Effective Field Theory for Resonant Wino Dark Matter

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Outline of this talk

- Wino WIMP dark matter
  - Direct wino-pair annihilation
  - Wino bound state production and subsequent annihilation
    - **New mechanism for monochromatic gamma ray signal**
- Nonrelativistic Effective Field Theory
- Zero-Range Effective Field Theory
- Wino bound state formation calculation
  - Through two photon transition (this talk)
  - Through three body recombination (future work)

Wino WIMP dark matter

- Extend the Standard Model to include one electroweak SU(2) x U(1) multiplet:
  - Triplet under SU(2) with zero hypercharge

$$\tilde{w} = \begin{pmatrix} \tilde{w}^{+} & \tilde{w}^{0} & \tilde{w}^{-} \end{pmatrix}$$

- Or, the MSSM in the region of parameters where the Lightest Supersymmetric Particle (LSP) is a wino-like neutralino
  - Wino: SUSY partner of the W boson

- Wino masses: neutral wino $M \sim$ few TeV, charged winos $M + \delta$
  - Electroweak radiative corrections give δ = 170 MeV and varies very little with M

Pierce et.al. NPB (1997)

- The neutral wino is the WIMP dark matter candidate
  - $M > 1$ TeV for the relic density to be compatible with observed dark matter density

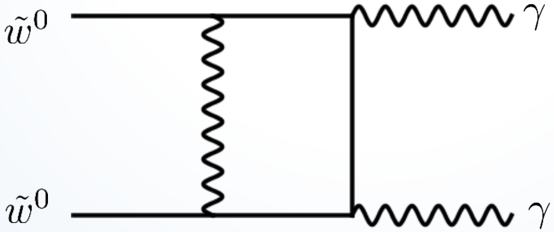
Wino interactions and nonperturbative effects

- A pair of neutral winos can annihilate into a pair of electroweak gauge bosons

$$\left.\begin{array}{rcl}\tilde{w}^0\tilde{w}^0 & \to & \gamma\gamma \\ & \to & \gamma Z^0\end{array}\right\}\text{Monochromatic } \gamma\text{-ray signal}$$

$$\left.\begin{array}{rcl}\tilde{w}^0\tilde{w}^0 & \to & Z^0 Z^0 \\ & \to & W^+W^-\end{array}\right\}\text{Continuous } \gamma\text{-ray and positron signals}$$

- Leading-order cross-section for $\tilde{w}^0\tilde{w}^0 \to \gamma\gamma$:



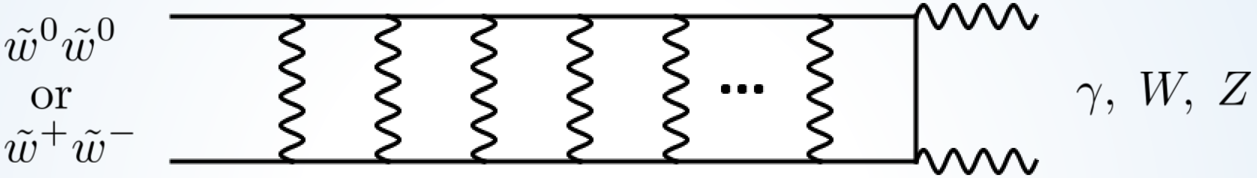
$$(\sigma_{\text{ann}} v)_{\text{LO}} \sim \frac{\alpha^2\alpha_2^2}{m_W^2}$$

Hisano et. al. PRD (2005)

- Exceeds unitarity bound $\sigma_{\text{ann}} v < 4\pi/vM^2$ for sufficiently large M
  - Higher-order diagrams must be included in the cross-section

Wino interactions and nonperturbative effects

- Transitions between wino pairs, $\tilde{w}^0\tilde{w}^0$ and $\tilde{w}^+\tilde{w}^-$, involve exchange of EW gauge bosons:



- The ladder diagrams must be summed to all orders
  - Each 'rung' of the ladder gives a factor of $\alpha_2 M/m_W$
  - For large enough M , $\alpha_2 M/m_W \sim 1$

Hisano et. al. PRD (2005)

  - The cross sections for $\tilde{w}^0\tilde{w}^0 \to \gamma\gamma$, γZ , ZZ , WW receive large corrections: "Sommerfeld Enhancement"
- Difficult to calculate in the fundamental field theory
- Instead, calculate with Nonrelativistic Effective Field Theory

Nonrelativistic Effective Field Theory (NREFT)

(momentum scales $<< M$)

- Ladder diagrams from exchange of electroweak gauge bosons between a pair of winos can be summed to all orders by solving the Schrödinger equation
    - Neutral wino pairs and charged wino pairs are coupled channels interacting through the potential

$$V(r) = \begin{pmatrix} 0 & -\sqrt{2}\alpha_2\, e^{-m_W r}/r \\ -\sqrt{2}\alpha_2\, e^{-m_W r}/r & -\alpha/r - \alpha_2 c_W^2\, e^{-m_Z r}/r \end{pmatrix}$$

Channel	Threshold Energy
$\tilde{w}^0\tilde{w}^0$	0
$\tilde{w}^+\tilde{w}^-$	2δ

- Sequence of critical masses where the neutral winos form a zero-energy resonance:
    - Resonance near the neutral wino scattering threshold: “Sommerfeld Enhancement”

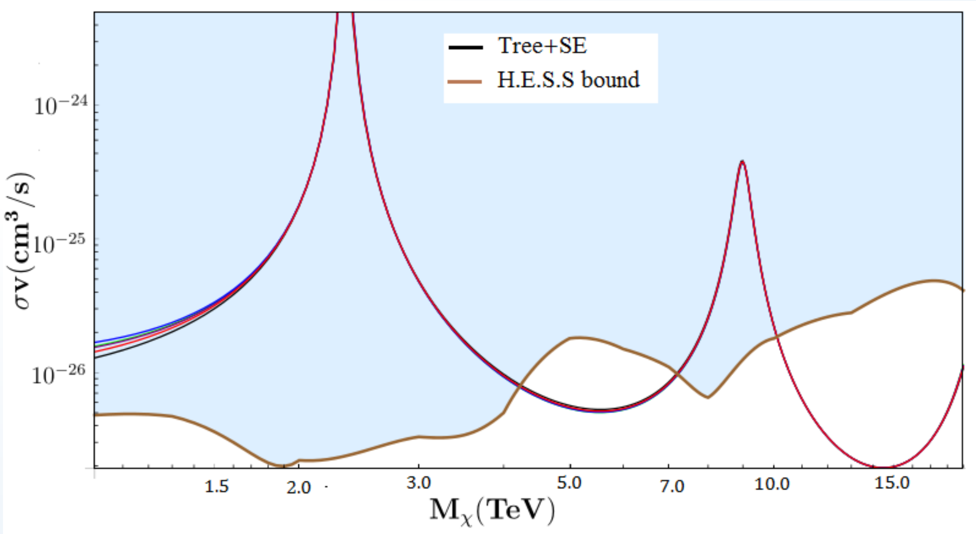
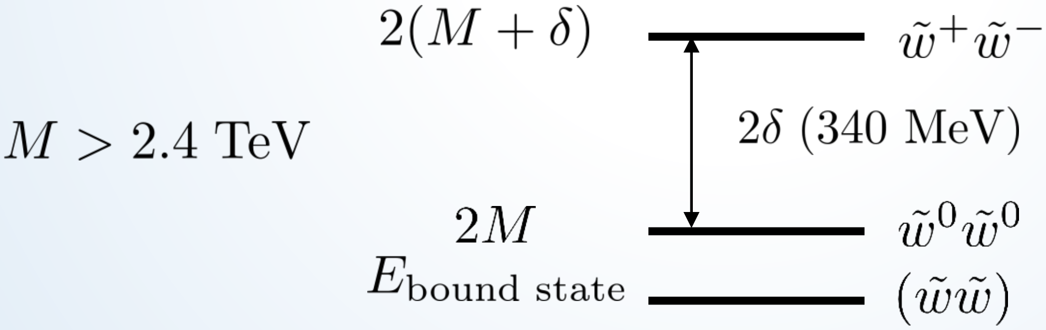


Figure from Baumgart et. al. JHEP (2015)

Resonant Wino DM and bound states

- There are critical values of the wino mass where a pair of neutral winos form a zero-energy resonance
  - Critical masses are determined by the mass splitting δ and electroweak parameters
  - For δ = 170 MeV, the first critical mass is M_* = 2.4 TeV

- When M is above M_* , the resonance is a bound state, denoted $(\tilde{w}\tilde{w})$



$M > 2.4\ \mathrm{TeV}$

$2(M+\delta)$ $\tilde{w}^+\tilde{w}^-$

$2\delta\ (340\ \mathrm{MeV})$

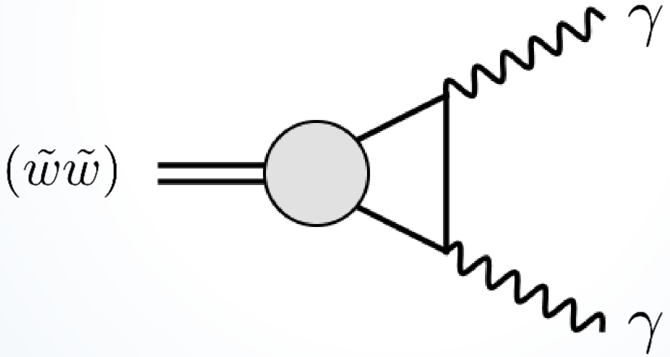
$2M$ $\tilde{w}^0\tilde{w}^0$

$E_{\text{bound state}}$ $(\tilde{w}\tilde{w})$

Bound state formation: New mechanism for monochromatic gamma ray signal

- If the wino mass is above the critical mass, the resonance in neutral wino scattering is a bound state, denoted by $(\tilde{w}\tilde{w})$

- Bound state production is followed by annihilation of the bound state into two hard photons $(\tilde{w}\tilde{w}) \to \gamma\gamma$

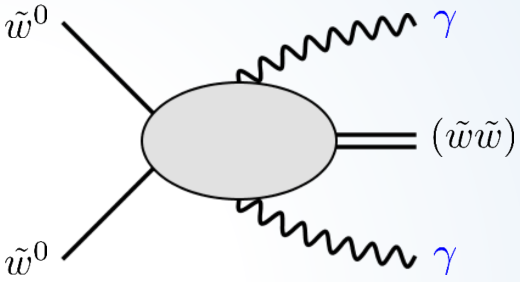


- Annihilation rate = bound state production rate
  - Wino bound state is unstable and decays with probability one

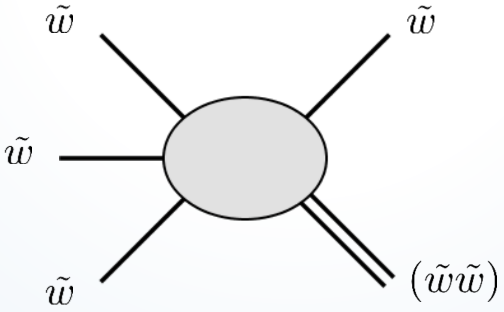
Bound state formation mechanisms

- Bound state can form in neutral wino scattering through double radiative transition

$$\tilde{w}^0\tilde{w}^0 \;\rightarrow\; (\tilde{w}\tilde{w}) + \text{soft photons}$$



- Bound state can also form through three body recombination



- Intractable to calculate using the Nonrelativistic Effective Field Theory
- Instead, calculate with Zero-Range Effective Field Theory

Zero-Range Effective Field Theory (ZREFT)

(momentum scales << m_W)

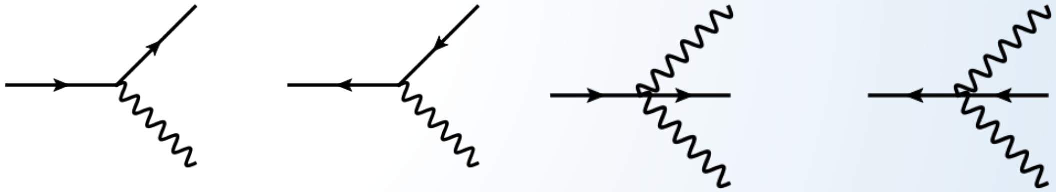
- Kinetic Lagrangian for wino fields:

$$\mathcal{L}_{\rm kinetic} \;=\; w_0^\dagger\left(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2M}\right)w_0 + \sum\nolimits_\pm w_\pm^\dagger\left(iD_0 + \frac{\boldsymbol{D}^2}{2M} - \delta\right)w_\pm$$

  - EM covariant derivative for charged winos: $D_0 w_\pm = (\partial_0 \pm ieA_0)w_\pm \quad , \quad \boldsymbol{D}w_\pm = (\boldsymbol{\nabla} \mp ie\boldsymbol{A})w_\pm$

  - Propagators for wino fields:

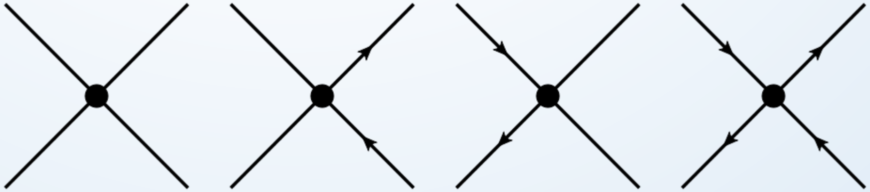
  - Photon emission vertices:



- Interaction Lagrangian for zero-range interactions between winos

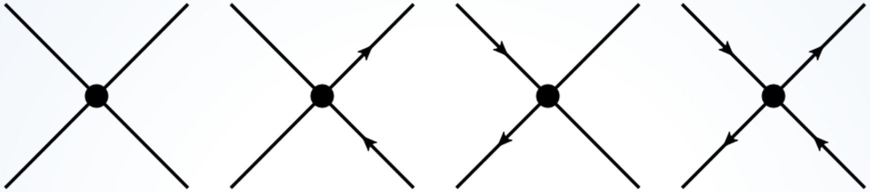
$$\begin{aligned}\mathcal{L}_{\rm zero-range} \;=\;& -\tfrac{1}{4}\lambda_{00}({w_0}^\dagger w_0)^2 - \lambda_{11}({w_+}^\dagger w_+)({w_-}^\dagger w_-)\\ & -\tfrac{1}{2}\lambda_{01}\left[({w_+}^\dagger w_0)({w_-}^\dagger w_0) + ({w_0}^\dagger w_+)({w_0}^\dagger w_-)\right]\end{aligned}$$

  - Zero-range vertices:

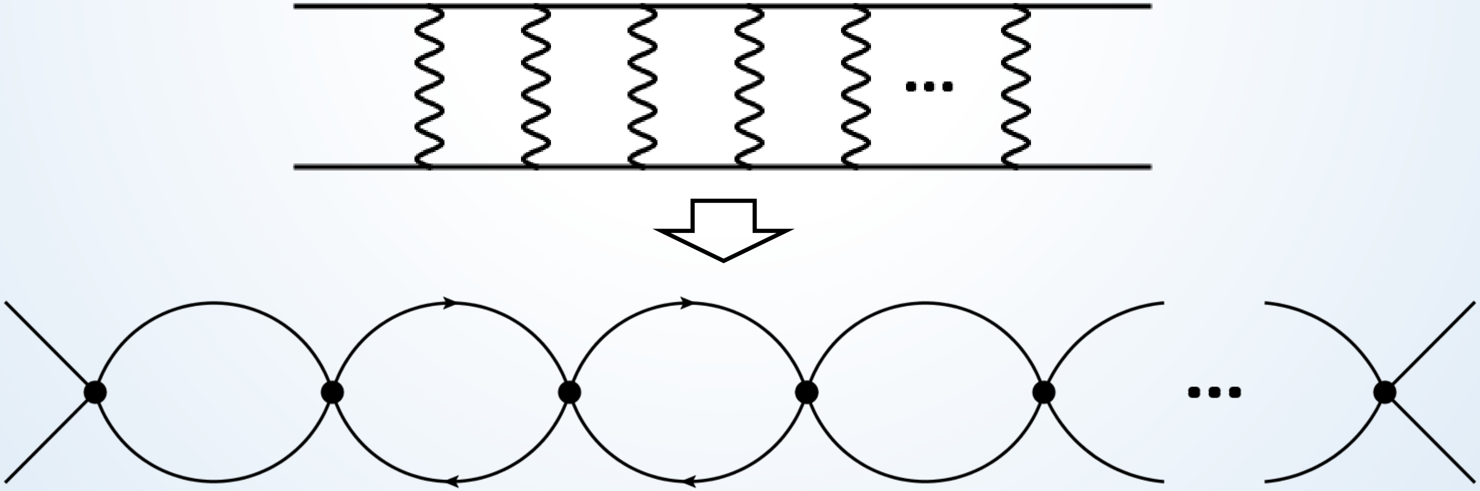


Zero-Range Effective Field Theory

- Wino pairs interact through local, zero-range contact interactions



- Non-perturbative W boson exchange in the fundamental theory included by summing bubble diagrams to all orders



Zero-Range Effective Field Theory

- The renormalization of the Zero-Range EFT is governed by UV RG fixed points

- Three fixed points corresponding to the number of fine-tuned parameters:
  - Trivial non-interacting case, no fine tuning
  - One resonance near the scattering thresholds requires a single fine tuning
  - Two resonances near the scattering thresholds requires a double fine tuning

Birse et. al. EPJ (2011)

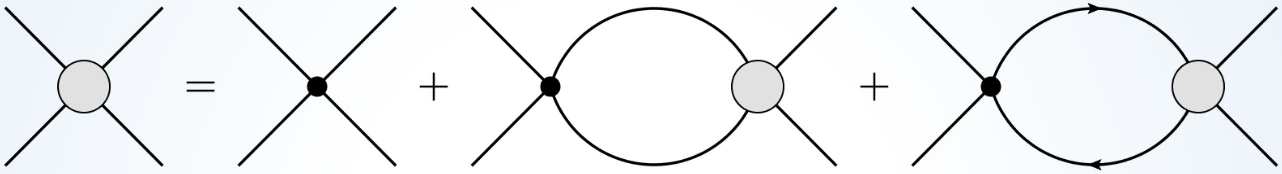
- Only the wino mass M is tuned to its critical mass, expect a single resonance

- The scattering channel for the single resonance can be a linear combination of $\tilde{w}^0\,\tilde{w}^0$ and $\tilde{w}^+\,\tilde{w}^-$

- Scattering channel: $\cos\phi\,\left(\tilde{w}^0\tilde{w}^0\right) + \sin\phi\,(\tilde{w}^+\tilde{w}^-)$

Zero-Range Effective Field Theory

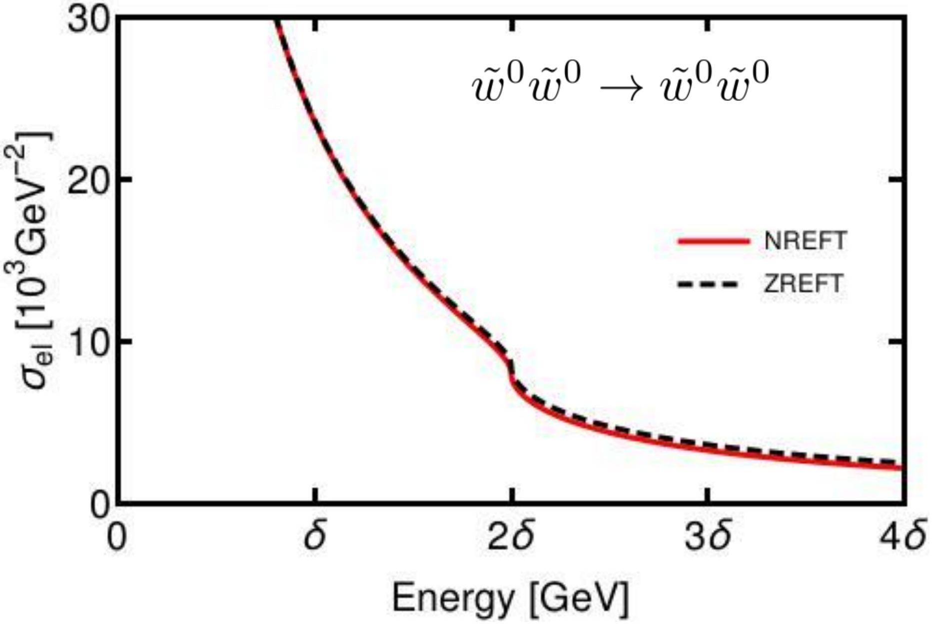
- Calculate scattering amplitudes analytically by solving the coupled-channel Lippmann-Schwinger integral equations

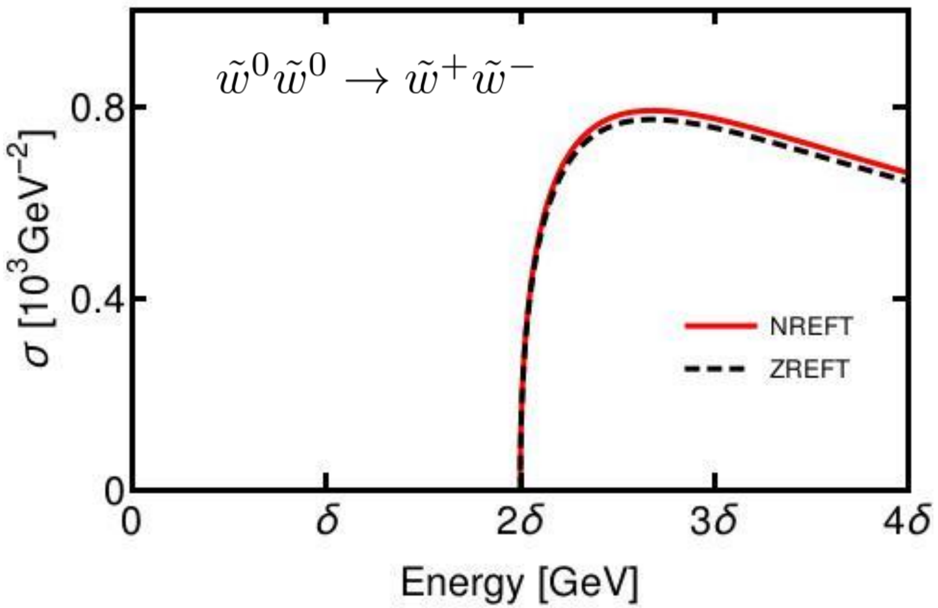


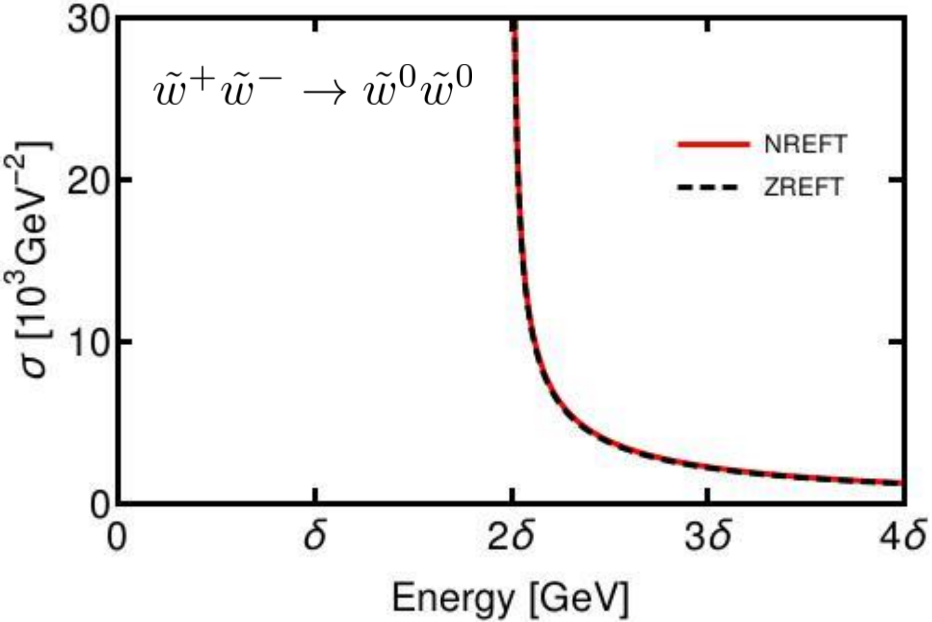
- Scattering amplitude in the neutral channel for $E < 2\delta$:

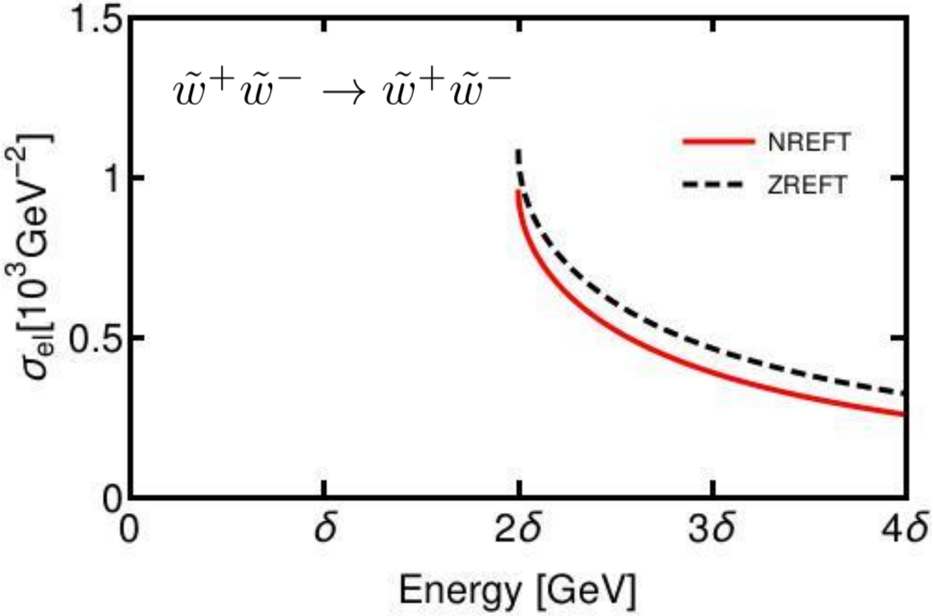
$$f_{00}(E) \;=\; \frac{\cos^2\phi}{-1/a_u - i\sqrt{ME}\cos^2\phi + \sqrt{M(2\delta - E)}\sin^2\phi}$$

- The scattering parameter a_u is determined by requiring the scattering amplitude to diverge at zero energy at the critical mass M_*
  - Fixes $1/a_u \;=\; \sqrt{2M_*\delta}\,\sin^2\phi$
- At the critical mass, the theory at leading order has **one** free parameter: ϕ
- The angle is determined by solving the coupled channel problem in the Non-Relatavistic EFT and fitting the elastic cross section at zero energy
  - Fitted value: $\phi = 39.8°$


$\tilde{w}^0\tilde{w}^0 \to \tilde{w}^0\tilde{w}^0$
NREFT
ZREFT
$\sigma_{\rm el}$ [10^3GeV^{-2}]
30
20
10
0
0
δ
2δ
3δ
4δ
Energy [GeV]


$\tilde{w}^0\tilde{w}^0 \to \tilde{w}^+\tilde{w}^-$
NREFT
ZREFT
σ [10^3GeV^{-2}]
0.8
0.4
0
0
δ
2δ
3δ
4δ
Energy [GeV]

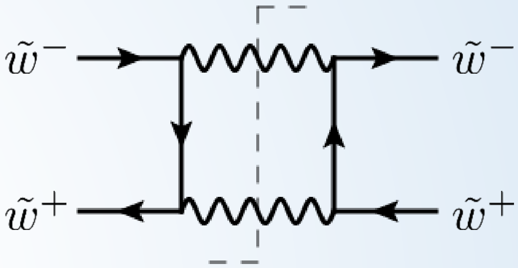

$\tilde{w}^+\tilde{w}^- \to \tilde{w}^0\tilde{w}^0$
NREFT
ZREFT
σ [10^3GeV^{-2}]
30
20
10
0
0
δ
2δ
3δ
4δ
Energy [GeV]


$\tilde{w}^+\tilde{w}^- \to \tilde{w}^+\tilde{w}^-$
NREFT
ZREFT
$\sigma_{\rm el}$[10^3GeV^{-2}]
1.5
1
0.5
0
0
δ
2δ
3δ
4δ
Energy [GeV]

Zero-Range Effective Field Theory

- Inelastic annihilation processes: $\tilde{w}^0\tilde{w}^0 \ \to \ \gamma\gamma \ , \ \gamma Z \ , \ ZZ \ , \ WW$
  - Annihilation contributions in the fundamental theory

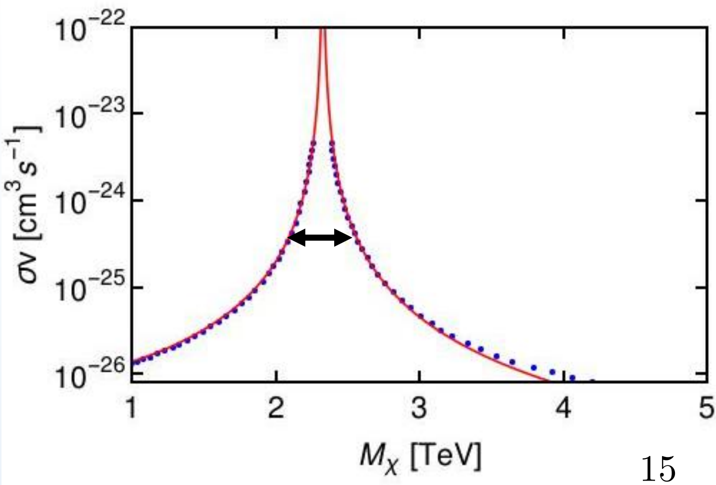
$$\mathrm{Im}\begin{bmatrix} \mathcal{M}[\tilde{w}^0\tilde{w}^0 \to \tilde{w}^0\tilde{w}^0] & \mathcal{M}[\tilde{w}^0\tilde{w}^0 \to \tilde{w}^+\tilde{w}^-] \\ \mathcal{M}[\tilde{w}^+\tilde{w}^- \to \tilde{w}^0\tilde{w}^0] & \mathcal{M}[\tilde{w}^+\tilde{w}^- \to \tilde{w}^+\tilde{w}^-] \end{bmatrix}$$



  - Account for in NREFT through a local imaginary term in the potential:

$$V(r) \ \to \ V(r) \, + \, i\,\Gamma\,\delta^3(\vec{r}) \qquad \Gamma \ = \ \frac{\alpha_2^2}{4M^2}\begin{pmatrix} 6 & \sqrt{2} \\ \sqrt{2} & 4 \end{pmatrix}$$

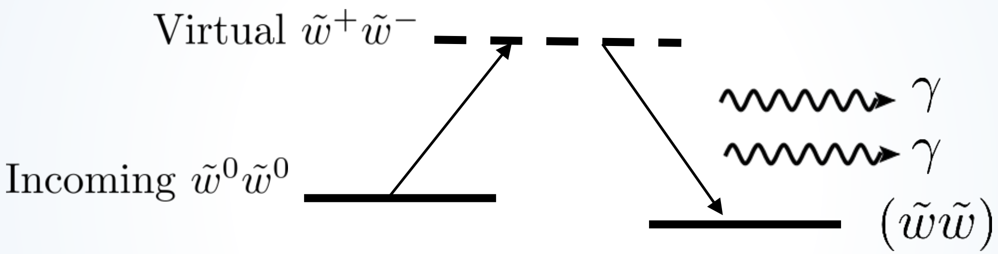
Hisano et. al. PRD (2005)



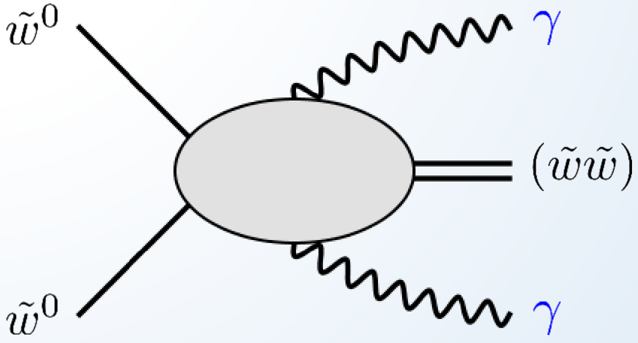
  - Account for in ZREFT by the imaginary part of the T-matrix element at zero energy of the form $\beta\,\Gamma$
  - Determine β by fitting the width of $\sigma_{\mathrm{ann}}v$ from NREFT
  - Best fit: $\beta \ = \ 0.5 \times 10^{-4}$

Example calculation: Production of wino-pair bound state via radiative transition

- To conserve energy, the wino pair must radiate photons
  - Single-photon emission: forbidden by parity
  - Double-photon emission: allowed

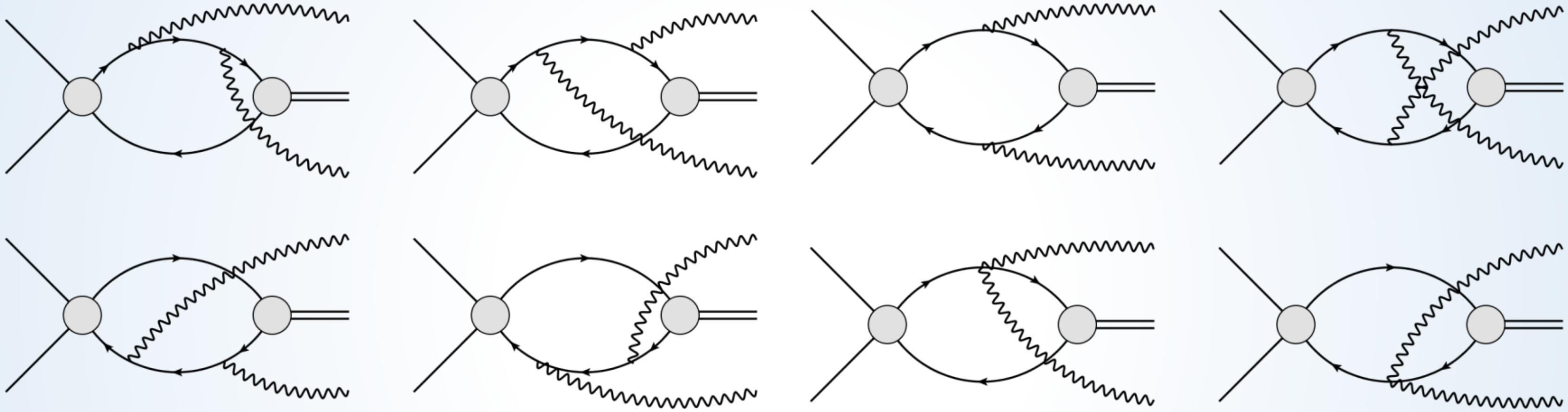


$$\tilde{w}^0\tilde{w}^0 \;\rightarrow\; (\tilde{w}\tilde{w}) + \text{soft photons}$$

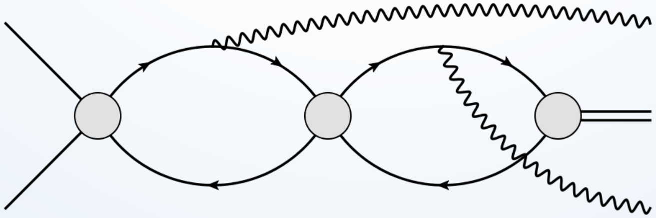


Bound state production

- At leading order in α, diagrams that contribute to $\tilde{w}^0\tilde{w}^0 \;\to\; (\tilde{w}\tilde{w}) + \gamma\gamma$ are

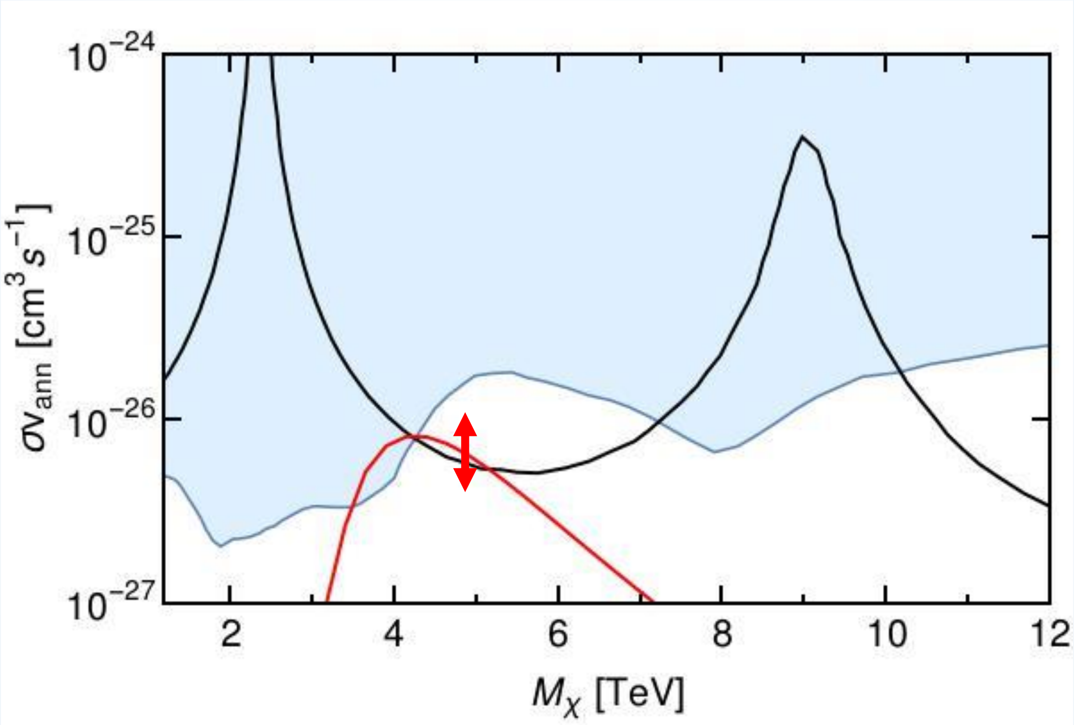


- Two-bubble diagrams with one photon attached to each bubble vanish by parity:



$= 0$

Bound state production rate



HESS PRL (2013)

- **Shaded region:** Excluded from recent bounds from H.E.S.S.
- **Black Curve:** Neutral-wino annihilation cross section to $\gamma\gamma$, γZ with Sommerfeld enhancement only .
- **Red curve:** Preliminary result for the shape of the bound state contribution to the annihilation cross-section above the first critical mass (normalization not yet finalized, but expected to be small)

Conclusion

- Zero-Range EFT describes low energy wino pair scattering processes very well
  - At LO, a single parameter fits the non-trivial behavior from NREFT
  - Can be systematically improved with two more scattering parameters at NLO

- Zero-Range EFT can also easily describe
  - Bound state production
  - Multi-body scattering
  - **...**

  These are very difficult to include in NREFT.