

Higher order effects in ε'/ε

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Abstract. The quantity ε'/ε measures direct CP violation in Kaon decays. Recent analysis of this ratio resulted in a 2.9 sigma discrepancy between the Standard Model predictions and the experimental data. Our ability to observe or constrain New Physics depends on the accuracy of determining the SM “background”, hence a precise evaluation of ε'/ε is particularly important. We discuss the Standard Model prediction and the relevant matching calculations at NNLO for this observable.

1. Introduction

The ratio ε'/ε parametrises the size of direct CP violation with respect to indirect CP violation in Kaon decays. Over the last decades, this observable has been subject to very intensive experimental and theoretical studies. After enormous efforts, on the experimental side the world average based on the recent results from NA48 [1] and KTeV [2, 3] collaborations reads

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}. \quad (1)$$

Theoretical predictions of this quantity are obtained through a weak effective Hamiltonian characterised by local operators and the corresponding Wilson coefficients. The latter can be computed in perturbation theory, whereas the matrix elements of the operators have to be evaluated within some non-perturbative approach like lattice QCD or the large-N approach [4, 5]. For Kaon decays, the Wilson coefficients are known at the Next-to-leading order (NLO) accuracy [7–12] and some pieces are available at the Next-to-Next-to-leading Order (NNLO) [13–15]. In the meantime an important progress has been achieved in the non-perturbative sector by the RBC-UKQCD lattice collaboration [16, 17]. Even though considerable improvement in calculating ε'/ε has been made, theoretical estimates of this ratio are still subject to very large hadronic uncertainties which interfere in the prediction of ε'/ε . A recent analysis [6] of this observable within the Standard Model has found a 2.9 sigma tension between theory and experiment. The main point here is the identification of a possible new anomaly in flavor physics, this time in the K sector. It is very important to disentangle the origin of this discrepancy. Kaon decays, within the SM, are loop induced and highly suppressed, consequently physics Beyond the Standard Model (BSM) can easily contribute. To be sure that this effect is due to the present of New Physics (NP) we need a reliable SM prediction.

Motivated by this inconsistency and the fact there are realistic prospects for improvements on the non-perturbative sector via Lattice QCD that would render NNLO accuracy essential,

we are working on the computation of these higher order corrections to the Wilson coefficients. These new contributions will have an impact on the theory prediction for ε'/ε . Although it seems greatly improbable that they can bring the SM prediction into reconciliation with experiment, it is still essential to study them in more detail. These perturbative corrections could be potentially important due to the large value of α_s at the low energy scale μ_c . In this talk, we cover the several steps involved in the calculation of higher order QCD corrections to this observable.

2. CP-Violation in Kaon Decays

CP violation emerges naturally in the three generation Standard Model. Its origin lies solely in Yukawa-type interactions of the quark fields with the complex Higgs fields. Within this model, CPV is only an effect of a complex phase. To explain matter-antimatter asymmetry in the Universe new sources are required.

In neutral K-meson decays CP violation has been accommodated in the Standard Model in a simple way. However, this phenomenon is one of the least tested aspects of the Standard Model. CP violating effects are of the order $\mathcal{O}(10^{-4})$. The small prediction for CPV in this framework is due to flavor suppression (CKM factors).

The study of Kaon decays can bring some light on this puzzle. However, in the framework of K mesons it is not easy to estimate CP violating observables with great precision, since strong interactions are in a non-perturbative regime. Nevertheless, the use of Effective Field Theories (EFT) simplifies the calculations. This framework introduces a natural way to separate the different energy scales and allows for a convenient method to sum large logarithms to all orders in perturbation theory. Below the charm mass scale, $\mu < m_c$, the effective Hamiltonian looks like [7–12],

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} (z_i(\mu) + \tau y_i(\mu)) \mathcal{O}_i, \quad \tau \equiv -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}}, \quad (2)$$

where the functions $z_i(\mu)$ and $y_i(\mu)$ are the Wilson coefficients which can be calculated perturbatively, V_{ij} are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, $\mathcal{O}_{1,2}$ are current-current operators, \mathcal{O}_{3-6} the QCD penguin operators, and \mathcal{O}_{7-10} the electroweak penguin operators.

In order to study this CP-violating observable, it is important to improve both the theoretical calculations of short-distance contributions [7–15] (the Wilson coefficients functions z_i and y_i) and the estimation of long-distance contributions (the hadronic matrix elements $\langle \mathcal{O}_i(\mu) \rangle$) [16,17]. For this reason we aim to complete the elaborate perturbative calculation. Our future results will address whether the complete NNLO corrections enhance or suppress the Wilson coefficients and they will have an impact on ε'/ε .

Theoretical predictions for CPV are quite involved. In terms of the isospin amplitudes this observable is parametrised as

$$\frac{\varepsilon'}{\varepsilon} = -i \frac{\omega_+}{\sqrt{2} |\varepsilon_K|} e^{i(\delta_2 - \delta_0 - \phi_{\varepsilon_K})} \left[\frac{\text{Im}(\mathcal{A}_0)}{\text{Re}(\mathcal{A}_0)} (1 - \Omega_{\text{eff}}) - \frac{1}{a} \frac{\text{Im}(\mathcal{A}_2)}{\text{Re}(\mathcal{A}_2)} \right], \quad (3)$$

where a and Ω_{eff} comprise isospin-breaking correction, $\delta_{0,2}$ denote the isospin strong phase-shifts, ϕ_{ε_K} stands for the phase of ε_K and ω_+ is determined from the charged decay mode.

For a precision SM prediction it is required to obtain the real and imaginary parts of the isospin amplitudes $\mathcal{A}_I \equiv \langle (\pi\pi)_I | \mathcal{H}_{\text{eff}} | K \rangle$ entering Eq.(3) in terms of the Wilson coefficients and hadronic matrix elements of the operators in the weak Hamiltonian, Eq.(2).

3. Computation of $\mathcal{I}m(\mathcal{A}_i)/\mathcal{R}e(\mathcal{A}_i)$

The formalism used to determine these important pieces is based on reference [6]. They work under the hypothesis that the amplitudes $\mathcal{R}e(\mathcal{A}_0)$ and $\mathcal{R}e(\mathcal{A}_2)$, in the SM, originate already at tree-level. Therefore these two quantities are expected to be only marginally affected by NP contributions,

$$\mathcal{R}e(\mathcal{A}_0) = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* (z_+ \langle \mathcal{O}_+ \rangle_0 + z_- \langle \mathcal{O}_- \rangle_0), \quad \mathcal{R}e(\mathcal{A}_2) = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* z_+ \langle \mathcal{O}_+ \rangle_2. \quad (4)$$

Assuming this dominance of SM dynamics in CP-conserving data, our determination of the contributions of $(V - A) \times (V - A)$ operators to ε'/ε is basically independent of the non-perturbative approach used. In this way a more accurate prediction for ε'/ε can be made than currently possible with direct lattice-QCD simulations:

$$\begin{aligned} \left(\frac{\mathcal{I}m(\mathcal{A}_0)}{\mathcal{R}e(\mathcal{A}_0)} \right)_{V-A} &= \mathcal{I}m(\tau) \frac{2y_4}{(1+q)z_-} + \mathcal{O}(p_3) \\ \left(\frac{\mathcal{I}m(\mathcal{A}_2)}{\mathcal{R}e(\mathcal{A}_2)} \right)_{V-A} &= \mathcal{I}m(\tau) \frac{3(y_9 + y_{10})}{2z_+}. \end{aligned} \quad (5)$$

In the above expression the factor q is defined in terms of the current-current Wilson coefficients and operators, $q \equiv (z_+(\mu) \langle \mathcal{O}_+(\mu) \rangle_0) / (z_-(\mu) \langle \mathcal{O}_-(\mu) \rangle_0)$. In addition, $\mathcal{O}(p_3)$ encodes subleading order contributions. Notice here that the $(V - A) \times (V - A)$ terms are dominated by short distance (Wilson coefficients).

The remaining contribution coming from the $(V - A) \times (V + A)$ operators depends on the two hadronic parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ and comprises the biggest uncertainty in ε'/ε :

$$\begin{aligned} \left(\frac{\mathcal{I}m(\mathcal{A}_0)}{\mathcal{R}e(\mathcal{A}_0)} \right)_{V+A} &= -\frac{G_F}{\sqrt{2}} \mathcal{I}m(\lambda_\tau) y_6 \frac{\langle \mathcal{O}_6 \rangle_0}{\mathcal{R}e(\mathcal{A}_0)} + \mathcal{O}(p_5) \\ \left(\frac{\mathcal{I}m(\mathcal{A}_2)}{\mathcal{R}e(\mathcal{A}_2)} \right)_{V+A} &= -\frac{G_F}{\sqrt{2}} \mathcal{I}m(\lambda_\tau) y_8^{\text{eff}} \frac{\langle \mathcal{O}_8 \rangle_2}{\mathcal{R}e(\mathcal{A}_2)}. \end{aligned} \quad (6)$$

It is easily to see that the terms with $(V - A) \times (V + A)$ chirality are controlled by long-distance (matrix elements).

To summarise the main ideas mentioned until now: in the SM the amplitudes $\mathcal{R}e(\mathcal{A})_{0,2}$ are mostly governed by the $\mathcal{O}_{1,2}$ current-current operators and ε'/ε by the QCD penguin \mathcal{O}_6 and electroweak penguin \mathcal{O}_8 operators.

In the coming years, RBC-UKQCD collaboration could reduce the statistical uncertainty of the non-perturbative sector. Consequently, the perturbative side would become the main source of uncertainty for ε'/ε .

4. Higher order QCD corrections

When a quark is integrated out as a dynamical degree of freedom its effects have to be taken into account. These threshold corrections are determined through a matching of the effective theories with n_f and $n_f + 1$ flavors. The respective calculation requires the equality of the Green's functions in the two theories at the matching scale $\mu_q = \mathcal{O}(m_q)$, where m_q is the quark mass.

In order to improve on the present NLO calculation, one needs to include higher order terms in the strong coupling expansion. The completion of this NNLO computation comprises the content of our project [19]. This task is more sophisticated. In particular the number of diagrams and structures that appear in the study, increases. In addition, there are diverse sources of matching

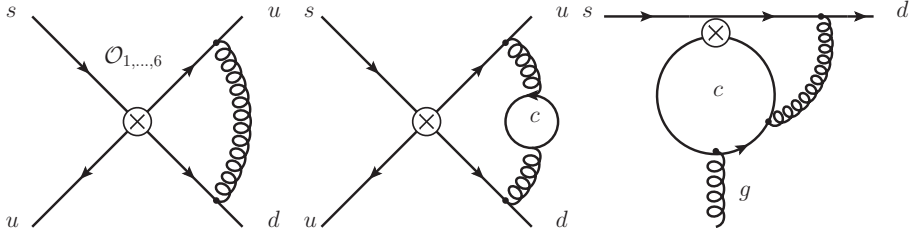


Figure 1. Feynman diagrams relevant for the threshold corrections at the charm quark scale. The one-loop current-current diagrams are identical in the four- and three-flavor theories, whereas at NNLO they receive non-trivial corrections from virtual charm quarks.

corrections. At one loop the penguin operators are already affected since they explicitly depend on light-quark fields. At NNLO, the matching of the current-current is also non-trivial. Two-loop matrix elements get extra contributions from virtual light quarks. Some relevant diagrams for this calculation are shown in Fig(1). In addition, also the strong coupling constant and the light-quark mass are discontinuous beyond leading order (LO).

An important piece in the calculation of the two-loop QCD corrections within the SM is the matrix elements. To simplify this computation we use the so called "modern basis" [18]. This solves the problems arising from the γ_5 matrix appearing in closed fermion loops in the framework of dimensional regularisation. We generate the effective diagrams and renormalise the effective amplitudes. To this end, we use `FeynArts` to find all topologies and a self-written Mathematica program to compute the corresponding amplitudes. We expand the external momenta up to $\mathcal{O}(k^2)$. In addition, we set the mass of the light quarks to zero. This introduces Infrared Divergences (Spurious) in the $n_f + 1$ theory amplitude which have to be cancelled by the Ultra-Violet divergences in the n_f flavor theory. The matching results in finite threshold corrections for the physical operators.

5. Scale cancellation

The aim here it is to show the log cancellation that occurs in this procedure and comment about the residual μ_q scale dependence. For a more exhaustive explanation refer to [14].

The calculation consists of several steps. First, the initial conditions for the Wilson coefficients at the electroweak scale are computed, $\bar{\mathcal{C}}(M_W)$. For this purpose we match the Standard Model Green's functions to those in the five-flavor theory, where the heavy particles have been integrated out. Subsequently, the Wilson coefficients are evolved down to the bottom-quark scale using the renormalisation group equations (RGE),

$$\bar{\mathcal{C}}(\mu_b) = \hat{U}(\mu_b, \mu_W) \bar{\mathcal{C}}(\mu_W), \quad (7)$$

where $\hat{U}(\mu_b, \mu_W)$ describes pure QCD evolution. Afterwards, we compute the threshold corrections at $\mu_b = \mathcal{O}(m_b)$, $\hat{M}(\mu_b)$. For this aim, we match the five-flavor theory onto an effective theory where also the bottom quark has been removed as a dynamical degree of freedom. The resulting Wilson coefficient $\bar{\mathcal{C}}(\mu_b)$ is then evolved down to the charm-quark scale using the RGE,

$$\bar{\mathcal{C}}(\mu_c) = \hat{U}(\mu_c, \mu_b) \hat{M}(\mu_b) \bar{\mathcal{C}}(\mu_b). \quad (8)$$

At this scale, $\mu_c = \mathcal{O}(m_c)$, the matching equation for the charm quark is now evaluated. Here, we match the four-flavor theory onto the three-flavor theory and find the new threshold correction

$\hat{M}(\mu_c)$. Finally, we incorporate these results and perform the renormalisation group evolution down to the scales where the hadronic matrix elements are computed:

$$\bar{\mathcal{C}}(\mu_{\text{Lattice}}) = \hat{U}(\mu_{\text{Lattice}}, \mu_c) \hat{M}(\mu_c) \bar{\mathcal{C}}(\mu_c). \quad (9)$$

The lines above provide us the general picture without giving much detail. However, to understand how the logarithms are cancelled we have to extend this formalism. In particular, the calculation can be organised in such a manner that the log cancellations occur separately at each scale. Following the notation used in [14] the evolution matrix is given by

$$\hat{U}(\mu_b, \mu_W) = \hat{K}(\mu_b) \hat{U}^{(0)}(\mu_b, \mu_W) \hat{K}^{-1}(\mu_W) \quad (10)$$

where,

$$\hat{U}^{(0)}(\mu, \mu_0) = \hat{V} \text{diag} \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right)^{a_i} \hat{V}^{-1} \quad (11)$$

stands for the LO term. The matrix \hat{V} and the magic numbers a_i are obtained from the diagonalisation of the LO anomalous dimension matrix (ADM), $\gamma^{(0)T}$:

$$\left(\hat{V}^{-1} \gamma^{(0)T} \hat{V} \right)_{ij} = 2\beta_0 a_i \delta_{ij}. \quad (12)$$

The evolution matrix $\hat{U}(\mu, \mu_0)$ depends on two different scales, with $\mu \ll \mu_0$. Therefore, we split it into two parts:

$$\begin{aligned} \hat{U}^{1/2}(\mu) &= \hat{K}(\mu) \hat{V} \text{diag} (\alpha_s(\mu))^{-a_i} \hat{V}^{-1} \\ \hat{U}^{-1/2}(\mu_0) &= \hat{V} \text{diag} (\alpha_s(\mu_0))^{a_i} \hat{V}^{-1} \hat{K}^{-1}(\mu_0), \end{aligned} \quad (13)$$

with

$$\hat{K}(\mu) = \hat{1} + \frac{\alpha_s(\mu)}{4\pi} \hat{J}^{(1)} + \mathcal{O}(\alpha_s^2), \quad \hat{K}^{-1}(\mu_0) = \hat{1} - \frac{\alpha_s(\mu_0)}{4\pi} \hat{J}^{(1)} + \mathcal{O}(\alpha_s^2). \quad (14)$$

From this factorisation we find that the following object appears in the intermediate states in our calculation:

$$\left[\hat{V} \text{diag} (\alpha_s(\mu))^{a_i} \hat{V}^{-1} \hat{K}^{-1}(\mu) \right]_{n_f} \left[\hat{M}(\mu) \right]_{n_f, n_{f+1}} \left[\hat{K}(\mu) \hat{V} \text{diag} (\alpha_s(\mu))^{-a_i} \hat{V}^{-1} \right]_{n_{f+1}}. \quad (15)$$

Note that the matching matrix depends on both $\alpha_s^{n_f}(\mu)$ and on $\alpha_s^{n_f+1}(\mu)$. Expressing α_s^{n+1} in terms of $\alpha_s^{n_f}$ and expanding the mass terms we observe that the scale dependence appearing in the evolution down to the n_f -flavor theory (third term in Eq.(15)) and the one emerging in the evolution up to this theory (first term in Eq.(15)) are cancelled by the $\log(\mu_q)$ from the light-quark matching, $\hat{M}(\mu_q)$. Therefore this object is individually scale- and scheme-independent. The log cancellation works order by order.

Once can then estimate higher order effects by varying the matching scale μ_q while not expanding the expression in α_s . This would result in a residual scale dependence that is of the size of higher order corrections.

6. Conclusions

The experimental data for ϵ'/ϵ is not well described by the SM. However, before establishing whether NP is present here many questions should be addressed. Answering them would also allow us to give an enhanced understanding of some particular NP models. In the near future, RBC-UKQCD collaboration could reduce the statistical uncertainty of the non-perturbative sector. Therefore, the perturbation side would become the main source of uncertainty for ϵ'/ϵ . A missing piece, in the last theoretical study [6], is the one due to the unknown matching corrections at $\mu = m_c$ which could be sizeable since the strong coupling is growing rapidly in this region. Our work is focused on covering this calculation at NNLO and checking the consistency of the perturbation theory. This new contribution will help to improve the SM predictions and potentially motivate to look into New Physics scenarios.

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