

Low-energy e^+e^- Hadronic Cross-Section Measurements and Implications for $(g-2)$ of the Muon

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Abstract. The *BABAR* Collaboration has performed an intensive program studying low energy hadronic cross sections in e^+e^- annihilations via initial-state radiation. These measurements are crucial to improve the precision of the Standard Model prediction of the muon anomalous magnetic moment, $a_\mu = \frac{1}{2}(g_\mu - 2)$, and could help shed light on the $> 3\sigma$ discrepancy between the Standard Model predicted value for a_μ and the experimental measurement of a_μ performed at Brookhaven National Laboratory. We have published results on a number of initial state radiative processes with two to six hadrons in the final state, at effective center-of-mass energies ranging from ~ 1 to 4.5 GeV. We report here on our most recent results obtained using the entire *BABAR* dataset.

1. Introduction and Motivation

We have entered an era of precision tests of the Standard Model of particle physics – any disagreement between the Standard Model and precise experimental measurements may be a hint of new physics beyond the Standard Model.

There is currently a discrepancy of more than 3σ between the theoretical Standard Model prediction[1] of the anomalous magnetic moment of the muon and the precise experimental measurement performed at the E-821 experiment at Brookhaven National Laboratory[2].

1.1. Anomalous Magnetic Dipole Moment of Muon

The magnetic moment of the muon depends upon its mass m , charge e , spin \vec{S} , and gyromagnetic ratio g_μ :

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c} \vec{S} \quad (1)$$

The gyromagnetic ratio for point-like Dirac particles is exactly $g_\mu = 2$. Deviations from 2 are a result of radiative corrections to the lepton-photon vertex. The anomalous magnetic moment of the muon, a_μ , is the deviation of the gyromagnetic ratio from the value $g_\mu = 2$, so that $a_\mu \equiv (g_\mu - 2)/2$.

The anomalous magnetic moment of the muon, a_μ , has been measured experimentally at Brookhaven's E-821 to a precision of better than 1 part in a million[2], and is precisely predicted theoretically within the framework of the Standard Model. The Standard Model calculations predicting a_μ involve QED, weak and hadronic contributions. The theoretical prediction of a_μ is

limited in precision by the hadronic contributions, and while they are small, their uncertainties dominate the uncertainty of the theoretical Standard Model prediction for a_μ .

Fortunately, the precision on the prediction of the hadronic terms contributing to a_μ may be improved by using low-energy experimentally measured e^+e^- hadronic cross-section data.

The theoretical Standard Model prediction of a_μ has QED, weak and hadronic contributions:

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{hadLO,VP} + a_\mu^{hadHO,VP} + a_\mu^{had,LBL} \quad (2)$$

These contributions are listed in Table 1. The QED contribution, a_μ^{QED} , is the dominant contribution to a_μ^{SM} . It has been calculated to 10th order in perturbation theory[3] and is very precisely known with a tiny uncertainty. The weak correction, a_μ^{weak} , is very small and precisely calculable, also with a small uncertainty. The hadronic corrections due to leading order hadronic vacuum polarization, $a_\mu^{hadLO,VP}$, higher order hadronic vacuum polarization, $a_\mu^{hadHO,VP}$, and hadronic light-by-light scattering, $a_\mu^{had,LBL}$, cannot be precisely calculated perturbatively at low energy. The leading order hadronic vacuum polarization, $a_\mu^{hadHO,VP}$ dominates the theoretical uncertainty in a_μ^{SM} , as seen in Table 1. While great progress may soon be made in lattice QCD, for now we use experimental cross-section data as input to precisely calculate hadronic contributions to a_μ^{SM} . Any discrepancy between the theoretical Standard Model prediction and experimental measurement would be an indication of new physics beyond the Standard Model.

2. Direct Measurement of the Anomalous Magnetic Moment of the Muon

The most precise direct measurement of a_μ was performed at experiment E-821[2] at Brookhaven National Laboratory. The spin precessions of 100 billion μ^+ and μ^- were measured in a uniform magnetic field, circulating in a storage ring, yielding the measurement:

$$a_\mu = (116\,592\,091 \pm 54_{stat} \pm 33_{syst}) \times 10^{-11} \quad (3)$$

There is a 3.6σ discrepancy between Standard Model theoretical prediction of a_μ and the BNL E-821 experimental measurement[2], as seen in Table 1.

3. Initial State Radiation and Hadronic Cross-Section Measurements

Contributions to the anomalous magnetic moment of the muon are shown in Figure 1. The dominant contribution to the uncertainty in the Standard Model prediction of a_μ^{SM} arises from the leading order hadronic vacuum polarization term, $a_\mu^{hadLO,VP}$. This term cannot be calculated

Table 1. Contributions to the Standard Model prediction[4] for a_μ ($\times 10^{-11}$) and comparison with experimental results .

QED	11658478.95	\pm	0.08
Weak	154.	\pm	1.
Leading Order Hadronic vacuum polarization	6923 .	\pm	42 .
Higher Order Hadronic vacuum polarization	-98.	\pm	1.
Hadronic light-by-light scattering	105.	\pm	26.
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Theory - Standard Model	116591803.	\pm	49.
Experiment - E-821 BNL	116592091.	\pm	54. \pm 33.
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Discrepancy (Theory-Experiment)	288.	\pm	80.

precisely at low energies. But we can estimate it with reasonable precision using experimental e^+e^- cross-section measurements in a dispersion relation[5] to evaluate the weighted integral of the hadronic cross-section $\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})$, over a range of effective e^+e^- center-of-mass energies, $\sqrt{s'}$:

$$a_\mu^{\text{hadLO,VP}} = \frac{1}{4\pi^3} \int_0^\infty \sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})(s) K(s') ds' \quad (4)$$

The kernel $K(s')$ [6] and $\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})(s')$ each $\propto 1/s'$, ensuring that low-energy contributions to $a_\mu^{\text{hadLO,VP}}$ will dominate. The low energy part of the integral, up to 1.8 GeV, is obtained using experimental cross-section measurements, and the high-energy part of the integral, above 1.8 GeV, is computed using perturbative QCD.

As seen in Equation 4, the hadronic vacuum polarization term requires integration over a range of e^+e^- center-of-mass energies. A convenient method to effectively vary the center-of-mass energy is the initial state radiation(ISR) method. An ISR event $e^+e^- \rightarrow f\bar{f}\gamma$ is shown in Figure 2. The ISR process effectively gives a variable center-of-mass energy, without ever having to vary the beam energies of the accelerator. The process $e^+e^- \rightarrow f\bar{f}$ with an initial state radiative photon allows us to measure the $e^+e^- \rightarrow f\bar{f}$ cross-sections over a range of lower effective e^+e^- center-of-mass energies. The virtual photon vertex in Figure 2 has contributions from QED, weak, and hadronic processes, as illustrated in Figure 1.

New physics beyond the Standard Model could make an additional vertex contribution to a_μ , and hence result in a discrepancy between the SM prediction and experimental measurement of a_μ . The $(g_\mu - 2)$ of the muon is much more sensitive to new physics than the electron's $(g_e - 2)$; the sensitivity $\propto (m_e/m_\mu)$, so $(g_\mu - 2)$ is a good probe for new physics.

4. BABAR Hadronic Cross-Section Measurements Using the ISR Method at BABAR

The BABAR detector at the Stanford Linear Accelerator Center (SLAC), on the PEP-II e^+e^- storage ring, collides 9 GeV electrons on 3.1 GeV positrons. The BABAR detector is described elsewhere[7]. Using the BABAR detector, we have performed ISR hadronic cross-section

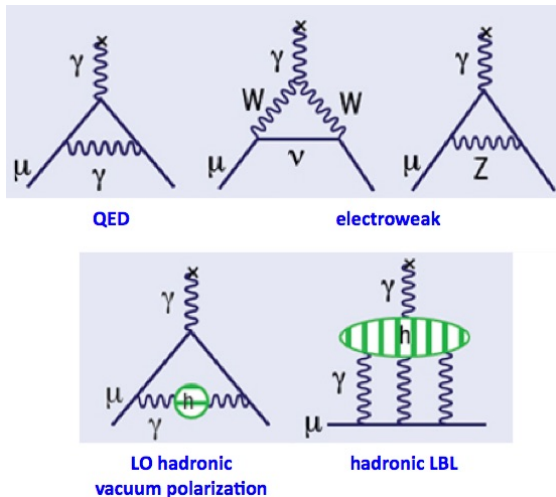


Figure 1. QED, weak, and hadronic contributions to a_μ^{SM} . The hadronic vacuum polarization contribution is the dominant contribution to the uncertainty in a_μ^{SM} .

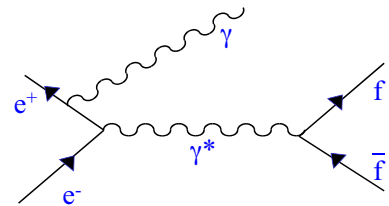


Figure 2. The Feynman diagram for $e^+e^- \rightarrow f\bar{f}\gamma$ with a photon radiated from the initial state is the basis for the ISR method to study $e^+e^- \rightarrow f\bar{f}$ at lower effective center-of-mass energies.

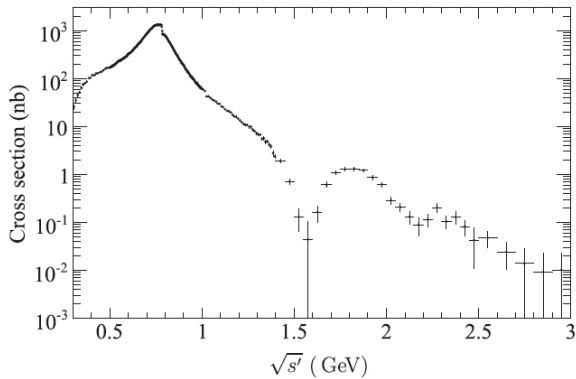


Figure 3. The measured cross section for $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ for effective center-of-mass energies, $\sqrt{s'}$, ranging from 0.3 to 3 GeV.[8]

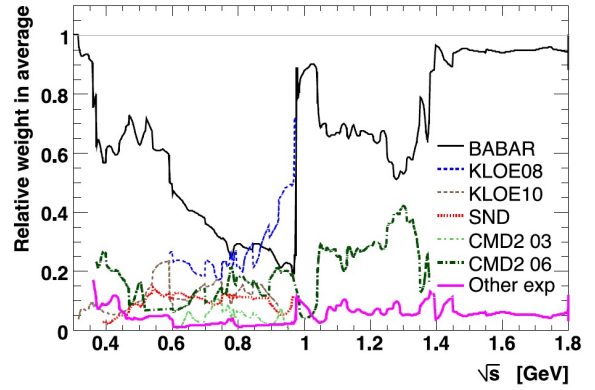


Figure 4. Relative local averaging weight per experiment versus effective center-of-mass energy for $e^+e^- \rightarrow \pi^+\pi^-$ [8][10][11][12]

measurements and hence critical tests of the muon's ($g_\mu - 2$) in the Standard Model. Until a few years ago, low energy hadronic cross-section data was dominated by the work at the VEPP machines, in the 1-2 GeV center-of-mass energy region, at Budker Institute for Nuclear Physics.

BABAR contributes significantly to a large number of exclusive ISR cross-section measurements at and just above the VEPP energies, from threshold up to 3-5 GeV, depending on backgrounds of each exclusive hadronic channel. In addition to hadronic cross-section measurements for ($g_\mu - 2$), *BABAR* studies investigate topics, ranging from spectroscopy to form-factor studies.

In *BABAR* ISR cross-section analyses, the initial state radiated photon is detected at a large angle, within the sensitive volume of the detector, with energy ≥ 3 GeV. The topology is such that the ISR photon is roughly back-to-back with the produced hadrons in the event, resulting in high acceptance and good particle identification. Kinematic fits reduce the backgrounds and give good energy resolution. Control samples are used to determine backgrounds, corrections and efficiencies. ISR event reconstruction and selection may be found in [8].

4.1. The $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ Channel

The $\pi\pi$ ISR channels account for approximately 75% of the contributions to a_μ^{had} . The ISR method is used at *BABAR* to measure the $\pi^+\pi^-$ cross-section from the $m_{\pi\pi}$ threshold up to 3 GeV, allowing the possibility of an additional radiated photon, denoted (γ). The $\pi^+\pi^-(\gamma)$ cross-section is shown in Figure 3. The ρ resonance, $\rho - \omega$ interference and other structures are evident. The *BABAR* $\pi^+\pi^-$ analysis yields a cross-section measurement which is used to compute the contribution of the $\pi^+\pi^-$ mode to the theoretical prediction of a_μ^{SM} using the dispersion integral in Equation 4 to obtain:

$$a_\mu^{\pi^+\pi^-(\gamma),LO}[\text{from threshold to 1.8 GeV}] = (514.1 \pm 2.2_{stat} \pm 3.1_{syst}) \times 10^{-10} [9]. \quad (5)$$

The contribution to $a_\mu^{\pi^+\pi^-}$ due to final state radiation is $a_\mu^{\pi^+\pi^-,FSR} = (0.26 \pm 0.12) \times 10^{-10}$. The weights of different experiments in combining results for $\pi^+\pi^-$ contributions to $a_\mu^{\pi^+\pi^-}$ are shown in Figure 4. *BABAR* dominates at all energies shown, except in the 0.75-0.93 GeV region where the KLOE results dominate.[1]. A summary of the leading order hadronic $a_\mu^{\pi\pi}$ contributions are in Figure 5, determined using e^+e^- cross-section measurements, as well as results using τ data.

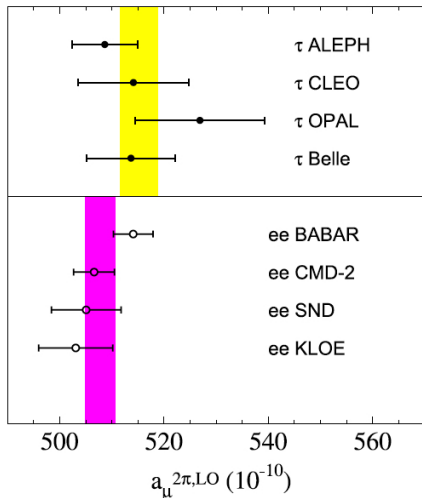


Figure 5. Leading order hadronic vacuum polarization 2π contributions to a_μ , evaluated from threshold to 1.8 GeV, for τ and e^+e^- data [8]

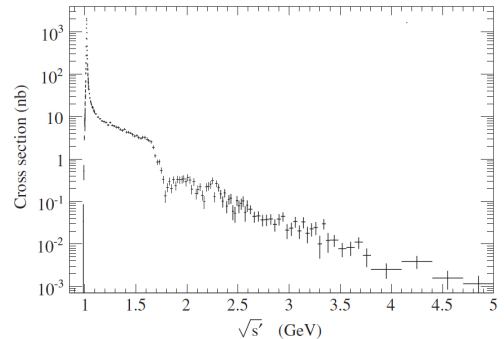


Figure 6. The measured cross section for $e^+e^- \rightarrow K^+K^-(\gamma)$ for effective center of mass energies, $\sqrt{s'}$, including final state radiation.[13]

4.2. The $e^+e^- \rightarrow K^+K^-(\gamma)$ Channel

Using the same selection and methods as for the $\pi^+\pi^-$ channel, and subtracting the J/ψ and $\psi(2S)$ resonances, we measured the $K^+K^-(\gamma)$ cross-section for effective center of mass energies ranging from threshold to 5 GeV[13], shown in Figure 6. Final state radiation is measured and included, but is negligible. The *BABAR* K^+K^- analysis yields a cross-section which can be used to compute the contribution of the K^+K^- mode to the theoretical prediction of a_μ^{SM} . The result of the dispersion integral (Equation 4) from threshold to 1.8 GeV is:

$$a_\mu^{K^+K^-(\gamma),LO}[\text{from threshold to 1.8 GeV}] = (22.93 \pm 0.18_{stat} \pm 0.22_{syst} \pm 0.03_{VP}) \times 10^{-10} [13] \quad (6)$$

4.3. Cross-sections of Other Channels Contributing to a_μ

The *BABAR* collaboration has measured many other low-energy single channel cross-sections using the ISR method, all relevant for the determination of the hadronic corrections to the $(g_\mu - 2)$ anomaly. These include recent measurements of $e^+e^- \rightarrow \pi^+\pi^-$, K^+K^- , $\pi^+\pi^-\pi^+\pi^-$, $K^+K^-K^+K^-$, $p\bar{p}$, $K_S^0K_L^0$, $K_S^0K_L^0\pi^+\pi^-$, $K_S^0K_S^0\pi^+\pi^-$, $K_S^0K_S^0K^+K^-$, $\pi^+\pi^-\pi^0\pi^0$, as well as other channels, as shown in Figure 7.[8, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]

5. Summary and Outlook

Hadronic cross-sections measurements using the ISR method have been performed at *BABAR* for many $e^+e^- \rightarrow \text{hadrons}$ channels, providing important input for the theoretical prediction of the anomalous magnetic moment of the muon. Measurements performed at *BABAR* have reduced the uncertainty in the hadronic contributions to the predicted a_μ , but a 3σ discrepancy between the experimental measurements at BNL E-821 and the Standard Model prediction for a_μ persists. With new $(g_\mu - 2)$ experiments coming online at FNAL (see talk by J. Holzbauer, in these proceedings) and JPARC, improvements in the precision of Standard Model prediction for $(g_\mu - 2)$ is crucially important. New BESIII results on this topic (see talk by C.F. Redmer) may also be found in these proceedings.

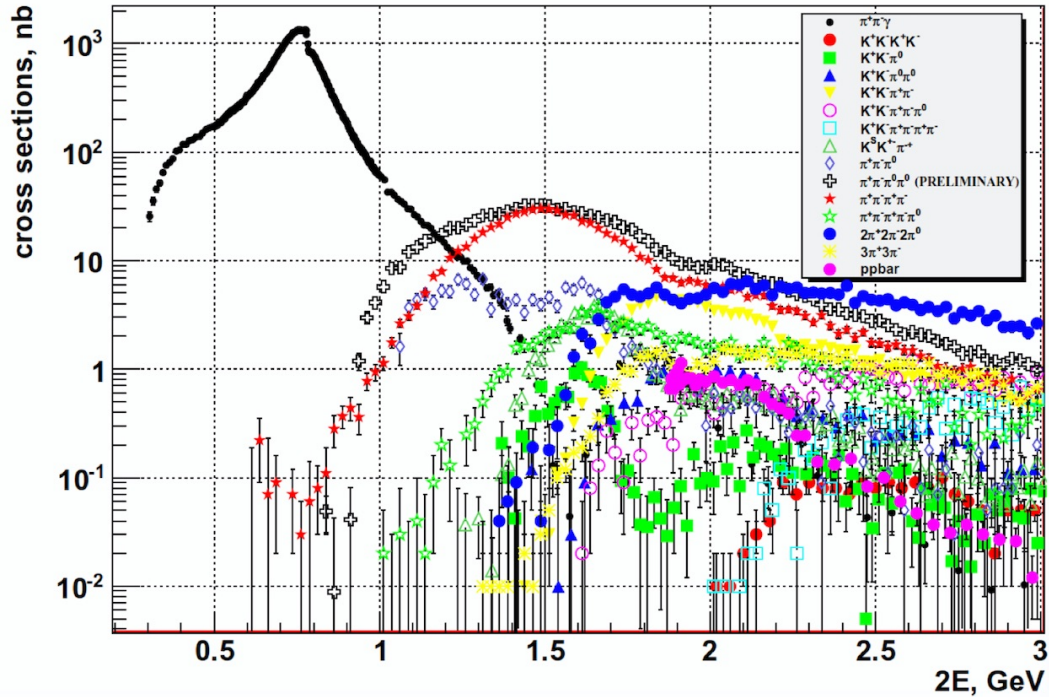


Figure 7. $e^+e^- \rightarrow$ hadronic cross-sections measured with the BABAR detector via ISR.[22]

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