On the Standard Model prediction for R_K and R_{K^*}

A Pattori [1,2]

[1] Physik-Institut, Universität Zürich, CH-8057 Zürich, Switzerland
[2] Dipartimento di Fisica e Astronomia 'G. Galilei', Università di Padova, Italy

E-mail: `pattori@phyisik.uzh.ch`

Abstract. In this article a recent work is reviewed, where we evaluated the impact of radiative corrections in R_K and R_{K^*}. We find that, employing the cuts presently applied by the LHCb Collaboration, such corrections do not exceed a few percent. Moreover, their effect is well described (and corrected for) by existing Montecarlo codes. Our analysis reinforces the interest of these observables as clean probe of physics beyond the Standard Model.

1. Introduction

The Lepton Flavor Universality (LFU) ratios

$$R_M[q^2_{\rm min},\ q^2_{\rm max}] = \frac{\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{{\rm d}\Gamma(B\to M\mu^+\mu^-)}{{\rm d}q^2}{\rm d}q^2}{\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{{\rm d}\Gamma(B\to M e^+e^-)}{{\rm d}q^2}{\rm d}q^2}\ , \tag{1}$$

where $q^2 = m^2_{\ell\ell}$, are very clean probes of physics beyond the Standard Model (SM): they have small theoretical uncertainties and are sensitive to possible new interactions that couple in a non-universal way to electrons and muons [1]. A strong interest in R_K has recently been raised by the LHCb result [2]

$$R_K\left[1\ {\rm GeV}^2,\ 6\ {\rm GeV}^2\right] = 0.745^{+0.090}_{-0.074} \pm 0.036\ , \tag{2}$$

that differs from the naïve expectation $R^{\rm (SM)}_{K^{(*)}} = 1$ by about 2.6σ. The interest in this anomaly is further raised by its combination with other $b \to s\ell^+\ell^-$ observables [3, 4], and by the independent hints of violations of LFU observed in $B \to D^{(*)}\tau\nu_\ell$ decays [5–7], presenting an overall 3.9σ tension with the SM predictions [8, 9].

While perturbative and non-perturbative QCD contributions cancel in $R_{K^{(*)}}$ (beside trivial kinematical factors), this is not necessarily the case for QED corrections. In particular, QED collinear singularities induce corrections of order $(\alpha/\pi)\log^2(m_B/m_\ell)$ to $b \to s\ell^+\ell^-$ transitions [10–12] that could easily imply 10% effects in $R_{K^{(*)}}$. In a recent paper [13] we have estimated these corrections to precisely quantify up to which level a deviation of R_K or R_{K^*} from 1 can be considered a clean signal of physics beyond the SM.

2. factorizable QED corrections in R_M

While discussing the $B \to M\ell^+\ell^-$ decay, the key observable we are interested in is the differential lepton-pair invariant-mass distribution

$$\mathcal{F}_M^\ell(q^2) = \frac{\mathrm{d}\Gamma(B \to M\ell^+\ell^-)}{\mathrm{d}q^2} \ , \tag{3}$$

since this is the observable involved in the R_M definition.

A complete evaluation of QED corrections to $\mathcal{F}_M^\ell(q^2)$ is a non-trivial task, due to the interplay of perturbative and non-perturbative dynamics (see e.g. [14]). However, the problem is drastically simplified if we are only interested in the low dilepton invariant mass region, and if interested in possible deviations from $R_{K^{(*)}}^{(\mathrm{SM})} = 1$ exceeding 1%. In this case the problem is reduced to evaluating $\log(m_\ell)$ enhanced terms, whose origin can be unambiguously traced to soft and collinear photon emission. The latter represents a universal correction factor [15, 16] that can be implemented, by means of appropriate convolution functions, irrespective of the specific short-distance structure of the amplitude.

Following the above observation, the $\log(m_\ell)$ enhanced terms in $\mathcal{F}_M^\ell(q^2)$ can be factorized and are independent from the spin of the meson M. In formulas:

$$\mathcal{F}_M^\ell(q^2) = \int_{q^2}^{q^2_{0,\max}} \frac{\mathrm{d}q_0^2}{q_0^2}\, \mathcal{F}_M^{(0)}(q_0^2)\, \omega\left(\frac{q^2}{q_0^2}, \frac{2m_\ell^2}{q_0^2}\right) \ . \tag{4}$$

$\mathcal{F}_M^{(0)}(q_0^2)$ is the differential distribution at tree level. $q^2_{0,\max}$ is the maximum value for the initial dilepton invariant mass squared (pre bremsstrahlung) compatible with the cut applied in the experimental analysis, namely that the reconstructed B-meson mass (from the measurement of leptons and hadron momenta) is above a minimum value m_B^{rec}. The function $\omega(x, x_\ell)$, which represents the probability density function that a dilepton system retains a fraction $\sqrt{x}$ of its original invariant mass after bremsstrahlung, includes both real and virtual QED corrections. Explicit expressions for $q^2_{0,\max}$ and $\omega(x, x_\ell)$ can be found in [13].

3. Numerical results

The relative impact of radiative corrections in $B \to K^+\ell^+\ell^-$, namely a plot of the ratio

$$\mathcal{R}_K^\ell(q^2) = \frac{\mathcal{F}_K^\ell(q^2)}{\mathcal{F}_K^{(0)}(q^2)} \ , \tag{5}$$

is shown in Figure 1 in the region $q^2 \in [1, 9]$ GeV^2. The different colors correspond to different lepton masses (red for the electron and blue for the muon). Dashed and full lines correspond to different choices of m_B^{rec}. We have choosen for the latter the two values used in Ref. [2] for the analysis of electron modes ($m_B^{\mathrm{rec}} = 4.880$ GeV, full lines) and muon modes ($m_B^{\mathrm{rec}} = 5.175$ GeV, dashed lines).

The first point to be noted in Figure 1 is that $\mathcal{R}_K^\ell(q^2)$ is a smooth function for sufficiently low values of q^2, while a sudden rise appear close to the resonance region. The latter is a manifestation of the radiative return from the J/Ψ peak. The position where the J/Ψ contamination appears depends only from the cut imposed on m_B^{rec}. Even for the looser cut applied in the electron case the region $q^2 \in [1, 6]$ GeV^2 is free from the J/Ψ contamination and can be estimated with good theoretical accuracy.

The second point to be noted is that in the regular region of the spectrum radiative corrections reach (or even exceed) the 10% level for the electrons (as naively expected); however, the net effect in R_K is significantly smaller. Indeed the magnitude of the corrections is larger for

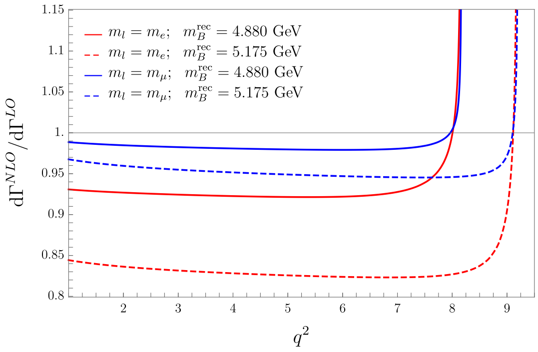

Figure 1. Relative impact of radiative correction in $B \to K^+\ell^+\ell^-$ decays for $q^2 \in [1, 9.5]$ GeV^2, with different cuts on the reconstructed mass and different lepton masses.

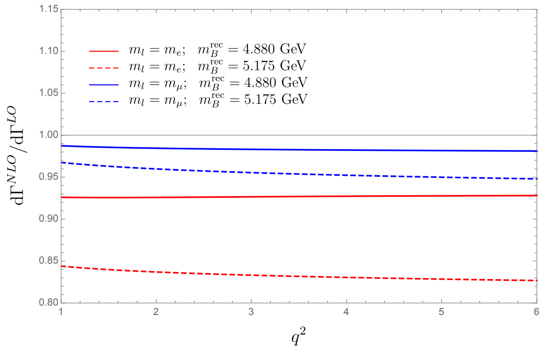

Figure 2. Relative impact of radiative correction in $B \to K^*\ell^+\ell^-$ for $q^2 \in [1, 6]$ GeV^2, with different cuts on the reconstructed mass and different lepton masses. .

Table 1. Relative impact on R_K (left) and $R_{K^{(*)}}$ (right) of radiative corrections for $q^2 \in [1, 6]$ GeV^2, with different cuts on the reconstructed mass and different lepton masses.

$B \to K\ell^+\ell^-$	$\ell = e$ (%)	$\ell = \mu$ (%)
$m_B^{\rm rec} = 4.880$ GeV	$-\ 7.6$	-1.8
$m_B^{\rm rec} = 5.175$ GeV	-16.9	-4.6

$B \to K^*\ell^+\ell^-$	$\ell = e$ (%)	$\ell = \mu$ (%)
$m_B^{\rm rec} = 4.880$ GeV	$-\ 7.3$	-1.7
$m_B^{\rm rec} = 5.175$ GeV	-16.7	-4.5

Table 2. Relative contribution of radiative corrections due emission from the meson leg, in the $B^+ \to K^+\ell^+\ell^-$ case, for $q^2 \in [1, 6]$ GeV^2.

$m_B^{\rm rec} = 4.880$ GeV	-0.02%
$m_B^{\rm rec} = 5.175$ GeV	-0.18%

electron vs. muons, but it increases for $m_B^{\rm rec} \to m_B$. This imply that the specific choice of $m_B^{\rm rec}$ cuts applied by the LHCb collaboration, i.e. a loose cut for the electrons and a tighter cut for the muons, give rise to a natural compensation of the QED corrections to R_K.

The integrated corrections that quantify the modifications to R_K are reported in Table 1. Given the choice of $m_B^{\rm rec}$ applied in Ref. [2], we estimate that radiative corrections induce a *positive* shift of the central value of R_K of a about $\Delta R_K = +3\%$. This effect is taken into account by the LHCb collaboration, who estimated the impact of radiative corrections with PHOTOS [17], and properly corrected for in the result reported. We have explicitly checked that our estimate of ΔR_K is in agreement with that obtained with PHOTOS up to differences within $\pm 1\%$.

In order to check the smallness of the non-log(m_ℓ) enhanced terms, in Table 2 we report the effect of the radiation from the meson leg, that is IR divergent but has no collinear singularities. We evaluated these terms developing the corresponding radiator function (see Ref. [18]), whose implementation depend only on $m_B^{\rm rec}$. As can be seen from Table 2, the results are well below the 1% level.

The impact of radiative corrections in the $B \to K^*\ell^+\ell^-$ decays is shown in Figure 2 and summarized by the integrated values reported in Table 1. The situation is very similar to the $B^+ \to K^+\ell^+\ell^-$: employing the same $m_B^{\rm rec}$ cuts for electron and muon modes as in Ref. [2], we find that the net impact of radiative corrections is $\Delta R_{K^*} = +2.8\%$. Also in this case this effect is well described by PHOTOS and therefore can be properly corrected for in future experimental analyses.

4. Conclusions

The experimental result in Eq. (2) ha stimulated a lot of theoretical activity. In view of this result and, especially, in view of possible future experimental improvements in the determination of R_K or R_{K^*}, we have re-examined the SM predictions of these LFU ratios.

As we have show, $\log(m_\ell)$-enhanced QED corrections may induce sizable deviations from $R_{K^{(*)}}^{(\rm SM)} = 1$, even up to 10%, depending on the specific cuts applied to define physical observables. In particular, a key role is played by the cuts on $q^2 = m_{\ell\ell}^2$ and on the reconstructed B-meson mass. The former is important to avoid rapidly varying regions in the dilepton spectrum (where the theoretical tools to compute QED corrections become unreliable), while the latter defines the physical IR cut-off of the rates. Employing the cuts presently applied by the LHCb Collaboration, the corrections in R_K do not exceed 3%. Moreover, their effect is well described (and corrected for in the experimental analysis) by existing Montecarlo codes.

According to our analysis, a deviation from $R_{K^{(*)}}^{(\rm SM)} = 1$ exceeding the 1% level, performed along the lines of Ref. [2] in the region $1~{\rm GeV}^2 < q^2 < 6~{\rm GeV}^2$, would be a clear signal of physics beyond the Standard Model.

Acknowledgments

The author would like to thank the organisers of the BEACH2016 conference for the invitation to give his talk. This research was supported in part by the Swiss National Science Foundation (SNF) under contract 200021-159720.

References

[1] Hiller G and Kruger F 2004 *Phys. Rev.* **D69** 074020 (*Preprint* hep-ph/0310219)
[2] Aaij R *et al.* (LHCb) 2014 *Phys. Rev. Lett.* **113** 151601 (*Preprint* 1406.6482)
[3] Altmannshofer W and Straub D M 2015 *Proceedings, 50th Rencontres de Moriond Electroweak Interactions and Unified Theories: La Thuile, Italy, March 14-21, 2015* pp 333–338 (*Preprint* 1503.06199) URL https://inspirehep.net/record/1353682/files/arXiv:1503.06199.pdf
[4] Descotes-Genon S, Hofer L, Matias J and Virto J 2016 *JHEP* **06** 092 (*Preprint* 1510.04239)
[5] Lees J P *et al.* (BaBar) 2012 *Phys. Rev. Lett.* **109** 101802 (*Preprint* 1205.5442)
[6] Huschle M *et al.* (Belle) 2015 *Phys. Rev.* **D92** 072014 (*Preprint* 1507.03233)
[7] Aaij R *et al.* (LHCb) 2015 *Phys. Rev. Lett.* **115** 111803 [Addendum: Phys. Rev. Lett.115,no.15,159901(2015)] (*Preprint* 1506.08614)
[8] Fajfer S, Kamenik J F and Nisandzic I 2012 *Phys. Rev.* **D85** 094025 (*Preprint* 1203.2654)
[9] Na H, Bouchard C M, Lepage G P, Monahan C and Shigemitsu J (HPQCD) 2015 *Phys. Rev.* **D92** 054510 [Erratum: Phys. Rev.D93,no.11,119906(2016)] (*Preprint* 1505.03925)
[10] Huber T, Hurth T and Lunghi E 2015 *JHEP* **06** 176 (*Preprint* 1503.04849)
[11] Huber T, Hurth T and Lunghi E 2008 *Nucl. Phys.* **B802** 40–62 (*Preprint* 0712.3009)
[12] Huber T, Lunghi E, Misiak M and Wyler D 2006 *Nucl. Phys.* **B740** 105–137 (*Preprint* hep-ph/0512066)
[13] Bordone M, Isidori G and Pattori A 2016 (*Preprint* 1605.07633)
[14] Gratrex J, Hopfer M and Zwicky R 2016 *Phys. Rev.* **D93** 054008 (*Preprint* 1506.03970)
[15] Yennie D R, Frautschi S C and Suura H 1961 *Annals Phys.* **13** 379–452
[16] Weinberg S 1965 *Phys. Rev.* **140** B516–B524
[17] Davidson N, Przedzinski T and Was Z 2016 *Comput. Phys. Commun.* **199** 86–101 (*Preprint* 1011.0937)
[18] Isidori G 2008 *Eur. Phys. J.* **C53** 567–571 (*Preprint* 0709.2439)