar{u}\ , \quad \pi^0 = u\bar{u} - d\bar{d}\ , \quad \Sigma = u\bar{u} + d\bar{d}\ .$$

- $\Rightarrow$ pions and the chiral condensate: the blueprint of spontaneous ChSB

The Dirac mode condensation

Banks-Casher relation

- $\rho(\lambda)$ eigenvalue distribution of Dirac operator
- Chiral condensate:

$$\Sigma = \frac{2m}{V} \sum \frac{1}{m^2 + \lambda^2} = 2m \int_0^\infty \frac{\rho(\lambda) d\lambda}{m^2 + \lambda^2}$$

- If Dirac modes condense at zero $\rho(0) \neq 0 \Rightarrow$ Banks-Casher relation:

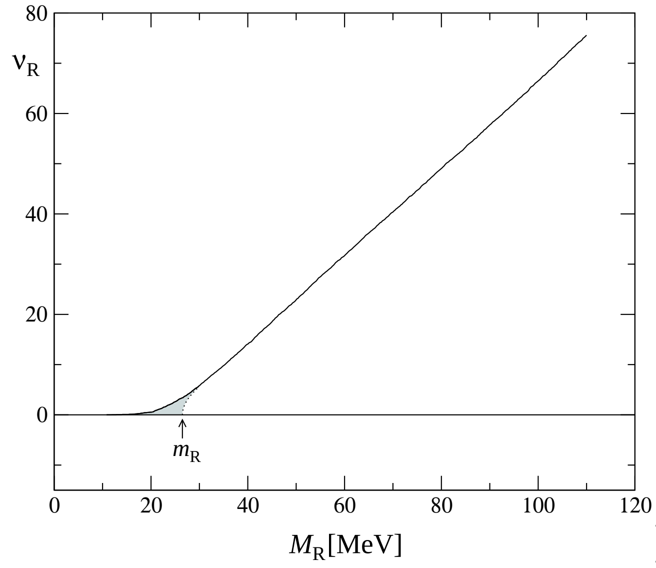
$$\Sigma = \pi \rho(0)$$

- $\Rightarrow$ Dirac mode condensation is SChSB equivalent
- $\Rightarrow$ a method to uncover the dynamics of ChSB on the lattice

The problem with Wilson fermions

- The number ν of eigenvalues as a function of a small cutoff:

$$\Lambda = \sqrt{M^2 - m^2}$$



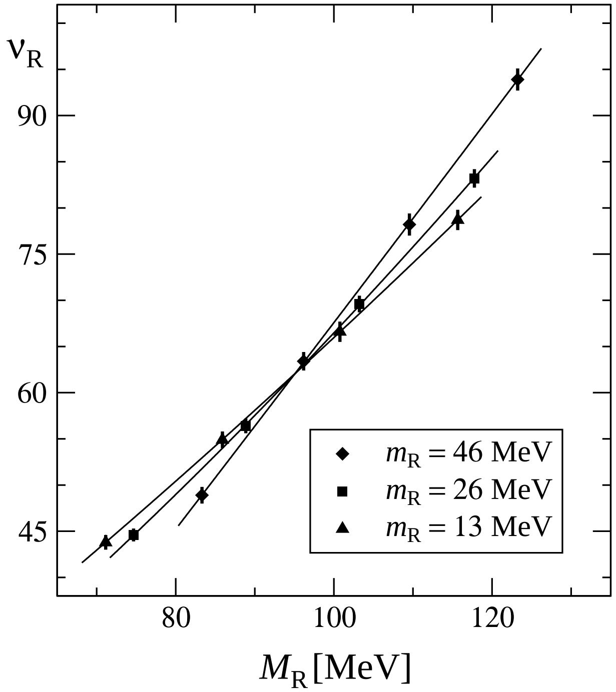
Lüscher 2008

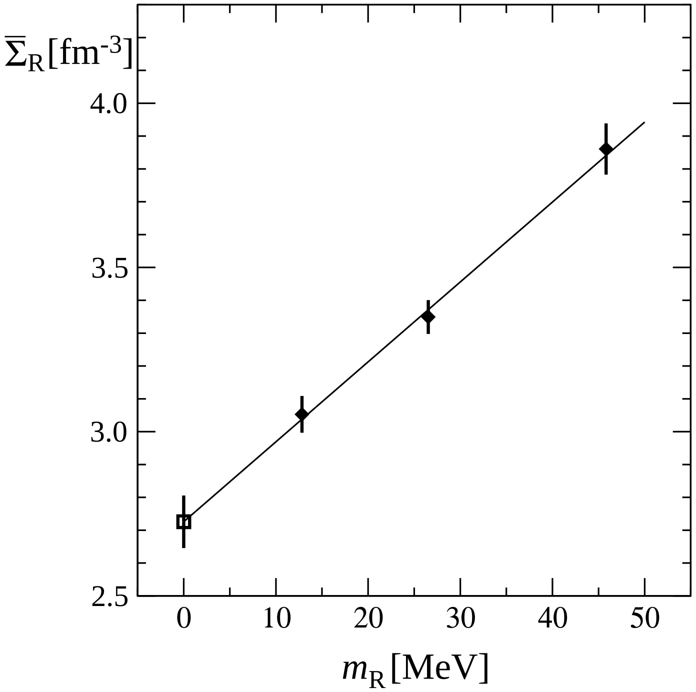
- $\Rightarrow$ estimate the slope around $\Lambda \sim 100\text{MeV}$

Counting modes of the Wilson operator

- Count small modes $\Lambda = \sqrt{M^2 - m^2}$:

$$\nu(M,m) = 2\int_0^{\Lambda} \rho(\lambda)d\lambda \qquad \Leftrightarrow \qquad \Sigma_{\text{eff}} = \frac{\pi}{2}\sqrt{1-\frac{m^2}{M^2}}\frac{\partial\nu(M,m)}{\partial M}$$



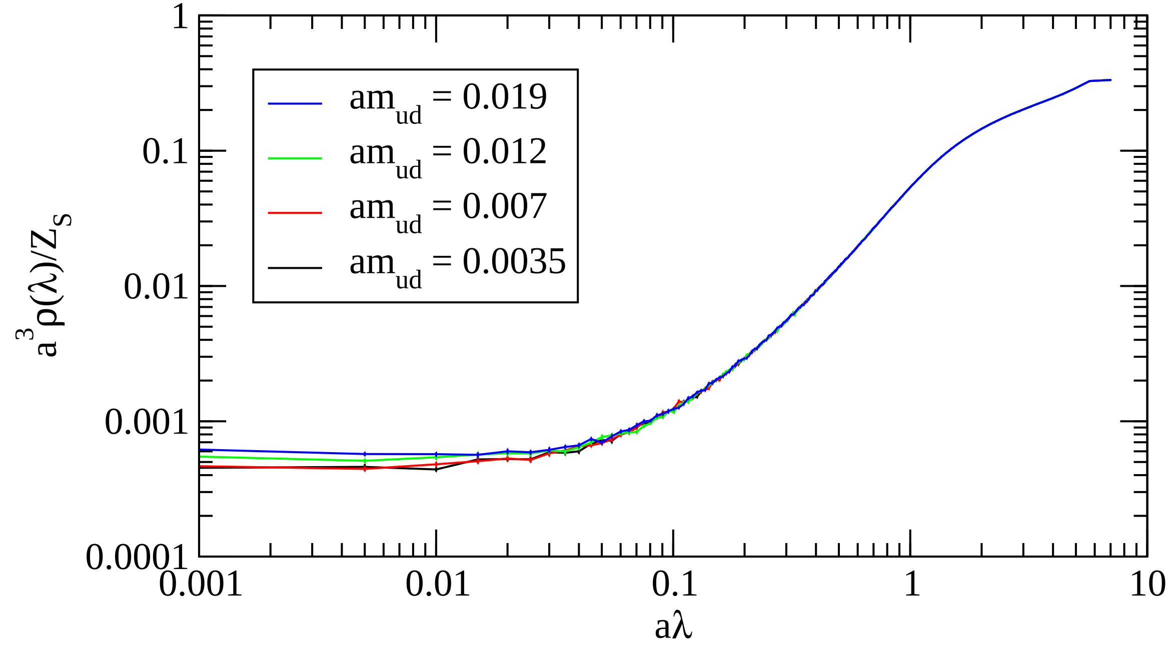


Lüscher 2008

Counting modes of the chiral operator

- Count modes using Chebyshev polynomials:

$$\nu(\lambda) = \sum_j \gamma_j T_j(\lambda)$$

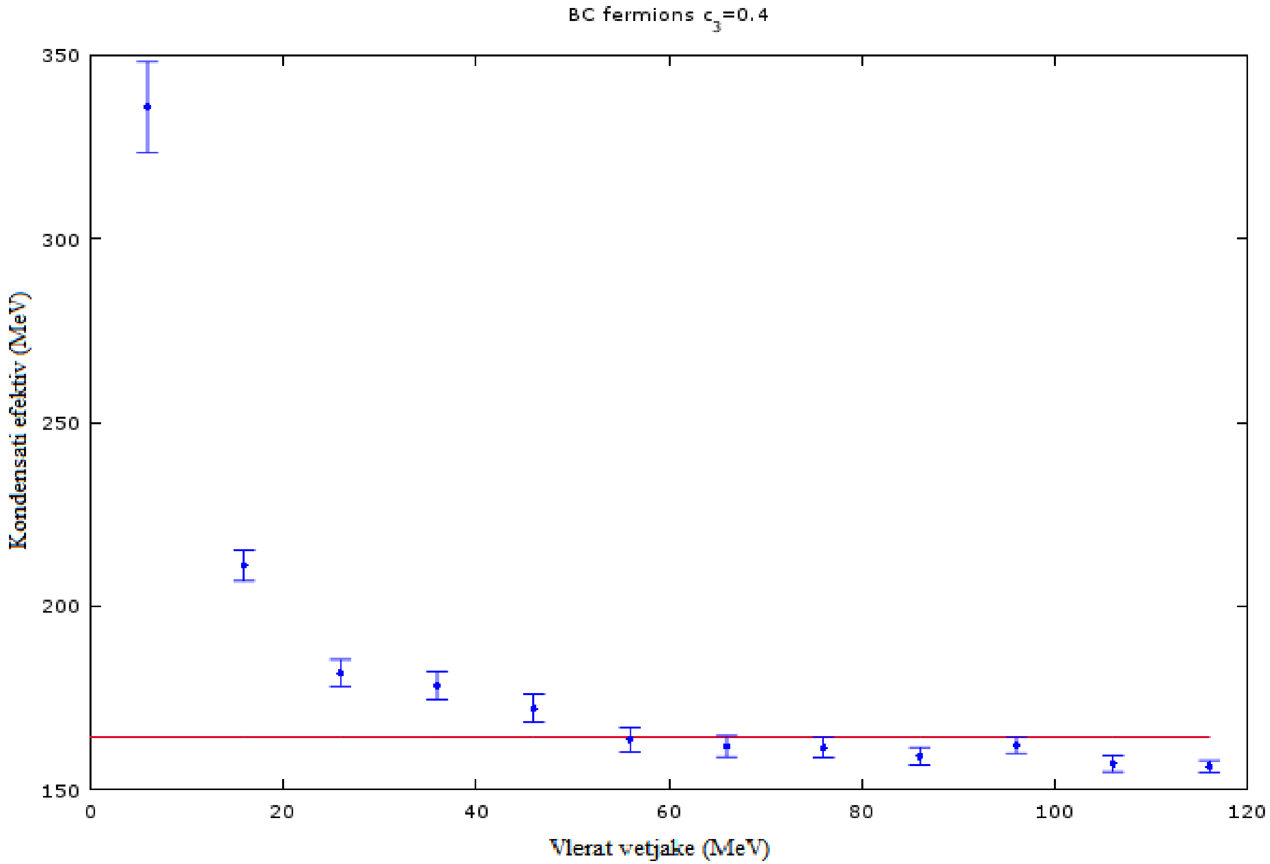


Cossu *et al* 2016

Counting modes of minimally doubled fermion

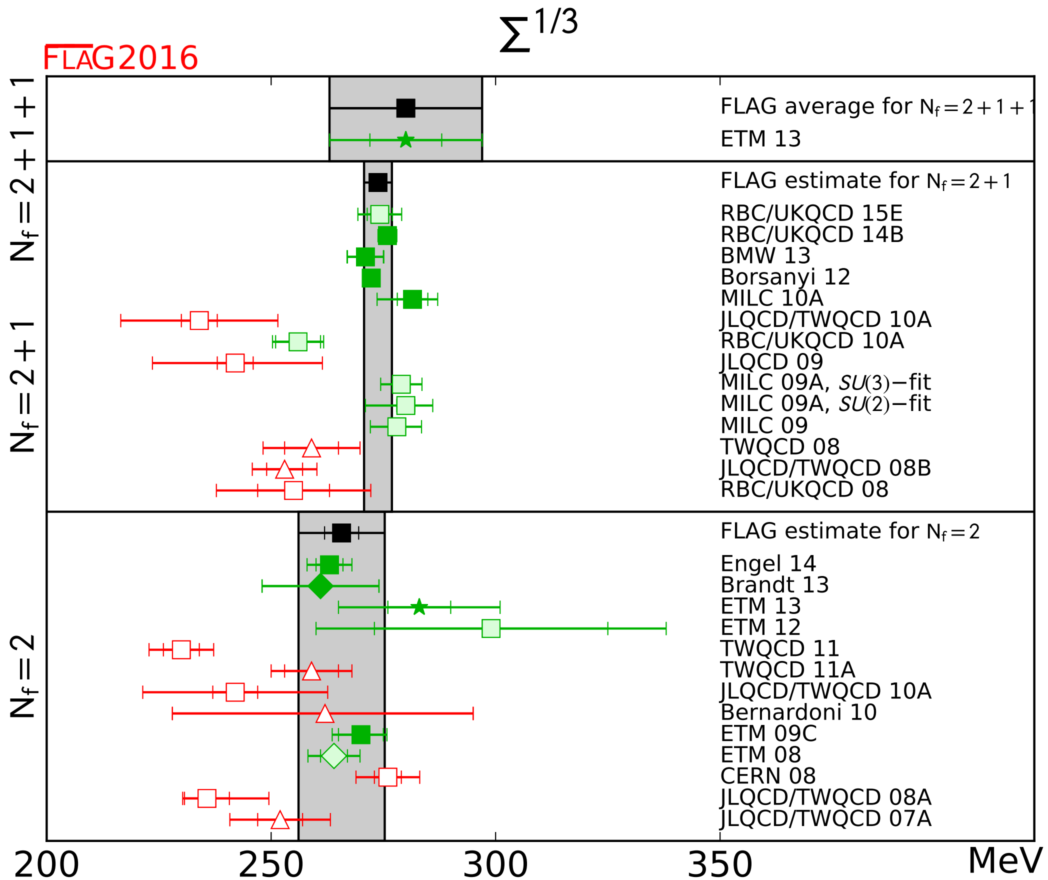
- Count modes using Gauss-Lanczos quadrature:

$$\nu(\Lambda) = \sum_j w_j [1 + \text{sgn}(\Lambda - \lambda_j)]/2 \ , \qquad \Sigma_{\text{eff}} = \frac{\pi}{2} \frac{\nu(\Lambda)}{\Lambda}$$

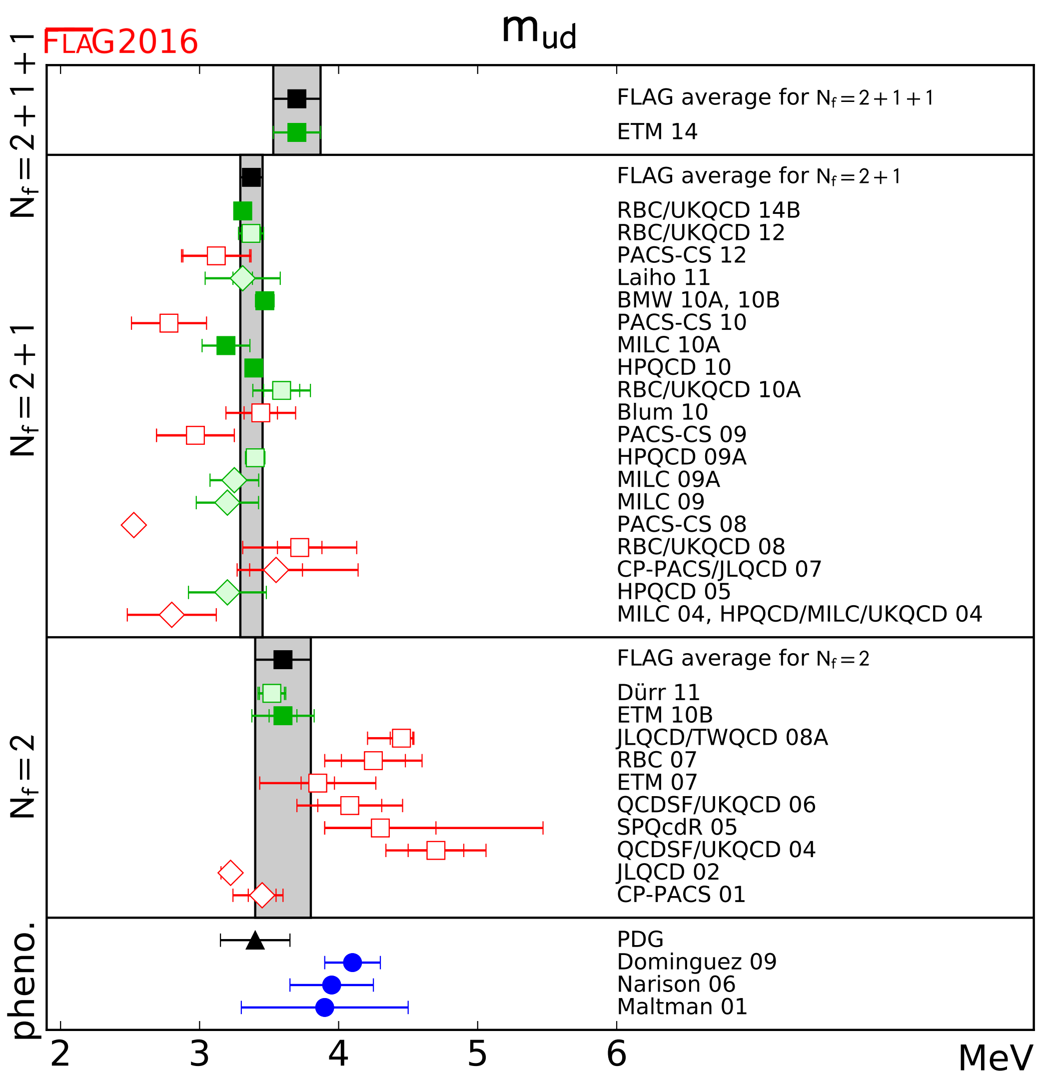


(Osmanaj talk)

Chiral condensate



Aoki et. al. 2016



Aoki et. al. 2016

Chiral dynamics on the lattice

Synthesis of lattice data

- $M_\pi = 134.8(3)\text{MeV}$
- $(m_u + m_d)/2 = 3.70(17)\text{MeV}$
- $\Sigma^{1/3} = 280(8)(15)\text{MeV}$
- $F_\pi / F = 1.076(2)(2)$
- $\sqrt{2} F_\pi^{\text{exp}} = 130.41(20)\text{MeV}$
- $\Rightarrow$ Gell-Mann-Oakes-Renner equation:

$$M_\pi^2 = (m_u + m_d)\langle 0|\bar{u}u|0\rangle \frac{1}{F_\pi^2}$$

is satisfied at one sigma level.

Conclusions

- Lattice computations have matured:
  - increased computer power
  - advances in algorithms
- QCD works in explaining the experiment
- The chiral symmetry breaking can be understood from the Standard Model
- Lattice calculation and LHC will help to falsify new theories

Lattice careers

Learning outcomes:

- Particle physics theory
- Numerical algorithms
- Parallel programming
- Collaborate in a multi-disciplinary and inter-disciplinary enviroment

Career paths:

- University and research labs
- Big data analytics and machine learning
- Computing related industries
- Risk management, investment fonds
- Private consultancy