Axionic Dark Matter and Axion Stars

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arXiv: 1412.3430[1], 1512.01709[2], 1608.06911[3]

PIKIO
The Ohio State University

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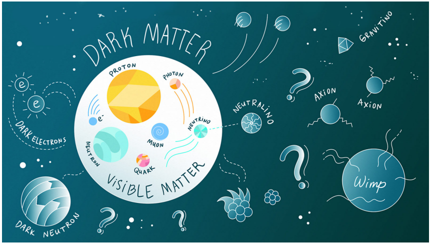
[1] JE, Peter Suranyi, Cenalo Vaz, and L.C.R. Wijewardhana. Journal of High Energy Physics **3** (2015) 1-15.

[2] JE, Peter Suranyi, and L.C.R. Wijewardhana. Modern Physics Letters A, Vol. 31, No. 15 (2016) 1650090.

[3] JE, Madelyn Leembruggen, Peter Suranyi, and L.C.R. Wijewardhana (submitted to JHEP)

Dark Matter

What is dark matter?



[0]Image Credit: Symmetry Magazine

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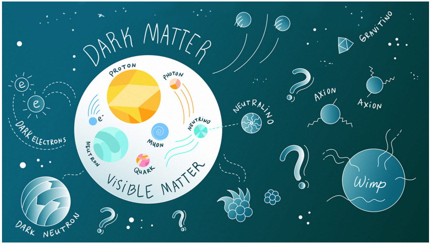
What is dark matter?

- WIMPs? SIMPs? ELDERs?
- MACHOs? PBHs?
- Asymmetric DM? Sterile νs?
- Axion-like particles?



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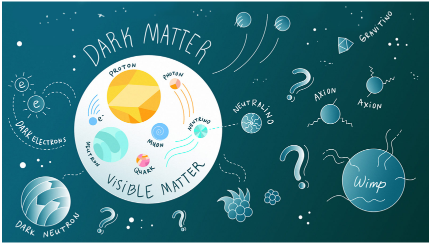

- QCD axion is well motivated by Strong CP problem, and axion-like particles are generic in BSM physics
- At low temperatures, can condense into "axion stars", which can be very large and very massive
- The existence of axion stars could have consequences in terrestrial DM detection, galactic dynamics, lensing, z_{eq}, ...

[0] Image Credit: Symmetry Magazine

Outline

- Weakly Bound Axion Stars
- Self-Interactions and Gravitational Collapse
- Strong(er) Binding and Radiation
- Current/Future Work

Different Approaches

Axion stars: macroscopic condensates of low-energy axion particles

- Fluid: Equation of state, hydrodynamics
- $N \sim 10^{60}$ axions confined to a sphere of radius R
- (Mostly) Non-relativistic, continuous wavefunction $\psi(r)$
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Of particular interest:

- Macroscopic properties (M, R)
- Stability
- If unstable, what happens during collapse?

Axion Star Self-Energy

The low-energy potential for axions is

$$V(\phi) = m^2\, f^2 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

$$m \in \{10^{-6} - 10^{-2}\}\ \text{eV}$$
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Expand and drop $\mathcal{O}(e^{\pm imt})$ terms:

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$$E(\psi) = \int d^3r \left[\underbrace{\frac{|\nabla\psi|^2}{2m}}_{\text{kinetic}} + \underbrace{\frac{1}{2} V_{grav}(|\psi|^2)}_{\text{gravitational}} + \underbrace{W(\psi)}_{\text{self-interaction}}\right]$$

Variational Method

Gaussian ansatz:

$$\psi(r) = \frac{\sqrt{N}}{\pi^{3/4}\sigma^{3/2}} e^{-r^2/2\sigma^2}$$

Rescale macroscopic parameters:

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Goal: Find minima of $E(\rho) \Rightarrow$ Stable bound states

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Leading-Order Interaction

$\delta = \mathcal{O}(10^{-14})$ in QCD, so the LO expansion in δ seems justified.

LO Energy:

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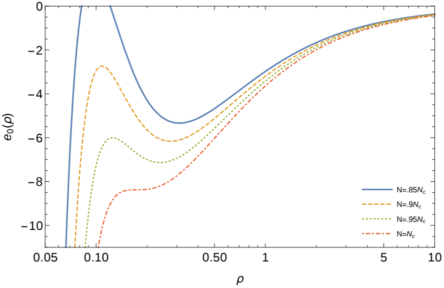

- $M_c \approx \left(\frac{10^{-5}\text{eV}}{m}\right) 10^{19}$ kg, $R_* \approx 200$ km
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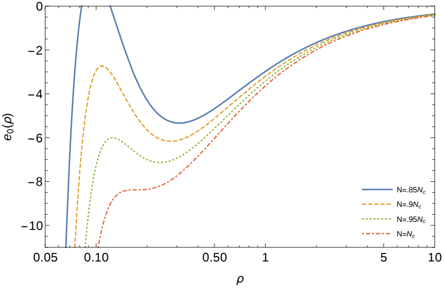

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! $E_0(\rho) \to -\infty$ as $\rho \to 0$

! What happens if $M > M_c$?

LO Collapse (Chavanis 1604.05904)

The variational method lends itself easily to a calculation of the collapse time by making $R \to R(t)$ dynamical

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- Thus collapse proceeds unhindered from dilute minimum R_* to R_S, the Schwarzschild radius
- Conclusion: $M > M_c$ boson stars with attractive self-interactions collapse to black holes

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During collapse, the condensate density increases. At some point, the truncation of the series at LO, $(\psi^*\psi)^2$, breaks down.

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$$E_1(\rho) = m\,N\,\delta\left[\frac{3}{4}\frac{1}{\rho^2} - \frac{1}{\sqrt{2\pi}}\frac{n}{\rho} - b_0\frac{n}{\rho^3} + b_1\frac{\delta\,n^2}{\rho^6}\right]$$

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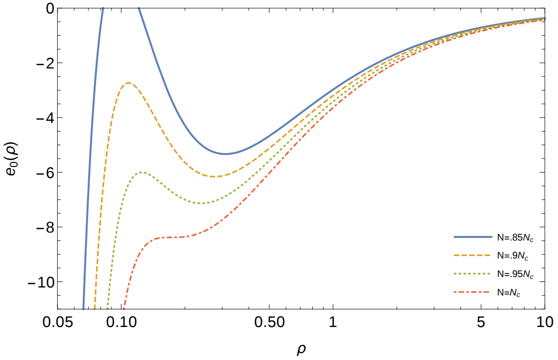
Upshot: Need NLO interaction in analysis of axion star collapse.

Beyond LO Energy

We thus include higher order $(\psi^*\psi)^K$ terms when $\rho \lesssim 10^{-5}$:

$$\frac{E_K(\rho)}{m\,N\,\delta} = \frac{3}{4}\frac{1}{\rho^2} - \frac{1}{\sqrt{2\pi}}\frac{n}{\rho} - \frac{1}{\delta}\sum_{k=0}^{K}(-1)^k\, b_k\left(\frac{n\,\delta}{\rho^3}\right)^{k+1}$$



No change in dilute minimum, $\rho = \mathcal{O}(1)$

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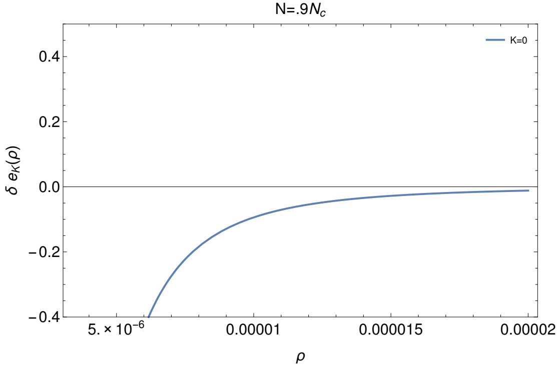

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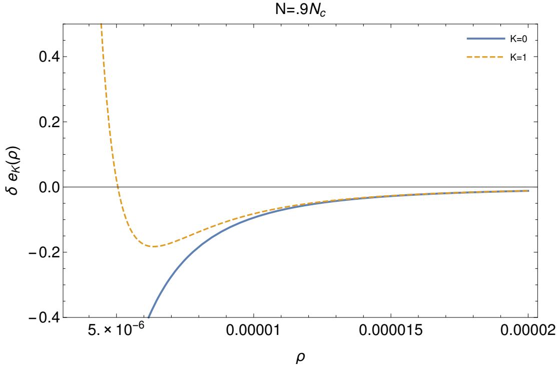

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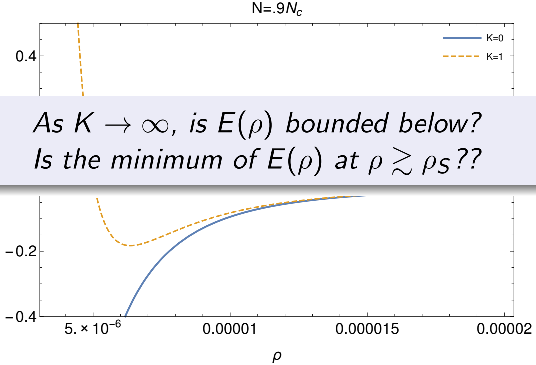

As $K \to \infty$, is $E(\rho)$ bounded below?
Is the minimum of $E(\rho)$ at $\rho \gtrsim \rho_S$??

Even K: No Global Minimum

Odd K: Global min. at $\rho_D \sim 10^{-6} - 10^{-5} \Rightarrow R_D \sim 10$ m

$N^{\infty}LO$ (1608.06911)

We prove without truncation in the Gaussian[1] ansatz that the full axion star energy has a global minimum at a radius $R_{GM} > R_S$.[2]

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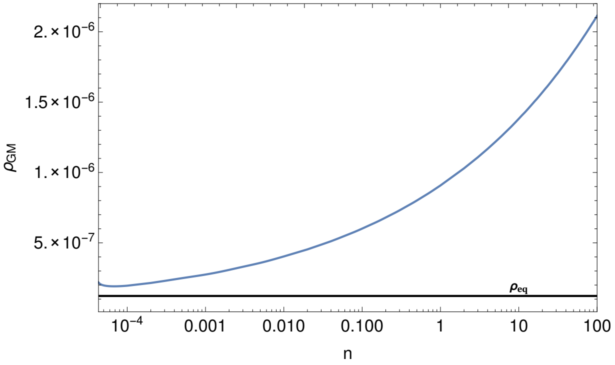

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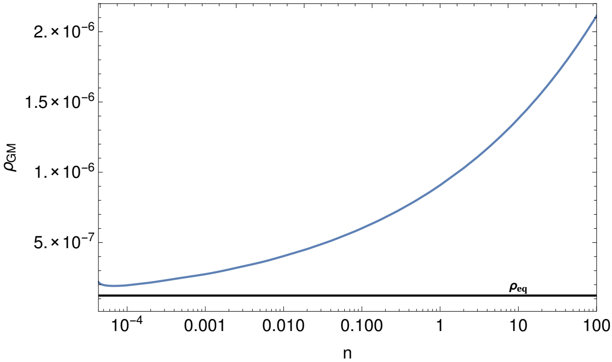

- Can approximate ρ_{GM} by truncating the potential on a repulsive term, but not on an attractive one

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We repeat the analysis of Chavanis including NLO interation and taking into account $\rho_D \gg \rho_S$. Collapse from $\mathcal{O}(R_*)$ to R_D:

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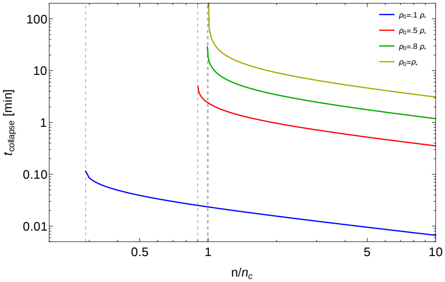

Collapse time $t \lesssim \mathcal{O}(\text{few hours})$

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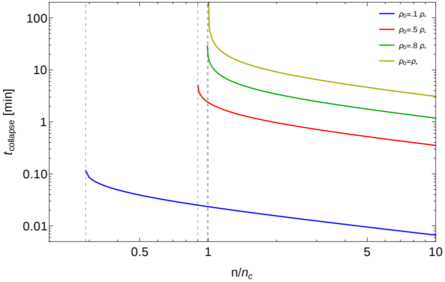

- Binding energy rises continuously during collapse
- The nonrelativistic expansion of ϕ *breaks down* at very strong binding

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- Include continuous spectrum in operator expansion:

$$\phi = R(r) e^{i\,E_0\,t} a_0 + \int \frac{d^3p}{\sqrt{2\,E_p}} a_p e^{i\,p\cdot r - i\,E_p\,t} + h.c.$$

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- Axion stars are quantum objects: They have a wavefunction with a coordinate and momentum uncertainty:
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  - $R \sim (m\,\Delta)^{-1} \Leftrightarrow \delta p \sim m\,\Delta$, with $\Delta \equiv \sqrt{1 - E_a^2/m^2}$ (binding energy parameter)
- We calculated the transition rate for

$$|N \text{ condensed}\rangle \to |(N-3) \text{ condensed} + 1 \text{ emitted}\rangle$$

which increases with Δ (decreases with M for dilute stars)

Transition $A_N \to A_{N-3} + a_p$

$$\mathcal{M}_3 = \int dt\, d^3r \, \langle N| \left[1 - \cos\left(\frac{\phi}{f}\right)\right] |N-3,\ p\rangle$$

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$$\sim \int d^3r \left(\frac{1}{6\,f^3}\phi(r)^3 - \frac{1}{24\,f^5}\phi(r)^5 + \ldots\right) e^{i\,p\cdot r}$$

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$$\left. - \frac{1}{6\, f^3} \prod_{i=1}^{5} d^3P_i\, \phi(P_i)\, e^{i\,(p-P_1-P_2-P_3-P_4-P_5)\cdot r} + \ldots\right)$$

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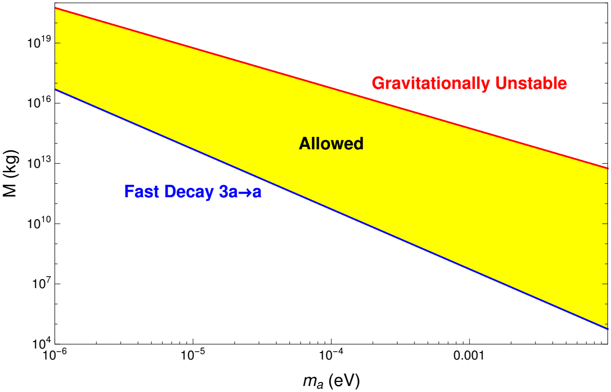
\begin{aligned}
\mathcal{M}_3 &= \int dt\, d^3r\, \langle N| \left[1 - \cos\left(\frac{\phi}{f}\right)\right] |N-3,\ p\rangle \\
&= -i\, \frac{m^2\, f}{\sqrt{2\, E_p}} \int d^3r\, dt\, J_3\left(\frac{2\phi(r)}{f}\right) e^{i\, p\cdot r}\, e^{i\, t\, (3E_0 - E_p)} \\
&\sim \int d^3r \left(\frac{1}{6\, f^3}\phi(r)^3 - \frac{1}{24\, f^5}\phi(r)^5 + \ldots\right) e^{i\, p\cdot r} \\
&\sim \int d^3r \left(\frac{1}{6\, f^3} \prod_{i=1}^{3} d^3P_i\, \phi(P_i)\, e^{i\, (p - P_1 - P_2 - P_3)\cdot r} \right. \\
&\quad \left. - \frac{1}{6\, f^3} \prod_{i=1}^{5} d^3P_i\, \phi(P_i)\, e^{i\, (p - P_1 - P_2 - P_3 - P_4 - P_5)\cdot r} + \ldots\right)
\end{aligned}
$$

Momentum conserved; absorbed by rest of axion star in “chunks” of 3,5,7,...

Leading interaction is “chunk” of 3; hence, “$3\, a \to a$”

Spectrum for Weakly Bound Axion Stars

Weakly Bound QCD Axion Stars have $M \ll M_\odot$



- Upper bound: Gravitational stability (1412.3430)
- Lower bound: Axion star decay through $A_N \to A_{N-3} + a_p$ (1512.01709)
  - "Weak binding" $\Leftrightarrow \Delta \equiv \sqrt{1 - E_a^2/m^2} \ll 1$

Collapse; Decay through $A_N \to A_{N-3} + a_p$

We find that for binding energies of $B \gtrsim 10^{-3}m$ ($\Delta \gtrsim .05$), decay occurs very fast: $\tau \ll \tau_U$

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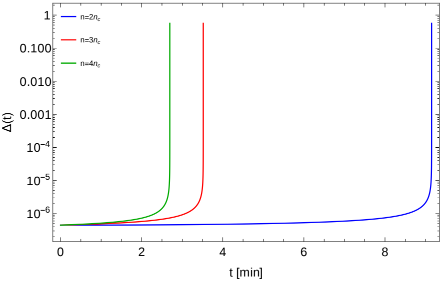

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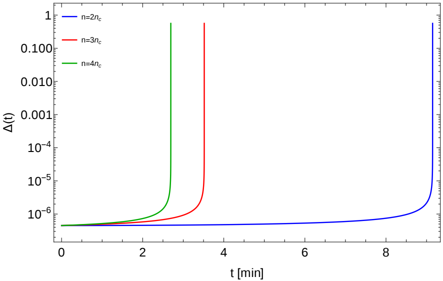

$\Rightarrow$ Rapid emission of relativistic bosons

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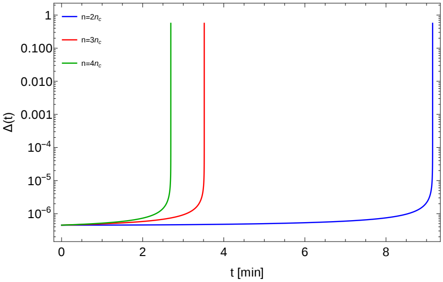

⇒ Rapid emission of relativistic bosons
⇒ “Bosenova”

A Flurry of Activity...

"Collapse of Axion Stars" 1608.06911 appeared on August 24

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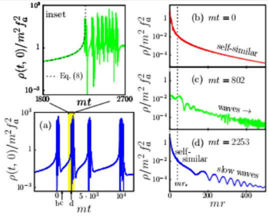

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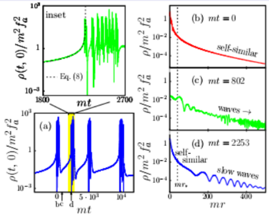

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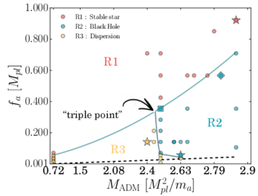

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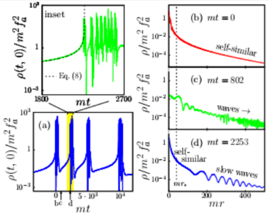

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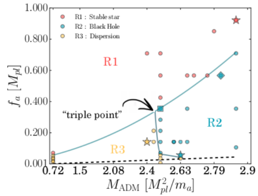

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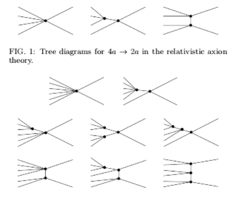

FIG. 1: Tree diagrams for $4a \to 2a$ in the relativistic axion theory.

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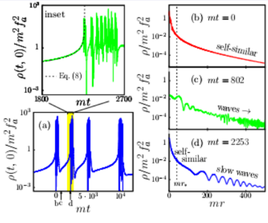

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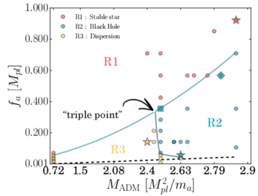

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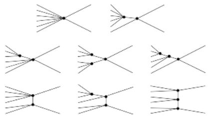

FIG. 2: Tree diagrams for $6a \to 2a$ in the relativistic axion theory.

Lots of good ideas, and work to be done!

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Collisions of axion stars with other bodies would change the effective energy functional

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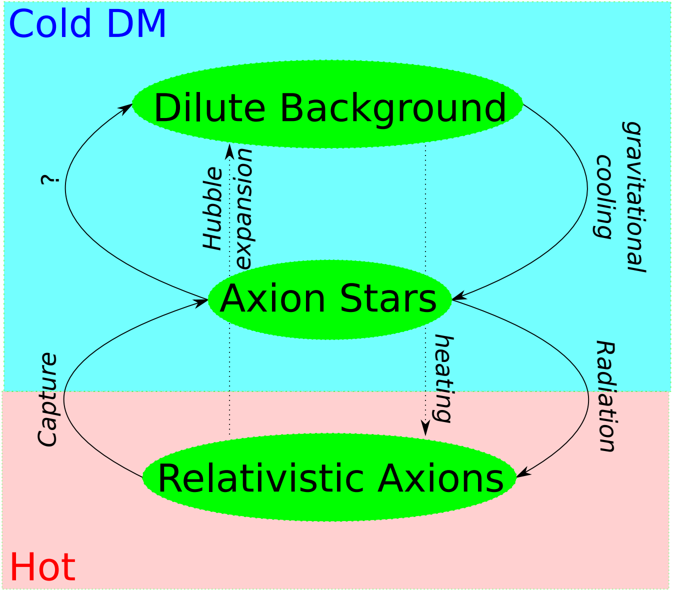
Such collisions can induce collapse!
(Work in progress.)

Axion Cosmology Revisited



Conclusions

- The mass of weakly bound axion stars is bounded above by gravitational stability, and below by decay through $3\,a \to a$
- A star with $M > M_c$ collapses to an energetically stable dense state, a global minimum of the energy *which is not a black hole*
- During collapse, increased binding energy leads to fast decay through $3\,a \to a \Rightarrow$ Bosenova
- Astrophysics: Can collapses be triggered by collisions of axion stars? Preliminary: Yes!
- Cosmology: Interplay between dilute axion background, axion stars, and relativistic axions in the early universe

Acknowledgements

- Madelyn Leembruggen, Joseph Leeney, Peter Suranyi, Cenalo Vaz, and Rohana Wijewardhana (University of Cincinnati)
- Support through Mary J. Hannah Fellowship (UC) and DOE

- Thanks for the invitation and for your attention!

Backup: The Infrared Limit

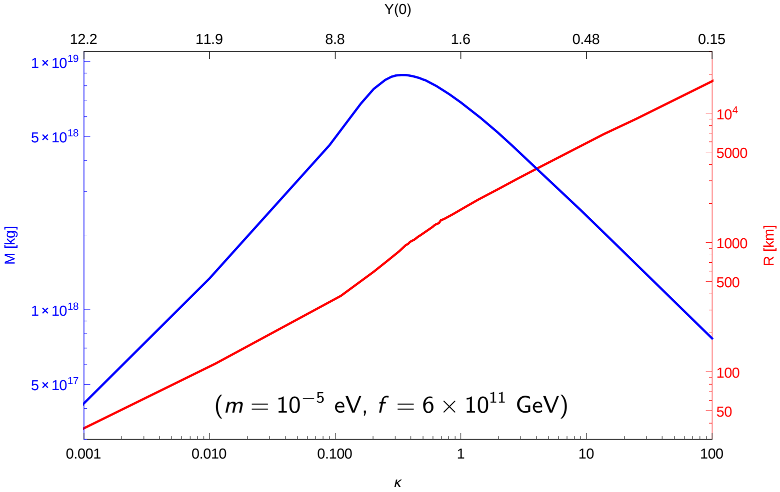
We extract powers of Δ corresponding to the engineering dimensions of y and $Z(y)$: Let $Y(x) = Z(y) \,/\, \Delta$ and $x = \Delta\, y$

$$\langle N|V(\mathcal{A})|N\rangle = 2\, m^2\, f^2 \Big[1 - J_0(Z)\Big]$$
$$= \frac{m^2\, f^2}{2}\left(Z^2 - \frac{1}{16} Z^4 + \mathcal{O}(Z^6)\right)$$

- Powers of $\Delta \Leftrightarrow$ operator dimension.
  Irrelevant operators are naturally suppressed by Δ in this framework.

Similar considerations for KG equation, where we have
$\langle N-1|V'(\mathcal{A})|N\rangle = m^2 f\, J_1(Z) = \frac{m^2 f}{2}\left(Z - \frac{1}{8} Z^3 + \mathcal{O}(Z^5)\right)$

Backup: Macroscopic Parameters

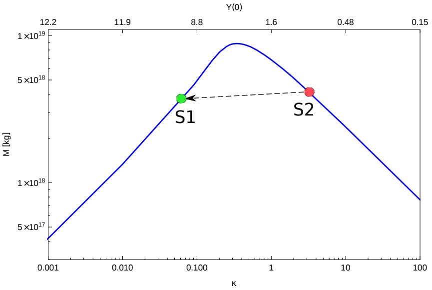


Nonrelativistic limit corresponds to $\kappa \gtrsim 1 \rightarrow$

- Larger $Y(0) \Leftrightarrow$ Smaller $R_{99} \Leftrightarrow$ Larger $\Delta \Leftrightarrow$ Smaller κ
- Maximum mass near $\kappa = .34$

Backup: Tunneling between Axion Star States

!: For a given N, $\exists$ two states S_1 and S_2 with $M_1 < M_2$
$\Rightarrow S_2$ can decay to S_1.



Using $M(\Delta) = Nm\sqrt{1-\Delta^2}$,

$$\frac{\delta M}{M} \approx \frac{1}{2}\left(\Delta_1^2 - \Delta_2^2\right) = \frac{1}{2\Lambda}\left(\kappa_1^{-1} - \kappa_2^{-1}\right)$$

Mass difference a small fraction of total mass, but still large amount of energy: typically $\gtrsim 1000$ kg!

Computing the $3a \to a$ Decay Rate

We modify the axion field expansion to include a free axion term

$$\mathcal{A} = R(r)e^{iE_0 t}a_0 + \int \frac{d^3p}{\sqrt{2E_p}} a_p e^{ip\cdot r - iE_p t} + h.c.$$

This leads to the leading-order matrix element

$$\begin{aligned}
\mathcal{M}_3 &\equiv \mathcal{M}[N \to (N-3) + 1 \text{ emitted}] \\
&= m^2 f^2 \int dt\, d^3r \langle N-3, p|1 - cos\left(\frac{\mathcal{A}}{f}\right)|N\rangle \\
&= -i\frac{f}{m}\frac{1}{\sqrt{2E_p}} \int dt\, d^3y\, J_3[Z(y)] e^{ip\cdot r} e^{i(3E_0 - E_p)t} \\
&= -i\frac{4\pi^2}{\sqrt{2E_p}}\frac{f}{p}\delta(3E_0 - E_p) \underbrace{\int_{-\infty}^{\infty} y\, sin\left(\frac{p\, y}{m}\right) J_3[Z(y)]\, dy}_{I_3(p)}
\end{aligned}$$

Backup: Equations of Motion

The self-gravity of the scalar field perturbs the metric

$$ds^2 = B(r)dt^2 - A(r)dr^2 - r^2 d\Omega^2$$

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Equations of motion:

$$\frac{A'(y)}{A(y)} = \frac{1 - A(y)}{y} + \frac{yA(y)}{4\Lambda}\left[\frac{E_0^2}{m^2}\frac{Z(y)}{B(y)} + \frac{Z'(y)^2}{A(y)} + 4\Big(1 - J_0[Z(y)]\Big)\right]$$

$$\frac{B'(y)}{B(y)} = -\frac{1 - A(y)}{y} + \frac{yA(y)}{4\Lambda}\left[\frac{E_0^2}{m^2}\frac{Z(y)}{B(y)} + \frac{Z'(y)^2}{A(y)} - 4\Big(1 - J_0[Z(y)]\Big)\right]$$

$$Z''(y) = -\left[\frac{2}{y} + \frac{B'(y)}{2B(y)} - \frac{A'(y)}{2A(y)}\right]Z'(y) - A(y)\left[\frac{E_0^2}{m^2}\frac{Z(y)}{B(y)} - 2J_1[Z(y)]\right]$$

with $Z(y) = \frac{2\sqrt{N}R(r)}{f}$, $y = m\,r$, and $\Lambda = \frac{M_P^2}{f^2}$