The summary of my beam dynamics studies for AWAKE

Alexander Gorn

Novosibirsk State University
Budker Institute for Nuclear Physics

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Outline

- Earth’s magnetic field issue in AWAKE Run I.
- Beam loading in AWAKE Run II.
- Ionization injection.

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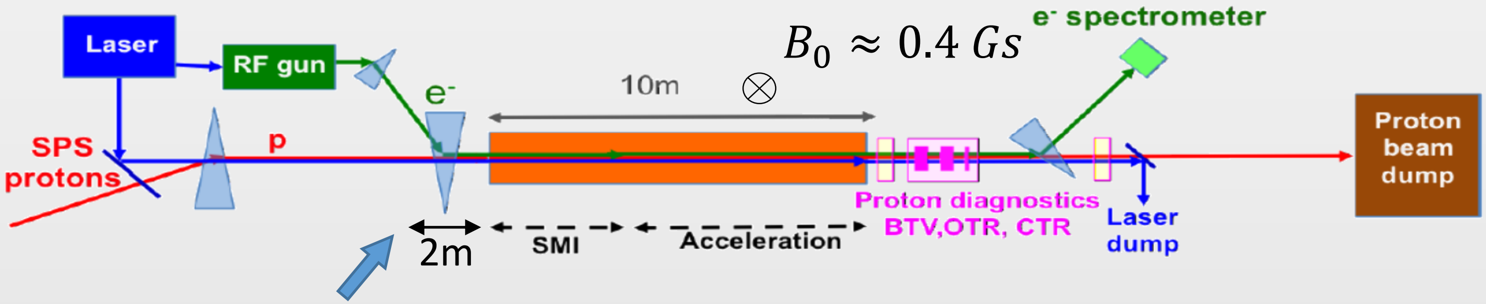
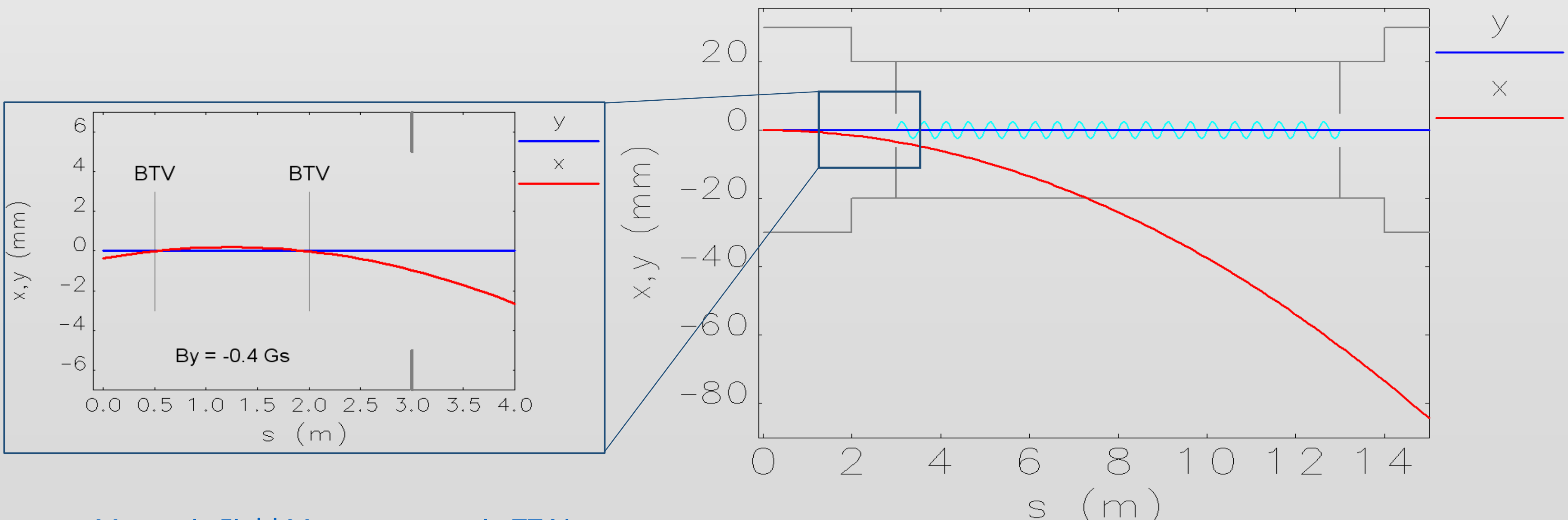
Earth’s magnetic field issue

Earth’s magnetic field rejects the electron beam inside empty plasma cell, so without any corrections it may be dumped to the wall. Also beam gains the significant unpredicted offset before the injection point.



The last diagnostics for the electron beam before plasma cell



[Magnetic Field Measurements in TT41]

Earth's magnetic field issue

$\vec{r}(t) - ?$
$\vec{p}(t) - ?$

Without proton beam:

$$R = \frac{pc}{eB_0} \approx 1.33\text{km}$$

With proton beam:

$$\begin{cases} \dfrac{d\vec{r}}{dt} = \vec{V}(\vec{r}) \\ \vec{p}(\vec{r}) = \gamma m_e \vec{V}(\vec{r}) \\ \dfrac{d\vec{p}(\vec{r})}{dt} = -e\vec{E}(\vec{r}) - \dfrac{e}{c}\left[\vec{V}(\vec{r}) \times \vec{B}(\vec{r})\right] \end{cases}$$

We solve this equation system numerically

Proton beam parameters:

$I_b = 50\text{A}$

$\sigma_r = 0.2\text{mm}$

$p_z = 400\text{GeV/c}$

Beam generate fields:

$$E_r(r) = \frac{2I_b}{rc}\left(1 - e^{-\frac{r^2}{2\sigma_r}}\right)$$

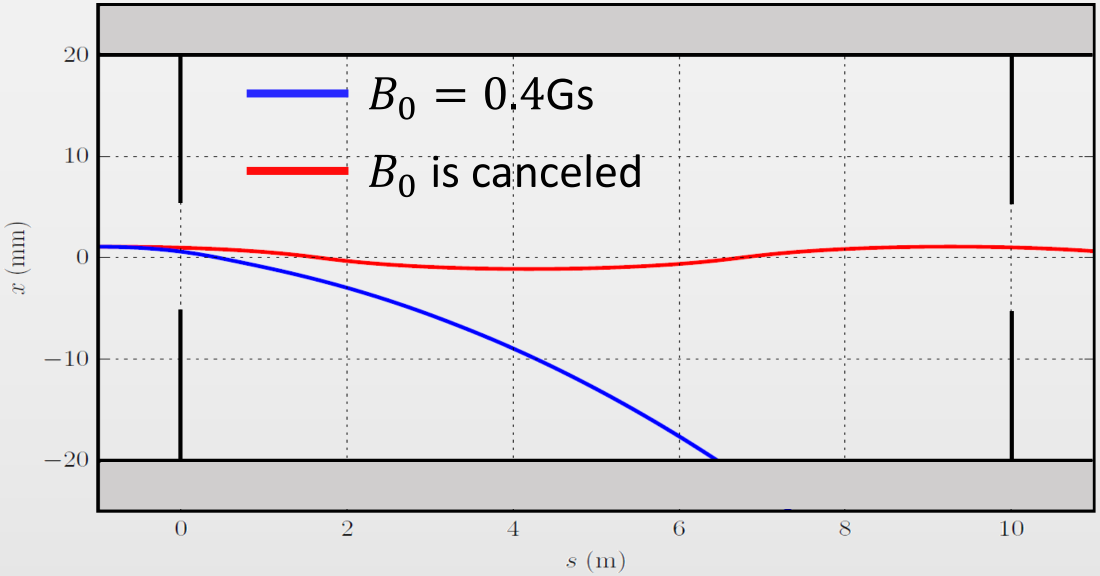
$$B_\varphi(r) = \frac{V_b}{c} E_r(r)$$

Earth's magnetic field:

$B_y = -B_0 = -0.4\text{Gs}$

See more details in my report.

Earth's magnetic field issue

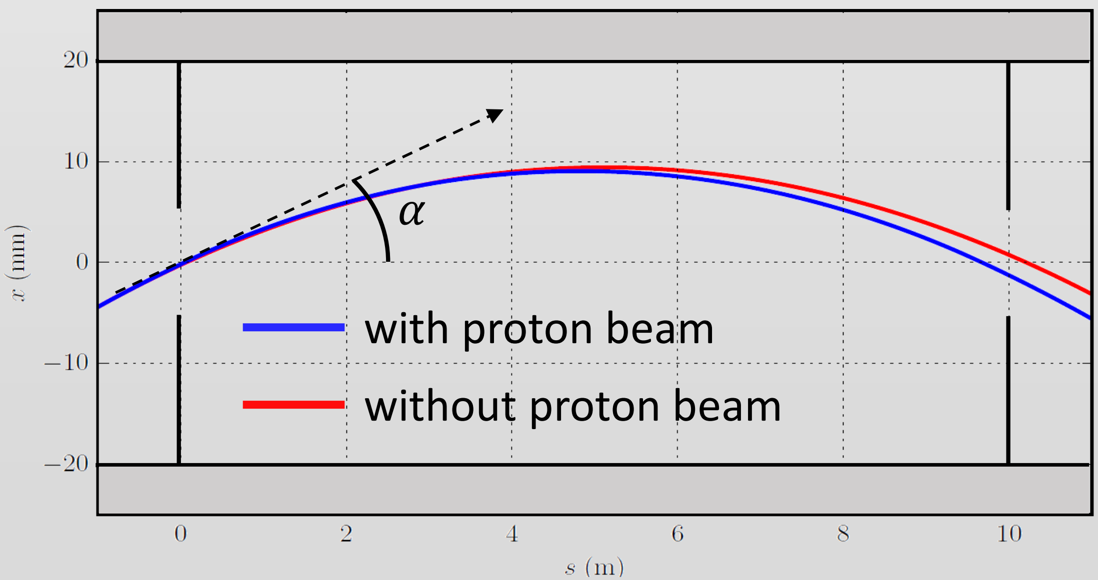


Option 1: to cancel the magnetic field

Proton beam field is not strong enough to trap the electrons.

Even for $r = \sigma_r$ ($\approx$ max E_r) B_0 must be less, than 0.3Gs.

Earth's magnetic field must be canceled by a significant factor.



Option 2: to launch the beam at some angle

The electron must have $\alpha \approx 3.8$mrad at the first orifice to pass through the center of second one.

Proton beam does not change the electron trajectory significantly.

See more details in my report.

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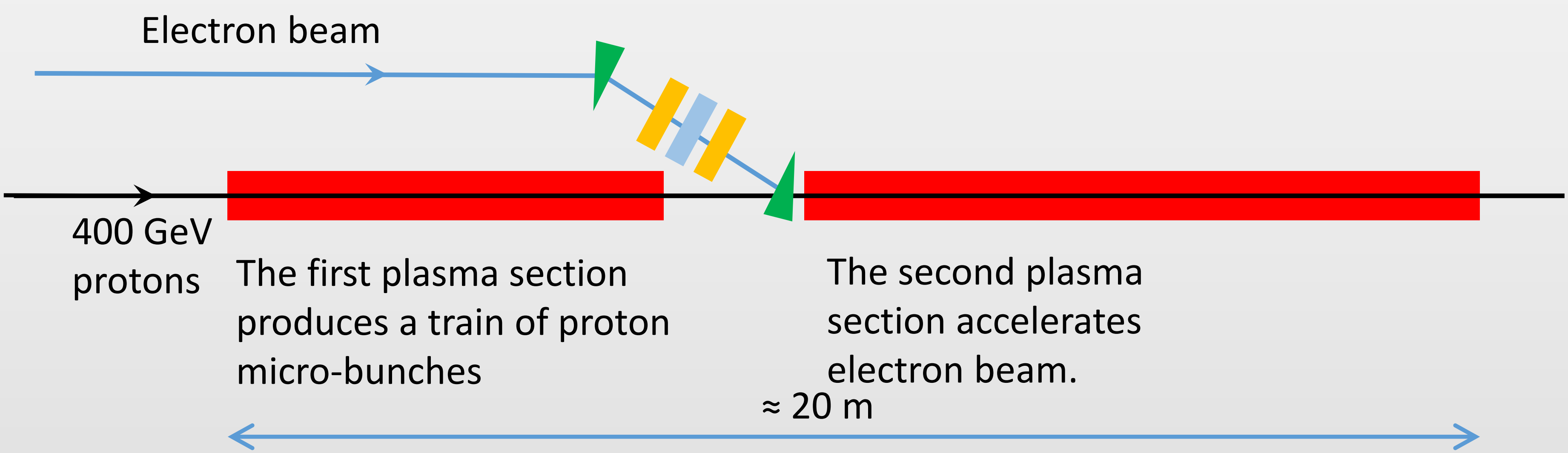
Beam loading in AWAKE Run II

Run II goal: accelerate significant electron charge with the best possible beam quality and the highest possible accelerating gradient (achievable with the SPS proton beam and SMI).

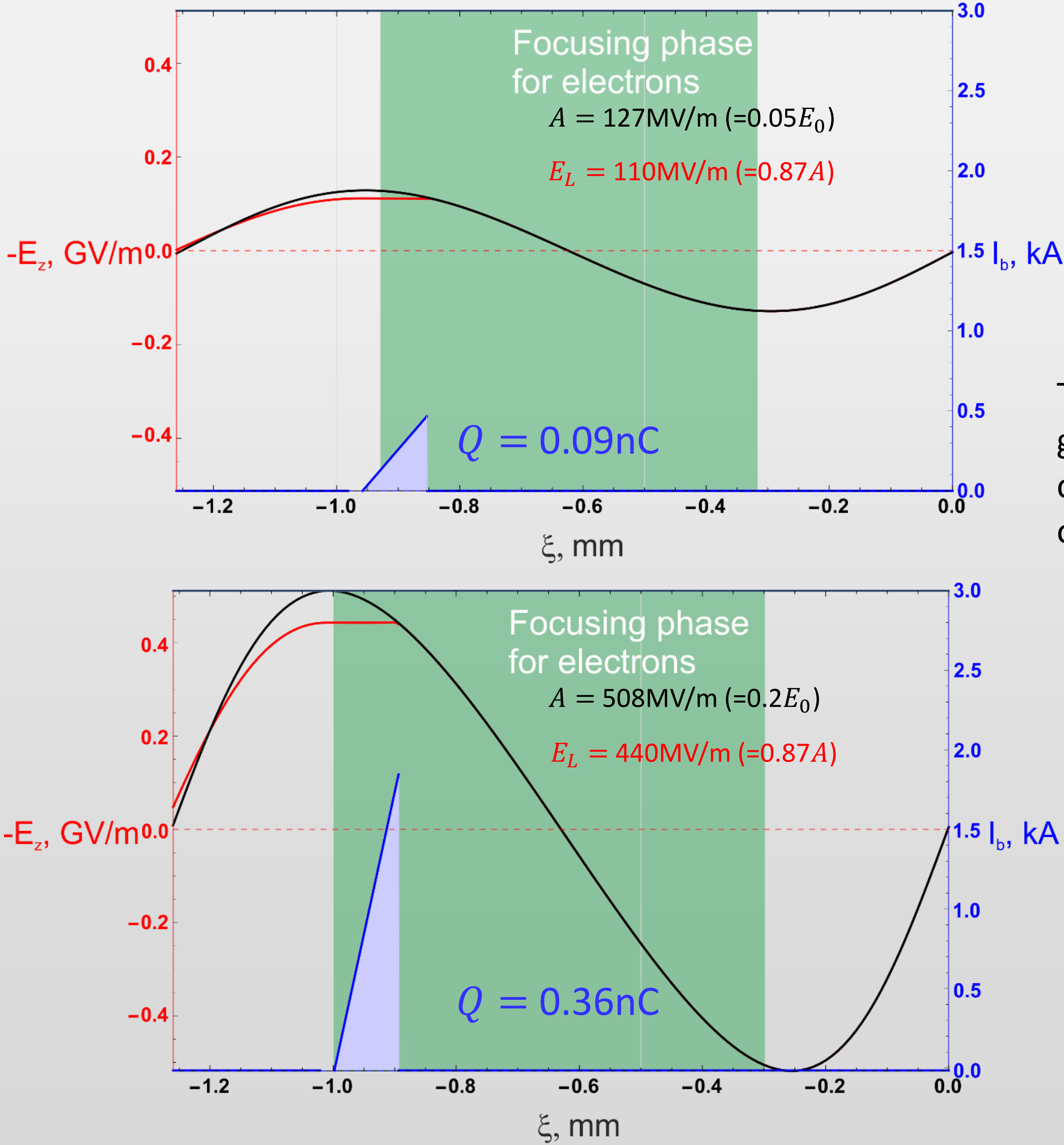


It was Simon van der Meer, who suggested to use triangular shaped beams to reach uniform accelerating gradient.

Ideal beam loading case shows us how much charge we can accelerate effectively.

See more details in my report.

Beam loading in AWAKE Run II



A – wakefield amplitude
n_0 =7·10^{14} cm^{-3} – plasma density
E_0 = 2.54GV/m – wavebreaking field
E_L – loaded wakefield magnitude
Q – beam charge

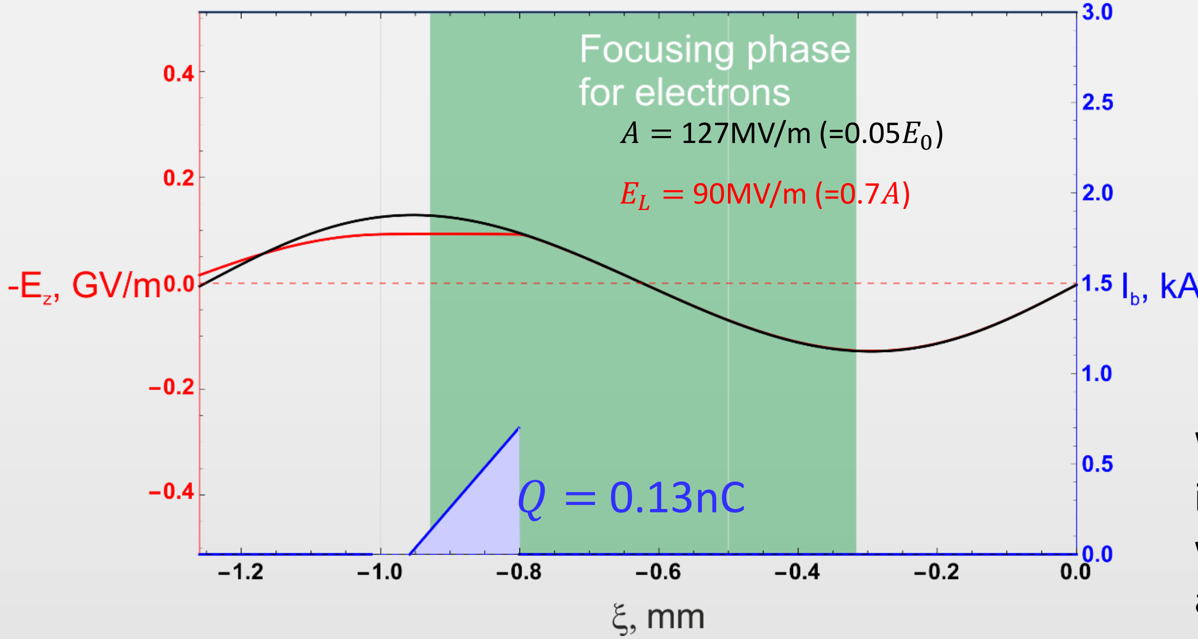
To conserve the right beam loading for given injection point ξ_0 we should change beam and plasma parameters in order to:

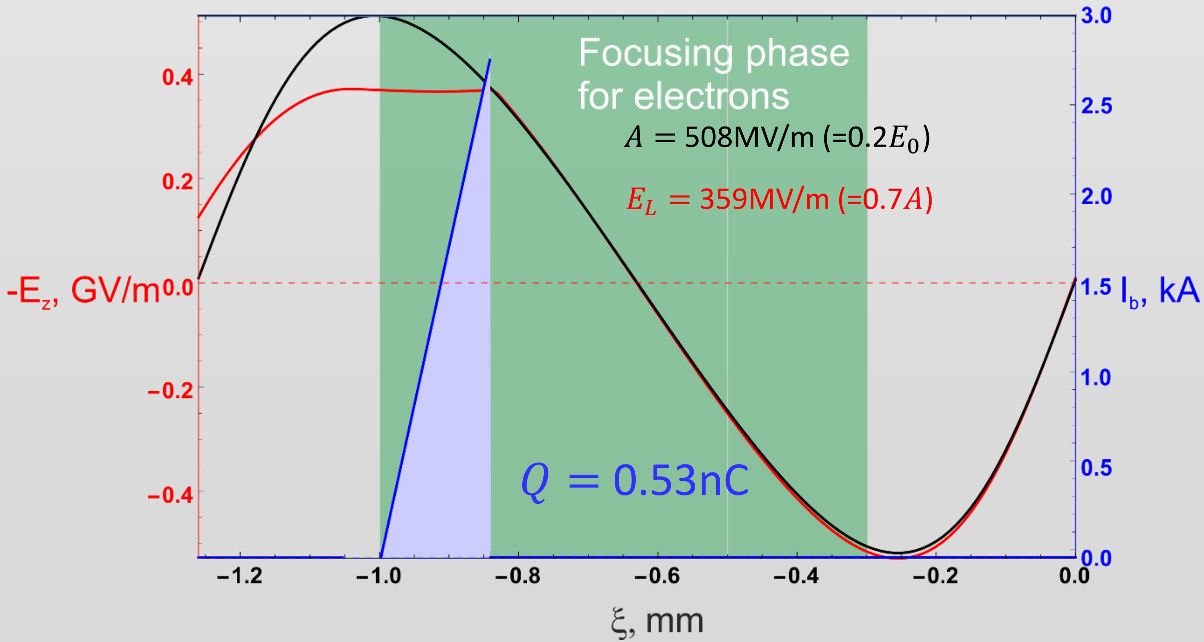
$$\frac{I_b E_0}{A \sigma_{x,y}^2 n_0} = \text{const}$$

Simulation was made for gaussian beam with $\sigma_{x,y}$ = 0.2mm (=1 c/ω_p)

See more details in my report.

Beam loading in AWAKE Run II





A – wakefield amplitude

n_0 =7·10^{14} cm^{-3} – plasma density

E_0 = 2.54GV/m – wavebreaking field

E_L – loaded wakefield magnitude

Q – beam charge

We can accelerate more charge by injecting beam further from the wakefield extremum, but it will reduce accelerating gradient.

$$I_b \sim \sin(k_p \xi_0)$$

$$l_b = \tan(k_p \xi_0)$$

$$E_L = A\cos(k_p \xi_0)$$

Simulation was made for gaussian beam with $\sigma_{x,y}$ = 0.2mm (=1 c/ω_p)

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Ionization injection

As recently suggested by Erdem ionization injection can be considered for AWAKE. Plasma wakefield of high enough amplitude can trap free-standing electrons.

How to create free standing electrons?

To add some buffer gas to rubidium vapor and ionize it by another laser pulse.

	Ionization Energy, eV	Threshold Field, GV/m	Threshold Laser Intensity, TW/cm^2
H	13.6	26.7 ($10.5E_0$)	100
Li	5.38	5.5 ($2.18E_0$)	4
Na	5.14	5.0 ($2E_0$)	4
K	4.34	3.8 ($1.5E_0$)	2
Rb	4.18	3.6 ($1.4E_0$)	1.7
Cs	3.38	3.1 ($1.23E_0$)	1

See more details in my report.

Ionization injection

$$E(z,t) = A\cos\left(k_p\,(z-ct)\right)$$ −1D wakefield

$$\begin{cases} \dfrac{dz}{dt} = V_z\,(z) \\ p_z(z) = \gamma m_e V_z\,(z) \\ \dfrac{dp_z(z)}{dt} = -eE(\mathrm{z}) \end{cases}$$

$$E(z,t) = -\frac{\partial\Phi(z,t)}{\partial(z-ct)}$$

$$\Phi(z,t) = -\frac{A}{k_p}\cos(k_p\,(z-ct))$$

See more details in my report.

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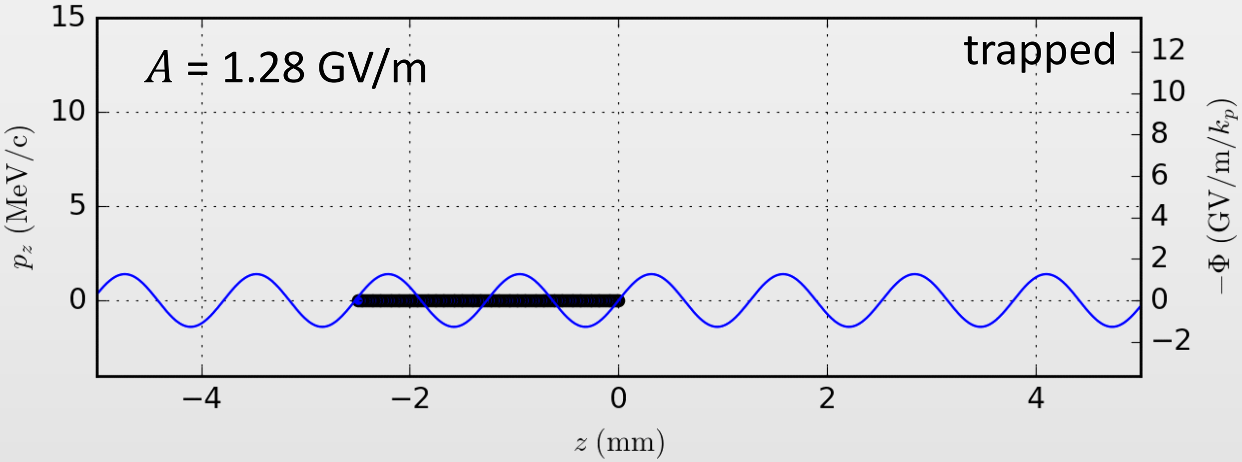

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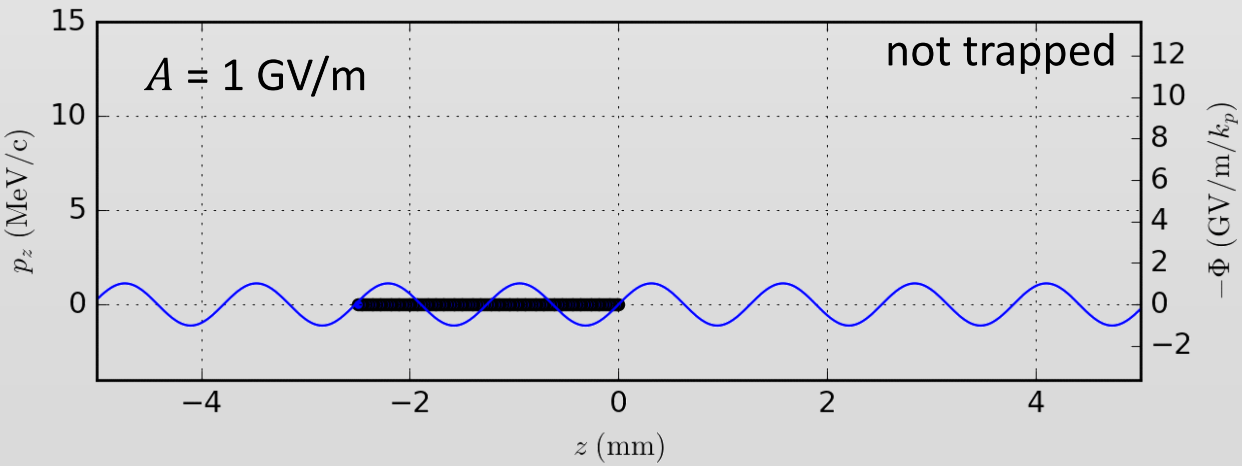

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Ionization injection

$$E(z,t) = A\cos\left(k_p\,(z-ct)\right) \quad -\text{1D wakefield}$$

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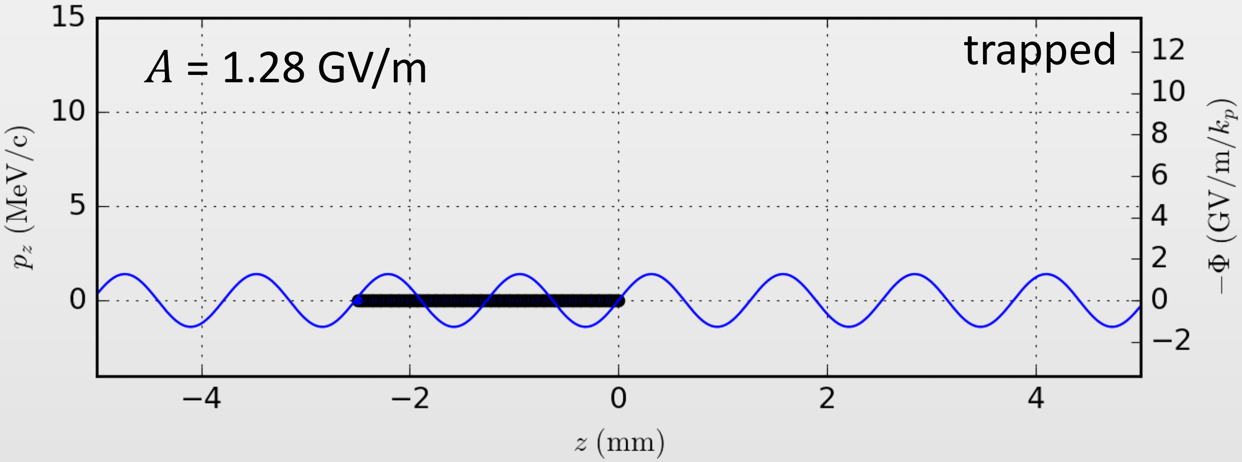

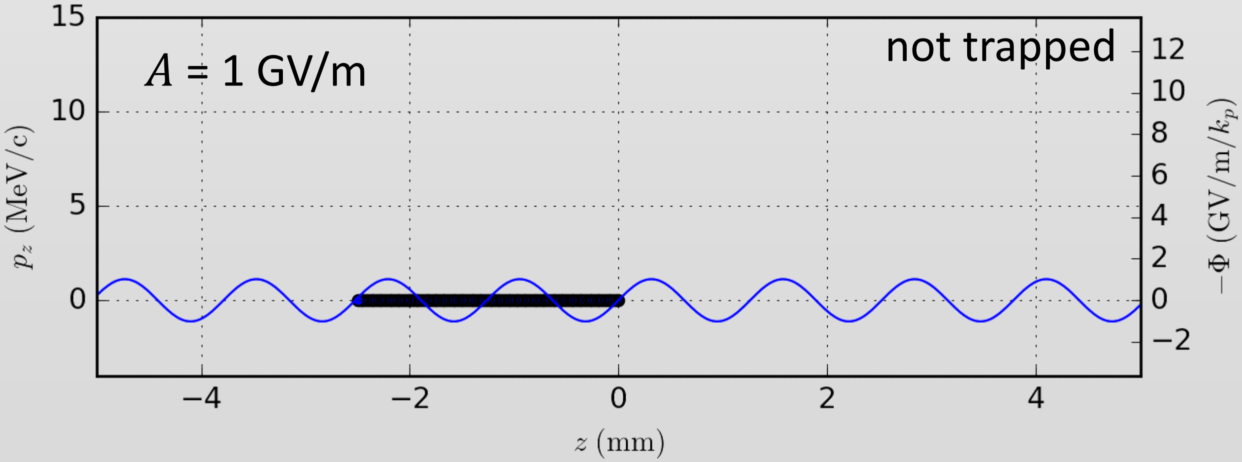

$$E(z,t) = -\frac{\partial\Phi(z,t)}{\partial(z-ct)}$$

$$\Phi(z,t) = -\frac{A}{k_p}\cos(k_p\,(z-ct))$$

What particles will be trapped?

$$\Phi_i \le \Phi_{max} - \frac{mc^2}{e}$$

See more details in my report.

Ionization injection

$$E(z,t) = A\cos\left(k_p\,(z-ct)\right) \quad \text{—1D wakefield}$$

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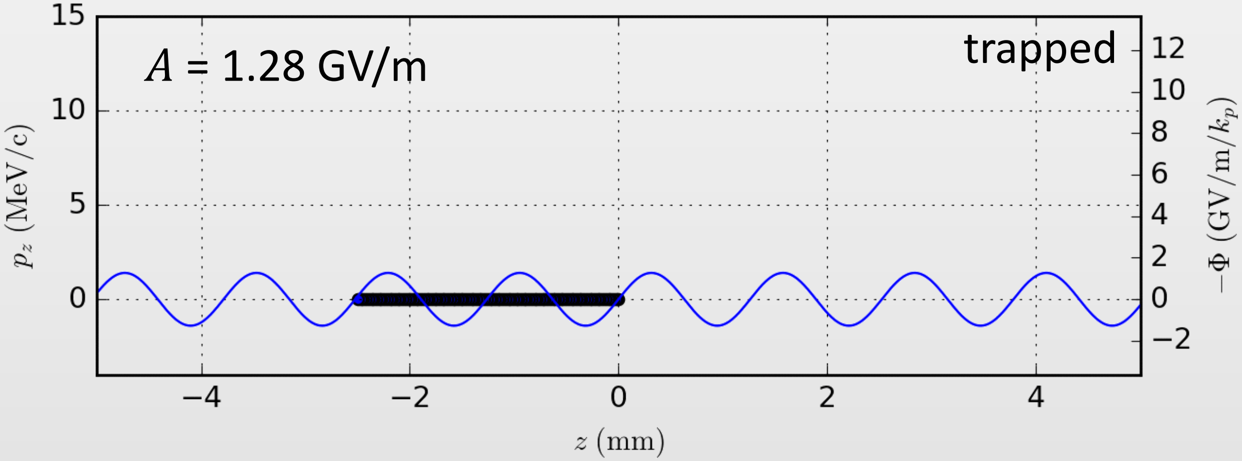

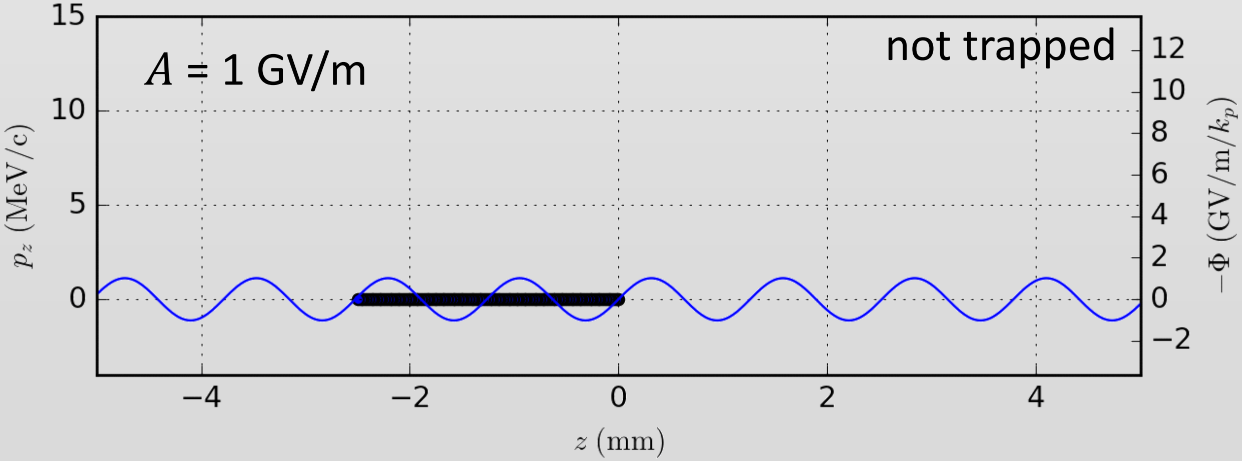

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What particles will be trapped?

$$\Phi_i \leq \Phi_{max} - \frac{mc^2}{e}$$

$$A \geq \frac{E_0}{2}$$

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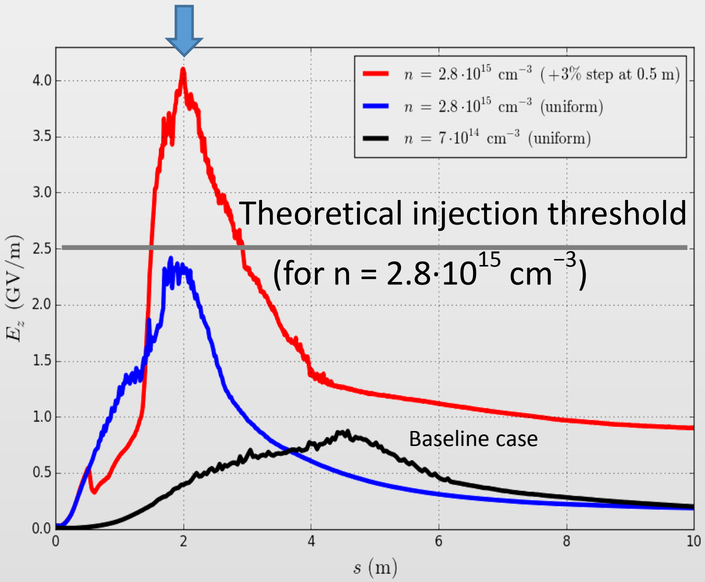
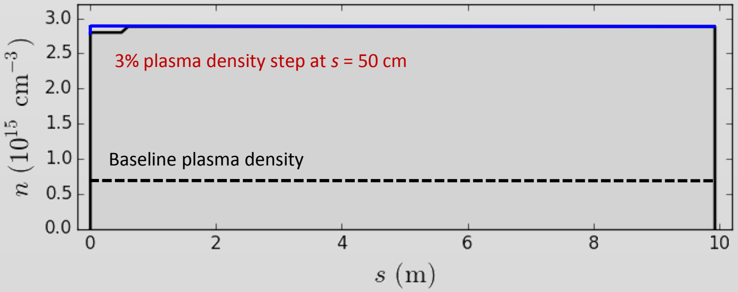
Ionization injection

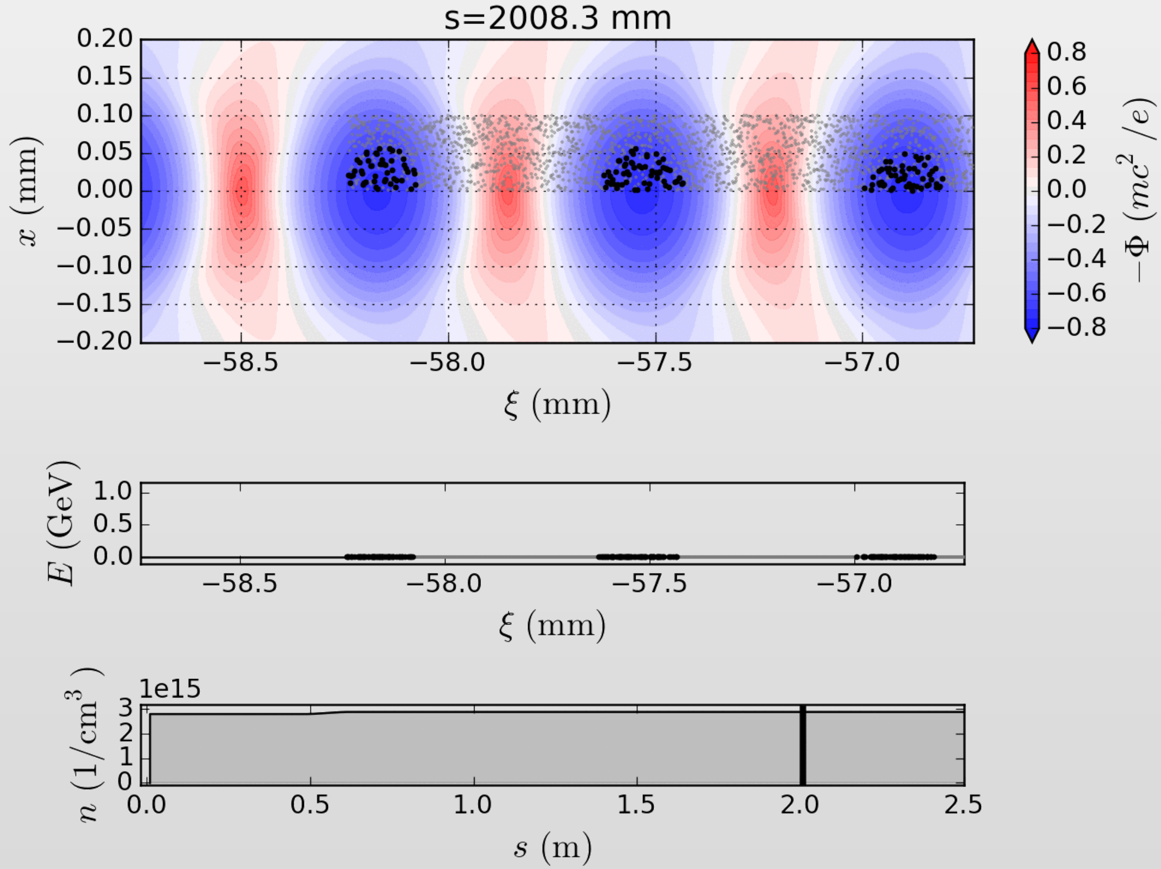
[Alexey's 2D simulation]

What particles will be trapped?

$$\Phi_i \leq \Phi_{max} - \frac{mc^2}{e}$$







Just 3 mm is necessary to reach 10 MeV!
(10 MeV / 3 mm = 3.3 GeV/m)

Conclusion

- It is possible to pass the electron beam through the empty plasma cell by canceling the Earth's magnetic field or launching it at the angle of 4 mrad at the first orifice. We must also take the Earth's magnetic field into account during the beam injection.

- In AWAKE Run II we should think about the beam loading. Its ideal case shows how much charge we can accelerate with the lowest energy spread.

- Trapping of the electrons during the ionization injection is a relatively simple 1D effect. Even not captured electrons gain significant energy, which strongly depends on the wakefield amplitude. That is why this effect can be used as a good diagnostics or even an alternative to direct beam injection.

Thanks for listening

Аналитические выражения для полей

$$\frac{\partial}{\partial \xi}\left(E_r - B_\varphi\right) = \frac{4\pi}{c} j_r$$

$$\frac{1}{r}\frac{\partial}{\partial r} r B_\varphi = \frac{4\pi}{c} j_z - \frac{\partial E_z}{\partial \xi}$$

$$\frac{\partial}{\partial \xi}\left(B_r + E_\varphi\right) = -\frac{\partial B_z}{\partial r}$$

$$\frac{\partial}{\partial \xi}\left(B_r + E_\varphi\right) = 0$$

$$\frac{\partial}{\partial \xi}\left(B_\varphi - E_r\right) = -\frac{\partial E_z}{\partial r}$$

$$\frac{1}{r}\frac{\partial}{\partial r} r E_\varphi = \frac{\partial B_z}{\partial \xi}$$

$$\frac{\partial V_z}{\partial \xi} = \frac{e E_z}{mc}$$

$$E_r - B_\varphi = -\frac{\partial \Phi}{\partial r}$$

$$E_z = -\frac{\partial \Phi}{\partial \xi}$$

$$-c\frac{\partial \delta n}{\partial \xi} + \frac{n_0}{r}\frac{\partial}{\partial r} r V_r + n_0 \frac{\partial V_z}{\partial \xi} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \Phi}{\partial r} - \frac{\omega_p^2}{c^2}\Phi = 4\pi q n_b$$

$$\Phi'' + \frac{\Phi'}{r'} - \Phi = 4\pi q n_b \qquad r' = r\frac{\omega_p}{c}$$

$$\delta n = \begin{cases} \frac{q}{e} n_b(\xi, r) \sin\left(\frac{\omega_p}{c}(\xi - \xi_0)\right), & r' \le R' \\ 0, & r' > R' \end{cases}$$