

Multiple Scattering Paper

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University of Glasgow

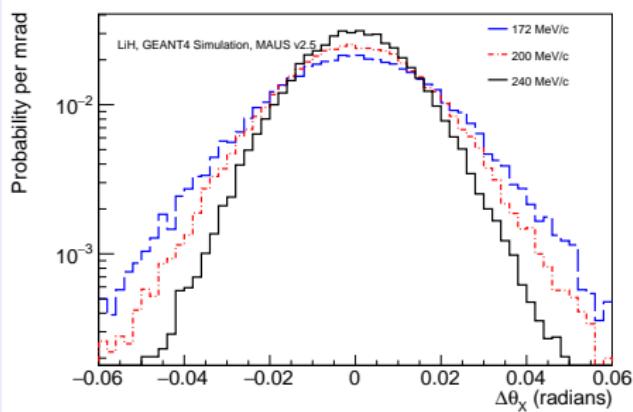
6 October, 2016

Since Last Time...

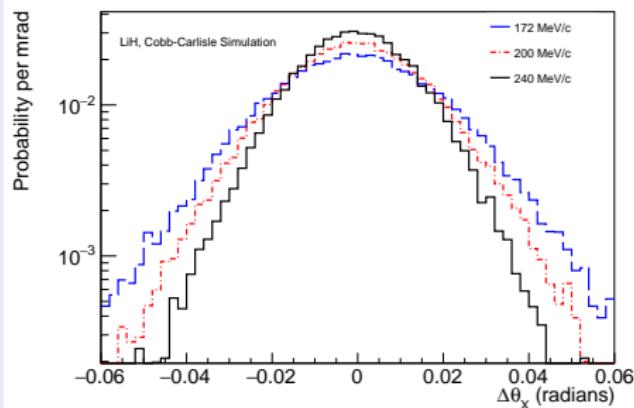
- Changed method of choosing the TOF selection to better match momenta.
- Created a momentum dependent measurement of scattering.
- Applied analysis to Xenon.
- Systematic studies have progressed.

Mulitple Scattering Models and Definitions

GEANT 4 Scattering in LiH



Carlisle-Cobb Model in LiH



- Width approximated by
$$\frac{d\sigma_{\Delta\theta^2}}{dz} = \left(\frac{13.6 \text{ MeV}/c}{p_\mu \beta} \right)^2 \frac{1}{X_0}$$
- $\Delta\theta$ is the projection about the X or Y axis.

$$\Delta\theta_X = \arctan y'_{US} - \arctan y'_{DS},$$

$$\Delta\theta_Y = \arctan x'_{US} - \arctan x'_{DS}$$

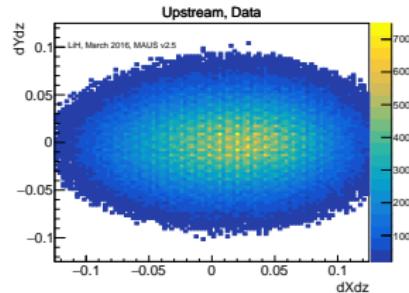
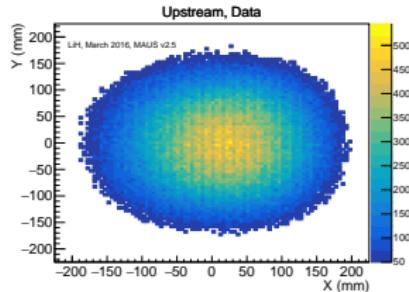
- Alternatively, define the 3-D scattering angle

$$\theta_{Scatt} = \frac{\vec{p}_{US} \cdot \vec{p}_{DS}}{|\vec{p}| |\vec{p}|} \approx \sqrt{\Delta\theta_X^2 + \Delta\theta_Y^2}$$

Data Collected

Collected Data

State	TOF1	TOF2
Xe 240 MeV/c, Pion	883118	75879
He 240 MeV/c, Pion	185983	16155
Empty 172 MeV/c, Muon	624577	94722
Empty 200 MeV/c, Muon	384909	56314
Empty 240 MeV/c, Muon	314739	62546
LiH 172 MeV/c, Muon	1282488	174405
LiH 200 MeV/c, Muon	1223560	177460
LiH 240 MeV/c, Muon	1239827	232982

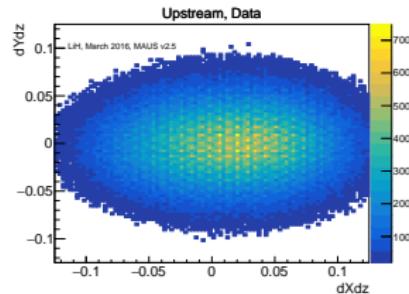
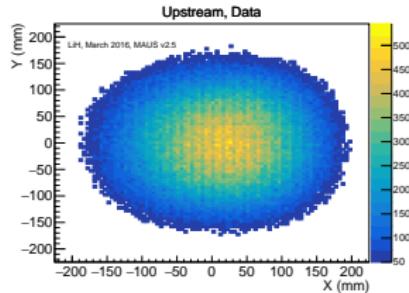


- Data recently re-analyzed with MAUS v2.5.0
- Collection of 100000 good muons targeted

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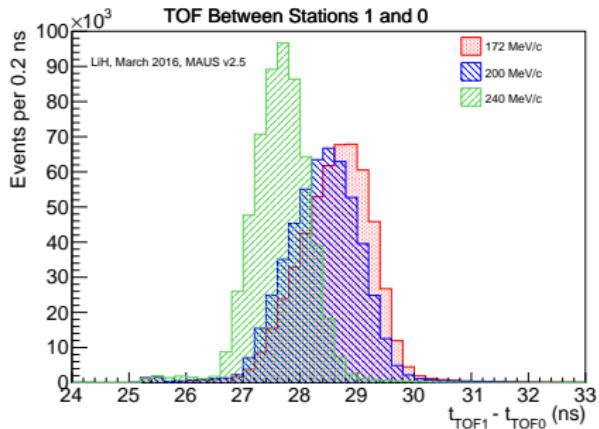
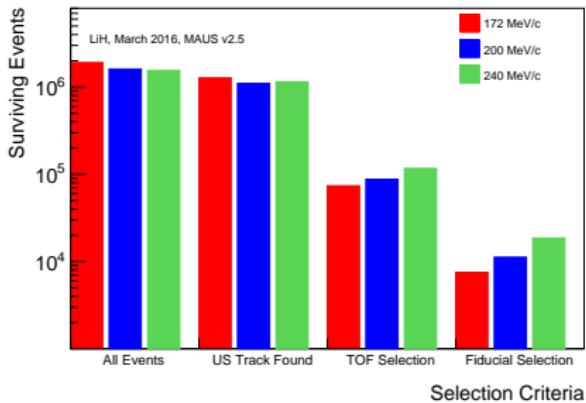
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Event Selection

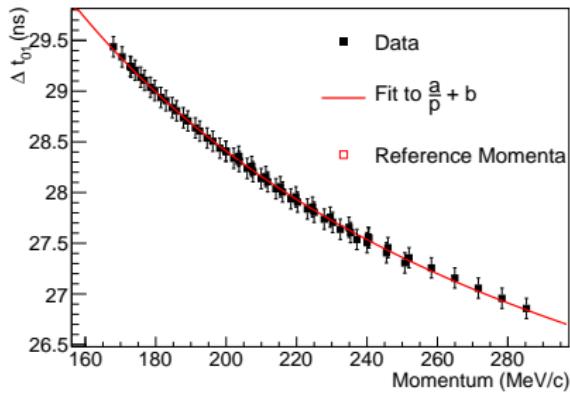


- Require a US track. If a DS track not extant, statistics are set to overflow values.
- Assumed a 200 ps selection;
 - $\Delta t_{01} \in \{29.15, 28.35\}$ for 172 MeV/c beams
 - $\Delta t_{01} \in \{28.31, 28.51\}$ for 200 MeV/c beams
 - $\Delta t_{01} \in \{27.45, 27.65\}$ for 240 MeV/c beams
 - $\Delta t_{01} \in \{27., 28.4\}$ for 240 MeV/c Pion beam
- Require projection of US tracks to appear within central 150 mm radius of DS plane 1 projected with 5 mrad dispersion added (18 mm).

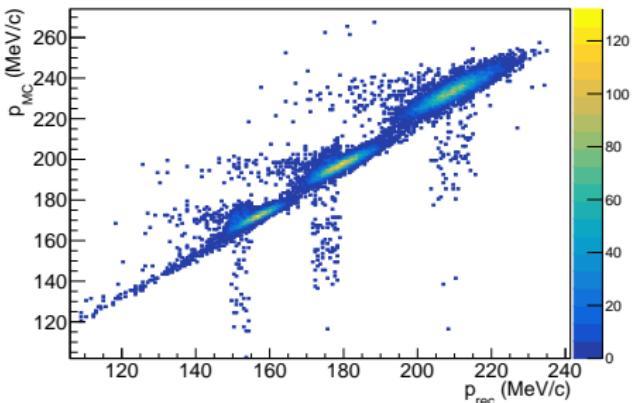
Motivating the TOF Selection

- Scan the TOF selection over the TOF spectrum
- Plot the TOF selection against the mean calc. momentum.
- Correct the momentum by fit to response of TOF calc. in MC.

The TOF vs. Mean Momentum



$|\vec{p}|$ Rec. Response from MC

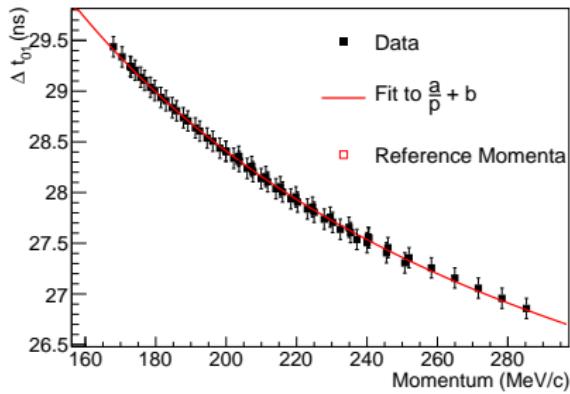


- Assume Rec. offset wrt to MC $p_{MC} = p_{rec} + 19.46(2)$ MeV/c.
- Deviations are systematic effects
- Eval. TOF at 172 MeV/c, 200 MeV/c, and 240 MeV/c

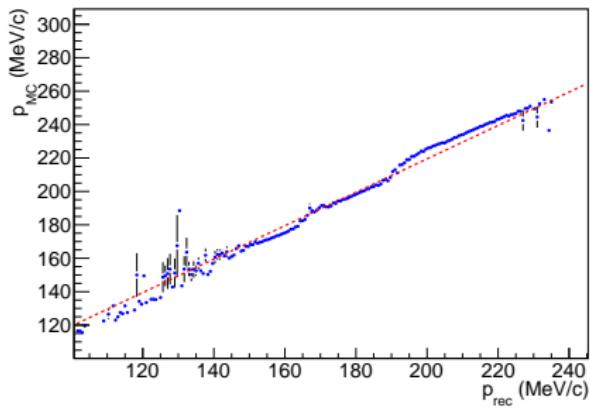
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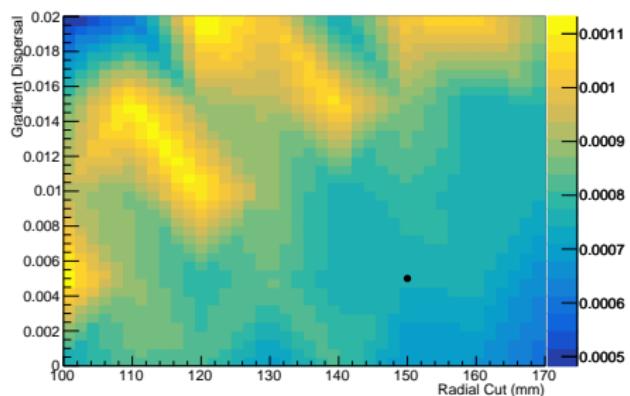


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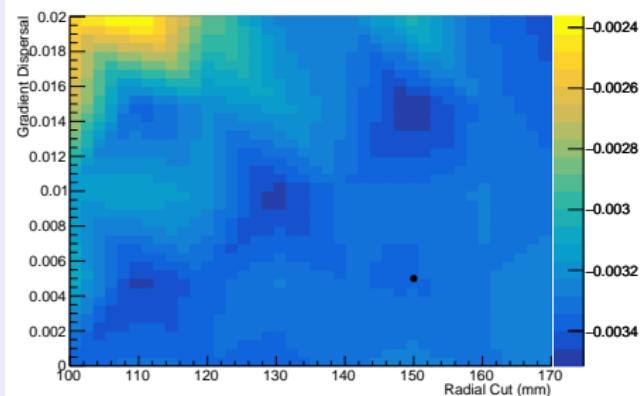
Motivating the Fiducial Selection

- Use to reduce scattering bias (to first order $\langle x'_{DS} \rangle - \langle x'_{US} \rangle$)
- Alternatively consider an asymmetry in the tails;
 $(N(\theta < \mu - \sigma/2) - N(\theta > \mu + \sigma/2))/N_{all}$

$\langle y'_{DS} \rangle - \langle y'_{US} \rangle$



$\langle x'_{DS} \rangle - \langle x'_{US} \rangle$

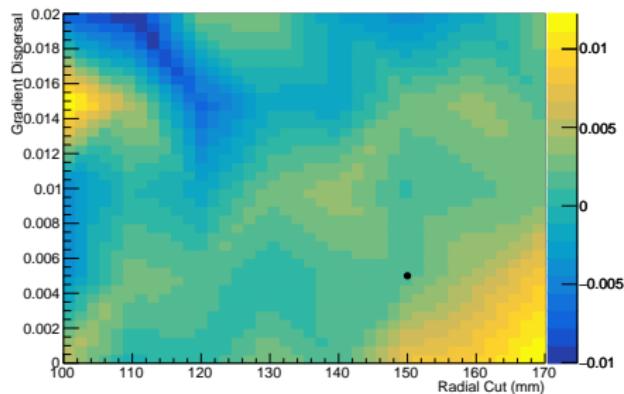


- Better behaviour for a larger radius and smaller projection dispersal.
- Largest radius allowed is 150 mm.

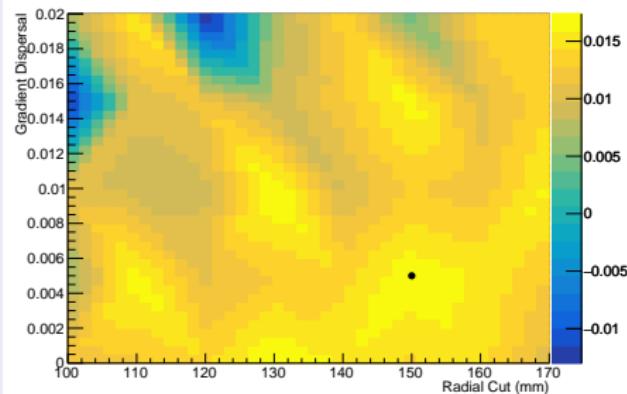
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Asymmetry of $\Delta\theta_X$ tails



Asymmetry of $\Delta\theta_Y$ tails

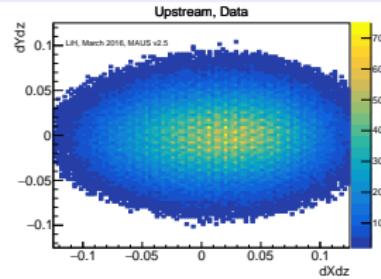
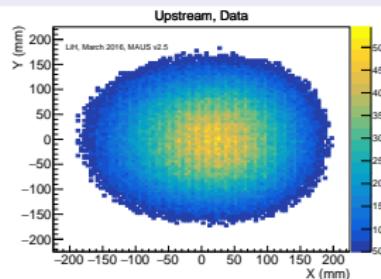


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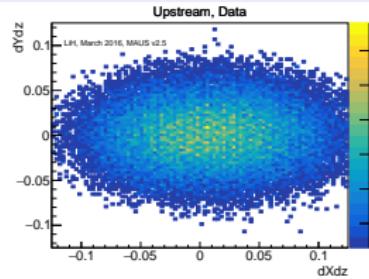
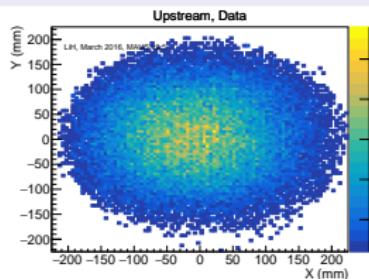
Beam After Selection Criteria

- 200 MeV/c muon beam at TkU Station 1 shown.

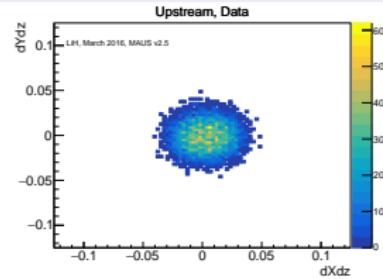
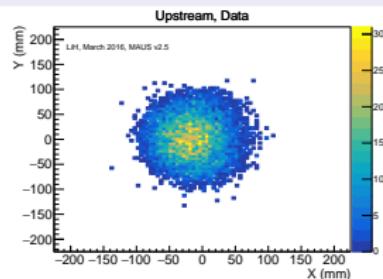
All TkU Events



After TOF Selection

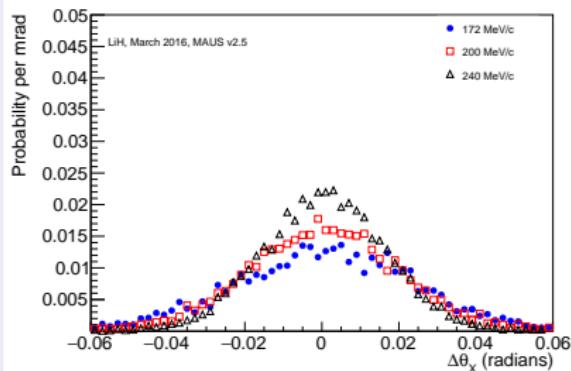


After Fid. Selection

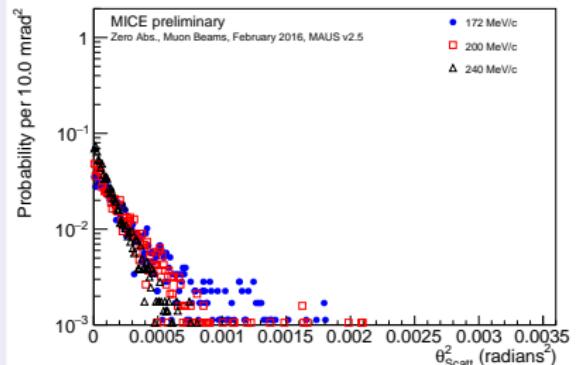
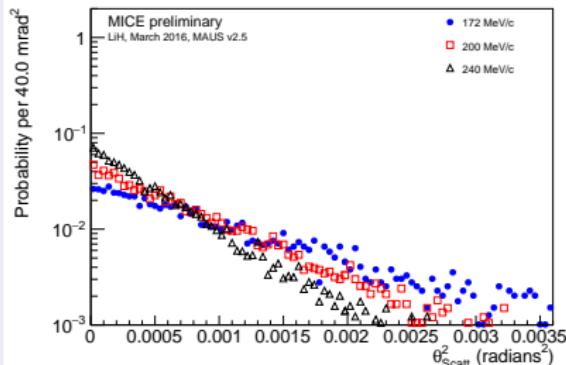
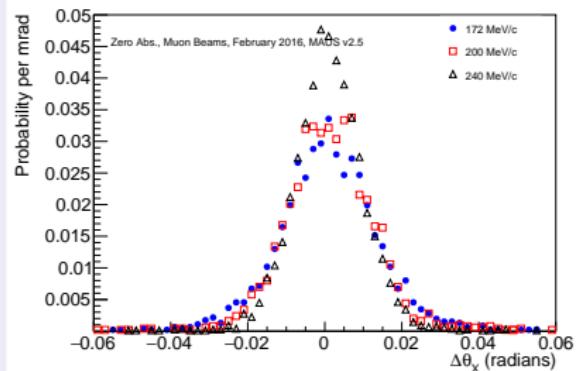


Scattering Distributions in Data

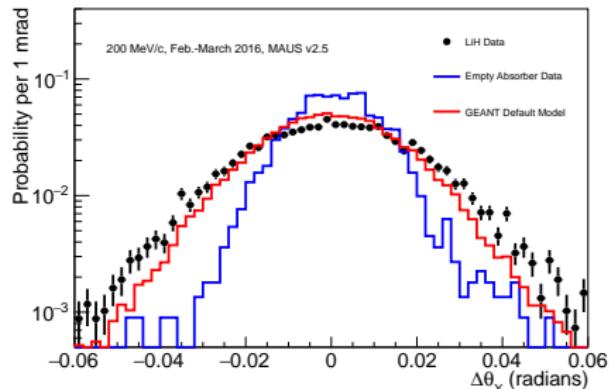
Scattering with LiH



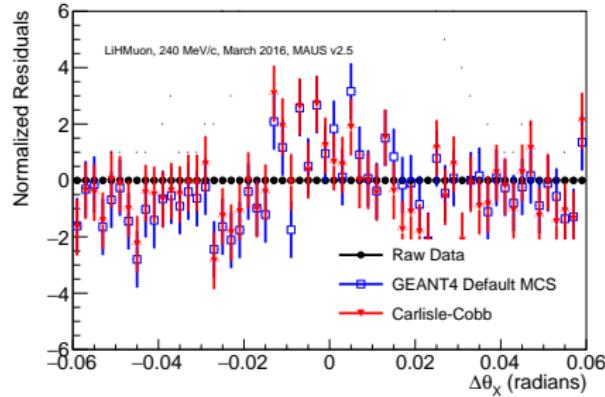
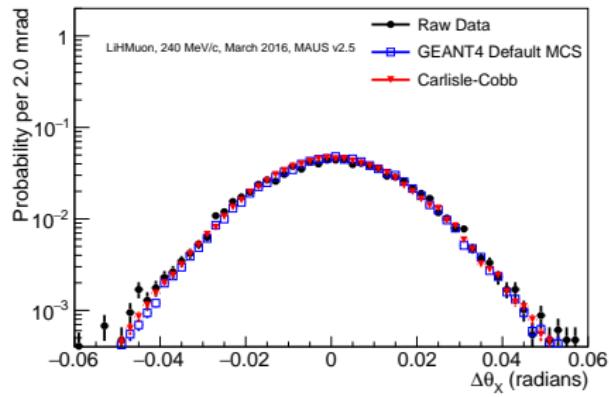
Scattering without LiH



Convolution Between Models and Empty AFC Data

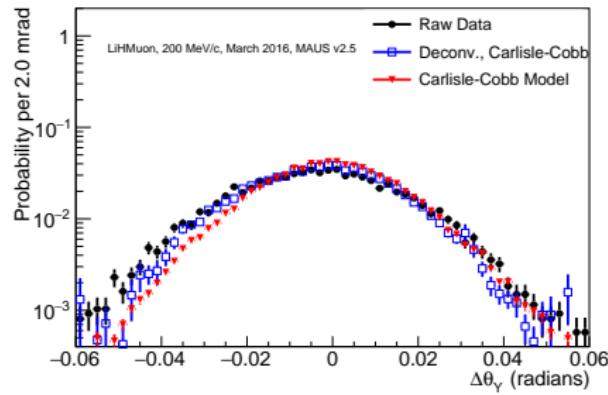
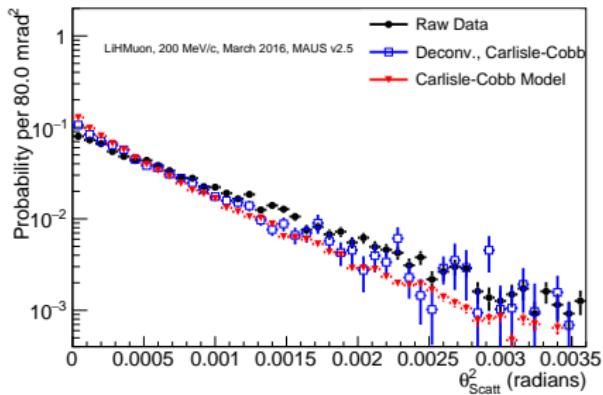


- Add model of scattering in LiH to the Empty AFC data.
- Model is sampled 10 times for each Empty AFC event to increase statistics.
- Ideal for model testing;
 - ▶ Calculate χ^2 from the data and the model convolution.

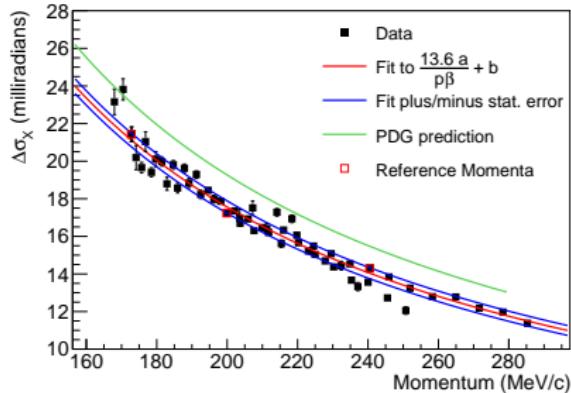
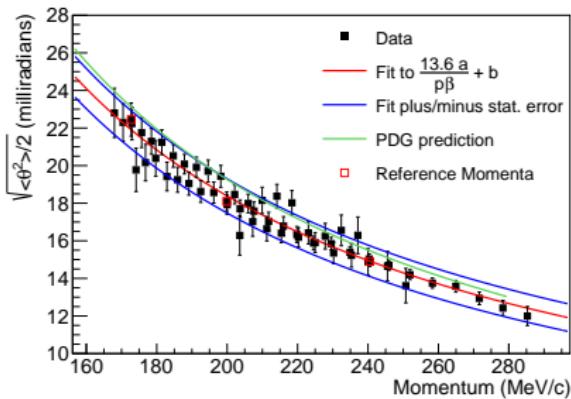


Deconvolution

- Use an iterative algorithm that uses the conditional probability to characterize the response of the reconstructed scattering angle to the true scattering angle.
- Requires a model of the true scattering angle in the absorber material.
- Convolution between Empty AFC data and scattering models used to provide response.

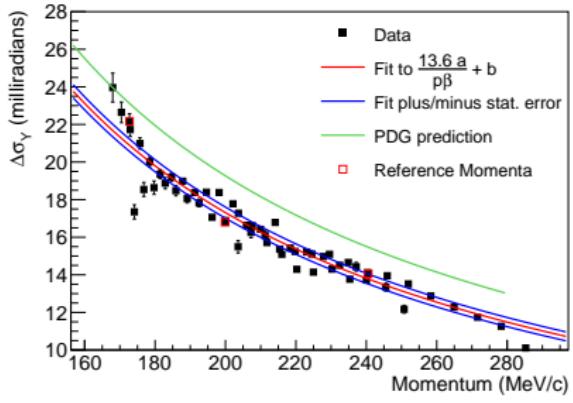


Momentum Dependence



Fit parameters

Angle	a (mrad)	b (mrad)
$\Delta\theta_X$	232 ± 2	-0.3 ± 0.2
$\Delta\theta_Y$	233 ± 2	-0.6 ± 0.1
$\sqrt{\langle \theta_{Scatt}^2 \rangle / 2}$	229 ± 6	0.8 ± 0.4
PDG	250	0



TOF and Momentum Systematic

- Add and subtract 400 ps to mean TOF of event selection.
- Scale difference by 129 ps/800 ps to reflect TOF resolution and momentum calibration.

Effect before Deconvolution

	$\Delta\theta_X$			$\langle\theta_{Scatt}^2\rangle$		
	Δ	σ	rel. σ	Δ	σ	rel. σ
171.95 ± 0.03	-1.83	0.29	0.01	-3.64	0.59	0.02
198.71 ± 0.04	-3.17	0.51	0.03	-3.39	0.55	0.02
240.01 ± 0.04	-3.31	0.53	0.03	-4.08	0.66	0.03

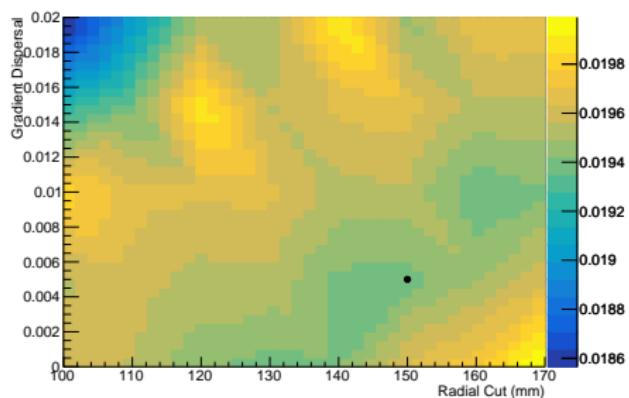
Effect After Deconvolution

	$\Delta\theta_X$			$\langle\theta_{Scatt}^2\rangle$		
	Δ	σ	rel. σ	Δ	σ	rel. σ
171.95 ± 0.03	5.38	0.87	0.04	-2.1	0.34	0.01
198.71 ± 0.04	-1.82	0.29	0.02	-3.56	0.57	0.03
240.01 ± 0.04	-3.89	0.63	0.04	-3.49	0.56	0.03

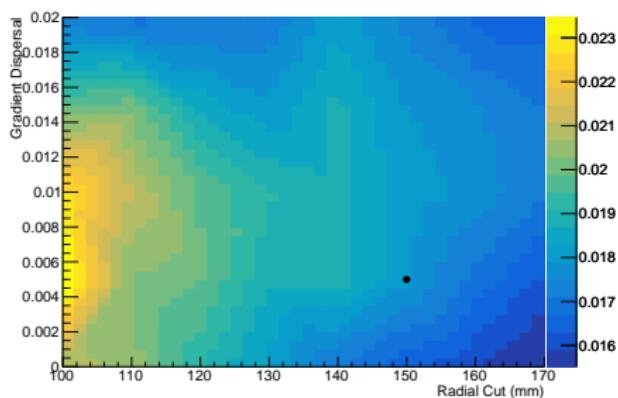
Fiducial Systematic

- Use the previous scan on the fiducial parameter to determine distribution sensitivity
- Select adjacent points about selected fiducial as limiting cases.
- Use the tracker resolution (0.478 mm) as a reference scale for changes.

Widths Post Selection



Widths Post Deconvolution



Fiducial Systematic Post Selection

Fid. Pitch Post Selection

	$\Delta\theta_X$			$\langle\theta_{Scatt}^2\rangle$		
	Δ	σ	rel. σ	Δ	σ	rel. σ
171.95 ± 0.03	-0.05	0.01	0.0	-0.11	0.03	0.0
198.71 ± 0.04	0.32	0.08	0.0	0.34	0.08	0.0
240.01 ± 0.04	0.27	0.07	0.0	0.21	0.05	0.0

Fid. Radius Post Selection

	$\Delta\theta_X$			$\langle\theta_{Scatt}^2\rangle$		
	Δ	σ	rel. σ	Δ	σ	rel. σ
171.95 ± 0.03	0.56	0.01	0.0	0.13	0.0	0.0
198.71 ± 0.04	-0.08	0.0	0.0	-0.09	0.0	0.0
240.01 ± 0.04	-0.16	0.0	0.0	-0.1	0.0	0.0

Material Systematic

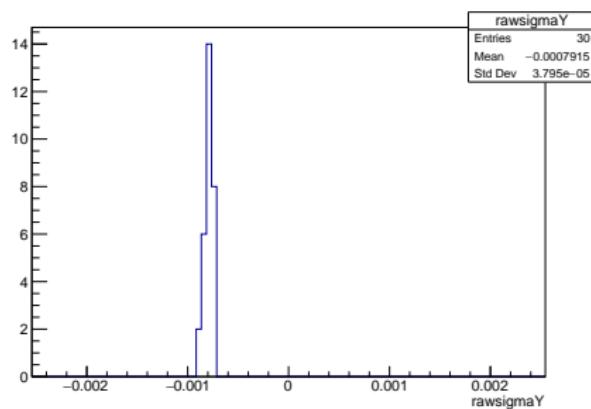
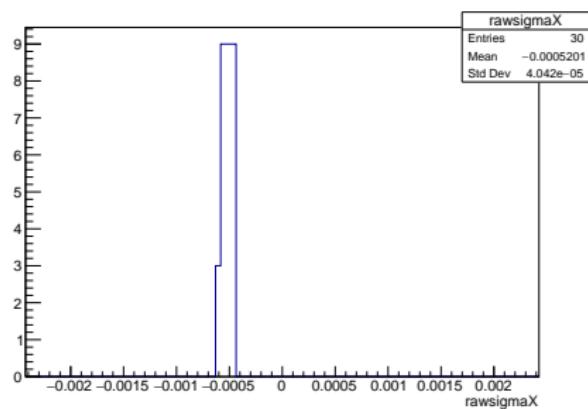
- Composition and density of LiH absorber has been verified
 - LiH composition: 81% ${}^6\text{Li}$, 4% ${}^7\text{Li}$, 14% ${}^1\text{H}$ (trace of C, O, and Ca).
 - Density: $0.693 \pm 0.004 \text{ g/cm}^3$.
- Exaggerate the effective material density or thickness by multiplying model by 1 ± 0.03 .

Effect Post Selection

	$\Delta\theta_X$			$\langle\theta_{Scatt}^2\rangle$		
	Δ	σ	rel. σ	Δ	σ	rel. σ
171.95 ± 0.03	-1.57	0.39	0.02	0.04	0.01	0.0
198.71 ± 0.04	0.05	0.01	0.0	-0.22	0.05	0.0
240.01 ± 0.04	-0.18	0.04	0.0	-0.36	0.09	0.0

Alignment Systematics

- Defined by a complicated set of parameters
 - Use a "bootstrap" procedure to determine uncertainty.
 - Throw random misalignments about the expected values and look at RMS.
- RMS of gaussian widths is less than 5×10^{-5}
- Needs to be updated and incorporated with other systematics studies.



Model Comparisons with GEANT4 and Carlisle-Cobb

Compile the widths and χ^2 after event selection

$\langle p \rangle$ LiH	Angle	Data (mrad)	G4 (mrad)	χ^2 100 dof	CC (mrad)	χ^2 100 dof
171.95(3)	$\Delta\theta_X$	$22.9 \pm 0.4 \pm 0.3$	21.04 ± 0.13	151.1	21.15 ± 0.13	157.0
171.95(3)	$\Delta\theta_Y$	$23.6 \pm 0.5 \pm 0.3$	20.82 ± 0.13	229.6	21.03 ± 0.13	196.4
198.71(4)	$\Delta\theta_X$	$19.2 \pm 0.2 \pm 0.5$	18.49 ± 0.09	141.7	18.36 ± 0.09	148.2
198.71(4)	$\Delta\theta_Y$	$19.2 \pm 0.2 \pm 0.5$	18.52 ± 0.09	186.8	18.33 ± 0.09	148.7
240.01(4)	$\Delta\theta_X$	$15.9 \pm 0.1 \pm 0.5$	15.15 ± 0.05	161.1	15.28 ± 0.05	153.7
240.01(4)	$\Delta\theta_Y$	$15.8 \pm 0.1 \pm 0.5$	14.99 ± 0.05	321.0	15.0 ± 0.05	271.5
Xe						
229.52(5)	$\Delta\theta_X$	$15.0 \pm 0.1 \pm 0.1$	14.87 ± 0.05	759.1	16.4 ± 0.05	364.0
229.52(5)	$\Delta\theta_Y$	$15.0 \pm 0.1 \pm 0.04$	14.76 ± 0.05	636.8	16.3 ± 0.05	291.9
$\langle p \rangle$ LiH	Angle	Data (mrad)	G4 (mrad)	χ^2 51 dof	CC (mrad)	χ^2 51 dof
171.95(3)	$\langle \theta_{Scatt}^2 \rangle$	$33.0 \pm 1.2 \pm 0.6$	29.45 ± 0.31	115.3	29.63 ± 0.31	132.4
198.71(4)	$\langle \theta_{Scatt}^2 \rangle$	$27.2 \pm 0.5 \pm 0.5$	25.93 ± 0.21	66.0	25.72 ± 0.21	80.7
240.01(4)	$\langle \theta_{Scatt}^2 \rangle$	$22.3 \pm 0.2 \pm 0.7$	21.2 ± 0.11	132.3	21.3 ± 0.11	124.4
Xe						
229.52(5)	$\langle \theta_{Scatt}^2 \rangle$	$20.9 \pm 0.2 \pm 0.06$	20.88 ± 0.12	622.0	22.99 ± 0.11	488.0

Deconvolution Widths

Assume that GEANT4 provides the scattering model

$\langle p \rangle$ LiH	Angle	Data (mrad)	G4 (mrad)	χ^2 100	CC (mrad)	χ^2 100
171.95(3)	$\Delta\theta_X$	$20.5 \pm 0.3 \pm 0.9$	18.81 ± 0.08	305.5	18.91 ± 0.08	349.0
171.95(3)	$\Delta\theta_Y$	$22.2 \pm 0.4 \pm 0.6$	18.72 ± 0.08	447.8	18.81 ± 0.08	425.8
198.71(4)	$\Delta\theta_X$	$17.1 \pm 0.2 \pm 0.3$	16.21 ± 0.06	209.5	16.07 ± 0.06	243.5
198.71(4)	$\Delta\theta_Y$	$16.8 \pm 0.1 \pm 0.3$	16.12 ± 0.06	379.6	15.96 ± 0.06	339.7
240.01(4)	$\Delta\theta_X$	$14.1 \pm 0.1 \pm 0.6$	13.2 ± 0.04	304.3	13.39 ± 0.04	361.3
240.01(4)	$\Delta\theta_Y$	$14.1 \pm 0.1 \pm 0.1$	13.22 ± 0.04	679.4	13.3 ± 0.04	565.9
Xe						
229.52(5)	$\Delta\theta_X$	$7.1 \pm 0.04 \pm 0.01$	6.61 ± 0.03	3974.5	11.93 ± 0.03	7295.7
229.52(5)	$\Delta\theta_Y$	$7.2 \pm 0.04 \pm 0.02$	6.66 ± 0.03	3657.7	11.86 ± 0.03	6359.0
$\langle p \rangle$ LiH	Angle	Data (mrad)	G4 (mrad)	χ^2 51 dof	CC (mrad)	χ^2 51 dof
171.95(3)	$\langle \theta_{Scatt}^2 \rangle$	$29.9 \pm 1.0 \pm 0.4$	25.98 ± 0.19	332.1	26.17 ± 0.2	318.6
198.71(4)	$\langle \theta_{Scatt}^2 \rangle$	$23.6 \pm 0.4 \pm 0.6$	22.61 ± 0.14	140.4	22.37 ± 0.14	200.9
240.01(4)	$\langle \theta_{Scatt}^2 \rangle$	$19.8 \pm 0.2 \pm 0.6$	18.62 ± 0.08	355.6	18.83 ± 0.08	305.8
Xe						
229.52(5)	$\langle \theta_{Scatt}^2 \rangle$	$10.2 \pm 0.1 \pm 0.03$	9.99 ± 0.07	960.4	16.82 ± 0.07	7759.3

Deconvolution Widths

Assume that Carlisle-Cobb provides the scattering model

$\langle p \rangle$ LiH	Angle	Data (mrad)	G4 (mrad)	χ^2 100 dof	CC (mrad)	χ^2 100 dof
171.95(3)	$\Delta\theta_X$	$20.9 \pm 0.3 \pm 0.9$	18.81 ± 0.08	351.5	18.91 ± 0.08	300.9
171.95(3)	$\Delta\theta_Y$	$22.2 \pm 0.4 \pm 0.9$	18.72 ± 0.08	496.9	18.81 ± 0.08	402.7
198.71(4)	$\Delta\theta_X$	$17.0 \pm 0.2 \pm 0.5$	16.21 ± 0.06	336.0	16.07 ± 0.06	265.6
198.71(4)	$\Delta\theta_Y$	$16.9 \pm 0.2 \pm 0.4$	16.12 ± 0.06	421.0	15.96 ± 0.06	293.8
240.01(4)	$\Delta\theta_X$	$14.1 \pm 0.1 \pm 0.7$	13.2 ± 0.04	372.1	13.39 ± 0.04	332.4
240.01(4)	$\Delta\theta_Y$	$14.1 \pm 0.1 \pm 0.5$	13.22 ± 0.04	811.0	13.3 ± 0.04	597.8
Xe						
229.52(5)	$\Delta\theta_X$	$9.9 \pm 0.04 \pm 0.03$	6.61 ± 0.03	11006.8	11.93 ± 0.03	2130.9
229.52(5)	$\Delta\theta_Y$	$9.9 \pm 0.04 \pm 0.02$	6.66 ± 0.03	10280.5	11.86 ± 0.03	1612.8
$\langle p \rangle$ LiH	Angle	Data (mrad)	G4 (mrad)	χ^2 51 dof	CC (mrad)	χ^2 51 dof
171.95(3)	$\langle \theta_{Scatt}^2 \rangle$	$29.6 \pm 1.0 \pm 0.4$	25.98 ± 0.19	265.2	26.17 ± 0.2	255.4
198.71(4)	$\langle \theta_{Scatt}^2 \rangle$	$23.7 \pm 0.4 \pm 0.5$	22.61 ± 0.14	133.9	22.37 ± 0.14	150.6
240.01(4)	$\langle \theta_{Scatt}^2 \rangle$	$20.0 \pm 0.2 \pm 0.7$	18.62 ± 0.08	418.7	18.83 ± 0.08	310.4
Xe						
229.52(5)	$\langle \theta_{Scatt}^2 \rangle$	$13.7 \pm 0.1 \pm 0.1$	9.99 ± 0.07	8840.2	16.82 ± 0.07	3058.3

Conclusions

- There has been progress since the summer conferences.
- Systematic method developed for the selection of TOF criteria.
 - ▶ Resulted in momentum bins that are more consistent between simulations.
 - ▶ An analysis of momentum dependence of scattering also resulted.
- Systematic errors have been identified and are nearly complete.
 - ▶ Leading systematic now reflects uncertainty in momentum scale.
- Conclusions of model test have changed
 - ▶ There is no clear difference between atomistic implementation and GEANT4.

To Do

- Scheme needed to incorporate alignment systematic into errors (find extrema?)
- Define momentum dependent systematics.
- Finalize systematics for Xenon data
- Update analysis note.