Google Summer of Code 2016

Alternative integration methods and utilisation of dense output for field propagation

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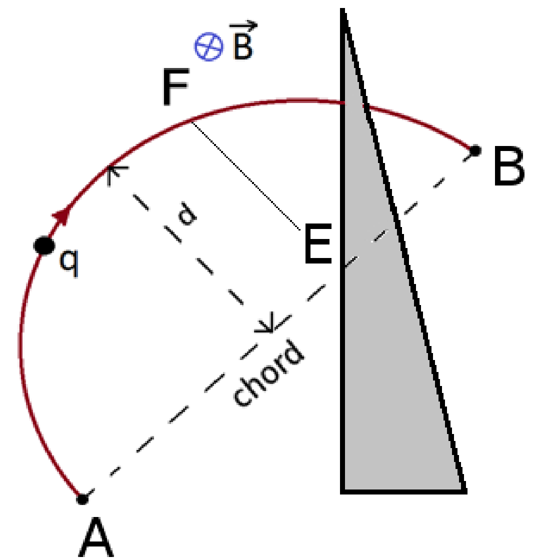
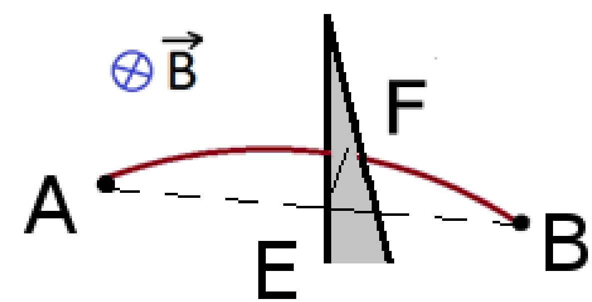
Outline

- Field propagation in Geant4
- The Bulirsch-Stoer method
- Alternative integration strategy for dense output methods
- Verification
  - Propagation in uniform magnetic field
  - NTST test

Field propagation in Geant4

- Uses Runge-Kutta integration with adaptive step size control
  1. d < deltaChord
  2. ΔB < deltaOneStep
- Locates boundary intersection using iterative intersections of chords





Until |F – E| < deltaIntersection

Project goals

- Implement the Bulisrch-Stoer method
- *Motivation: For smooth functions and large steps, extrapolation methods are more efficient than the Runge-Kutta methods. One of the most efficient extrapolation methods is the Bulirsch-Stoer method.*
- Develop the alternative integration strategy for dense output methods
- *Motivation: Integration methods may need a fixed number of extra field evaluations to provide dense output. Using dense output the solution can be evaluated for any point within the integration interval.*

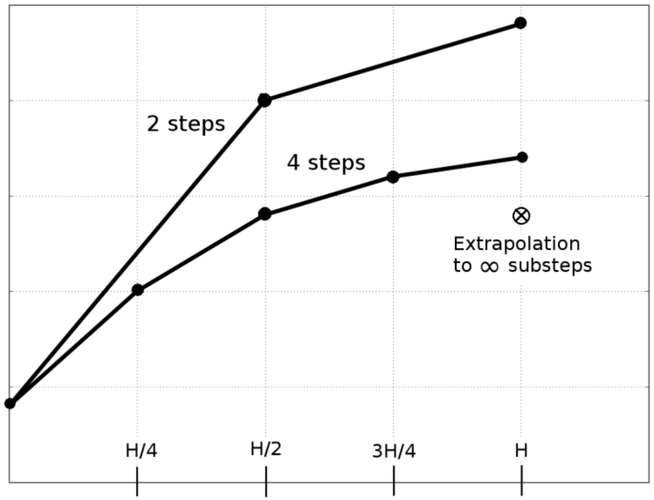
The Bulirsh-Stoer method

Idea:

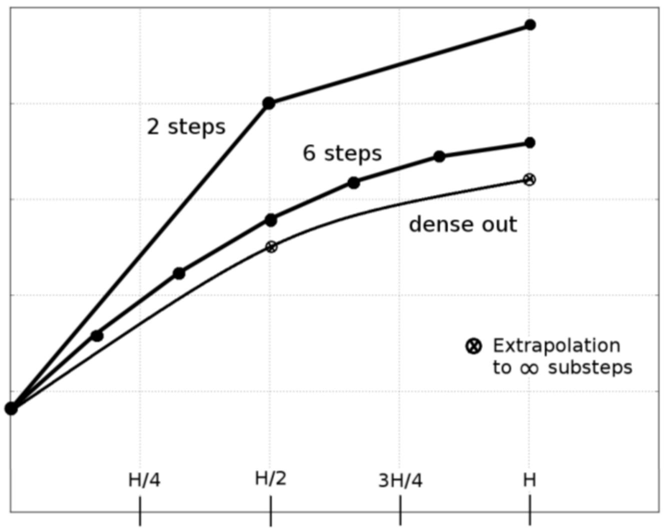
- Use midpoint method to estimate integral
- Vary number of intermediate points
- Approximate the integral using rational functions & extrapolate to n = ∞

Advantages:

- Step size and order control
- Very good for smooth problems and large steps
- Can provide dense output



$n = 2i+2$, for $i = 0,1,2\ldots.$



$n = 4i+2$, for $i = 0,1,2\ldots.$

Alternative integration strategy for dense output methods

Old strategy

- Make series of steps without error control to predict the step size (satisfying d < deltaChord)
- Make a step with error control to improve the accuracy

  (ΔB < deltaOneStep)
- If the chord intersects:

  make a series of substeps with error control to locate the intersection point

New strategy

- Make one step with error control
- Use dense output to divide the step to substeps (satisfying d < deltaChord)
- For each substep:

  if the chord intersects:

  Use dense output to locate the intersection point

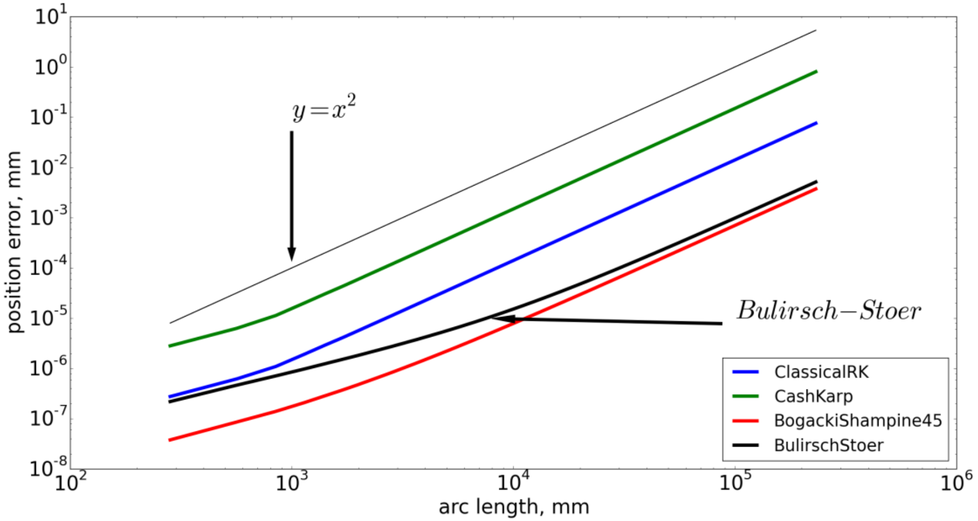
Pros: A lot fewer field evaluations required for large steps

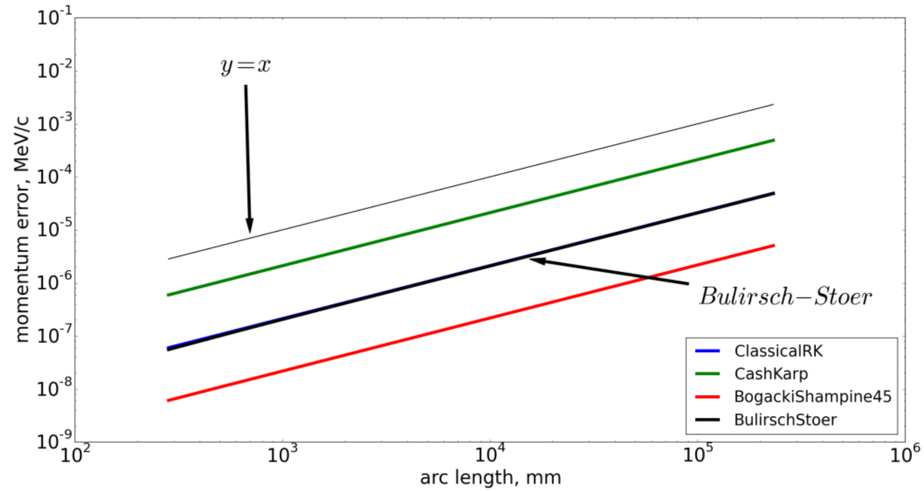
Cons: Dense output is less accurate than the solution

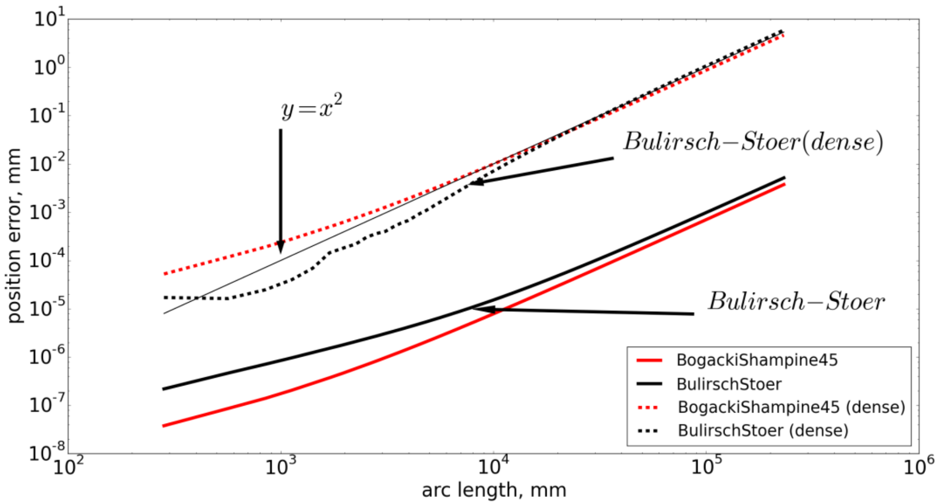
Propagation in uniform magnetic field

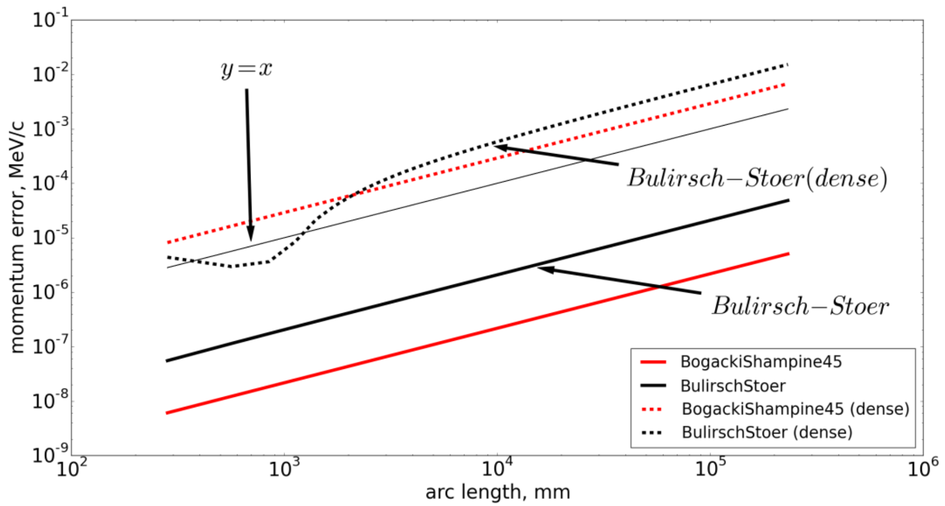
1 MeV proton in the uniform 1 tesla magnetic field. Radius of the circle in xz plane is 102.20 mm

Momentum is 43.33 MeV/c. deltaOneStep = 1e-4 mm, deltaIntersection = 1e-5 mm

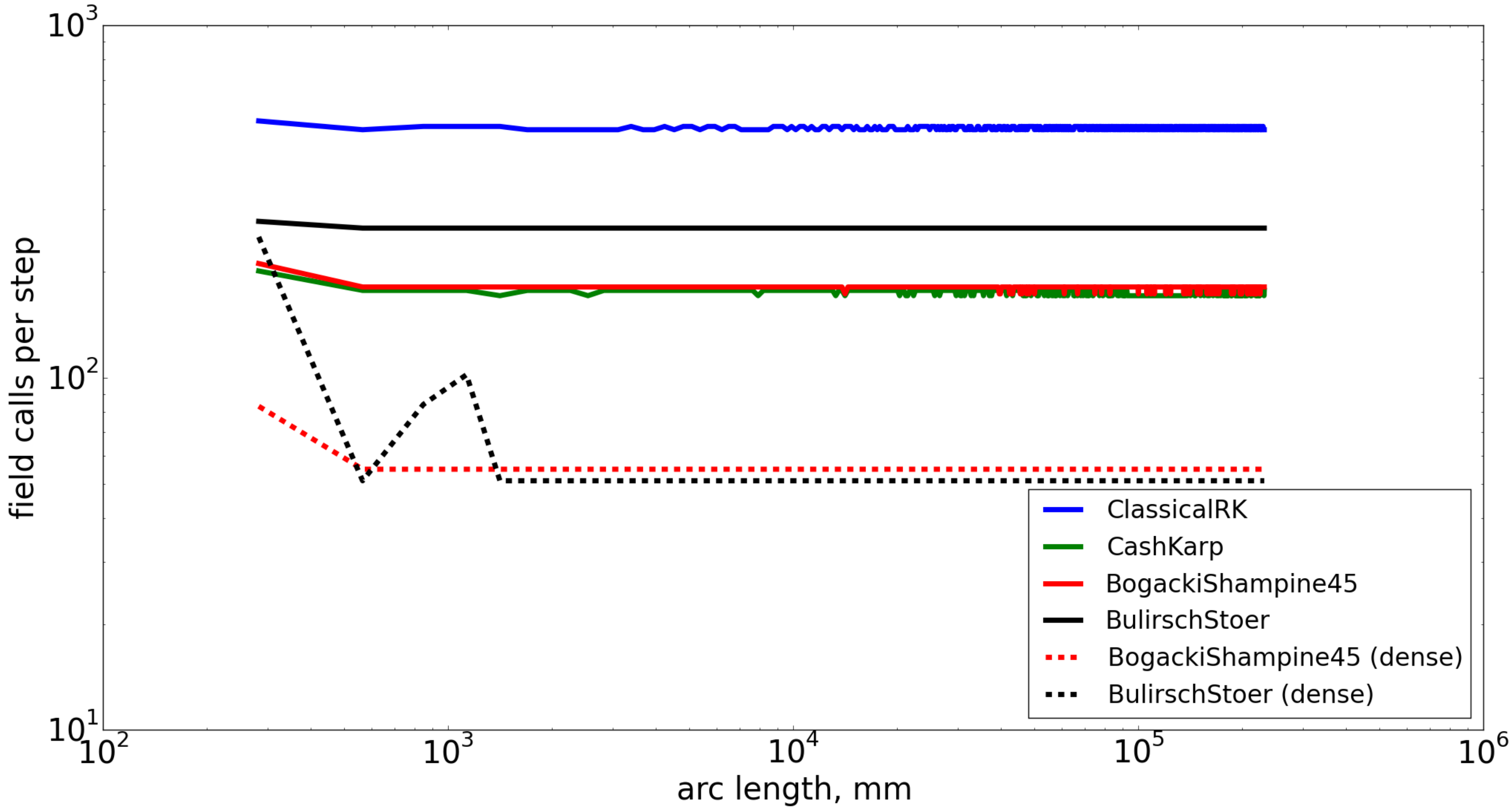




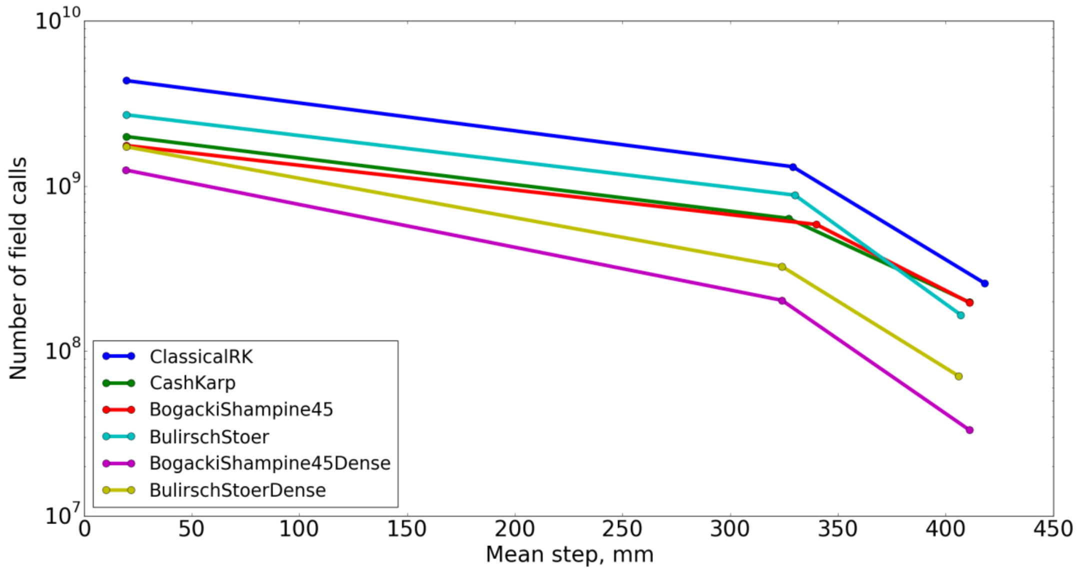


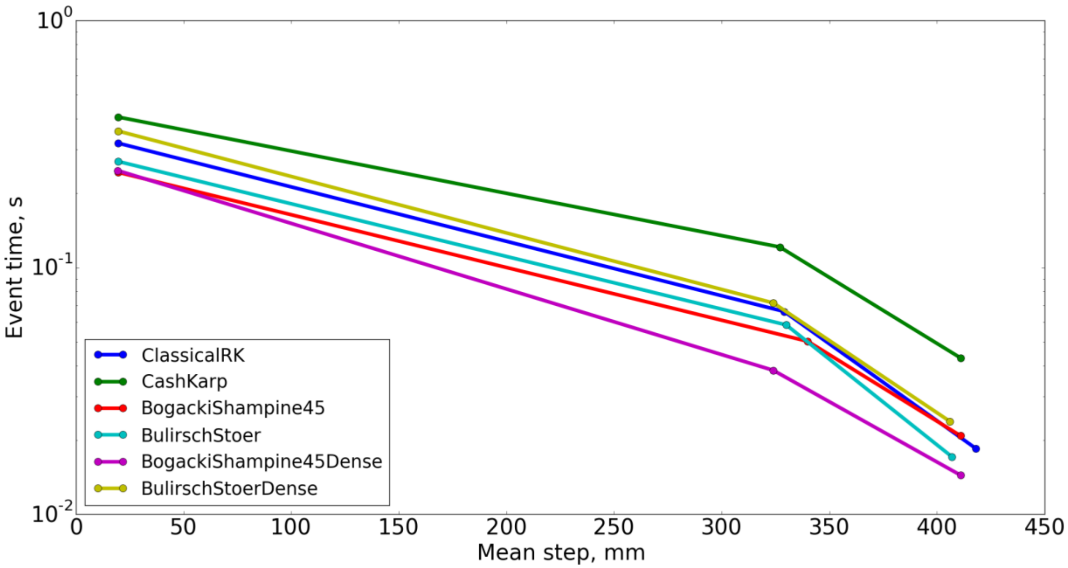


Propagation in uniform magnetic field



NTST test





Geant4 benchmark test. Simulates the BaBar silicon vertex tracker and 40-layer drift chamber.

File	range cut	looper cut	min Ecat
run2a. mac	1 mm	200 MeV	1 MeV
run2b. mac	1 mm	Not applied	1 MeV
run2c. mac	1 mm	Not applied	Not applied

Thank you