Multiple Scattering Analysis

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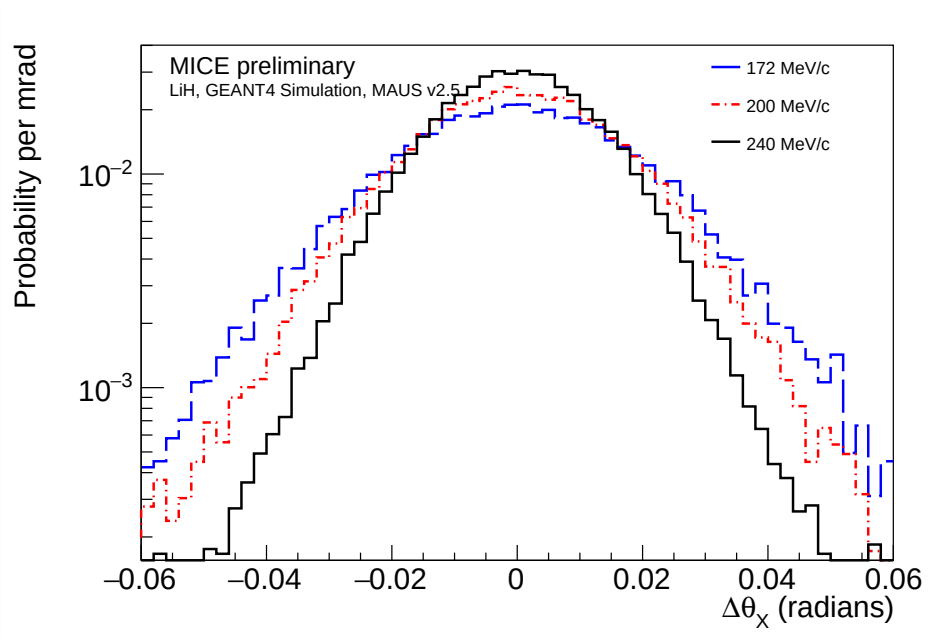
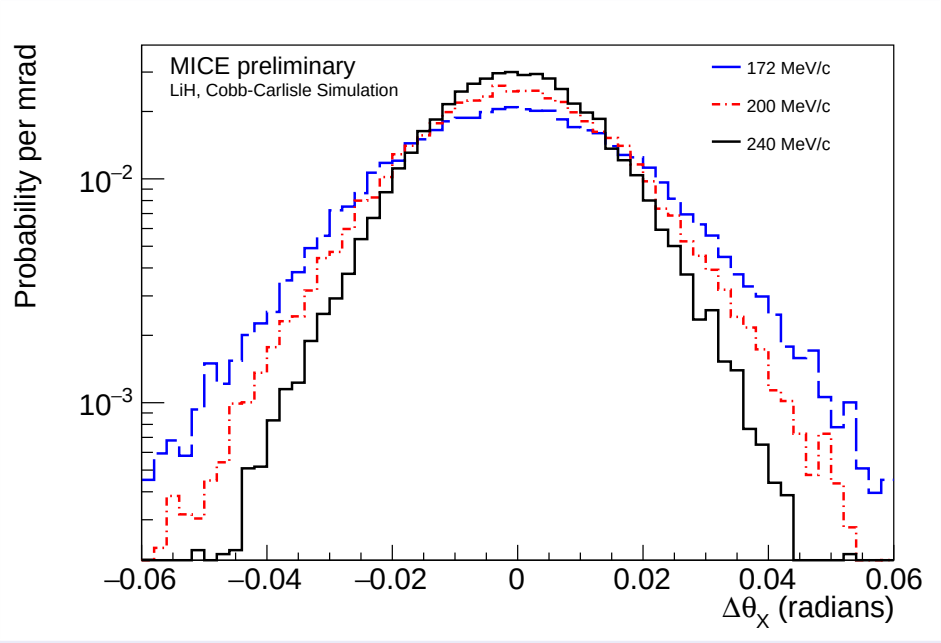
5 October, 2016

Mulitple Scattering Models and Definitions

GEANT 4 Scattering in LiH



Carlisle-Cobb Model in LiH



$$\Delta\theta_X = \arctan y'_{US} - \arctan y'_{DS},$$
$$\Delta\theta_Y = \arctan x'_{US} - \arctan x'_{DS}$$

- Width approximated by

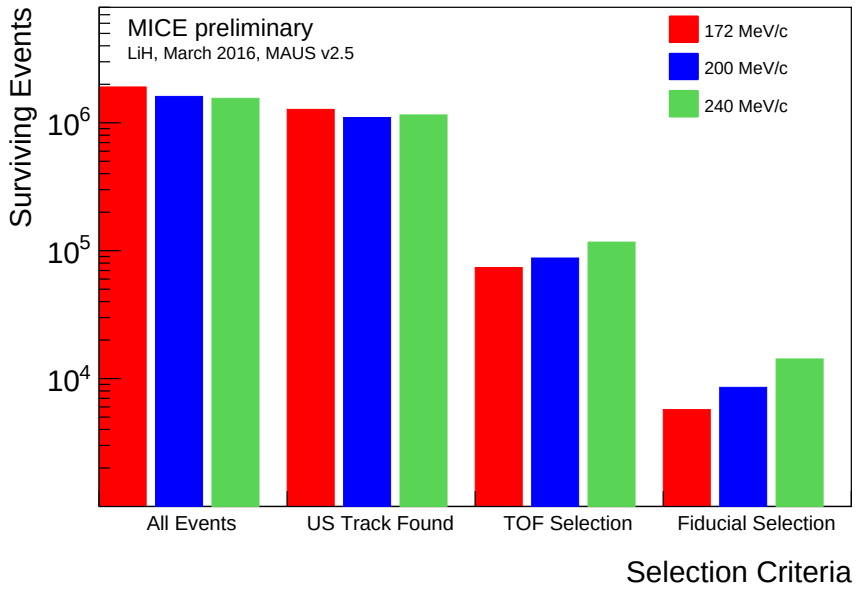
$$\frac{d\sigma_{\Delta\theta^2}}{dz} = \left(\frac{13.6\ \mathrm{MeV/c}}{p_\mu \beta}\right)^2 \frac{1}{X_0}$$

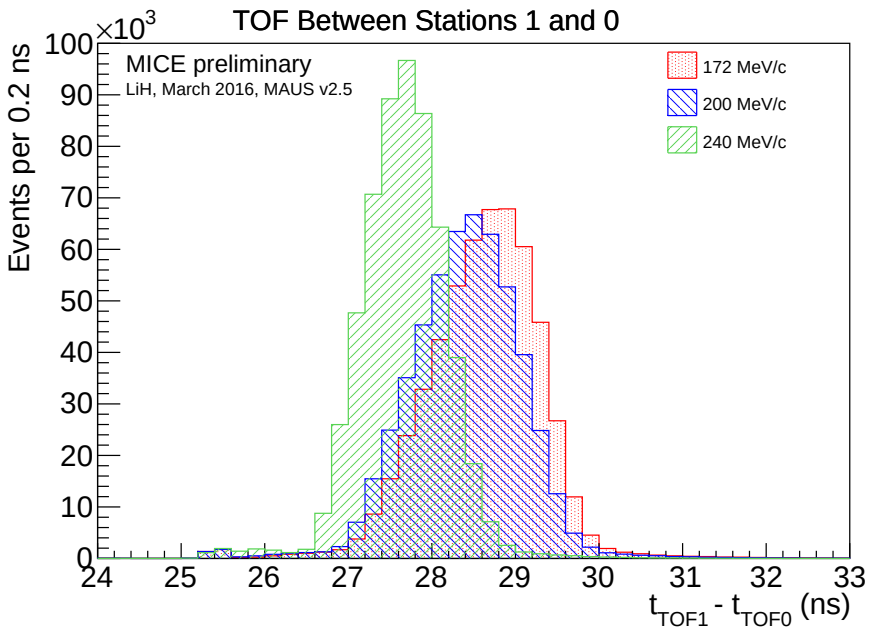
- $\Delta\theta$ is the projection about the X or Y axis.

- Alternatively, define the 3-D scattering angle

$$\theta_{Scatt} = \frac{\vec{p}_{US} \cdot \vec{p}_{DS}}{|\vec{p}||\vec{p}|} \approx \sqrt{\Delta\theta_X^2 + \Delta\theta_Y^2}$$

Event Selection



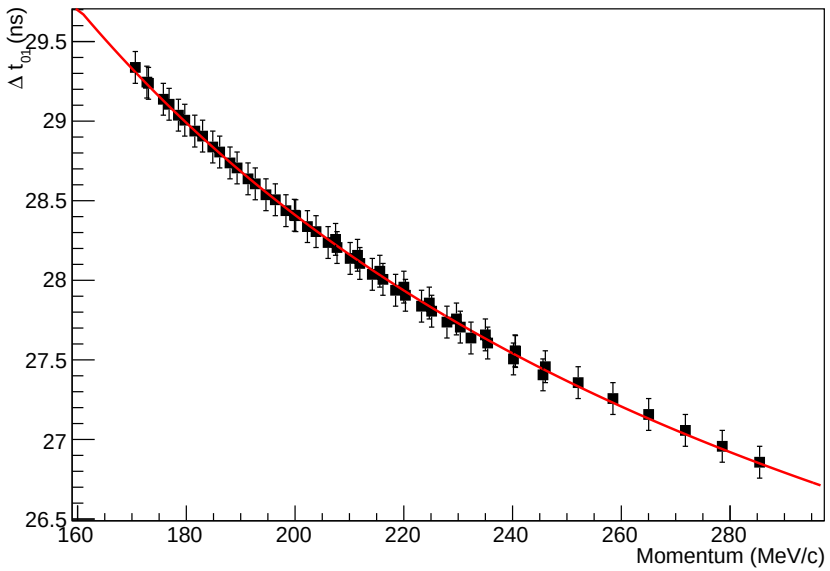


- Require a US track. If a DS track not extant, statistics are set to overflow values.
- Assumed a 200 ps selection;
  - $\Delta t_{01} \in \{29.15, 28.35\}$ for 172 MeV/c beams
  - $\Delta t_{01} \in \{28.31, 28.51\}$ for 200 MeV/c beams
  - $\Delta t_{01} \in \{27.45, 27.65\}$ for 240 MeV/c beams
- Require projection of US tracks to appear within central 150 mm radius of DS plane 1 projected with 5 mrad dispersion added (18 mm).

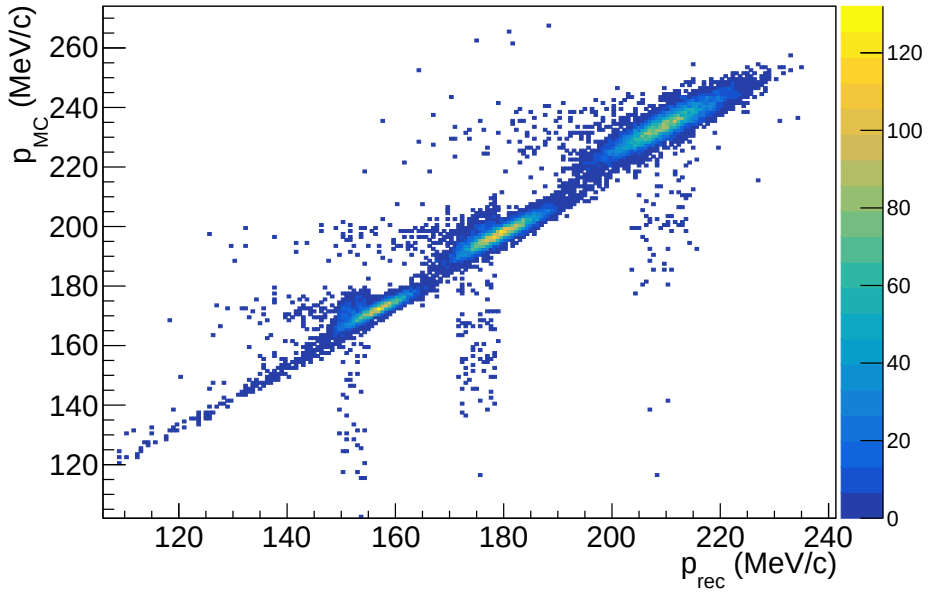
Motivating the TOF Selection

- Scan the TOF selection over the TOF spectrum
- Plot the TOF selection against the mean calc. momentum.
- Correct the momentum by fit to response of TOF calc. in MC.

The TOF vs. Mean Momentum



Momentum Rec. Response from MC



- Assume Rec. offset wrt to MC $p_{MC} = p_{rec} + 19.46(2)$ MeV/c.
- Deviations are systematic effects

Motivating the TOF Selection

- Scan the TOF selection over the TOF spectrum
- Plot the TOF selection against the mean calc. momentum.
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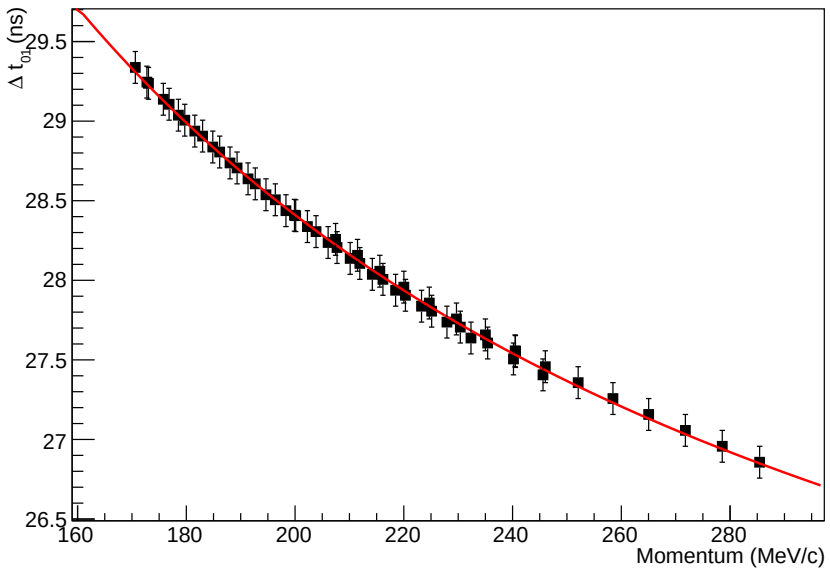
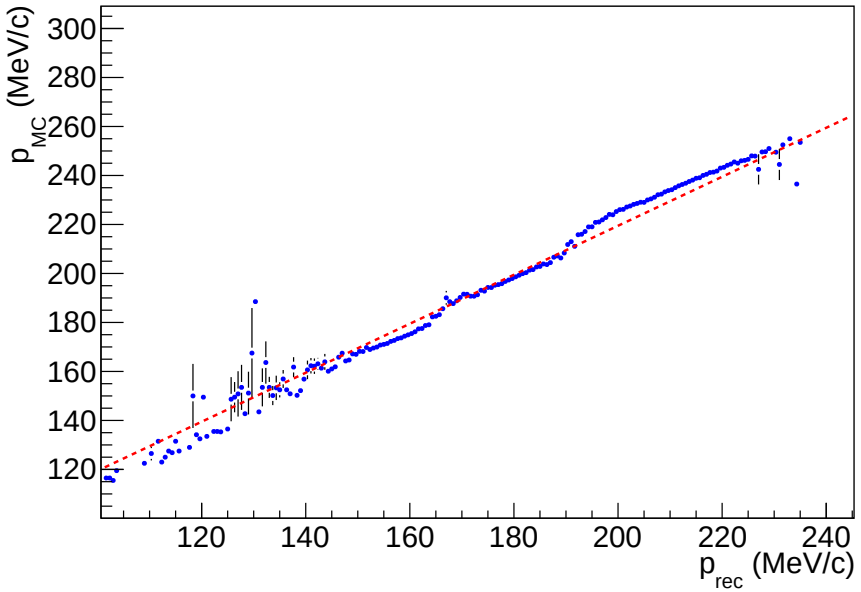
The TOF vs. Mean Momentum



Momentum Rec. Response from MC

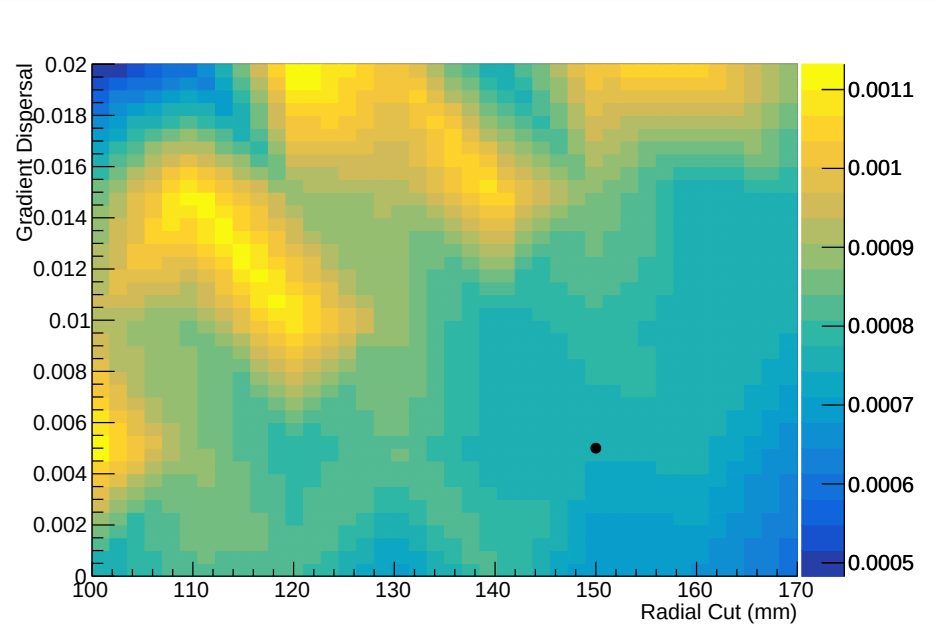


- Assume Rec. offset wrt to MC $p_{MC} = p_{rec} + 19.46(2)$ MeV/c.
- Deviations are systematic effects

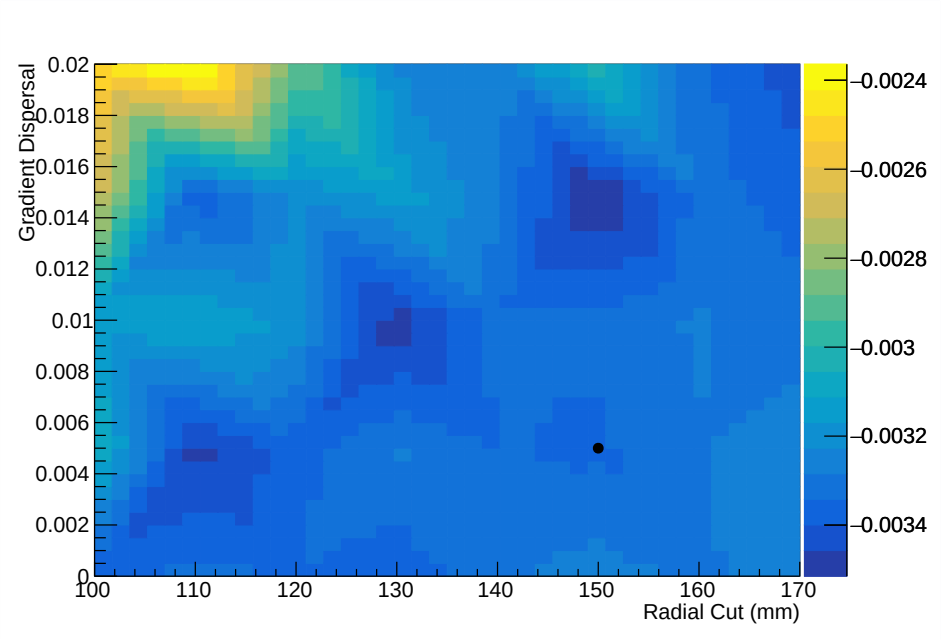
Motivating the Fiducial Selection

- Use to reduce scattering bias (to first order $\langle x'_{DS} \rangle - \langle x'_{US} \rangle$)

$\langle x'_{DS} \rangle - \langle x'_{US} \rangle$



$\langle y'_{DS} \rangle - \langle y'_{US} \rangle$

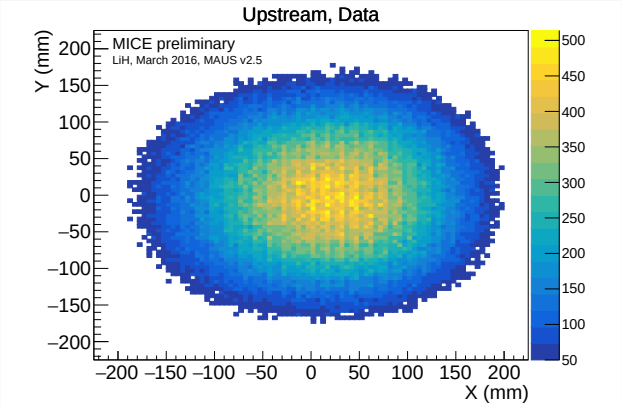


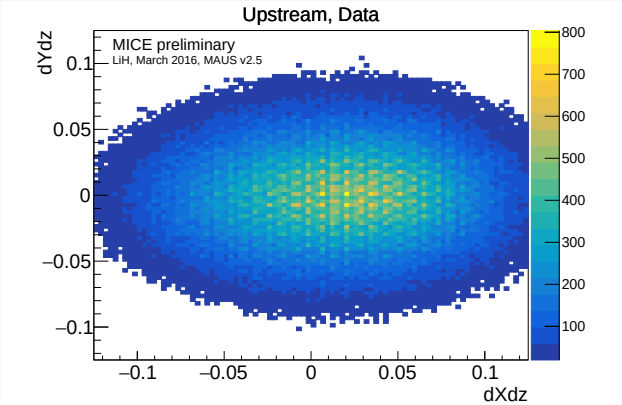
- Better behaviour for a larger radius and smaller projection dispersal.
- Largest radius allowed is 150 mm.

Beam After Selection Criteria

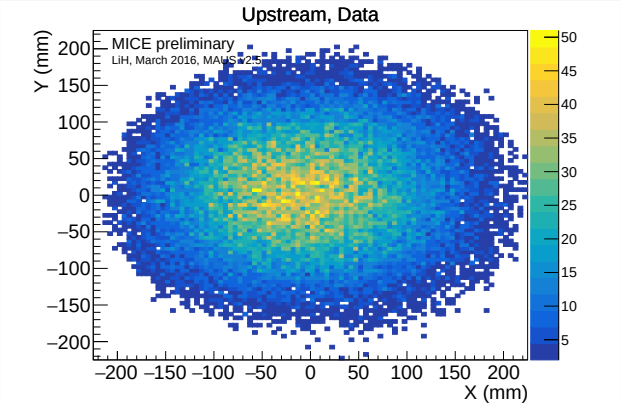
- 200 MeV/c muon beam at TkU Station 1 shown.

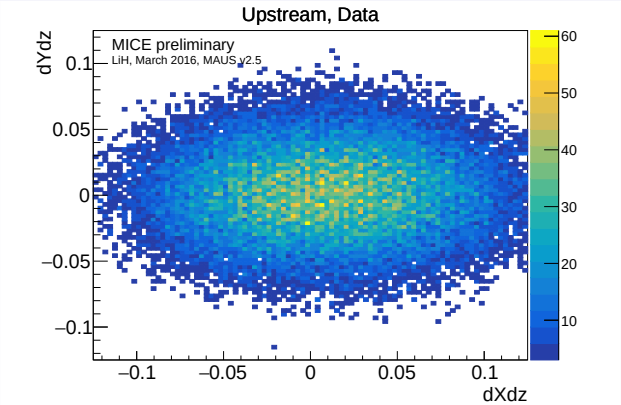
All TkU Events



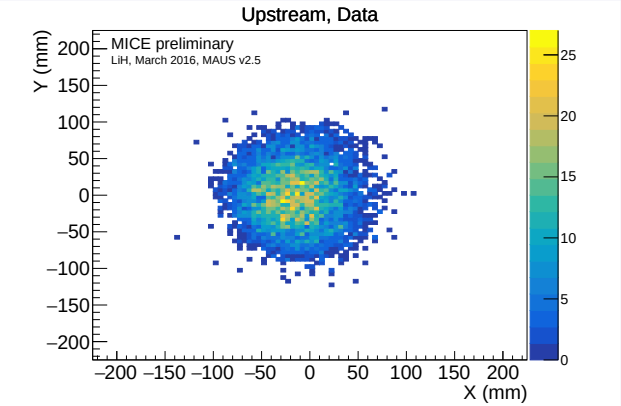


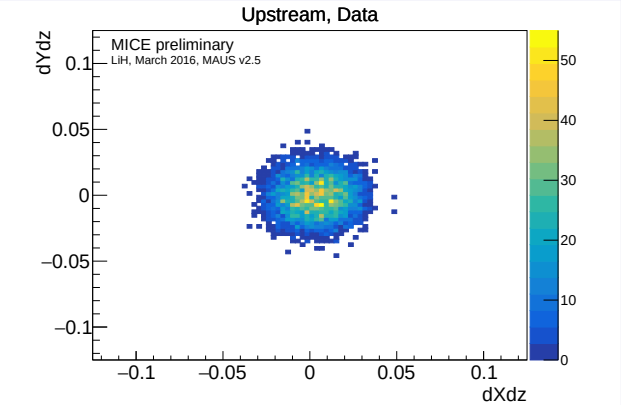
After TOF Selection





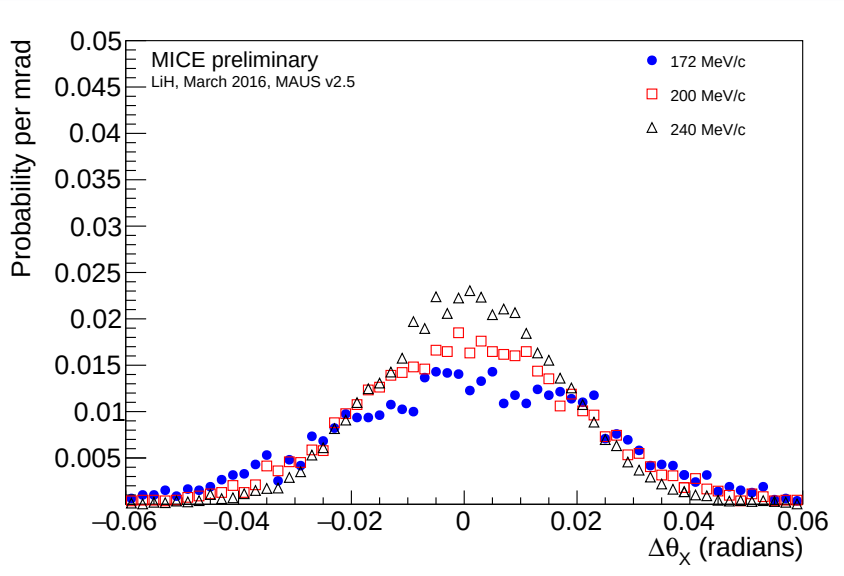
After Fid. Selection

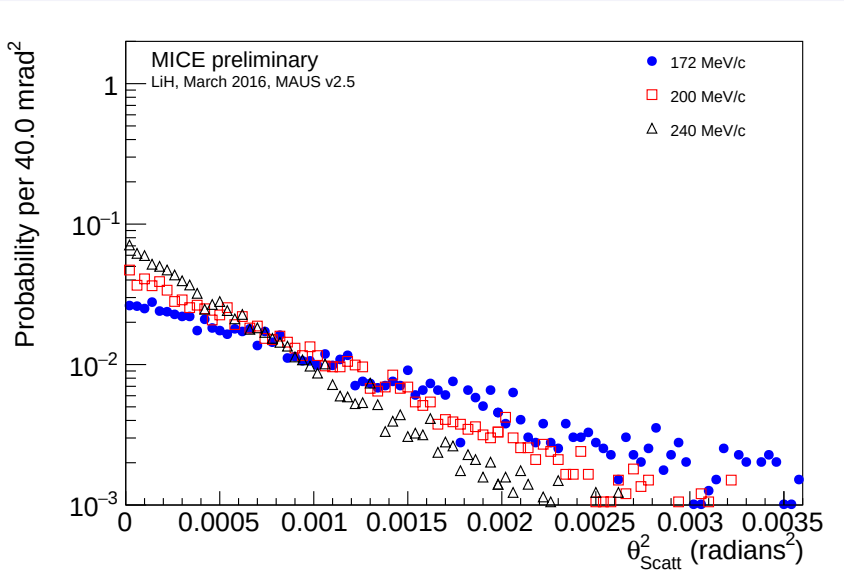




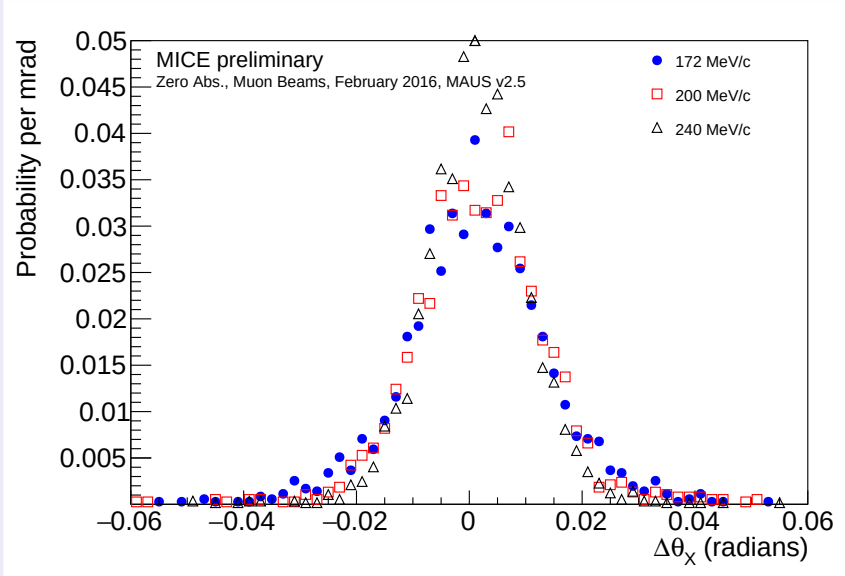
Scattering Distributions in Data

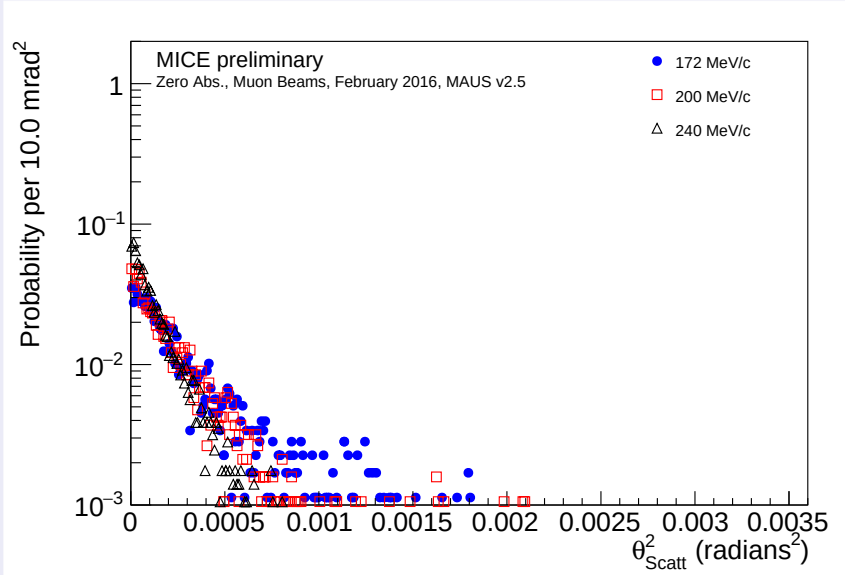
Scattering with LiH



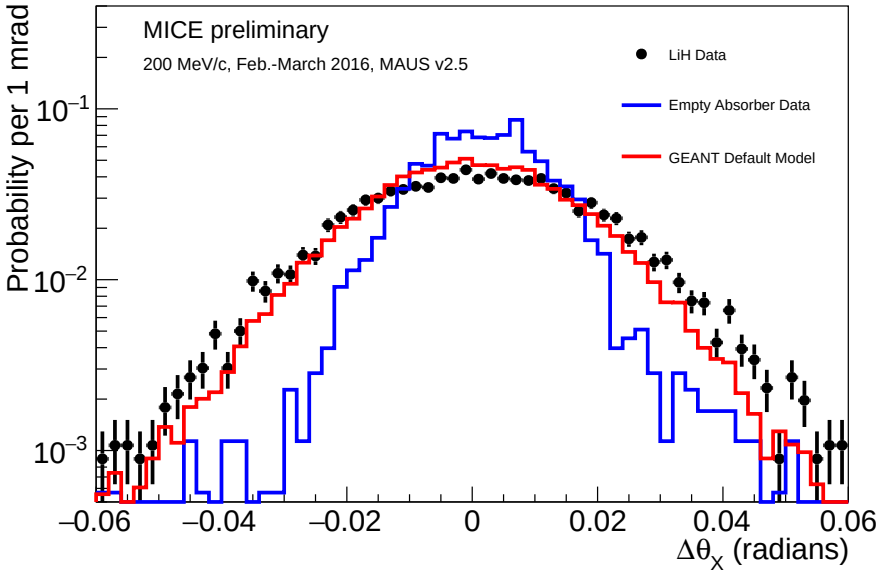


Scattering without LiH

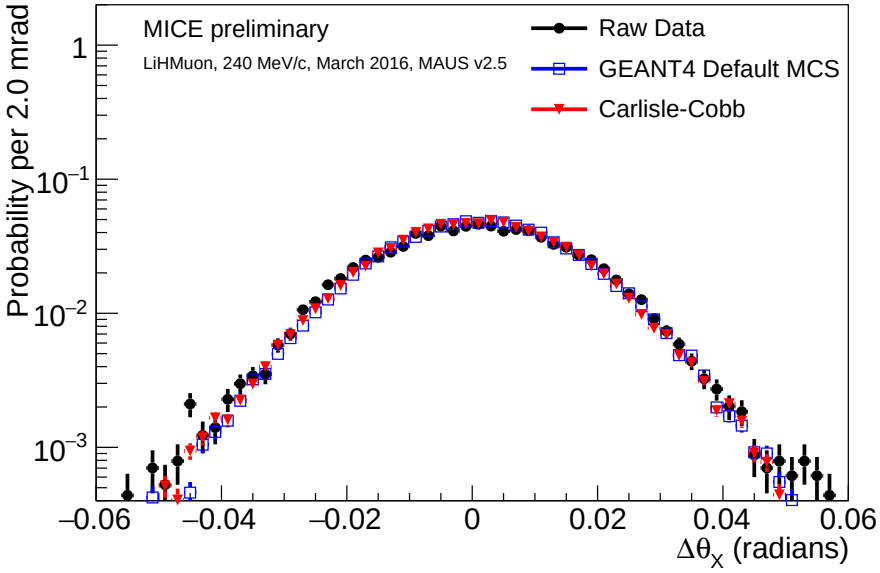


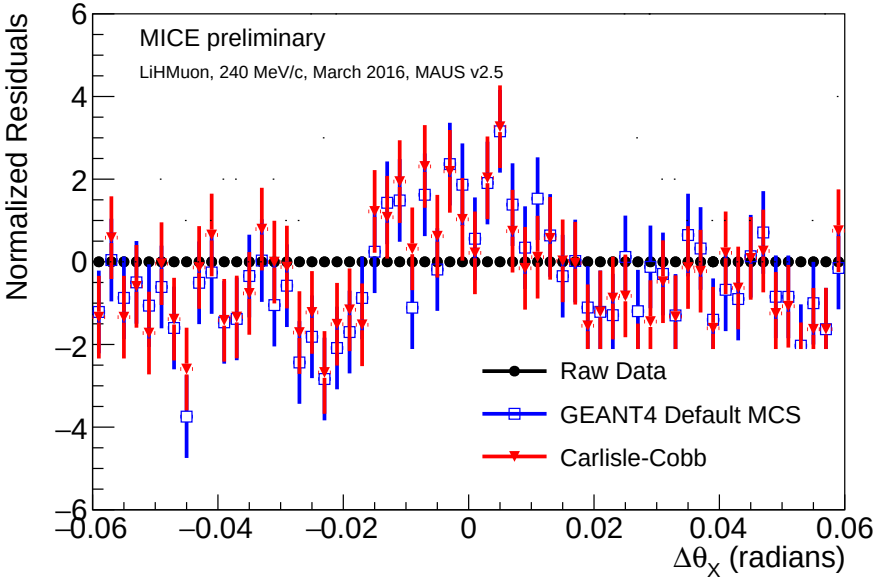


Convolution Between Models and Empty AFC Data



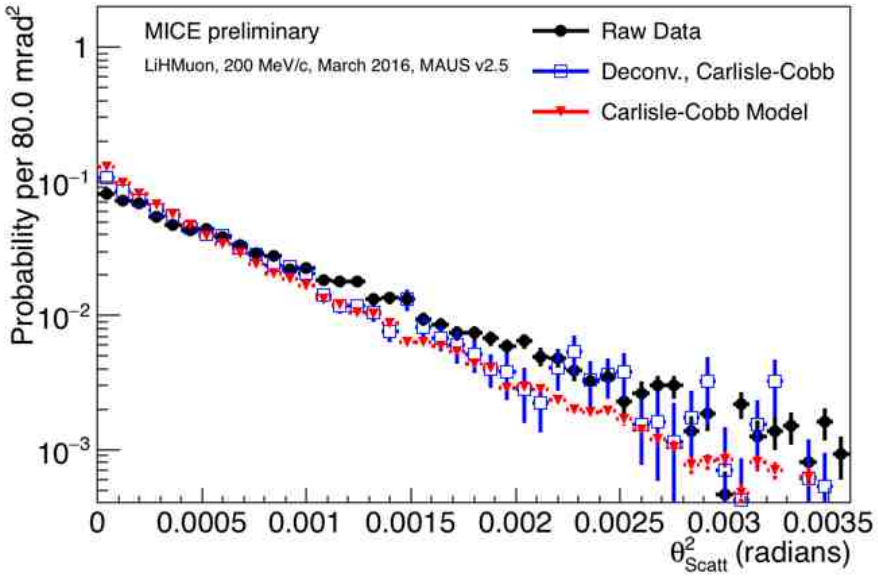
- Add model of scattering in LiH to the Empty AFC data.
- Model is sampled 10 times for each Empty AFC event to increase statistics.
- Ideal for model testing;
  - Calculate χ^2 from the data and the model convolution.

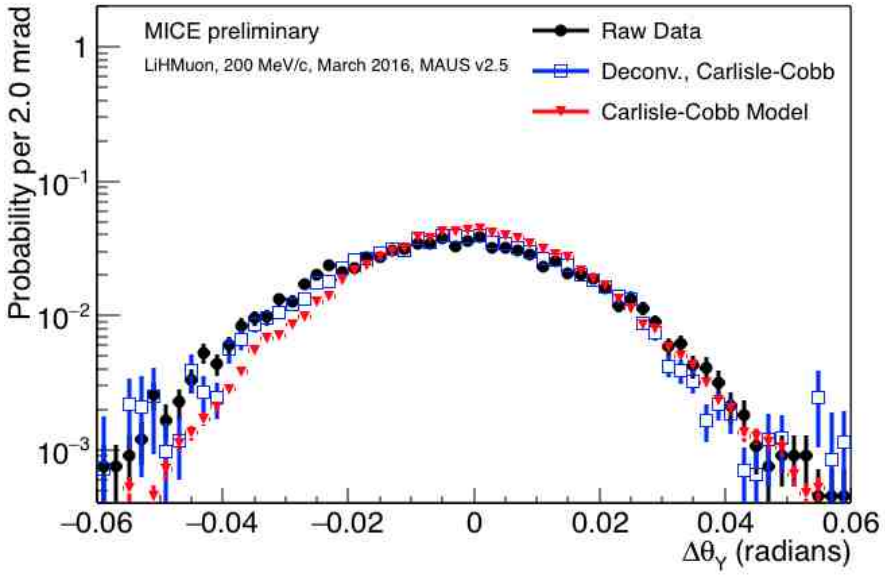




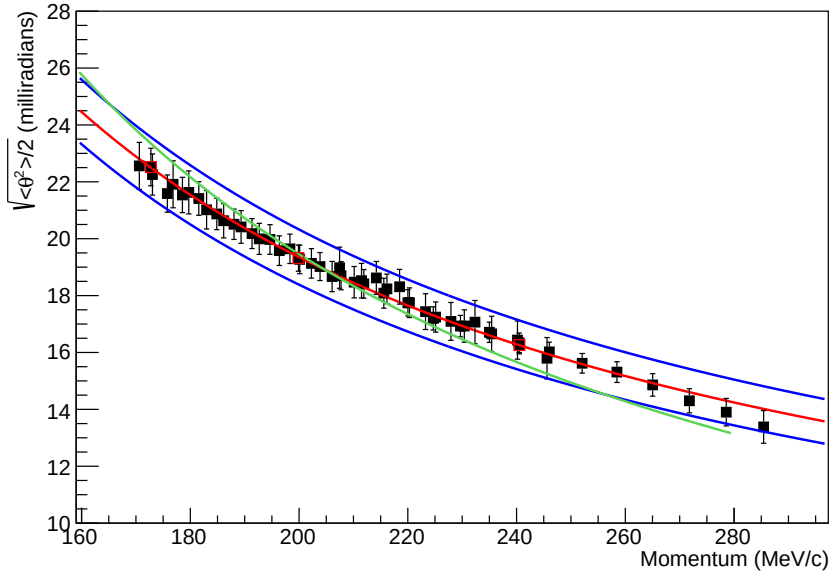
Deconvolution

- Use a iterative algorithm that uses the conditional probability to characterize the response of the reconstructed scattering angle to the true scattering angle.
- Requires a model of the true scattering angle in the absorber material.
- Convolution between Empty AFC data and scattering models used to provide response.

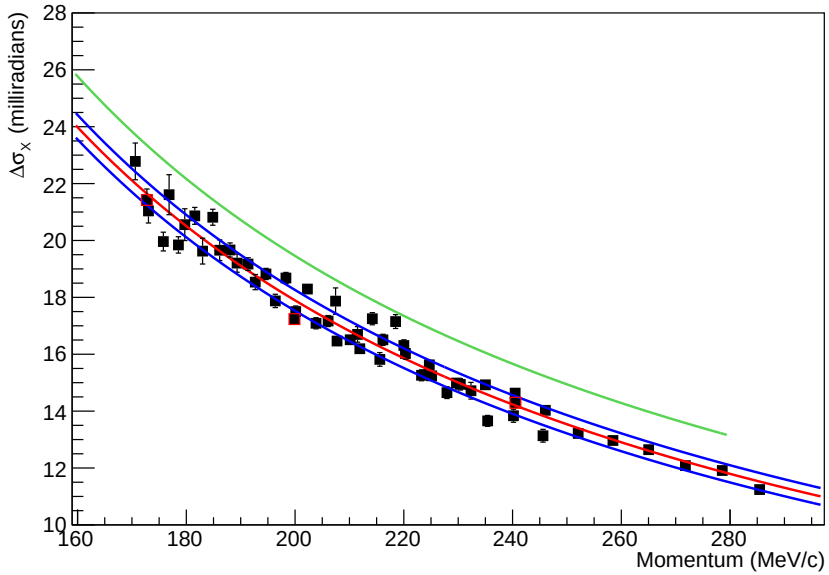


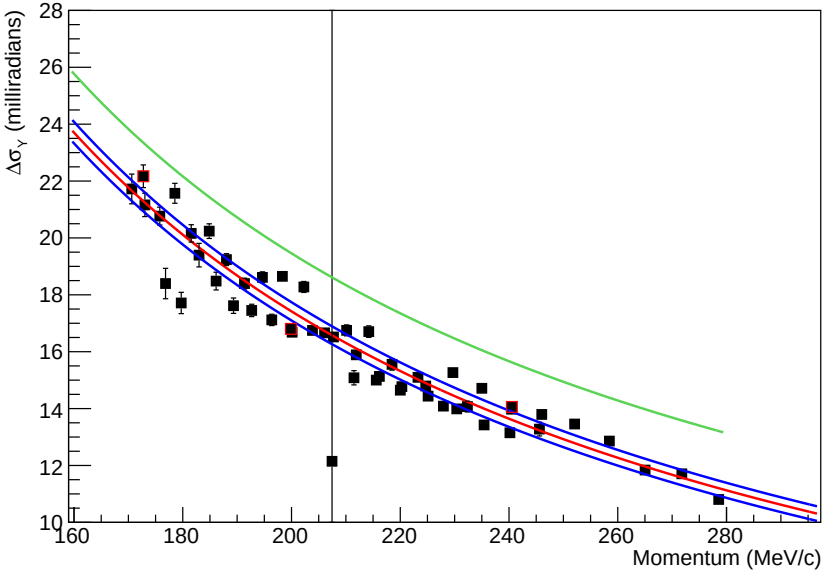


Momentum Dependence and Greisen and Rossi



- Widths of the deconvolution versus $\langle p \rangle$
- Red line: fit to $f(p) = a/(p\beta) + b$.
- Blue lines: uncertainty of fit.
- Green line: $a = \sqrt{z/X_0}$, $b = 0$.





TOF and Momentum Systematic

- Add and subtract 400 ps to mean TOF of event selection.
- Scale difference by 129 ps/800 ps to reflect TOF resolution and momentum calibration.

Effect before Deconvolution

	$\Delta\theta_X$			$\langle\theta^2_{Scatt}\rangle$		
	Δ	σ	rel. σ	Δ	σ	rel. σ
171.95±0.03	-1.83	0.29	0.01	-3.64	0.59	0.02
198.71±0.04	-3.17	0.51	0.03	-3.39	0.55	0.02
240.01±0.04	-3.31	0.53	0.03	-4.08	0.66	0.03

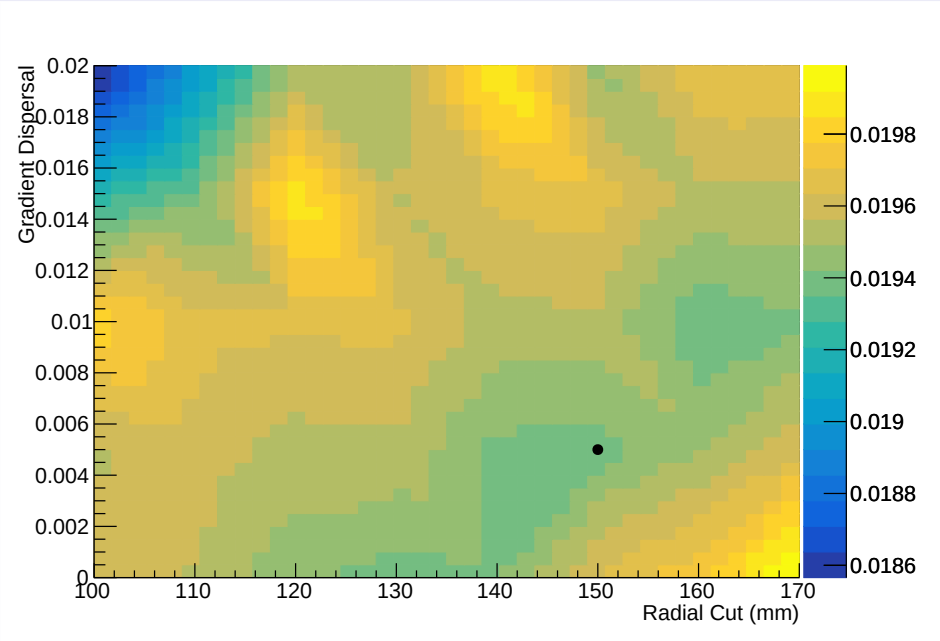
Effect After Deconvolution

	$\Delta\theta_X$			$\langle\theta^2_{Scatt}\rangle$		
	Δ	σ	rel. σ	Δ	σ	rel. σ
171.95±0.03	5.38	0.87	0.04	-2.1	0.34	0.01
198.71±0.04	-1.82	0.29	0.02	-3.56	0.57	0.03
240.01±0.04	-3.89	0.63	0.04	-3.49	0.56	0.03

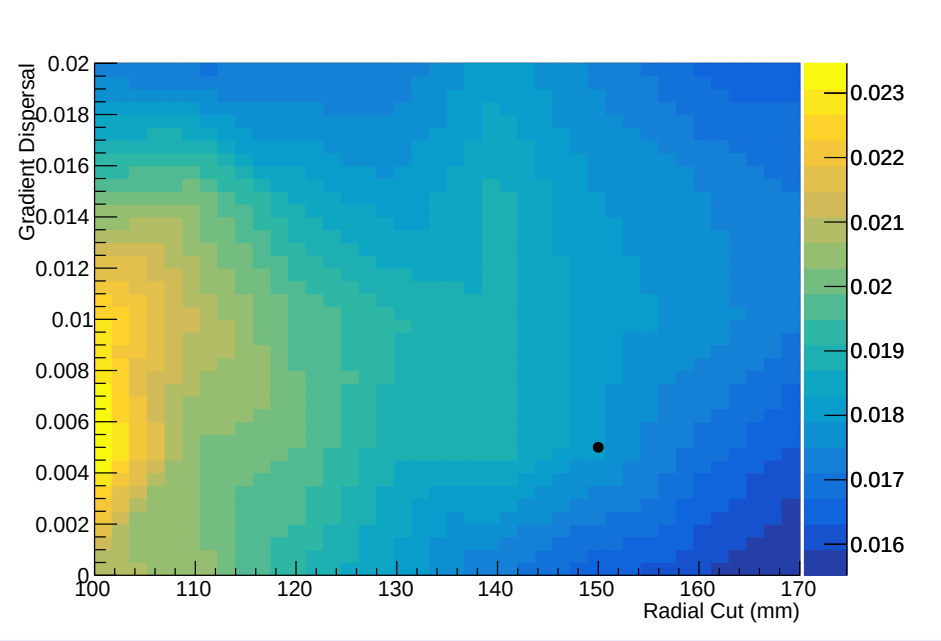
Fiducial Systematic

- Use the previous scan on the fiducial parameter to determine distribution sensitivity
- Select adjacent points about selected fiducial as limiting cases.
- Use the tracker resolution (0.478 mm) as a reference scale for changes.

Widths post selection



Deconvolved width



Fiducial Systematic

Fid. Pitch

	$\Delta\theta_X$			$\langle\theta^2_{Scatt}\rangle$		
	Δ	σ	rel. σ	Δ	σ	rel. σ
171.95±0.03	-0.05	0.01	0.0	-0.11	0.03	0.0
198.71±0.04	0.32	0.08	0.0	0.34	0.08	0.0
240.01±0.04	0.27	0.07	0.0	0.21	0.05	0.0

Fid. Radius

	$\Delta\theta_X$			$\langle\theta^2_{Scatt}\rangle$		
	Δ	σ	rel. σ	Δ	σ	rel. σ
171.95±0.03	0.56	0.01	0.0	0.13	0.0	0.0
198.71±0.04	-0.08	0.0	0.0	-0.09	0.0	0.0
240.01±0.04	-0.16	0.0	0.0	-0.1	0.0	0.0

Material Systematic

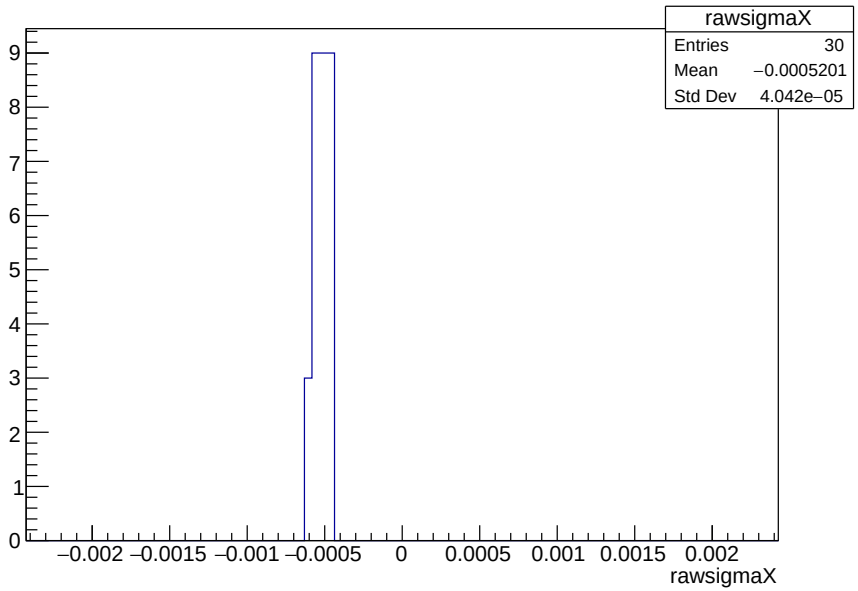
- Composition and density of LiH absorber has been verified
  - LiH composition: 81% ^{6}Li, 4% ^{7}Li, 14% ^{1}H (trace of C, O, and Ca).
  - Density: 0.693±0.004 g/cm^3.
- Exaggerate the effective material density or thickness by multiplying model by 1 ± 0.03.

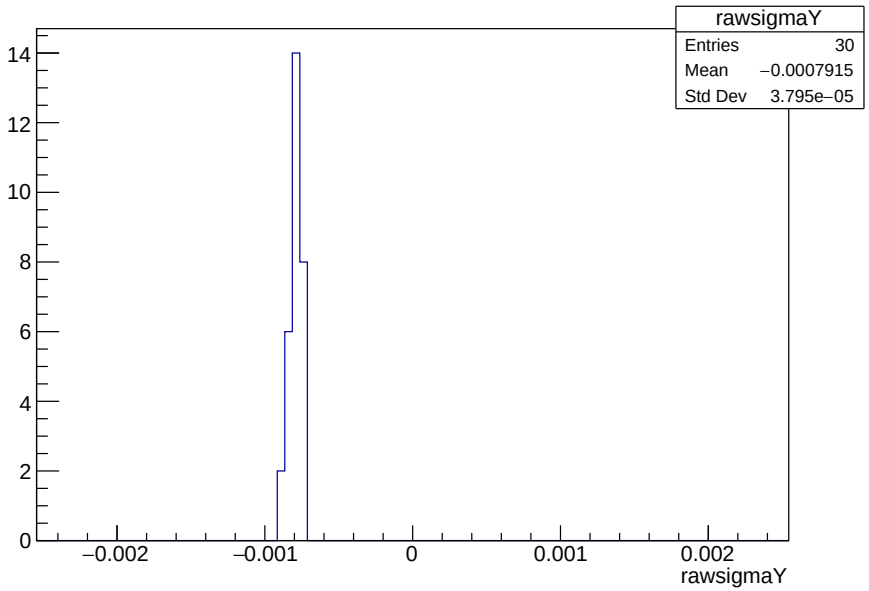
Effect before Deconvolution

	$\Delta\theta_X$			$\langle\theta^2_{Scatt}\rangle$		
	Δ	σ	rel. σ	Δ	σ	rel. σ
171.95±0.03	-1.57	0.39	0.02	0.04	0.01	0.0
198.71±0.04	0.05	0.01	0.0	-0.22	0.05	0.0
240.01±0.04	-0.18	0.04	0.0	-0.36	0.09	0.0

Alignment Systematics

- Defined by a complicated set of parameters
  - Use a "bootstrap" procedure to determine uncertainty.
  - Throw random misalignments about the expected values and look at RMS.
- RMS of gaussian widths is less than 10^{-5}





Model Comparisons with GEANT4 and Carlisle-Cobb

Compile the widths and χ^2 after event selection

$\langle p \rangle$ LiH	Angle	Data (mrad)	G4 (mrad)	χ^2 100 dof	CC (mrad)	χ^2 100 dof
171.95(3)	$\Delta\theta_X$	22.9±0.4±0.3	21.04±0.13	151.1	21.15±0.13	157.0
171.95(3)	$\Delta\theta_Y$	23.6±0.5±0.3	20.82±0.13	229.6	21.03±0.13	196.4
198.71(4)	$\Delta\theta_X$	19.2±0.2±0.5	18.49±0.09	141.7	18.36±0.09	148.2
198.71(4)	$\Delta\theta_Y$	19.2±0.2±0.5	18.52±0.09	186.8	18.33±0.09	148.7
240.01(4)	$\Delta\theta_X$	15.9±0.1±0.5	15.15±0.05	161.1	15.28±0.05	153.7
240.01(4)	$\Delta\theta_Y$	15.8±0.1±0.5	14.99±0.05	321.0	15.0±0.05	271.5
Xe						
229.52(5)	$\Delta\theta_X$	15.0±0.1±0.0	14.87±0.05	779.3	16.4±0.05	369.8
229.52(5)	$\Delta\theta_Y$	15.0±0.1±0.0	14.76±0.05	652.7	16.3±0.05	297.2

$\langle p \rangle$ LiH	Angle	Data (mrad)	G4 (mrad)	χ^2 51 dof	CC (mrad)	χ^2 51 dof
171.95(3)	$\langle\theta^2_{Scatt}\rangle$	33.0±1.2±0.6	29.45±0.31	115.3	29.63±0.31	132.4
198.71(4)	$\langle\theta^2_{Scatt}\rangle$	27.2±0.5±0.5	25.93±0.21	66.0	25.72±0.21	80.7
240.01(4)	$\langle\theta^2_{Scatt}\rangle$	22.3±0.2±0.7	21.2±0.11	132.3	21.3±0.11	124.4
Xe						
229.52(5)	$\langle\theta^2_{Scatt}\rangle$	20.9±0.2±0.0	20.88±0.12	641.8	22.99±0.11	499.8

Deconvolution Widths

Assume that GEANT4 provides the scattering model

$\langle p \rangle$ LiH	Angle	Data (mrad)	G4 (mrad)	χ^2 100	CC (mrad)	χ^2 100
171.95(3)	$\Delta\theta_X$	20.5±0.3±0.9	18.81±0.08	305.5	18.91±0.08	349.0
171.95(3)	$\Delta\theta_Y$	22.2±0.4±0.6	18.72±0.08	447.8	18.81±0.08	425.8
198.71(4)	$\Delta\theta_X$	17.1±0.2±0.3	16.21±0.06	209.5	16.07±0.06	243.5
198.71(4)	$\Delta\theta_Y$	16.8±0.1±0.3	16.12±0.06	379.6	15.96±0.06	339.7
240.01(4)	$\Delta\theta_X$	14.1±0.1±0.6	13.2±0.04	304.3	13.39±0.04	361.3
240.01(4)	$\Delta\theta_Y$	14.1±0.1±0.1	13.22±0.04	679.4	13.3±0.04	565.9
Xe						
229.52(5)	$\Delta\theta_X$	7.1±0.0±0.0	6.61±0.03	4039.1	11.93±0.03	7311.5
229.52(5)	$\Delta\theta_Y$	7.2±0.0±0.0	6.66±0.03	3704.5	11.86±0.03	6373.2

$\langle p \rangle$ LiH	Angle	Data (mrad)	G4 (mrad)	χ^2 51 dof	CC (mrad)	χ^2 51 dof
171.95(3)	$\langle \theta^2_{Scatt} \rangle$	29.9±1.0±0.4	25.98±0.19	332.1	26.17±0.2	318.6
198.71(4)	$\langle \theta^2_{Scatt} \rangle$	23.6±0.4±0.6	22.61±0.14	140.4	22.37±0.14	200.9
240.01(4)	$\langle \theta^2_{Scatt} \rangle$	19.8±0.2±0.6	18.62±0.08	355.6	18.83±0.08	305.8
Xe						
229.52(5)	$\langle \theta^2_{Scatt} \rangle$	10.2±0.1±0.0	9.99±0.07	970.8	16.82±0.07	7800.7

Deconvolution Widths

Assume that Carlisle-Cobb provides the scattering model

$\langle p \rangle$ LiH	Angle	Data (mrad)	G4 (mrad)	χ^2 100 dof	CC (mrad)	χ^2 100 dof
171.95(3)	$\Delta\theta_X$	20.9±0.3±0.9	18.81±0.08	351.5	18.91±0.08	300.9
171.95(3)	$\Delta\theta_Y$	22.2±0.4±0.9	18.72±0.08	496.9	18.81±0.08	402.7
198.71(4)	$\Delta\theta_X$	17.0±0.2±0.5	16.21±0.06	336.0	16.07±0.06	265.6
198.71(4)	$\Delta\theta_Y$	16.9±0.2±0.4	16.12±0.06	421.0	15.96±0.06	293.8
240.01(4)	$\Delta\theta_X$	14.1±0.1±0.7	13.2±0.04	372.1	13.39±0.04	332.4
240.01(4)	$\Delta\theta_Y$	14.1±0.1±0.5	13.22±0.04	811.0	13.3±0.04	597.8
Xe						
229.52(5)	$\Delta\theta_X$	9.9±0.0±0.0	6.61±0.03	11163.1	11.93±0.03	2141.7
229.52(5)	$\Delta\theta_Y$	9.9±0.0±0.0	6.66±0.03	10385.6	11.86±0.03	1621.0

$\langle p \rangle$ LiH	Angle	Data (mrad)	G4 (mrad)	χ^2 51 dof	CC (mrad)	χ^2 51 dof
171.95(3)	$\langle \theta^2_{Scatt} \rangle$	29.6±1.0±0.4	25.98±0.19	265.2	26.17±0.2	255.4
198.71(4)	$\langle \theta^2_{Scatt} \rangle$	23.7±0.4±0.5	22.61±0.14	133.9	22.37±0.14	150.6
240.01(4)	$\langle \theta^2_{Scatt} \rangle$	20.0±0.2±0.7	18.62±0.08	418.7	18.83±0.08	310.4
Xe						
229.52(5)	$\langle \theta^2_{Scatt} \rangle$	13.7±0.1±0.0	9.99±0.07	8962.9	16.82±0.07	3074.4

To Do

- Scheme needed to incorporate alignment systematic into errors (find extrema?)
- Finalize fiducial selection.
- Tune TOF selection based on fiducial selection.
- Define momentum dependent systematics.
- Finalize systematics for Xenon data
- Update analysis note.