The Equation of State of cold quark matter

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Introduction to Thermal Field Theory

Quantum field theory

- QCD (quarks, gluons, ghosts)
- Lagrangian $\mathcal{L}$, generating functional Z
- Feynman Diagrams

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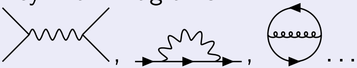

Thermodynamics

- Observables: pressure, entropy, etc. via partition function Z

$$F = -T \ln Z\,, \quad S = \frac{\partial(T \ln Z)}{\partial T} \ldots$$

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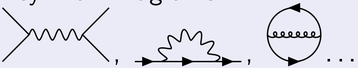

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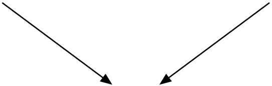
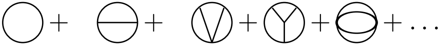


$\Rightarrow \ln Z = \ln$ 

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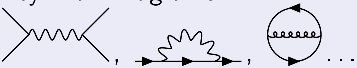

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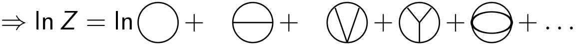

- Compute thermodynamical observables by perturbative (Taylor) expansion in the coupling g.
- Translation: Solve many, many Feynman integrals.
- Powerful method: Reduce 10^7 to 10^1 integrals

Why Thermal Field Theory?

- Big difference to collider physics ($T = 0$, $\mu = 0$):

$$\int \mathrm{d}^{4-2\epsilon} p \to \sum_{p_0=-\infty}^{\infty} \int \mathrm{d}^{3-2\epsilon} p$$

- Difficult to solve due to nested infinite sums / Riemann-ζ regularization.

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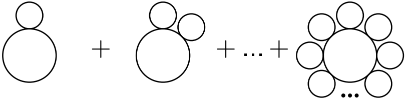
- PhD Bielefeld:

  Y. Schröder: "Want to try solving ?"
  Me: "Sure, why not."
  ... three years later

⇒ Gluon Debye mass to 3-loop order.

More problems

- So called IR divergences: Due to soft gluon modes in a quark-gluon plasma.
- In fact soft gluon modes are screened by thermal mass.
- Gluon mass nowhere in the Lagrangian, dynamically generated.
  $\Rightarrow$ Resummation of infinite number of diagrams to every order in the Taylor expansion



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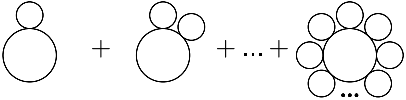
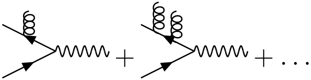


- Particle production rates: multiple scatterings in a plasma: Higher order processes are parametrically as large as first order:



- $\Rightarrow$ Postdoc in Bern with Mikko Laine: Higher order corrections to dilepton production rate using LPM resummation.

Cold nuclear matter in Helsinki

with A. Vuorinen, T. Gorda, A Kurkela, P. Romatschke, M. Säppi

- Goal: Equation of State of cold ($T = 0$) nuclear matter (finite μ) to Order $g^6 \ln g$:

$$\Omega = -T \ln Z$$

- Important to understand
  - The QCD phase diagram.
  - Neutron stars.

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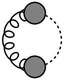
$\sum$  $= \ln\Big(1 +$  $/P^2\Big)$

- Integrands contain an imaginary part: $\int \frac{1}{(p_0+i\mu)^2+\mathbf{p^2}}$ $\rightarrow$ contour integrals $\rightarrow$ residue theorem.

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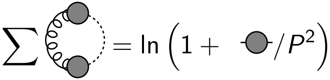

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- "Cutting rules" considerably simplify integration. They work! How? $\Rightarrow$ Proof comes out next week. Check arXiv.
- soon EoS to follow