CERN

Tutorial:
Impedance

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Tutorial

- Goals
  - Get acquainted with the main concepts related to beam impedance
  - Understand the effect of different materials, and different geometries
- Means
  - Experimental measurements with Vector Network Analyser (VNA)
  - Sincere thanks to Jean-Jacques Gratier from Keysight / Computer Controls for kindly providing a E5080A VNA for free for the school
  - Computational exercises with CST Studio Suite and ImpedanceWake2D (IW2D)
  - Sincere thanks to Monika Balk from CST / 3DS for providing several free full licenses for Studio Suite for the duration of the school
  - Short explanations of theoretical background
- The students group will be divided in two for the experimental work
- Computer work will be done in teams of 2.

Plan of the tutorial

- Introduction
  - Surface impedance (10' Sergio)
  - Recap on beam impedance, introduction to CST studio Suite and IW2D (20' Benoît)
- Work with network analyser 1 (60' Sergio)
  - Measurement of induction of different materials with solenoid
  - Identification of materials and coatings
- And in parallel: first practice with CST Studio Suite (60' Benoît)
  - Different runs on predefined models: tubes, cavities, bellows…

← Ice-cream break

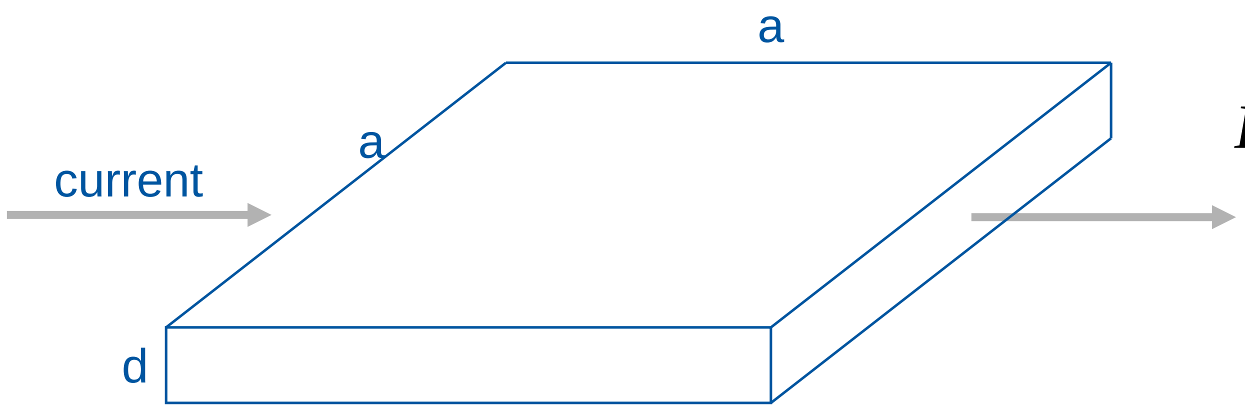
- Guided simulations with IW2D (30')
  - Dependence of impedance on chamber diameter, bunch length, coating, …
- Work with network analyser 2 (30')
  - Measurement of resonances (with/without RF fingers)
- And in parallel: simulations à la carte (30')

What is impedance?

- A term with many meanings, depending on context
  - Impedance in electrical circuits
  - Impedance of materials
  - Surface impedance
  - Beam coupling impedance
- There is a common rationale: $$Z = \frac{V}{I}$$
  -
  - Voltage and current flowing in the circuit, the material, the surface
  - The beam current, and the potential it generates on the vacuum pipe

Square resistance and surface resistance

Consider a square sheet of metal and calculate its resistance to a transverse current flow:

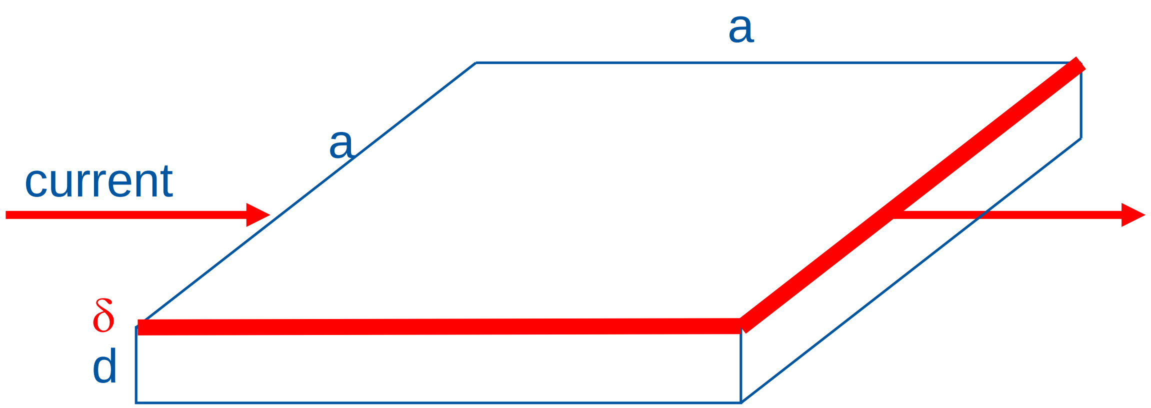


$$R = \frac{\rho \cancel{a}}{d \cancel{a}} = \frac{\rho}{d}$$

This is the so-called square resistance often indicated as $R_{\blacksquare}$

Square resistance and surface resistance

And now imagine that instead of DC we have RF, and the RF current is confined in a skin depth: $\delta = \sqrt{\dfrac{2\rho}{\mu_0 \omega}}$

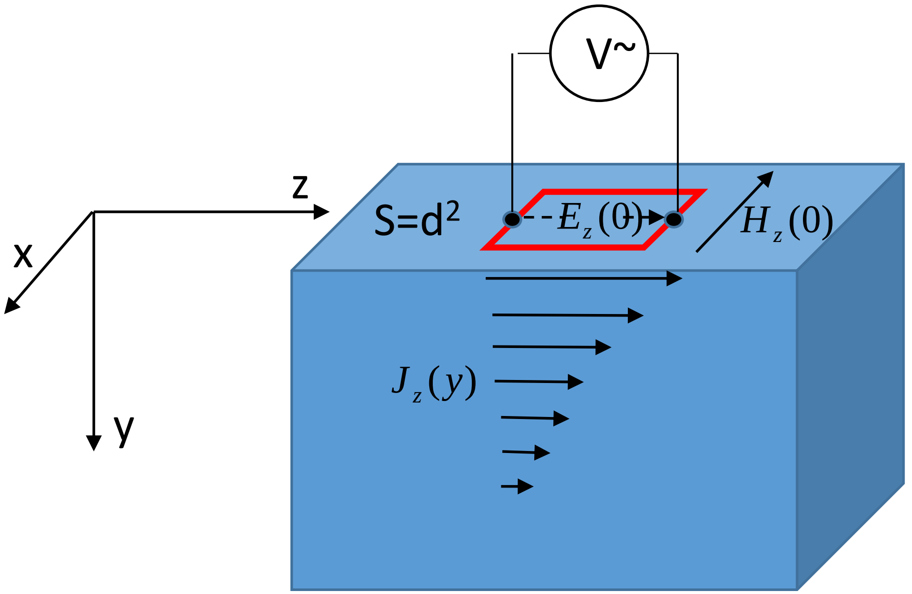


$$R = \frac{\rho\, \cancel{a}}{d\, \cancel{a}} = \frac{\rho}{d}$$

$$R_s = \frac{\rho}{\delta} = \sqrt{\frac{\rho \mu_0 \omega}{2}}$$

This is a (simplified) definition of surface resistance R_s

Surface impedance



$$Z_s = \frac{V}{I}$$

$$V = dE_z(0)\ ;\ I = d\int_0^{+\infty} J_z(y)dy$$

$$\delta = \frac{1}{J_z(0)}\int_0^{+\infty} J_z(y)dy$$

$$\boxed{Z_s = R_s + iX_s = \frac{E_z(0)}{\int_0^{+\infty} J_z(y)dy}} = \frac{E_z(0)}{\delta J_z(0)} = \boxed{\frac{E_z(0)}{H_x(0)}}$$

$$\overline{P_{tot}(t)} = \frac{1}{2}R_s I^2 = \frac{1}{2}R_s d^2 H_x^2 \qquad \overline{P}/S = \boxed{P_{rf} = \frac{1}{2}R_s H_{rf}^2} = \frac{1}{2}R_s\left(\frac{B_{rf}}{\mu_o}\right)^2$$

Normal metals in the local limit

$$J_z(y) = J_z(0)e^{-\frac{y}{\delta}} \qquad \delta = \sqrt{\frac{2\rho}{\omega\mu_o}} \qquad \left(\vec{J}(t) = \vec{J}(0)e^{i\omega t}\right)$$

$$Z_n = R_n + iX_n = \frac{E_z(0)}{\delta J_z(0)} = \frac{\rho}{\delta}(1+i) = \sqrt{\frac{\mu_o\omega\rho}{2}}(1+i)$$

$$\boxed{R_s = X_s = \frac{\rho}{\delta} = \frac{1}{\delta\sigma} = \sqrt{\frac{\mu_o\omega\rho}{2}} = \sqrt{\frac{\mu_o\omega}{2\sigma}}} \qquad (R_n \propto \sqrt{\omega})$$

$$R_{square} = \frac{\rho}{d}$$ (in DC)

Surface impedance practice

- For a bulk metal: $$R_s = X_s = \sqrt{\frac{\mu_o \omega \rho}{2}} \qquad \delta = \sqrt{\frac{2\rho}{\omega \mu_o}}$$

- Thin film over a substrate (metal)?
  - Depends on substrate
  - Depends on thickness wrt skin depth

$$\boxed{\delta = \sqrt{\frac{2\rho}{\omega \mu_o}} \quad \leftrightarrow \quad d}$$

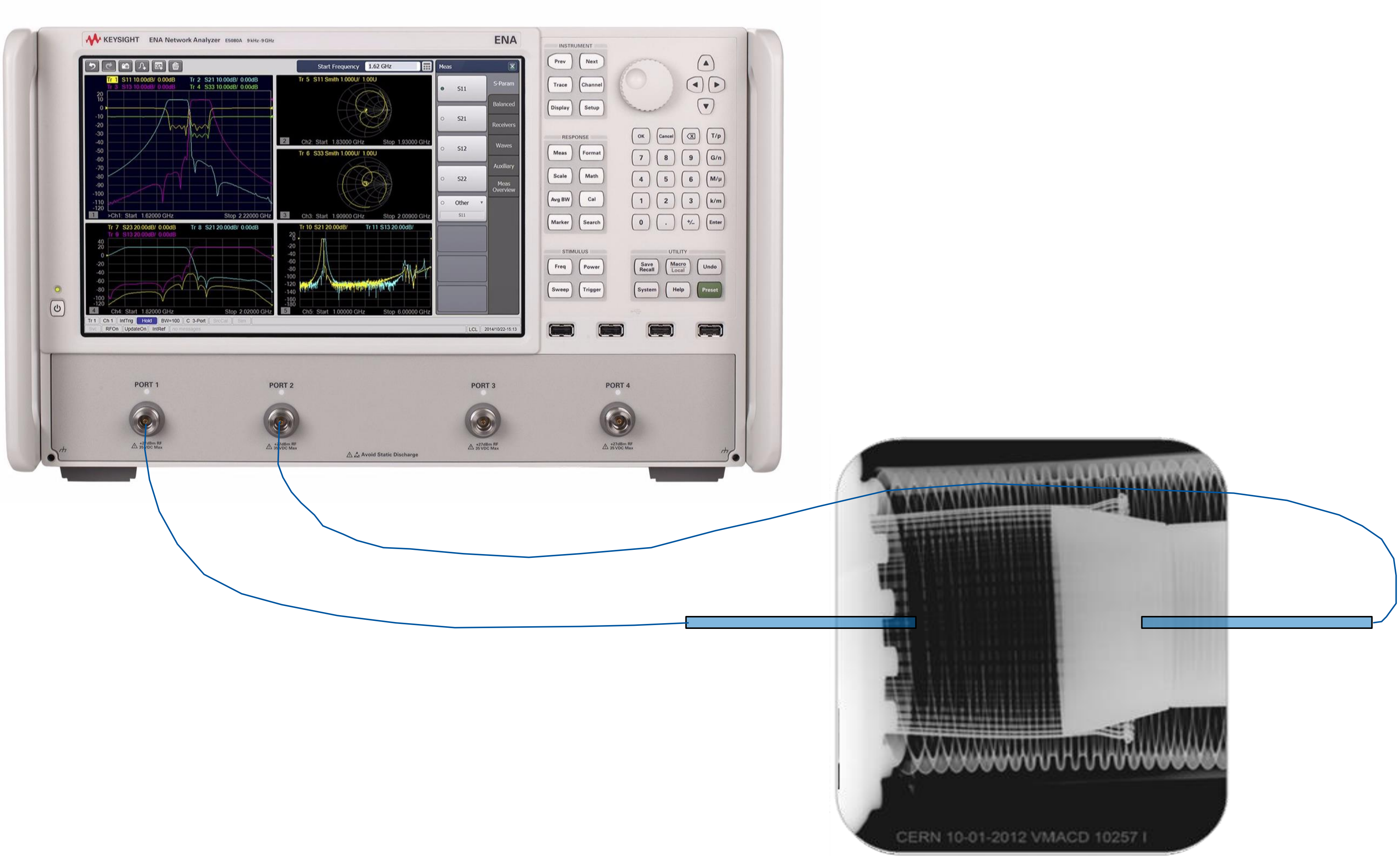
- Practice: open two-layers.xlsx
  - Thanks to Patrick Krkotić and Uwe Niedermayer for the paper “Iterative Model for Calculating the Surface Impedance of Multilayers”, 2016

Coil measurement from 100 kHz to 2 MHz

How the measurement is done

- We'll measure S_{11}
- (i.e. $S_{1\leftarrow 1}$) power from port 1 going back into port 1
- $S_{11} = \frac{P_{REF}}{P_{FWD}}$
- If no power dissipated by the coil: $S_{11} = 1$
- Output from VNA is always in dB = $-10 \times log_{10}(S_{11})$
- What happens when we put a material (with or without coating!) facing the coil?

Cavity measurement with Keysight VNA up to 1.5 GHz



How the measurement is done

- We'll measure S_{21}
- (i.e. $S_{2\leftarrow 1}$) power from port 1 going into port 2
- $S_{21} = \frac{P_{TRANS}}{P_{FWD}}$
- Output from VNA is always in dB = $-10 \times log_{10}(S_{21})$
- Compare modules with perfect and damaged fingers
- How to identify a resonance?

Impedance in accelerators

Impedance: image charges

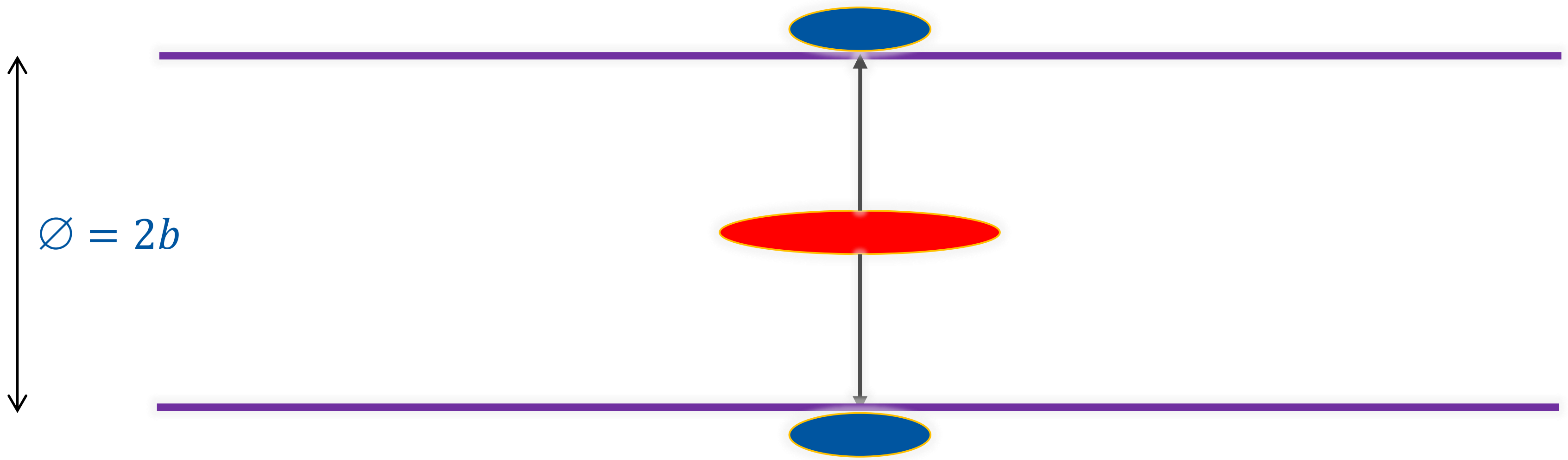


Image charges flow on the surface of the beampipe

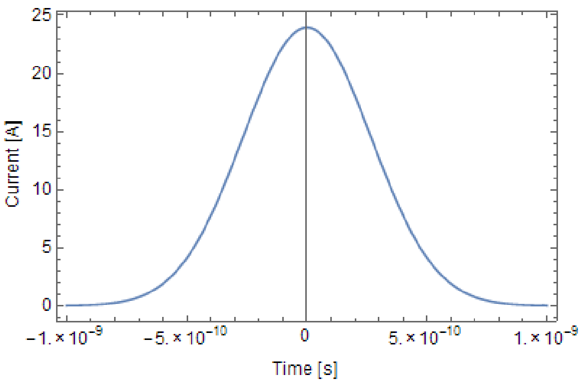
The wakefields potential is proportional to surface impedance of the vacuum chamber surface $Z_s = \frac{V}{I_b}$

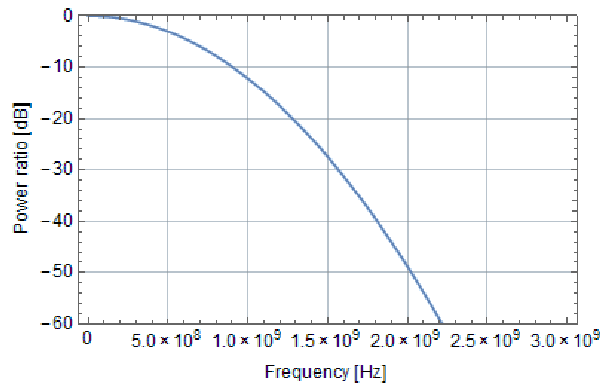
Bunch frequency spectrum: LHC case

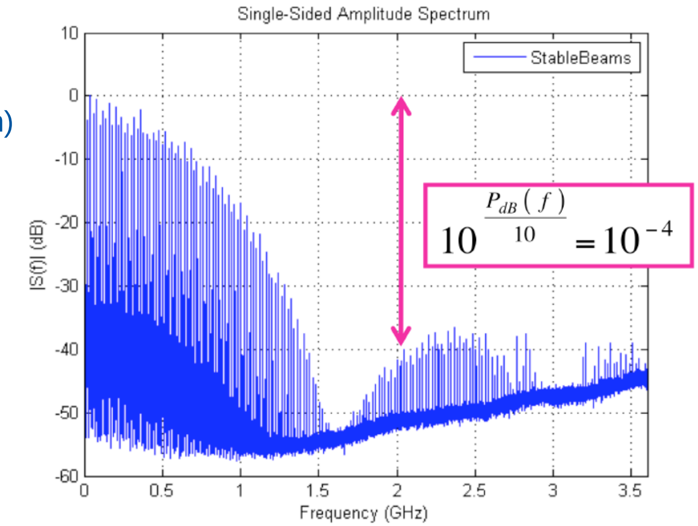
Real bunches

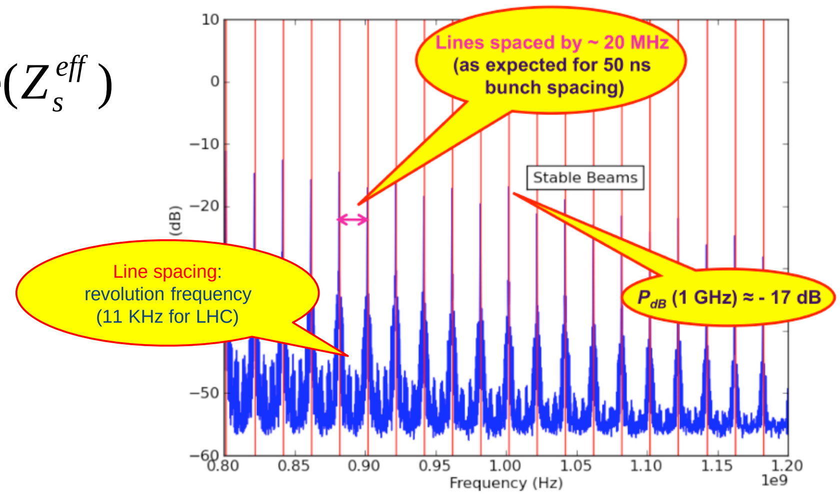
Gaussian bunches of 10^{11} protons, 8 cm long

Beam instantaneous image current → Frequency spectrum (Fourier transform)









Power dissipation from wakes is $P_{loss} = M I_b^2 \,\mathrm{Re}(Z_s^{eff})$

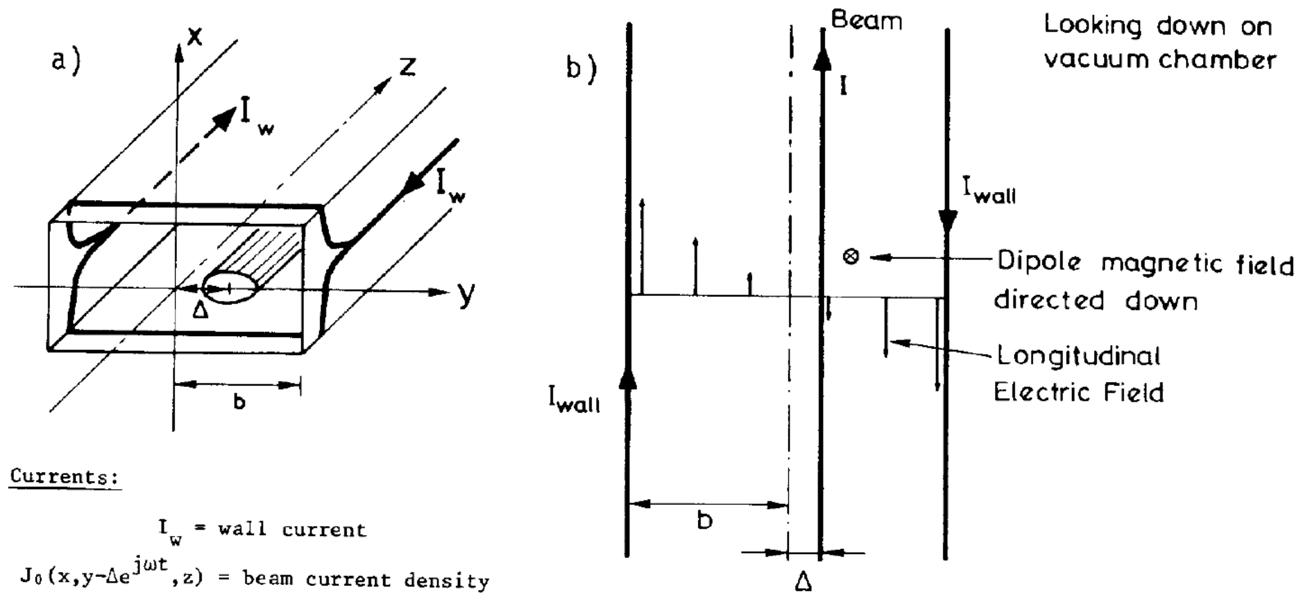
where Z_s^{eff} is a summation of $(2\pi\, R / 2\pi\, b) Z_s$

over the bunch frequency spectrum

($2\pi\, R$ is the accelerator circumference)
($2\pi\, b$ is the vacuum chamber circumference)

From: E. Métral

Transverse impedance



Currents:

I_w = wall current

$J_0(x,y-\Delta e^{j\omega t},z)$ = beam current density

$\simeq J_0(x,y,z) - \frac{\partial J_0}{\partial y}\Delta e^{j\omega t}$

with

I = beam current = $\int J_0(x,y,z)\,dx\,dy$.

Fields:

$E_z = E_0 \frac{y}{b} e^{j\omega t}$ (median plane)

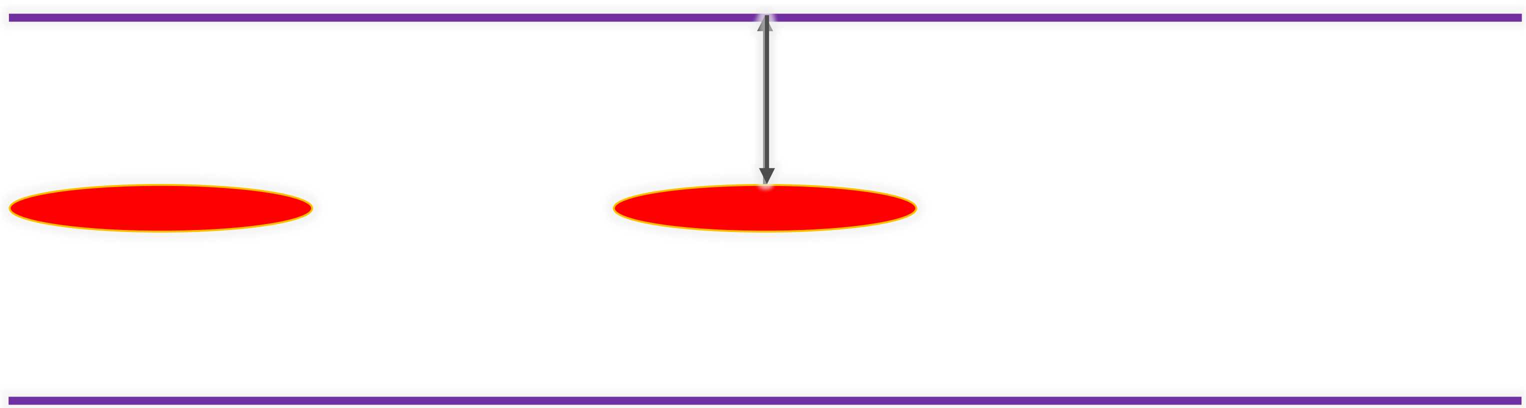
$B_x = \frac{j}{\omega}\frac{E_0}{b} e^{j\omega t}$ (from $\nabla \times E + \dot{B} = 0$).

$$Z_\perp = j\frac{\int_0^{2\pi R}\left[E + c\times B\right]_\perp ds}{I\Delta} \quad [\Omega/m]$$

Fig. 5

F. Sacherer in: Proceedings of the First Course of the International School of Particle Accelerators, CERN 77-13, p. 198

From wakefields to impedance



Wakefields have an effect on beam stability, in particular the transverse plane
Risetime of resistive-wall instability depends on the transverse impedance

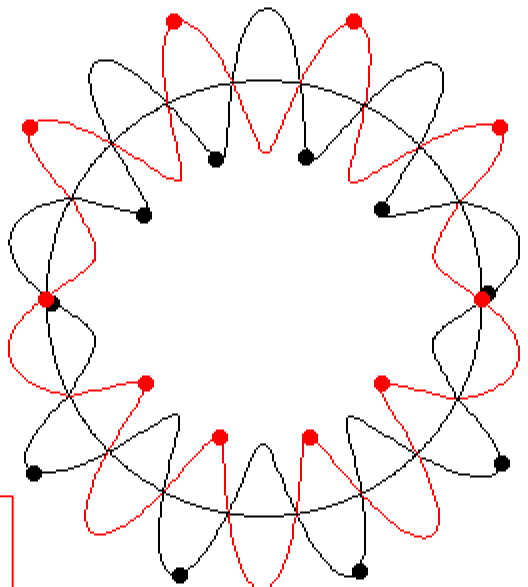
$$\frac{1}{\tau} \propto \frac{I_b M}{EL} \mathrm{Re}(Z_T^{eff}) \quad \text{with} \quad Z_T = \frac{2\pi R\, c}{\pi b^3\, \omega} Z_s$$ at the frequency of the unstable mode

Betatron motion – simplified model

$\boxed{\ddot{x}+\omega_{\beta,0}^2 x = 0}$ Unperturbed motion of single particle with $\boxed{\omega_{\beta,0} = Q_x \omega_0}$ and $\boxed{\omega_0 = \frac{c}{R}}$ $\quad Q_x = R/\beta_x$

Quadrupoles give the restoring (focussing) force -> harmonic oscillations

$$x(t) = x_0 e^{j(\omega_{\beta,0} t + \varphi)}$$



Adding a perturbation to the beam that changes focussing:

$$\boxed{\ddot{x}+\omega_{\beta,0}^2 x = -Fx}$$

$$\omega_\beta^2 = \left(\omega_{\beta,0}^2 + F\right) = \left(\omega_{\beta,0} + \Delta\omega\right)^2 \Rightarrow \boxed{\Delta\omega \approx F/2\omega_{\beta,0}}$$

$$\boxed{x(t) = x_0 e^{j(\omega_\beta + \mathrm{Re}|\Delta\omega| t + \varphi)} e^{-\mathrm{Im}|\Delta\omega|\tau}}$$

$\boxed{\frac{1}{\tau} = -\mathrm{Im}|\Delta\omega|}$ ← Growth rate of instability

Anomalous skin effect

Limits for conductivity and skin effect

$$\delta = \sqrt{\frac{2\rho}{\omega\mu_0}} \qquad \ell \sim \frac{1}{\rho}$$

1. Normal skin effect if: $\ell << \delta$ e.g.: high temperature, low frequency

2. Anomalous skin effect if: $\ell >> \delta$ e.g.: low temperature, high frequency

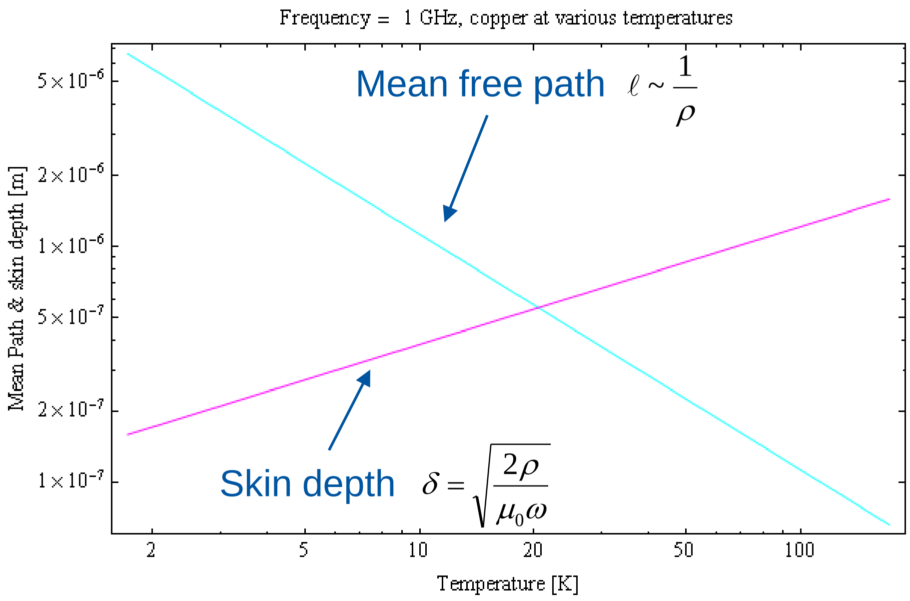
Note: 1 & 2 valid under the implicit assumption $\omega\tau << 1$

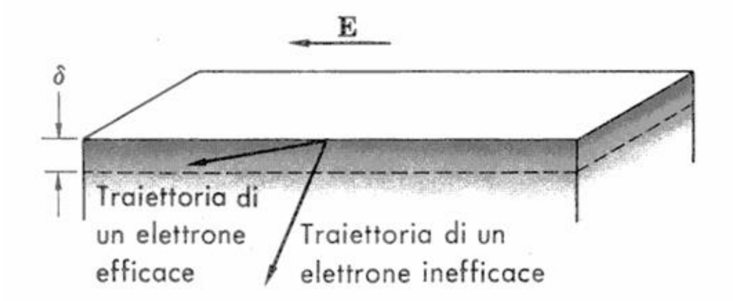
1 & 2 can also be rewritten (in advanced theory) as:

$$\frac{\ell}{\delta} << \left(1 + \omega^2\tau^2\right)^{3/4}$$

It derives that 1 can be true for $\omega\tau << 1$ and also for $\omega\tau >> 1$

Mean free path and skin depth



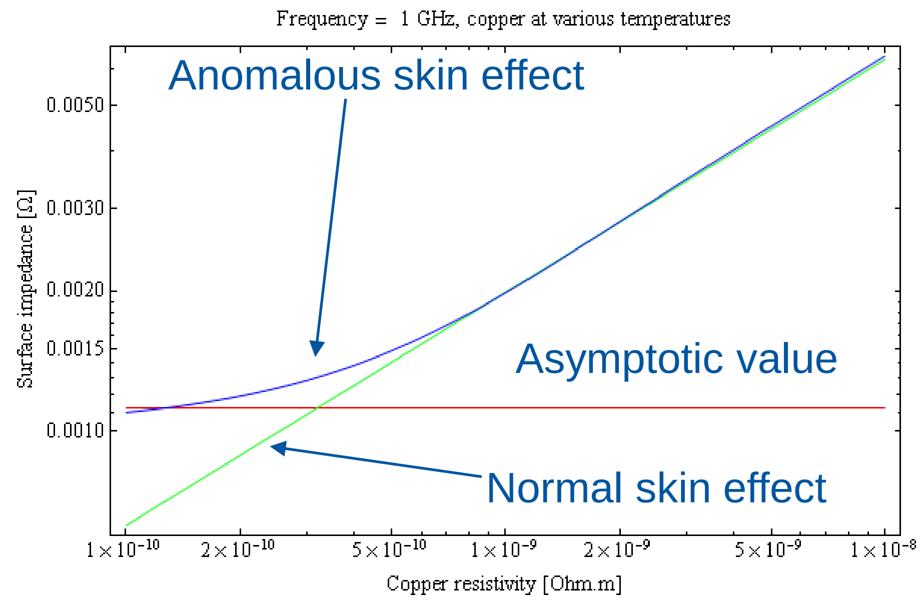


$$n_{eff} \approx n_0 \frac{\delta}{\ell}$$

$$\sigma_0 = \frac{ne^2 \ell}{m_e v_F} \Rightarrow \sigma_{eff} \approx \frac{ne^2 \delta}{m_e v_F}$$

$$\delta = \sqrt{\frac{2}{\sigma \omega \mu_0}} \Rightarrow \delta \xrightarrow[\tau \to \infty]{} const.$$

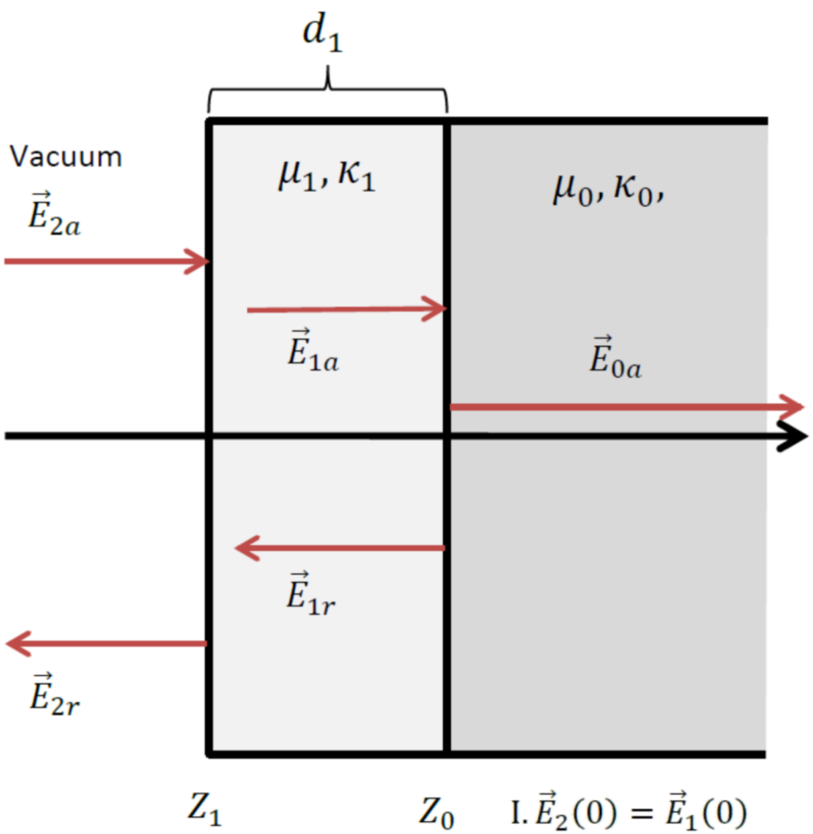
Anomalous skin effect



Understood by Pippard, Proc. Roy. Soc. A191 (1947) 370
Exact calculations Reuter, Sondheimer, Proc. Roy. Soc. A195 (1948) 336

Two-layers

Iterative model



$$\text{I. } \vec{E}_2(0) = \vec{E}_1(0)$$

$$\text{II. } \vec{H}_2(0) = \vec{H}_1(0)$$

$$\text{III. } \vec{E}_1(d_1) = \vec{E}_0(d_1)$$

$$\text{IV. } \vec{H}_1(d_1) = \vec{H}_0(d_1)$$

$$Z_1(Z_0, \omega, \kappa_1, \delta_1, d_1) = \frac{1 + A_1}{1 - A_1} \cdot \alpha_1$$

$$A_1 = \frac{[Z_0 - \alpha_1] \cdot \exp\{-D_1\}}{[Z_0 + \alpha_1] \cdot \exp\{+D_1\}}$$

$$D_1 = (1 + i) \cdot \frac{d}{\delta_1}$$

$$\delta_1 = \sqrt{\frac{2}{\mu_1 \kappa_1 \omega}}$$

$$\alpha_1 = \frac{(1 + i)}{2} \cdot \omega \delta_1 \mu_1$$

$$Z_0 = \frac{(1 + i)}{\delta_0 \cdot \kappa_0}$$