Emittance reduction in MICE

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1) Emittance definition

The RMS normalised emittance is expressed as

$$\epsilon_n = \frac{1}{m}\sqrt[4]{D} \tag{1}$$

with D the determinant of the covariance matrix defined by

$$D = \det \begin{bmatrix} V_{xx} & V_{xp_x} & V_{xy} & V_{xp_y} \\ V_{p_x x} & V_{p_x p_x} & V_{p_x y} & V_{p_x p_y} \\ V_{yx} & V_{yp_x} & V_{yy} & V_{yp_y} \\ V_{p_y x} & V_{p_y p_x} & V_{p_y y} & V_{p_y p_y} \end{bmatrix} = \sum_{\beta} V_{\alpha\beta} C_{\alpha\beta},\ \forall\alpha \tag{2}$$

with $V_{\alpha\beta}$ the covariance of α and β defined as

$$V_{\alpha\beta} = \frac{1}{N}\sum_{i=1}^{N}(\alpha_i - \langle\alpha\rangle)(\beta_i - \langle\beta\rangle) = \langle\alpha\beta\rangle - \langle\alpha\rangle\langle\beta\rangle, \tag{3}$$

and $C_{\alpha\beta}$ the (α, β)-cofactor of the covariance matrix.

Measurement error propagation

The covariances error correlation can be expressed as a rank-4 tensor,

$$\Sigma^V = A\Sigma A^T, \tag{4}$$

with $\Sigma_{i\alpha\beta j} = \delta_{ij}\delta_{\alpha\beta}\sigma^2_{\alpha_i}$ and A the derivative tensor:

$$A_{\alpha\beta\eta k} = \frac{\partial V_{\alpha\beta}}{\partial \eta_k} = \frac{1}{N}\left[\delta_{\eta\alpha}\left(\beta_k - \langle\beta\rangle\right) + \delta_{\eta\beta}\left(\alpha_k - \langle\alpha\rangle\right)\right]. \tag{5}$$

Inputting equation 5 into equation 4 yields

$$\begin{aligned}
\Sigma_{\alpha\beta\kappa\lambda} \quad = \quad & \frac{1}{N^2}\sum_{i=1}^{N}\Big[\delta_{\alpha\kappa}\sigma^2_{\alpha_i}\left(\beta_i - \langle\beta\rangle\right)\left(\lambda_i - \langle\lambda\rangle\right) \\
& +\delta_{\alpha\lambda}\sigma^2_{\alpha_i}\left(\beta_i - \langle\beta\rangle\right)\left(\kappa_i - \langle\kappa\rangle\right) \\
& +\delta_{\beta\kappa}\sigma^2_{\beta_i}\left(\alpha_i - \langle\alpha\rangle\right)\left(\lambda_i - \langle\lambda\rangle\right) \\
& +\delta_{\beta\lambda}\sigma^2_{\beta_i}\left(\alpha_i - \langle\alpha\rangle\right)\left(\kappa_i - \langle\kappa\rangle\right)\Big]
\end{aligned} \tag{6}$$

Measurement error propagation (2)

This error tensor propagates into the determinant error through

$$\begin{aligned}\sigma_D^2 &= \sum_{\alpha\beta\kappa\lambda} \frac{\partial D}{\partial V_{\alpha\beta}} \Sigma^V_{\alpha\beta\kappa\lambda} \frac{\partial D}{\partial V_{\kappa\lambda}} \\ &= \frac{4}{N^2} \sum_{i=1}^{N} \sum_{\alpha\beta} \left[\left(C^T \hat{\sigma}^i C \right)_{\alpha\beta} \left(\alpha_i - \langle \alpha \rangle \right) \left(\beta_i - \langle \beta \rangle \right) \right] \end{aligned} \quad (7)$$

with $\hat{\sigma}^i_{\alpha\beta} = \delta_{\alpha\beta}\sigma^2_{\alpha_i}$, the diagonal matrix that contains the errors. This eventually yields a measurement error on the emittance of

$$\sigma_{\epsilon_n} = \left| \frac{\partial \epsilon_n}{\partial D} \right| \sigma_D = \frac{D^{-3/4}}{4m} \sigma_D \quad (8)$$

Other quantities of interest

→ 4D transverse beta function:

$$\beta_\perp = \frac{V_{xx} + V_{yy}}{2\epsilon}$$

$$\text{with} \qquad \epsilon = \det{}^{\frac{1}{4}} \begin{bmatrix} V_{xx} & V_{xx'} & V_{xy} & V_{xy'} \\ V_{x'x} & V_{x'x'} & V_{x'y} & V_{x'y'} \\ V_{yx} & V_{yx'} & V_{yy} & V_{yy'} \\ V_{y'x} & V_{y'x'} & V_{y'y} & V_{y'y'} \end{bmatrix}, \quad q' = p_q/p_z \tag{9}$$

→ Mean total momentum:

$$|\vec{p}| = \sqrt{p_x^2 + p_y^2 + p_z^2} \tag{10}$$

→ Transmission in the cooling channel

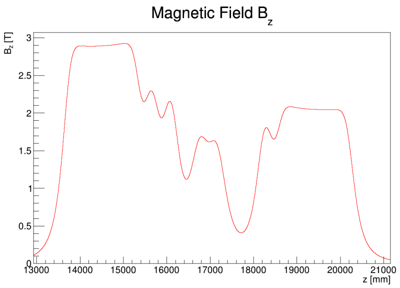
$$T_i = \frac{N_i}{N_0} \tag{11}$$

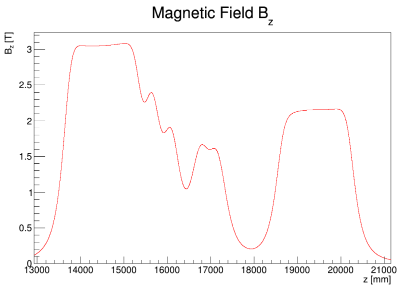
2.1) 200 MeV/c solenoid: 8 config. under investigation

- Two solenoid modes 200 MeV/c magnet settings (from A. Liu):

	ECE_U [%]	$M2_U$	$M1_U$	FC	$M1_D$	$M2_D$	ECE_D [%]
w/ $M2_D$	0.72	219.8	162.7	55.9	0	205.66	0.51
w/o $M2_D$	0.76	236.8	135.2	56	0	0	0.54

- 3 mm and 6 mm input normalised emittance
- With or without absorber (65 mm of LiH in this study)



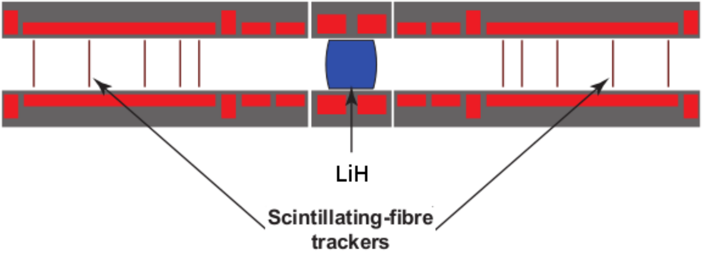


Geometries

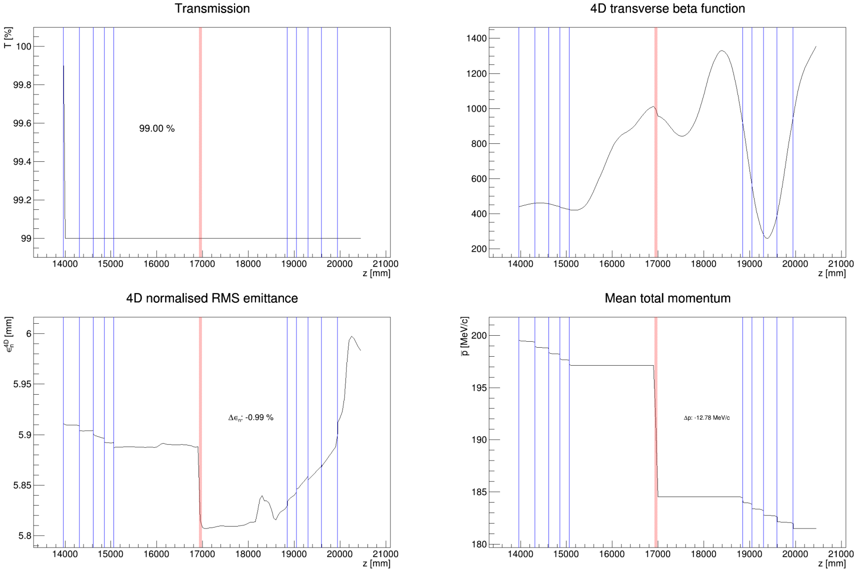
In first approximation, a simplified geometry was used

- → Two trackers in, 5 stations/tracker, 3 planes/station, full geometry
- → A simple 65 mm-thick, 225 mm in radius cylinder of LiH (or not)
- → Field maps generated in MAUS from the cooling channel currents
- → Fixed emittance input beam at 13800 m (just before TKUS5)
- → No momentum spread in the beam

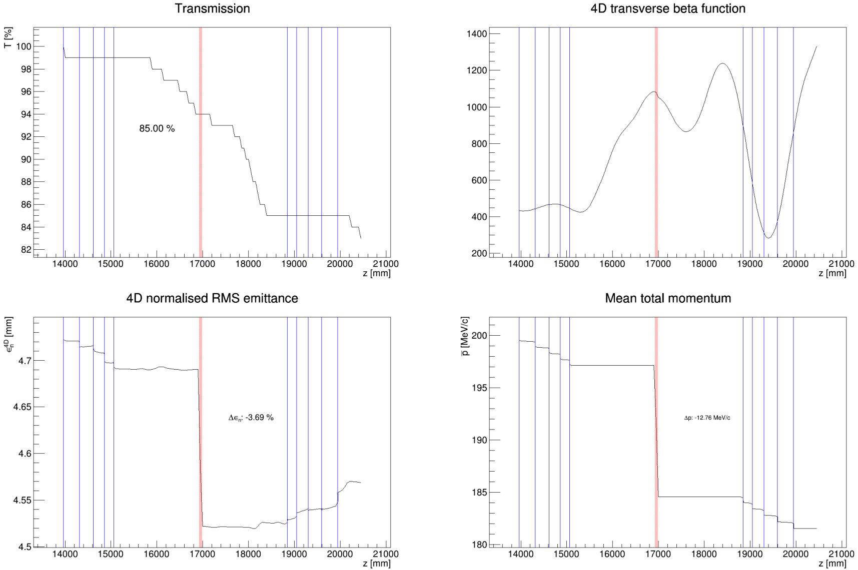
The simulations were also run with the full MAUS geometry and the same input beam, it did not have any significant effect on the measurements.



6mm, M2-on, LiH (300 mm fiducial+through)



6mm, M2-on, LiH (**150 mm fiducial+through**)



Summary of all 200 MeV/c solenoid configurations

M2 ON

3 mm, LiH	**Thru**
$\Delta\epsilon_n^{4D}$	+1.68%
Δp [MeV/c]	-12.73
Trans.[%]	97

6 mm, LiH	**Thru**
$\Delta\epsilon_n^{4D}$	-3.69%
Δp [MeV/c]	-12.76
Trans.[%]	85

3 mm, empty	**Thru**
$\Delta\epsilon_n^{4D}$	+0.11%
Δp [MeV/c]	-0.19
Trans.[%]	98

6 mm, empty	**Thru**
$\Delta\epsilon_n^{4D}$	+0.03%
Δp [MeV/c]	-0.20
Trans.[%]	86

M2 OFF

3 mm, LiH	**Thru**
$\Delta\epsilon_n^{4D}$	+2.14%
Δp [MeV/c]	-12.72
Trans.[%]	94

6 mm, LiH	**Thru**
$\Delta\epsilon_n^{4D}$	-4.87%
Δp [MeV/c]	-12.74
Trans.[%]	77

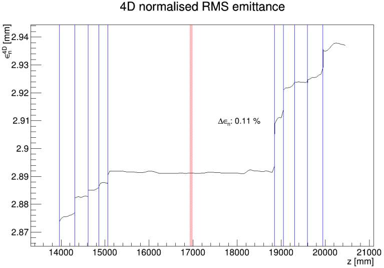
3 mm, empty	**Thru**
$\Delta\epsilon_n^{4D}$	+0.28%
Δp [MeV/c]	-0.19
Trans.[%]	96.

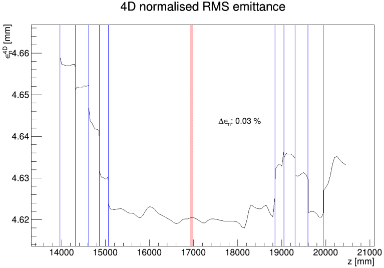
6 mm, empty	**Thru**
$\Delta\epsilon_n^{4D}$	-0.13%
Δp [MeV/c]	-0.20
Trans.[%]	78

Emittance reduction in the M2$_D$ **on** configurations

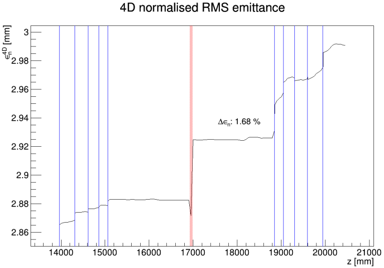
3 mm | **6 mm**

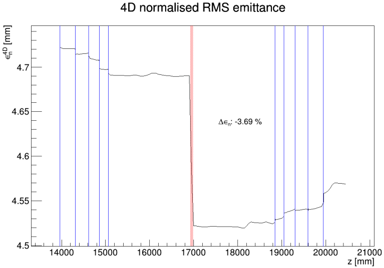
∅





LiH

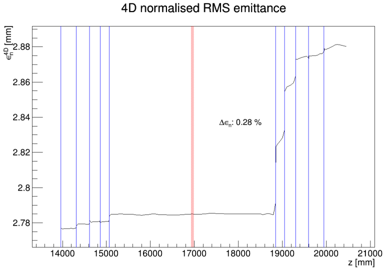


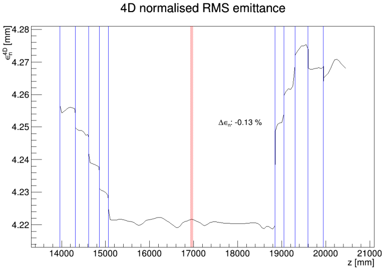


Emittance reduction in the M2$_D$ **off** configurations

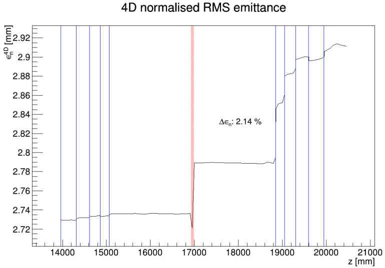
3 mm | **6 mm**

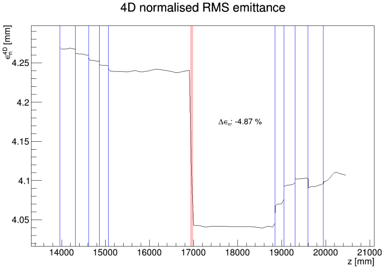
∅





LiH



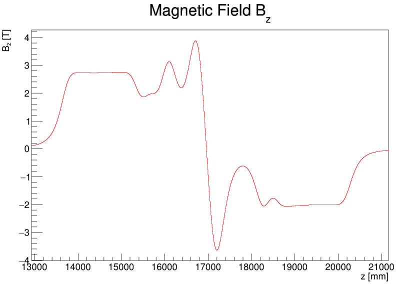


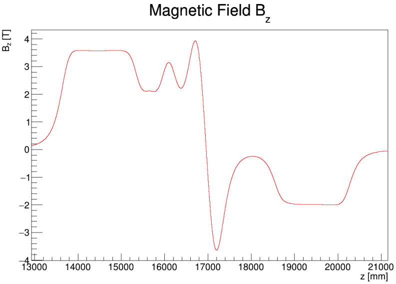
2.2) **200 MeV/c flip**: 8 config. under investigation

- Flip modes 200 MeV/c magnet settings (from A. Liu):

	ECE_U [%]	$M2_U$	$M1_U$	FC	$M1_D$	$M2_D$	ECE_D [%]
$M2_D$	0.68	150.40	253.18	$\pm$222.94	0	-244	-0.5
$\not{M}2_D$	0.89	153.19	251.15	$\pm$224.99	0	0	-0.5

- 3 mm and 6 mm input normalised emittance
- With absorber (65 mm of LiH in this study)





Summary of all 200 MeV/c flip configurations

M2 ON

3 mm, LiH	**Thru**	Recon.
$\Delta\epsilon_n^{4D}$	-2.75%	+0.53%
Δp [MeV/c]	-12.77	-14.36
Trans.[%]	99.00	100

6 mm, LiH	**Thru**	Recon.
$\Delta\epsilon_n^{4D}$	-4.20%	-1.65%
Δp [MeV/c]	-12.83	-13.64
Trans.[%]	91.21	100

3 mm, empty	**Thru**	Recon.
$\Delta\epsilon_n^{4D}$	+0.85%	+3.34%
Δp [MeV/c]	-0.19	-1.73
Trans.[%]	98.75	100

6 mm, empty	**Thru**	Recon.
$\Delta\epsilon_n^{4D}$	+1.28%	+3.36%
Δp [MeV/c]	-0.20	-1.10
Trans.[%]	89.80	100

M2 OFF

3 mm, LiH	**Thru**	Recon.
$\Delta\epsilon_n^{4D}$	-3.57%	+4.02%
Δp [MeV/c]	-12.76	-13.14
Trans.[%]	90.59	100

6 mm, LiH	**Thru**	Recon.
$\Delta\epsilon_n^{4D}$	-6.03%	-1.69%
Δp [MeV/c]	-12.84	-13.20
Trans.[%]	71.77	100

3 mm, empty	**Thru**	Recon.
$\Delta\epsilon_n^{4D}$	+0.93%	+7.39%
Δp [MeV/c]	-0.20	-0.67
Trans.[%]	87.95	100

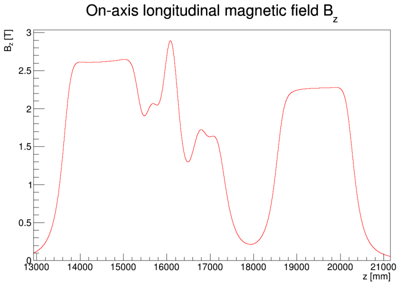
6 mm, empty	**Thru**	Recon.
$\Delta\epsilon_n^{4D}$	+0.74%	+4.20%
Δp [MeV/c]	-0.20	-0.47
Trans.[%]	67.55	100

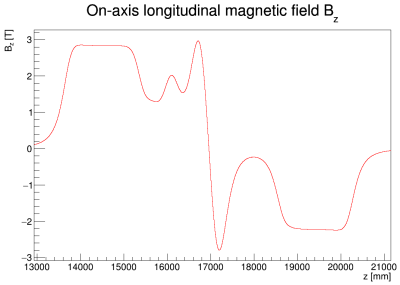
2.3) 140 MeV/c: 4 configurations under investigation

- Flip and solenoid modes 140 MeV/c magnet settings (from A. Liu):

	ECE_U [%]	$M2_U$	$M1_U$	FC	$M1_D$	$M2_D$	ECE_D [%]
Sol.	0.65	172.39	242.20	56.15	0	0	0.57
Flip	0.71	80.0	158.14	$\pm$172.05	0	0	-0.56

- 3 mm and 6 mm input normalised emittance reduction
- With absorber (65 mm of LiH in this study)





Summary of all 140 MeV/c configurations

Solenoid

3 mm	**Thru**	Recon.
$\Delta\epsilon_n^{4D}$	$+1.11\%$	$+9.98\%$
Δp [MeV/c]	-17.84	-16.76
Trans.[%]	89.03	69.75

6 mm	**Thru**	Recon.
$\Delta\epsilon_n^{4D}$	-11.54%	-2.88%
Δp [MeV/c]	-17.93	-16.39
Trans.[%]	65.76	58.18

Flip

3 mm	**Thru**	Recon.
$\Delta\epsilon_n^{4D}$	-9.89%	-0.26%
Δp [MeV/c]	-17.86	-16.22
Trans.[%]	91.45	69.12

6 mm	**Thru**	Recon.
$\Delta\epsilon_n^{4D}$	-14.26%	-5.49%
Δp [MeV/c]	-18.03	-16.37
Trans.[%]	65.86	52.22

→ Solenoid mode has a defocus at the absorber, poor cooling

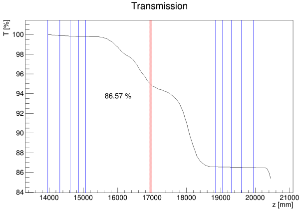
→ Flip mode has a tight focus

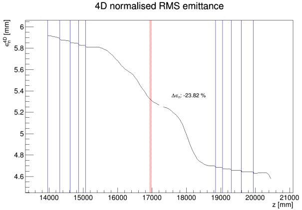
→ Very poor transmission at 6 mm, should maybe try to optimize the input emittance, M2 would also help...

→ Strong bias from scraping, but seems to be real cooling in flip mode

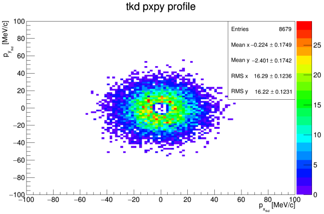
Main sources of bias on the emittance

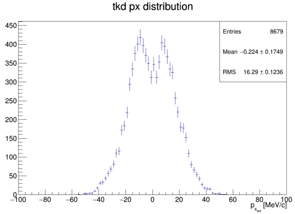
1 **Poor transmission**: scraping gives a seemingly reduced emittance





2 **Reconstruction inefficiencies**: The reconstruction produces a seemingly higher emittance due to the poor low p_T efficiency





3) Toy MC of the scraping bias

- To simplify, take the input beam to be an uncorrelated 2D Normal:

$$x_i \sim \mathcal{N}(0,\, \sigma_x^2) \quad p_{x,i} \sim \mathcal{N}(0,\, \sigma_{p_x}^2) \quad V_i = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_{p_x}^2 \end{bmatrix} \tag{12}$$

$\rightarrow \epsilon_i = \sqrt{\det V}/m$

$\rightarrow$ With the same uncorrelated dist. of y: $\epsilon_n^{4D} = \epsilon_i$

- Fully deterministic energy loss using the BB formula:

$$-\frac{dE}{dx} = \frac{k_1}{\beta^2}(\ln(k_2\beta^2\gamma^2) - \beta^2) \tag{13}$$

- Normal scattering: $\mathcal{N}(0,\, \theta_0^2)$
- The output distributions read:

$$x_o \sim \mathcal{N}(0,\, \sigma_x^2) \quad p_{x,o} \sim \mathcal{N}(0,\, p_o^2(\frac{\sigma_{p_x}^2}{p_i^2} + \theta_0^2)) \quad V_o = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & p_o^2(\frac{\sigma_{p_x}^2}{p_i^2} + \theta_0^2) \end{bmatrix}$$

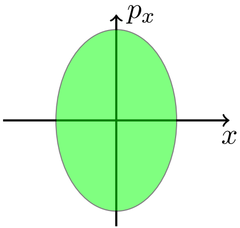
Effect of a drift on the distribution

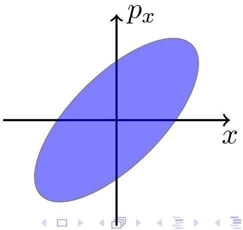
The effect of a drift of Δz on the covariance matrix is a transfer matrix:

$$V_p = M V_o M^T \qquad M = \begin{bmatrix} 1 & \Delta z/p_o \\ 0 & 1 \end{bmatrix}$$
$$\rightarrow V_p = \begin{bmatrix} \sigma_x^2 + \sigma_{p_{x,o}}^2 \Delta z^2/p_o^2 & \sigma_{p_{x,o}}^2 \Delta z/p_o \\ \sigma_{p_{x,o}}^2 \Delta z/p_o & \sigma_{p_{x,o}}^2 \end{bmatrix} \qquad \det V_p = \det V_o \tag{14}$$

In terms of distribution this is equivalent to a correlated 2D gaussian:

$$f_{x_p p_{x,p}}(x,p) = \frac{1}{2\pi\sigma_x\sigma_{p_{x,o}}} \exp\left[-\frac{(x-(p/p_o)\Delta z)^2}{2\sigma_x^2}\right] \exp\left[-\frac{p^2}{2\sigma_{p_{x,o}}^2}\right] \tag{15}$$





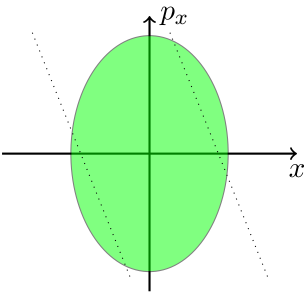
Scraping after a drift

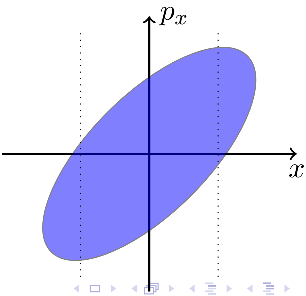
If we produce a radial cut, this is equivalent to cutting the edges in x:

$$f(x,p) = \frac{\Gamma}{2\pi\sigma_x\sigma_{p_{x,o}}}\exp\left[-\frac{(x-(p/p_o)\Delta z)^2}{2\sigma_x^2}\right]\exp\left[-\frac{p^2}{2\sigma_{p_{x,o}}^2}\right]\chi_{[-x_L,x_L]}(x)$$

If we operate the variable change $\xi = x - p\Delta z/p_o$, we get:

$$f(x,p) = \frac{\Gamma}{2\pi\sigma_x\sigma_{p_{x,o}}}\exp\left[-\frac{\xi^2}{2\sigma_x^2}\right]\exp\left[-\frac{p^2}{2\sigma_{p_{x,o}}^2}\right]\chi_{[-x_L+\frac{p}{p_o}\Delta z,x_L+\frac{p}{p_o}\Delta z]}(x)$$





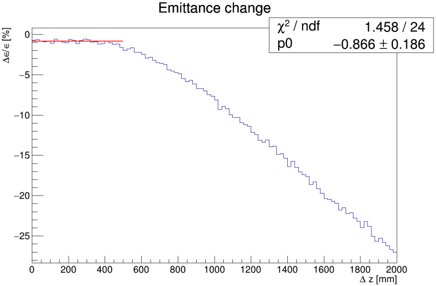
Emittance change of a sampled drifted distribution

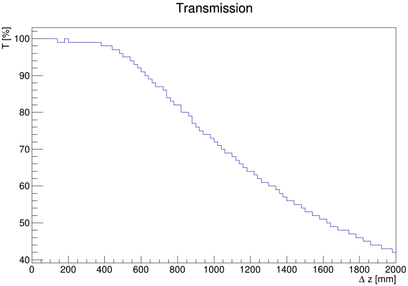
Toy MC settings:

- 200 MeV$/c$ muons, no p spread
- 6 mm input normalized emittance ϵ_n
- 200 mm β function at the absorber, ($V_{xx} \simeq 600$ mm)
- No correlation in the input

Results:

- Transmission drops as the drift length increases
- A loss in transmission causes a loss in high amplitude scatters first, hence the bias on the emittance change

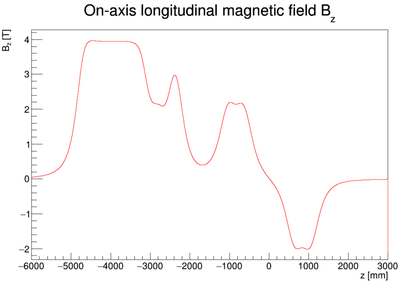


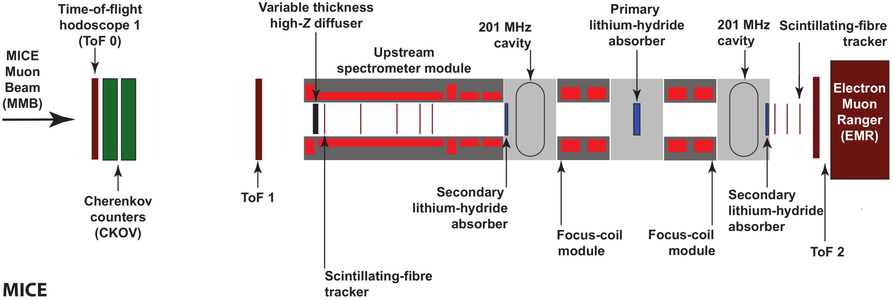


4) MICE Cooling Demo: SSD-less descope lattice

- EMR for DS momentum, tracker planes give x', y;
- Flip mode 200,MeV/c magnet settings (from the CD baseline);
- One primary (65 mm) and two secondary (32.5 mm) LiH abs.

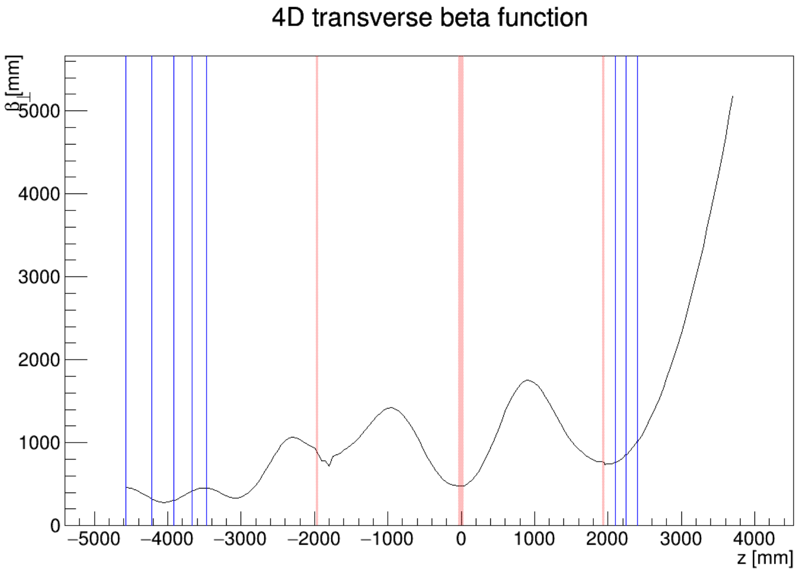
→ Cooling? Transmission? **Bias**?



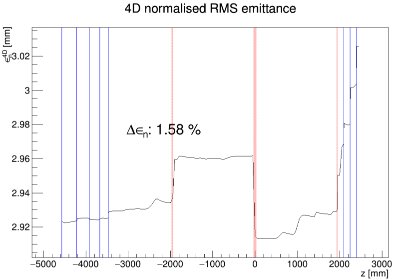


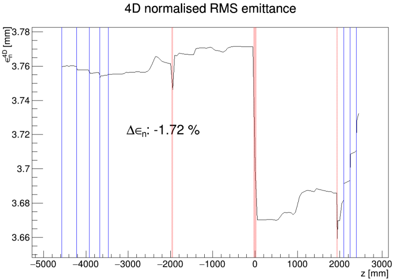
MICE

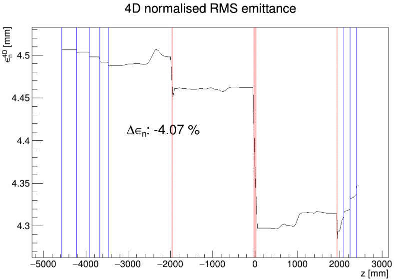
Beam envelope in the descope lattice

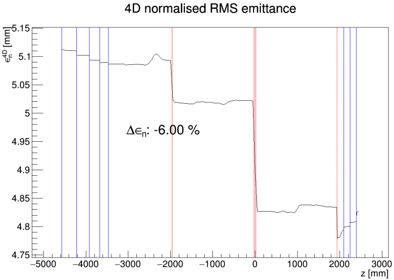


RMS emittance 3–6 mm

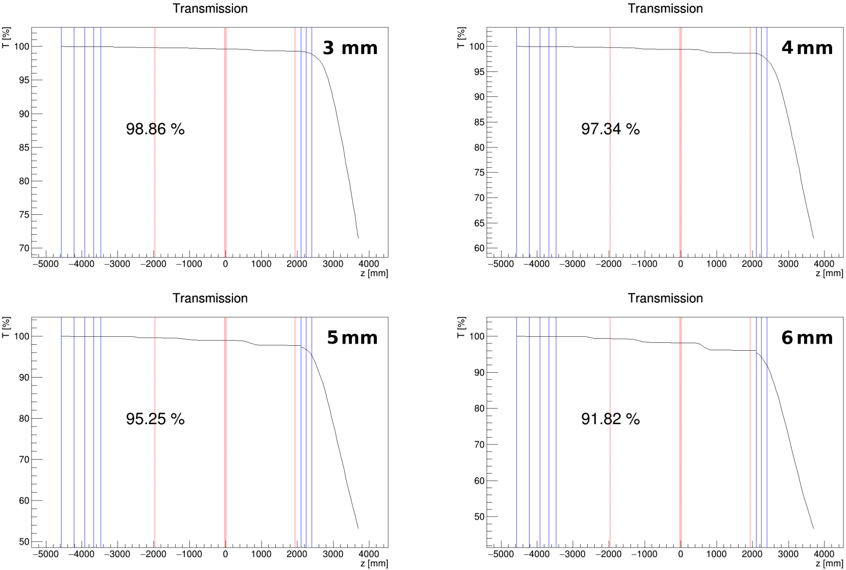




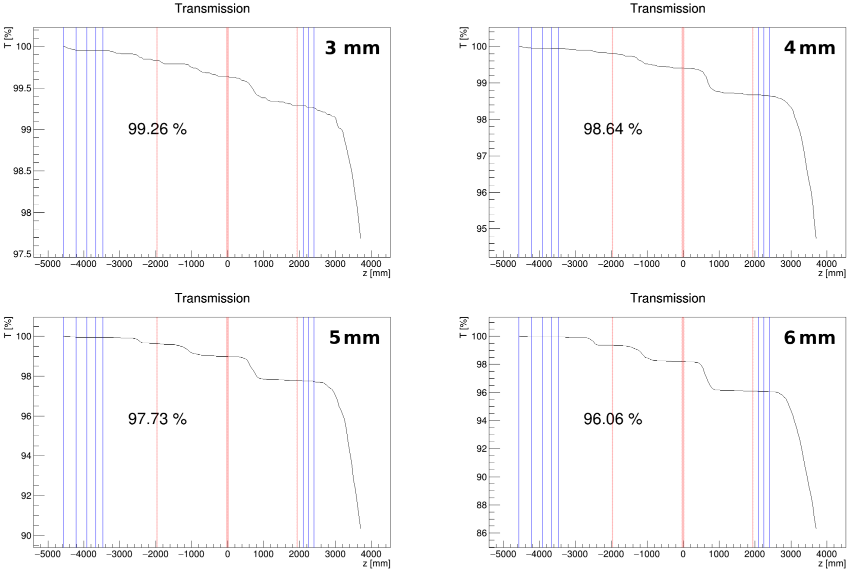




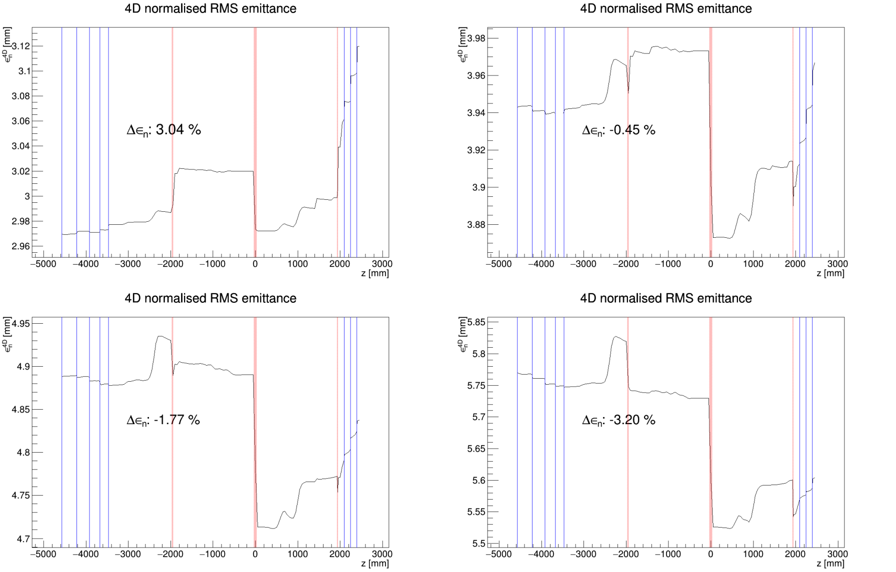
Transmission in a 150 mm cylinder (tracker dimensions)



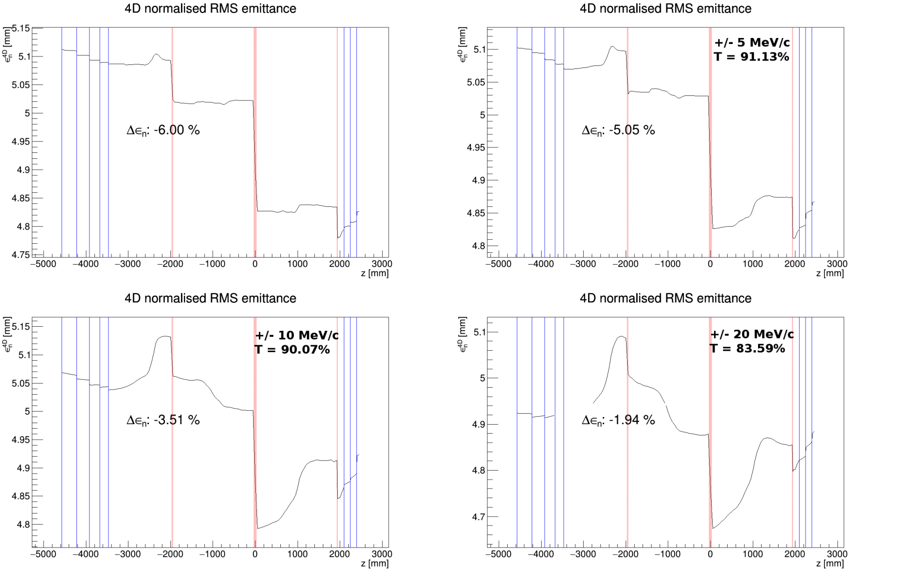
Transmission in a 300 mm cylinder (TOF2 dimensions)



RMS emittance, no fiducial, 3–6 mm



5) **Momentum spread**, deal with non-linearities

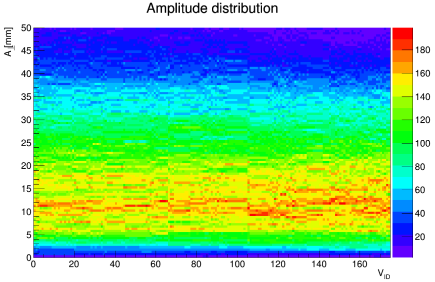


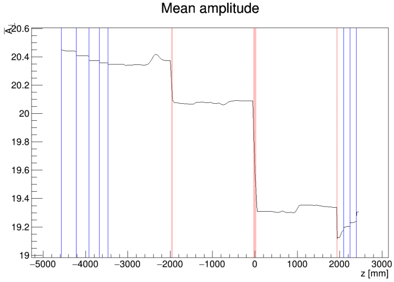
Single particle emittance

Instead of focusing on the single figure of the RMS emittance, one can look at individual particle amplitudes

$$\epsilon_i = \epsilon_n u_i^T \Sigma u_i \qquad \text{with} \quad u_i^T = \begin{pmatrix} x_i & p_{x,i} & y_i & p_{y,i} \end{pmatrix} \tag{16}$$

with ϵ_n the RMS emittance and u_i the phase-space vector.
$\rightarrow$ One can build fractional samples and calculate the volume occupied by a more central subset of particles (remove tail effects)

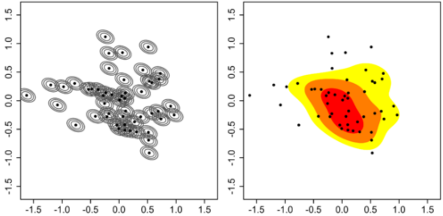




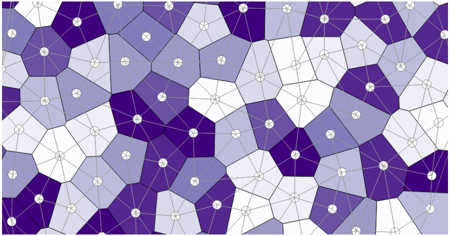
Alternative figures of merit of cooling

The RMS emittance is strongly prone to bias due to non linearities. There are alternative ways to quantify the phase volume

- **KDE** based
  - Place a kernel at each point, gives the probability density function everywhere
  - Compute a contour



- **Tessellation** of the space
  - Use extended Voronoi paving to weight the points in terms of density
  - Compute volume



Conclusions and looking ahead

Observations on the bias in MICE CC simulations

→ Simulations at 200 MeV/c sol/flip and 140 MeV/c flip showed strong **emittance reduction bias** correlated with transmission

→ Some Step IV settings see **true cooling** (w/ no fiducial)

→ Without M1, **M2 helps** to manage transmission greatly

E.g. 6 mm–200 MeV/c flip: 91.21% with and 71.77% without

Toy of the the effect of scraping on a drift in 2D:

→ The propagation of particles in the CC introduces $x - p_x$ correlation and hence preferential **scraping of high scatter angle** particles

Bias in the SSD-less MICE descope option:

→ Beam envelope grows very quickly after the DS secondary absorber, current baseline gives $>90\,\%$ transmission up to 6 mm input

→ **True cooling** to be observed, manageable bias

Alternative **cooling figures of merit** being looked into, more to come...